CONSORZIO INTERUNIVERSITARIO PER LA FISICA SPAZIALE

UNIVERSITA' DEGLI STUDI DELL'AQUILA

IL MIO VOLER CERCAR OLTRE LA META

INTERNATIONAL SCHOOL OF SPACE SCIENCE

L'Aquila - ITALY

Web School on: Dynamical Systems and Machine Learning Approaches to Sun-Earth Relations

Dynamical systems approaches and chaos in Sun-Earth relations

Part II. Results from the solar wind-magnetosphere-ionosphere system

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Outline

A. The solar wind-magnetosphere-ionosphere dynamical system

B. Global features: phase-space trajectories

C. Complexity measures: a scale-to-scale approach

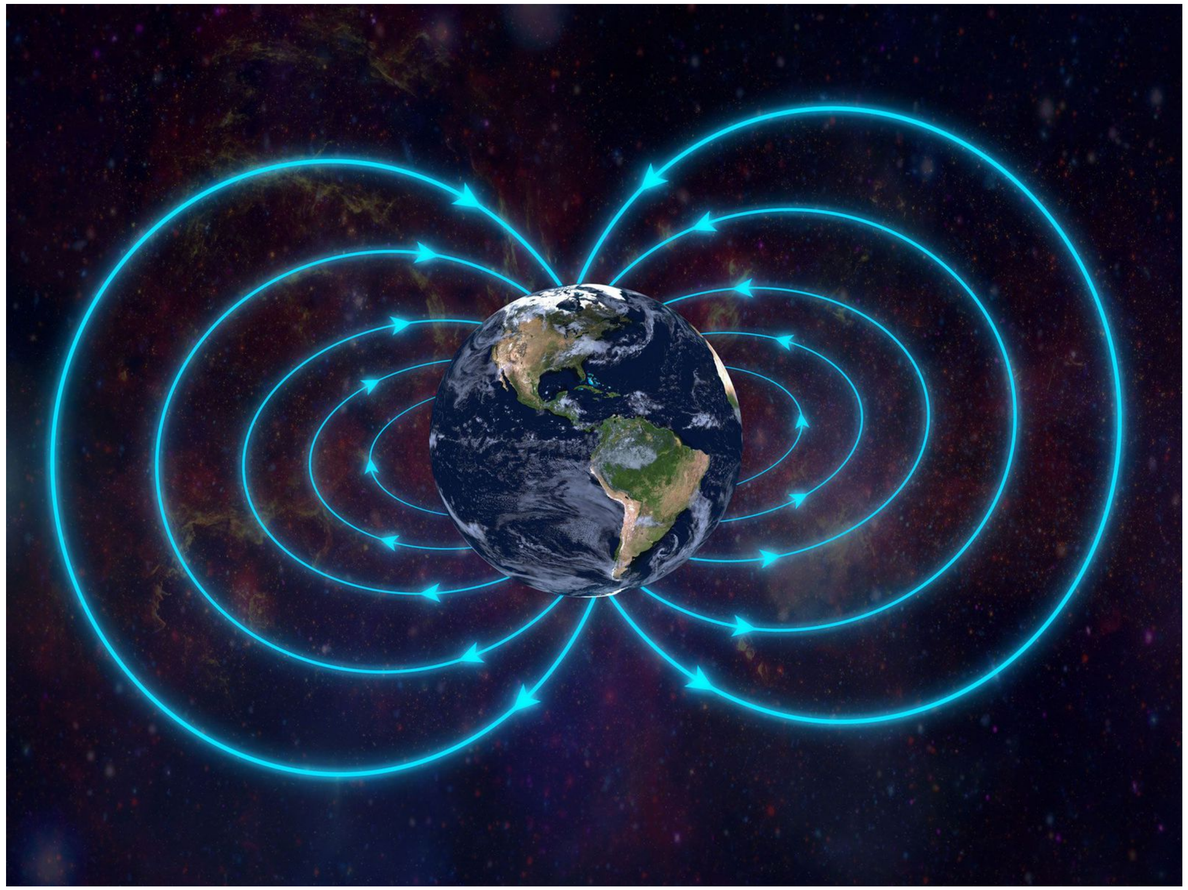
D. Chaotic measures: can we forecast the whole dynamics?

E. Final remarks

A. The solar wind-magnetosphere-ionosphere dynamical system

The solar wind-magnetosphere-ionosphere dynamical system

The Earth's magnetic field



Credits: https://www.smithsonianmag.com/science-nature/earths-magnetic-field-could-take-longer-flip-previously-thought-180972843/

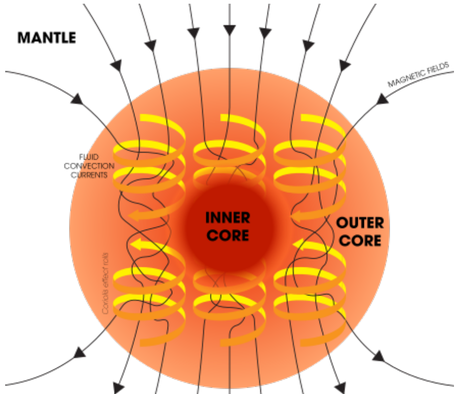
* A solid inner core and a fluid outer core

+

* a rotating planet

↓

* an α^2 dynamo mechanism generating the **magnetic field**

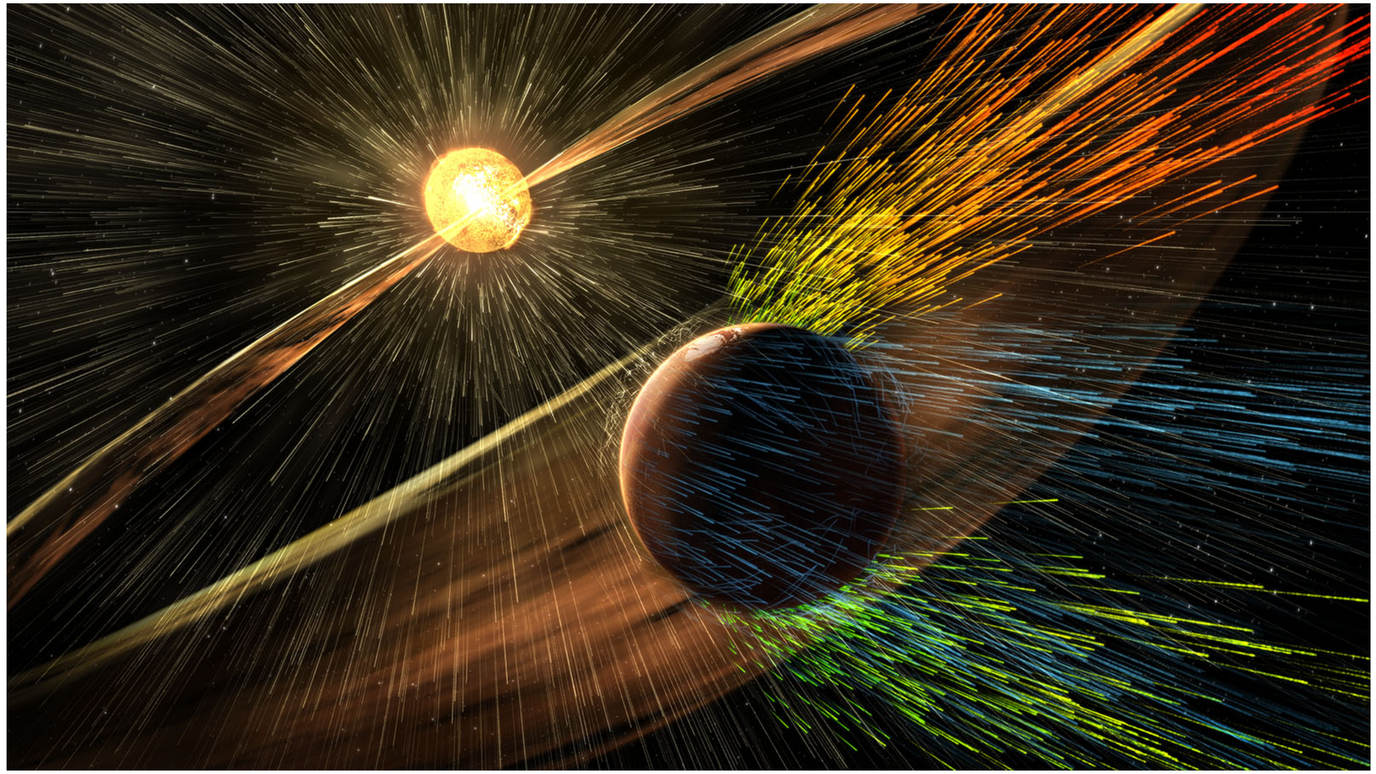


[Merrill+ (1996)]

Credits: https://web.archive.org/web/20150118213104/http://www.usgs.gov/faq/?q=categories%2F9782%2F2738

The solar wind-magnetosphere-ionosphere dynamical system

The solar wind



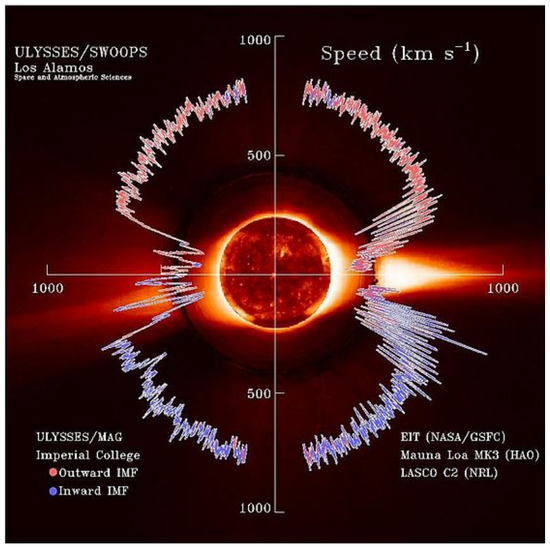
Credits: https://science.howstuffworks.com/dictionary/astronomy-terms/solar-wind-info.htm

* a stream of charged particles released from the upper atmosphere of the Sun

* mostly electrons, protons and alpha particles

* an embedded interplanetary magnetic field

* varying in density, temperature and speed over time

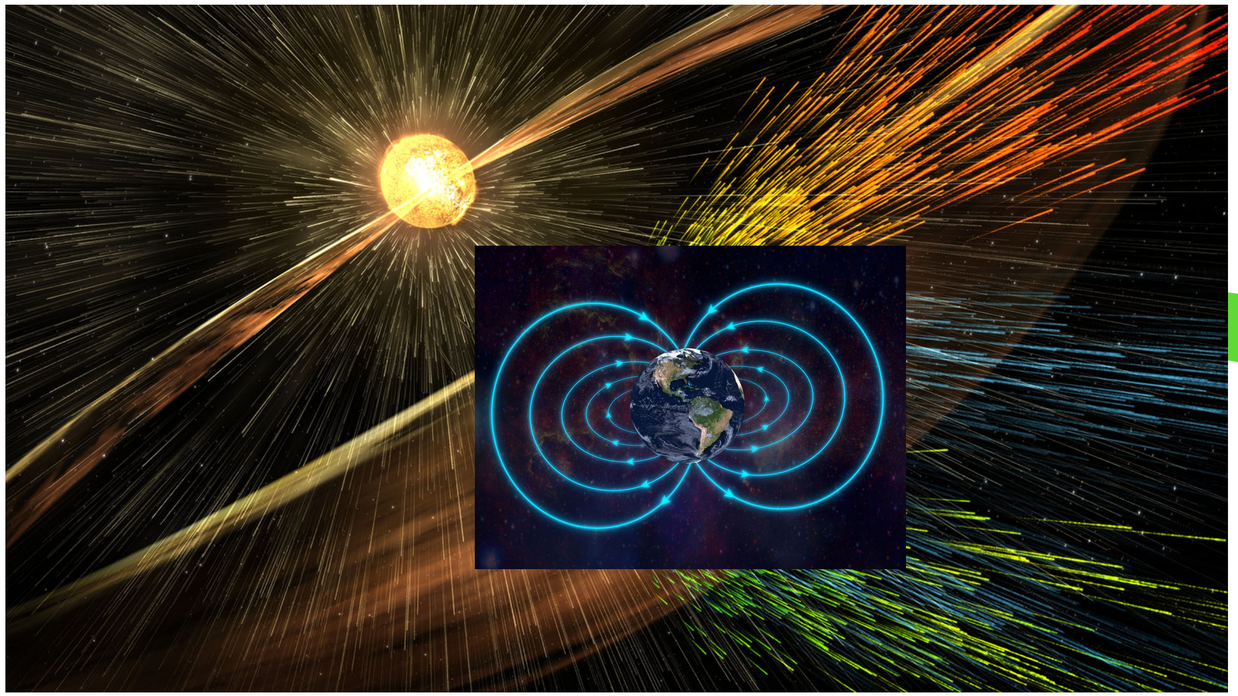


[Parker (1958), Bruno & Carbone (2016)]

[McComas+ (2003)]

The solar wind-magnetosphere-ionosphere dynamical system

The solar wind vs. the Earth's magnetosphere



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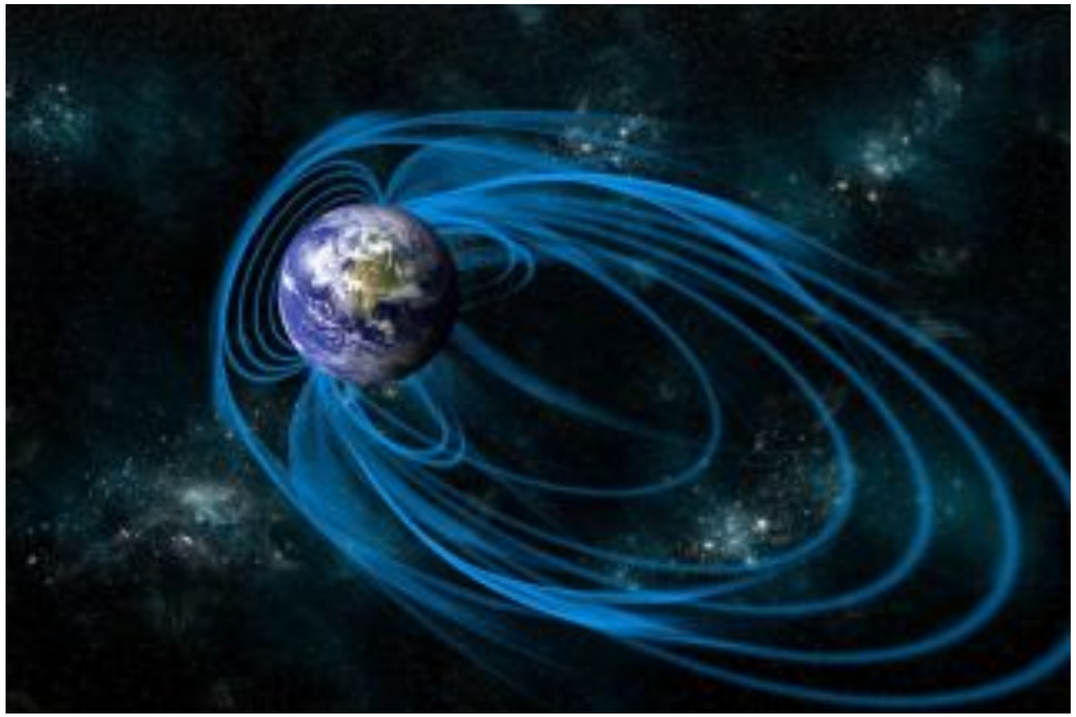
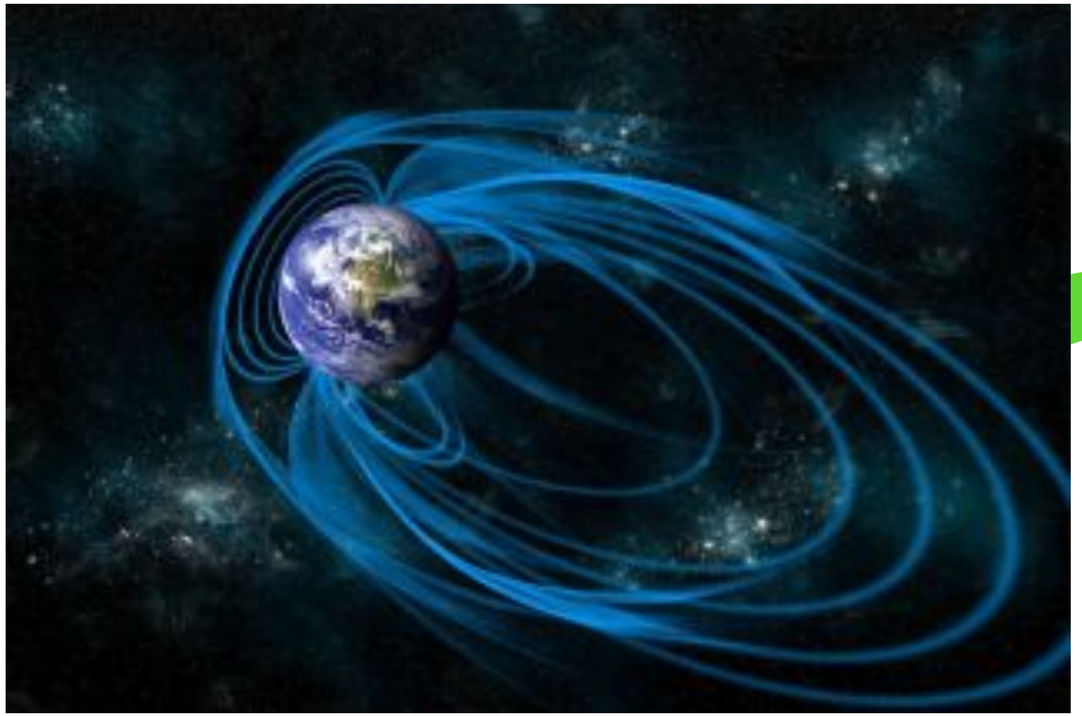
A "deformed" comet-like shape magnetosphere



Credits: https://www.livescience.com/65018-human-brain-senses-magnetic-field.html

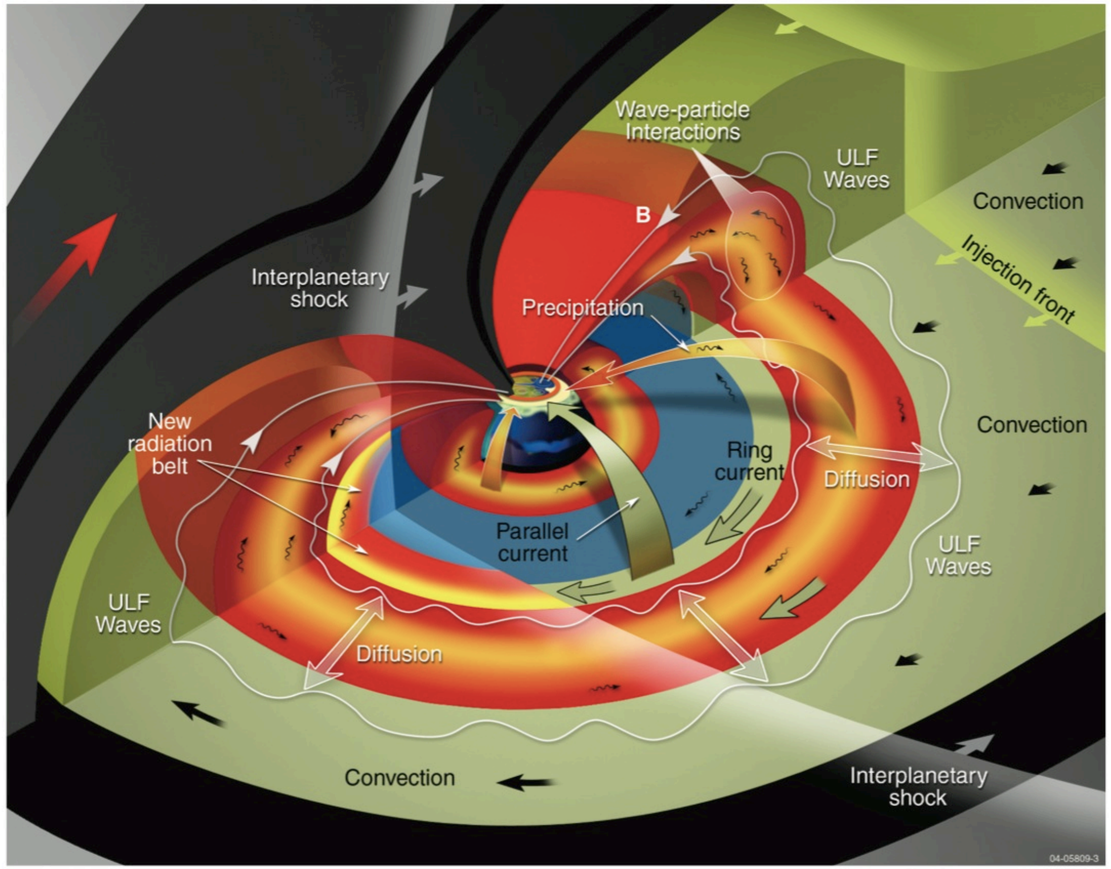
The solar wind-magnetosphere-ionosphere dynamical system

The solar wind vs. the Earth's magnetosphere



Credits: https://www.livescience.com/65018-human-brain-senses-magnetic-field.html

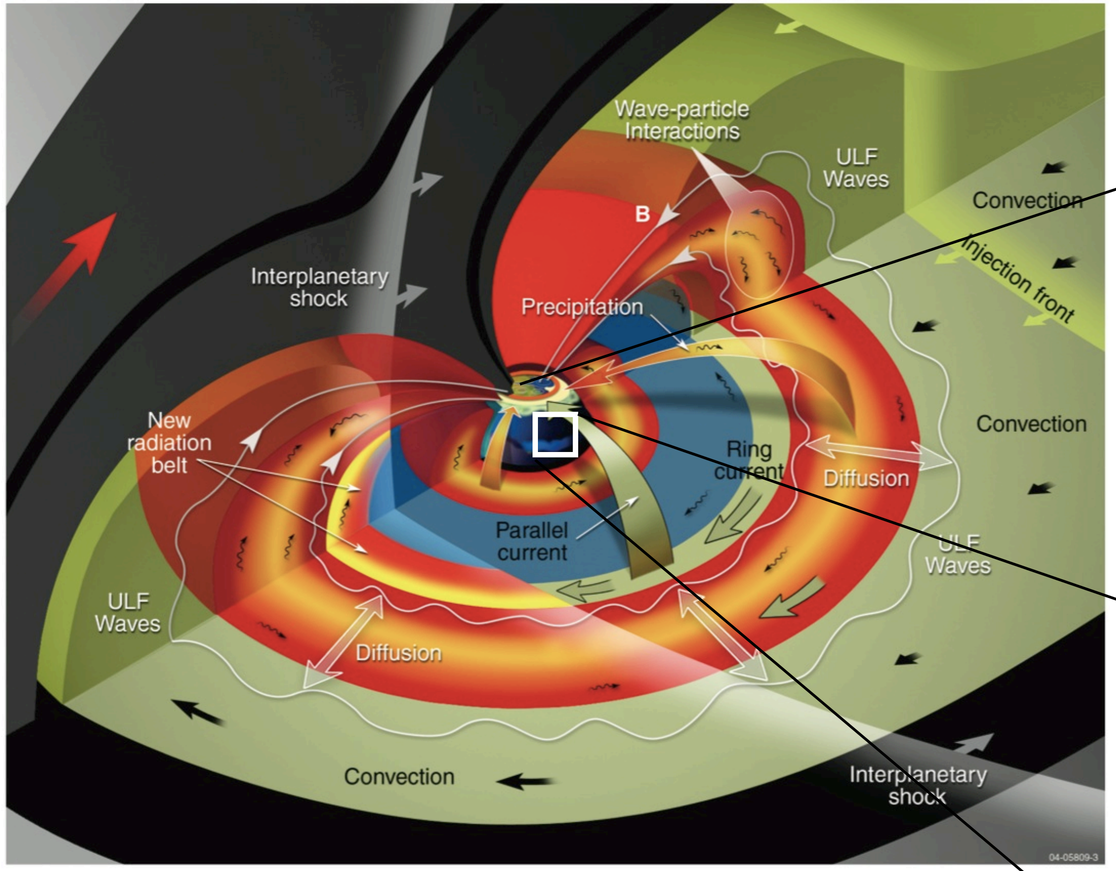
Many physical processes at different scales and locations



Credits: https://rbspgway.jhuapl.edu/sites/default/files//20161026/1_Wed/3_Q12/Foster_SWG_Q1_2_shock.pdf

The solar wind-magnetosphere-ionosphere dynamical system

The magnetosphere-ionosphere system



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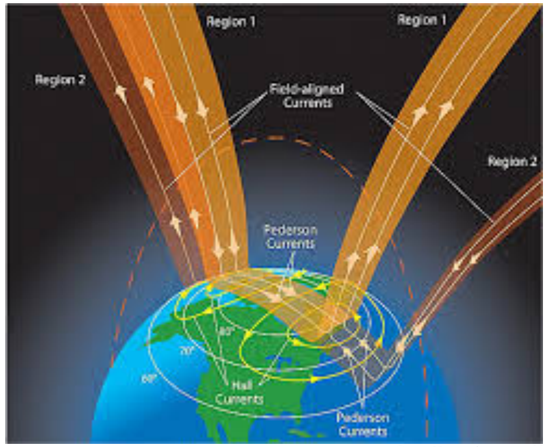
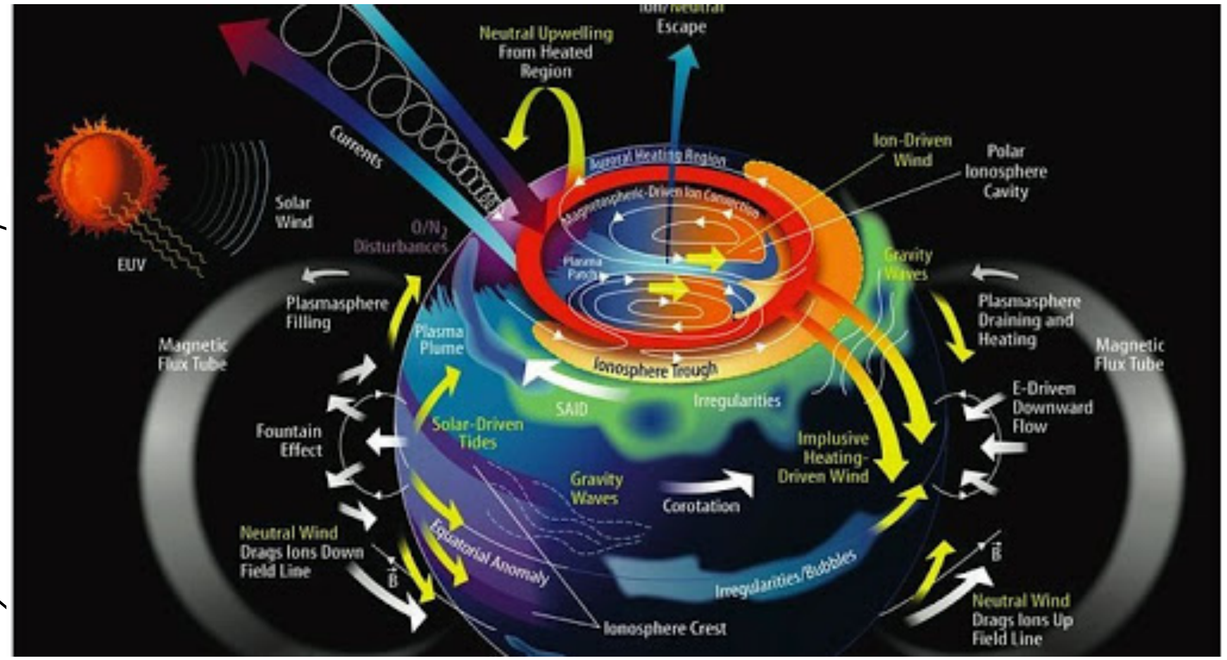


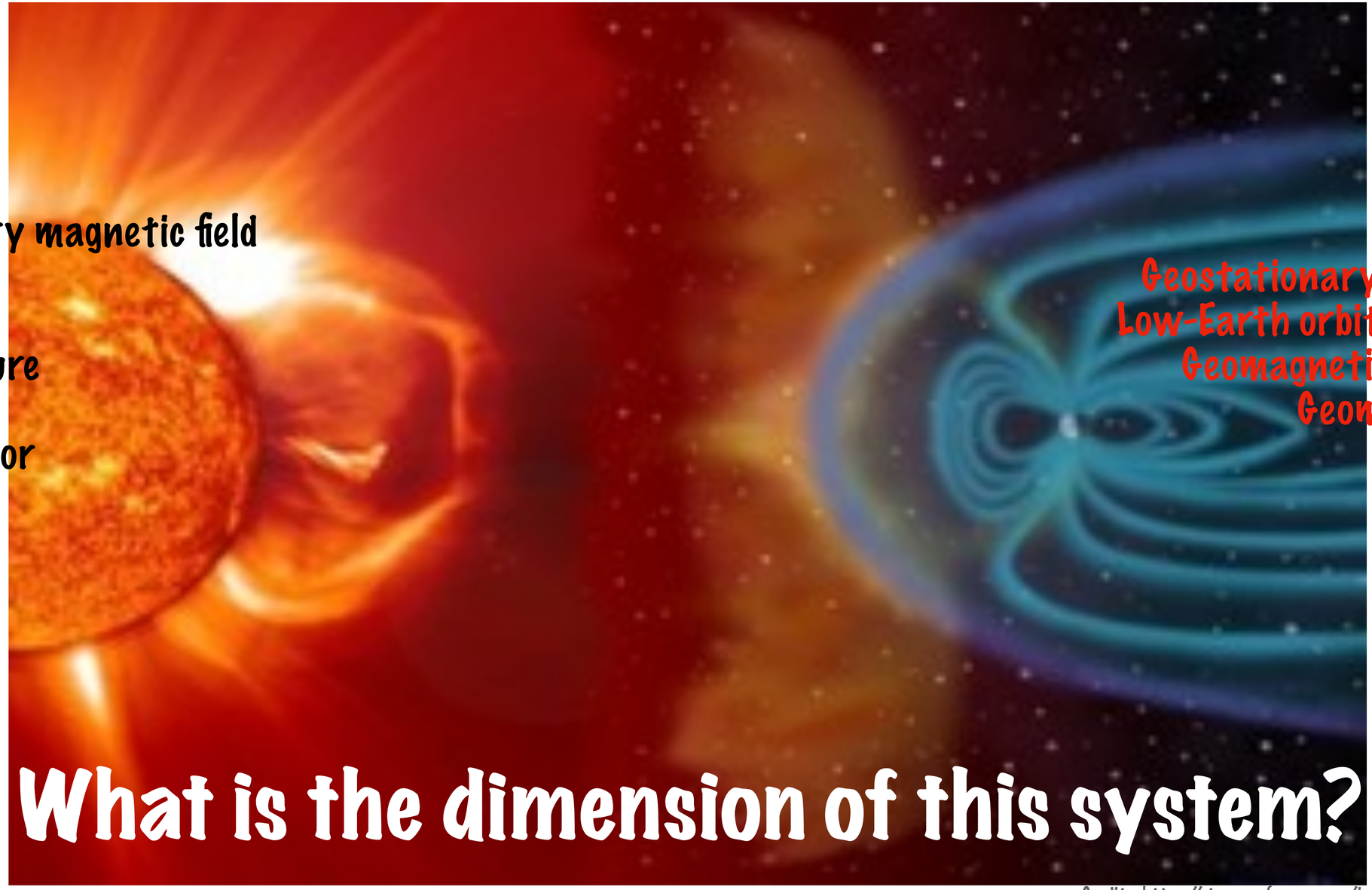
Credits: http://www.marspapers.org/paper/Davidson_2017_3.pdf



Credits: https://www.esa.int/ESA_Multimedia/Images/2015/06/Current_complexity

The solar wind-magnetosphere-ionosphere dynamical system

*Interplanetary magnetic field
*Ion density
*Ion velocity
*Ion temperature
*Electric field
*Poynting vector
*...

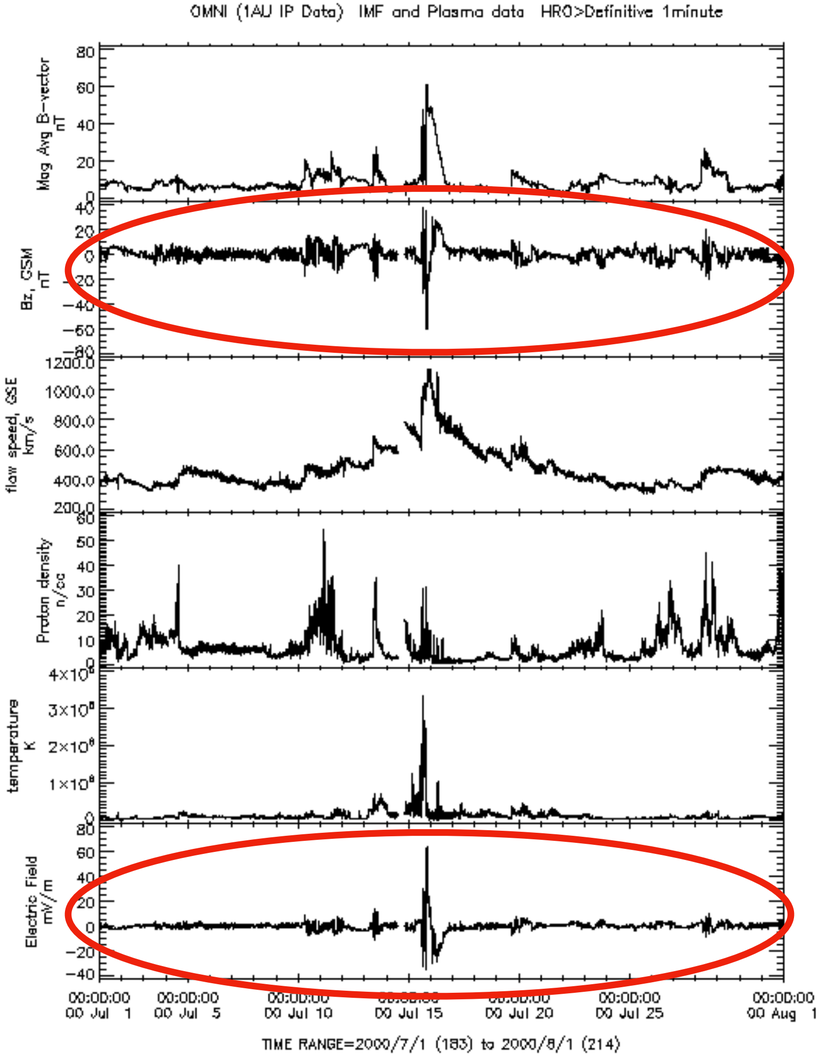


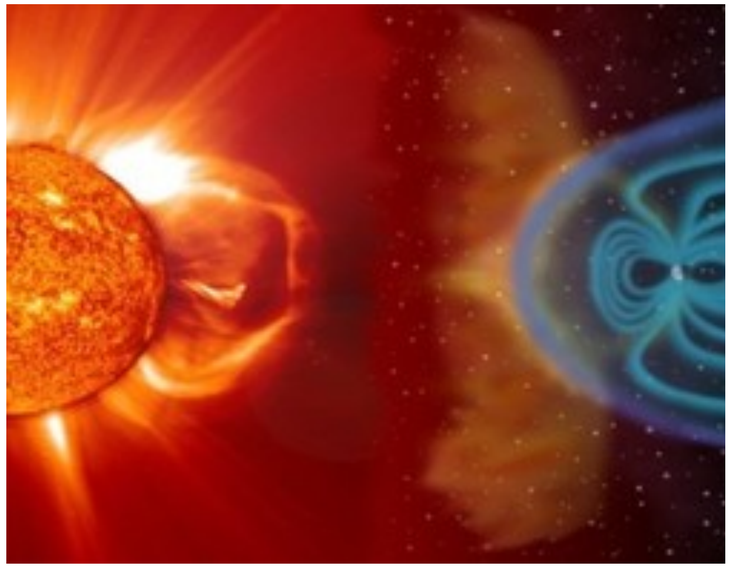
Geostationary measurements*
Low-Earth orbit measurements*
Geomagnetic observatories*
Geomagnetic indices*
...*

What is the dimension of this system?

Credits: https://stereo.gsfc.nasa.gov/img/3Dsun_web.pdf

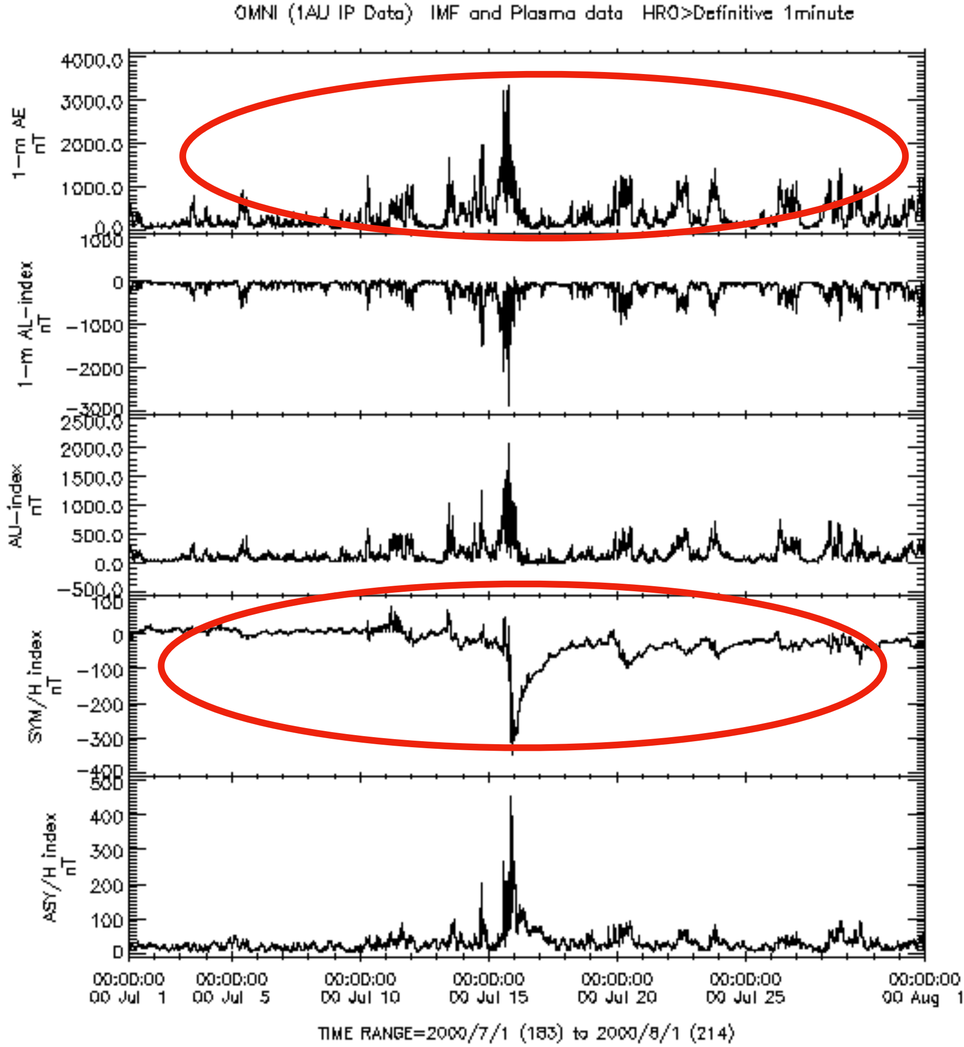
The solar wind-magnetosphere-ionosphere dynamical system



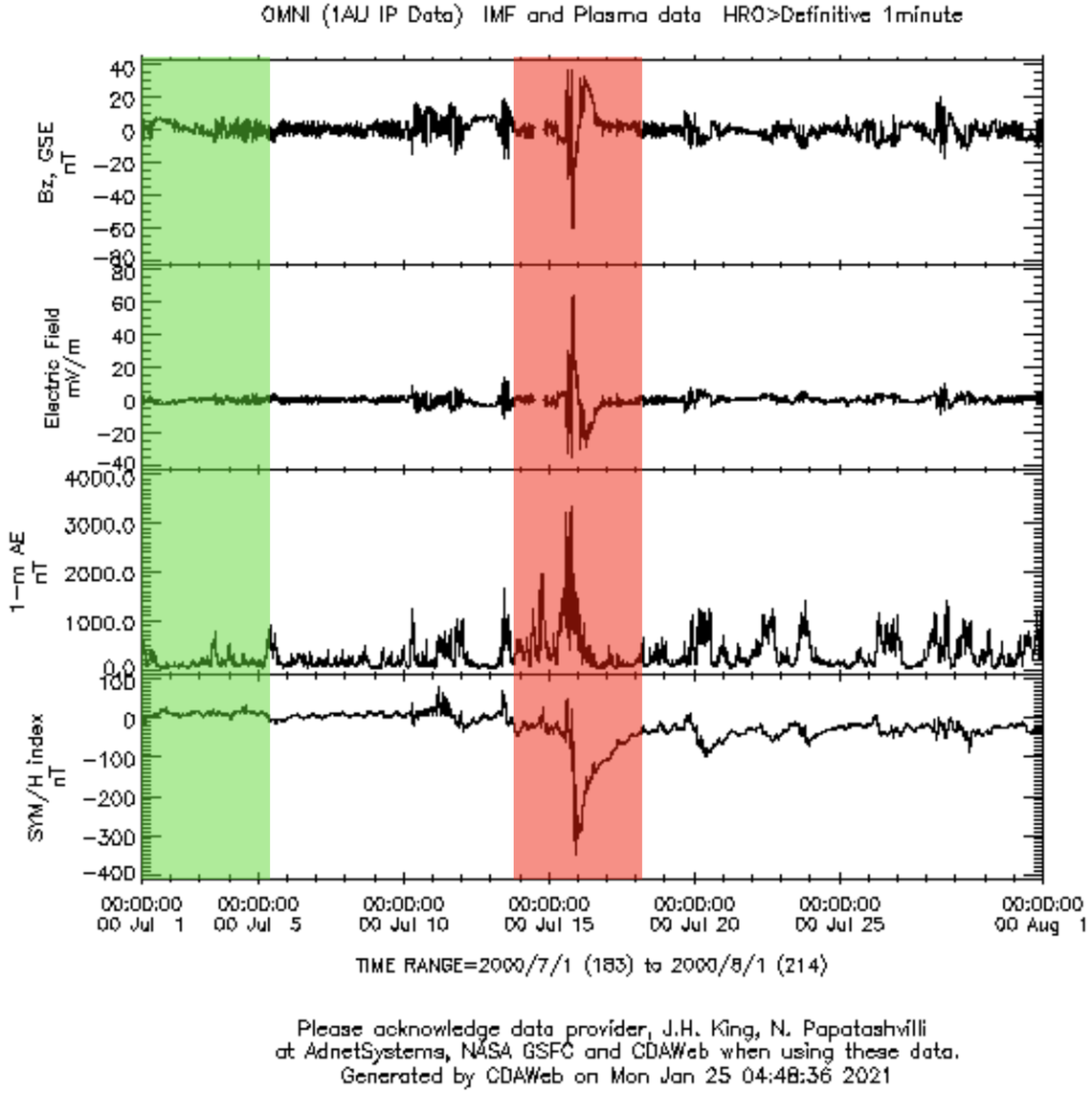


Credits: https://stereo.gsfc.nasa.gov/img/3Dsun_web.pdf

$$N \gg 1 \quad \rightarrow \quad N \sim 3-4$$



The solar wind-magnetosphere-ionosphere dynamical system



* Reduce the full phase-space of variables to a subset of them

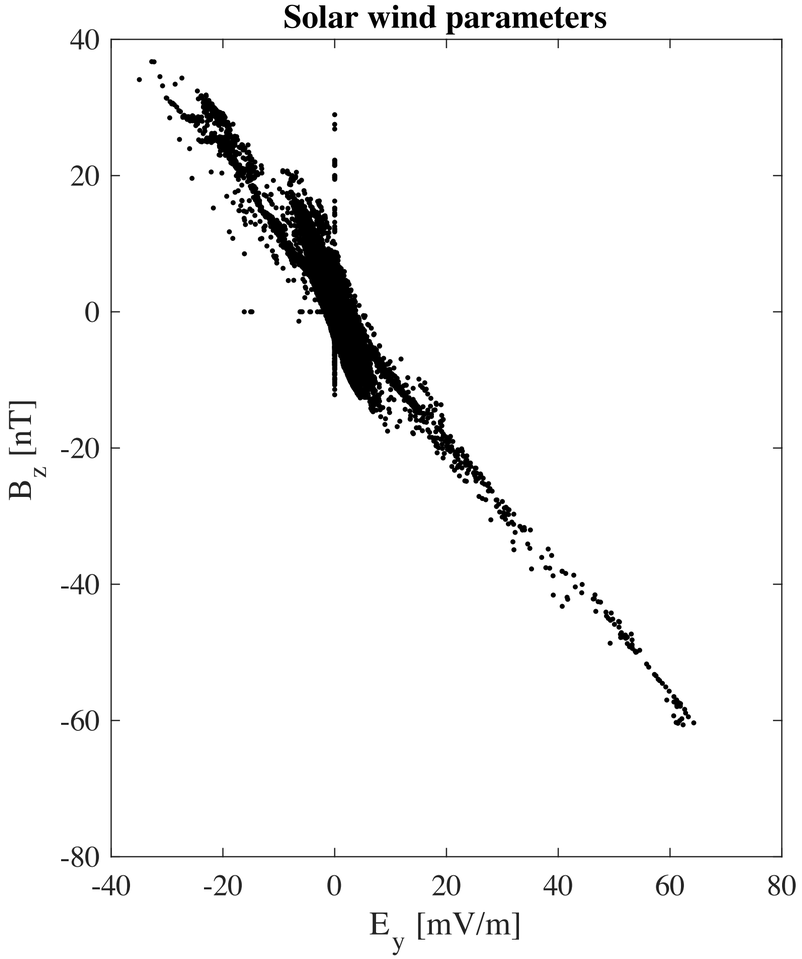
* Monitoring SW energy input and M-I response

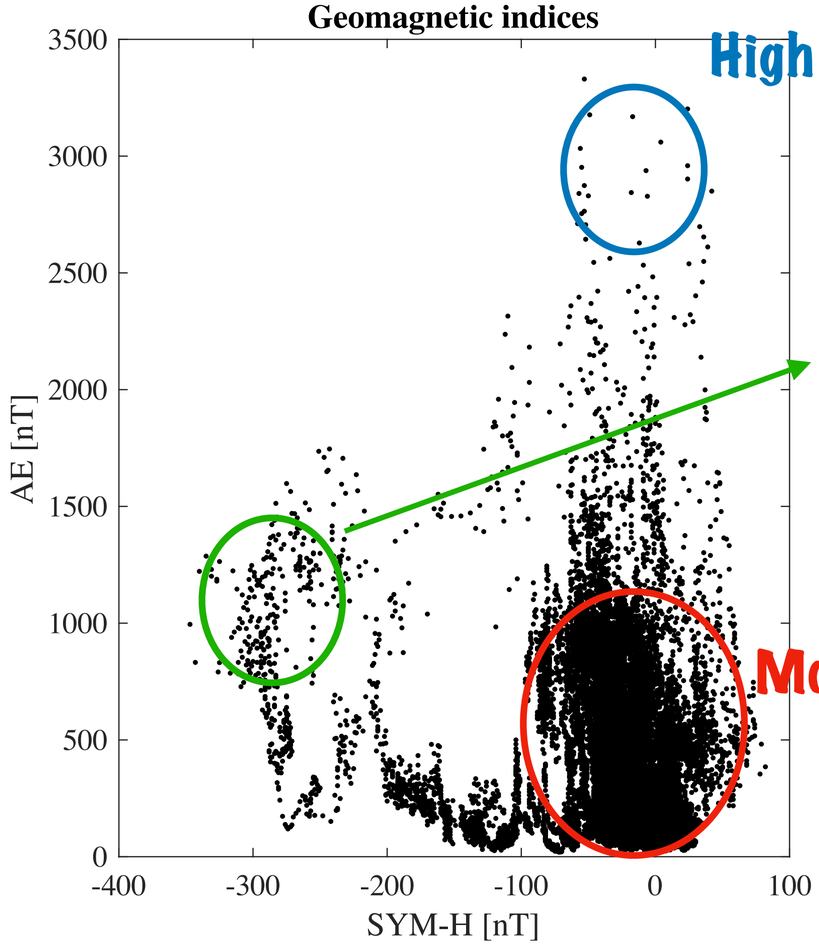
* Monitoring high- and low-latitude current systems

* Monitoring quiet vs. disturbed periods

B. Global features: phase-space trajectories

The SW vs. the MI reduced phase-space





High positive values of AE

[Davis & Sugiura (1966), Ahn+ (1983)]

Low negative values of SYM-H

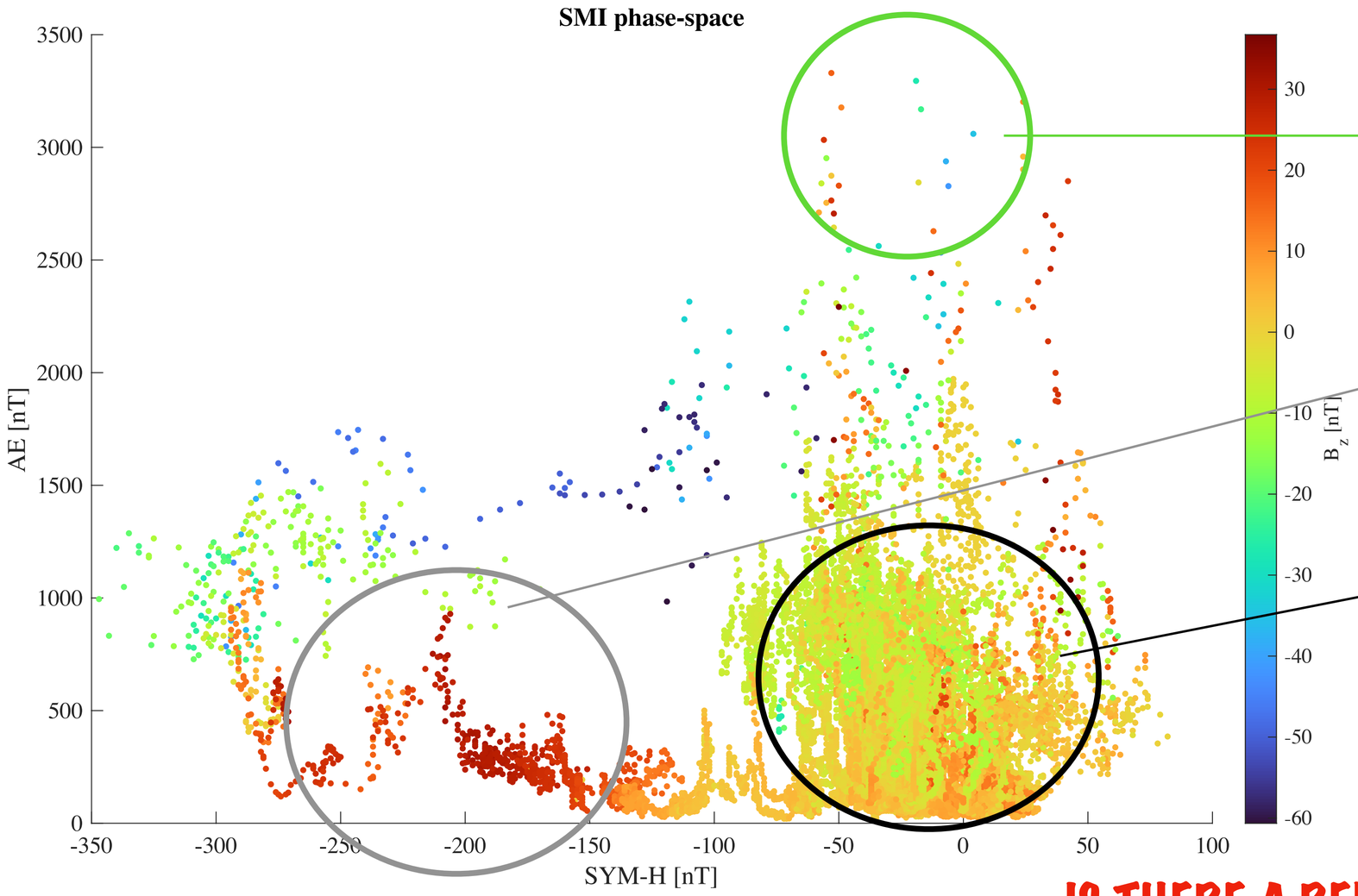
[Iyemori (1990)]

Most dense region of the phase-space

[Sharma (1995), Klimas+ (1996)]

* Close linear relationship between B_z and E_y
* No clear dependence between AE and SYM-H
* Safely move the subset of variables from 4 to 3

The SMI reduced phase-space (SYM-H, AE, B_z)



What happens here?

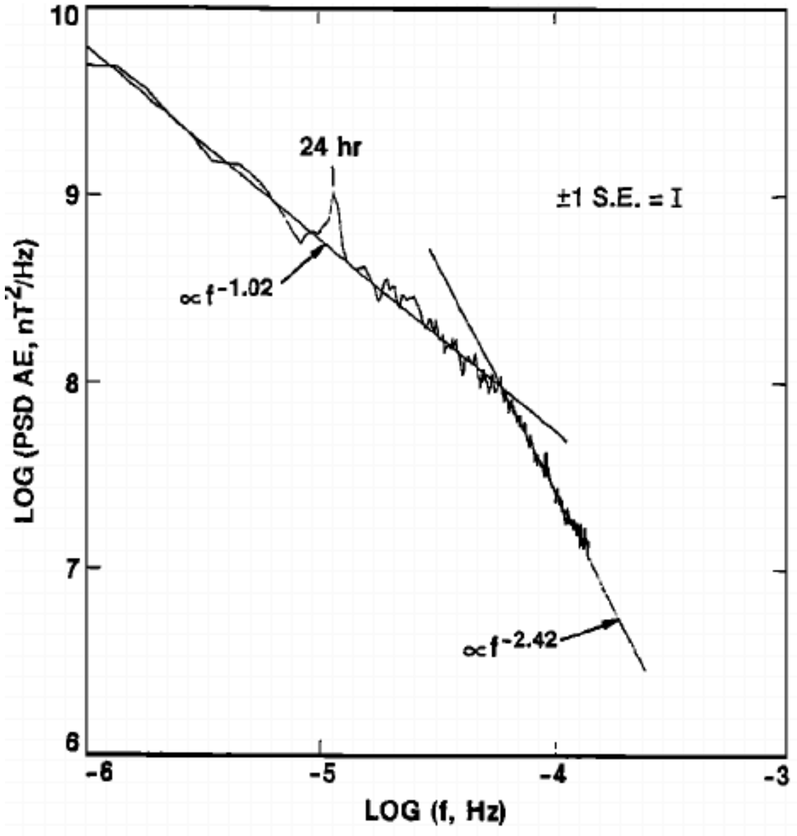
Bz is characterized by larger positive values

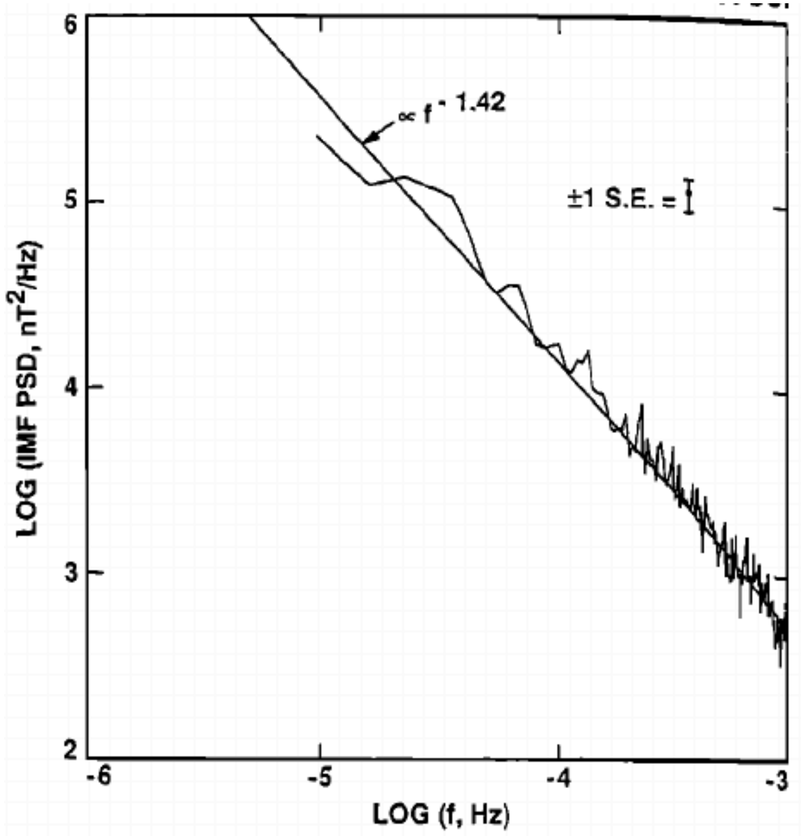
Bz fluctuates around 0 nT

IS THERE A RELATION BETWEEN Bz and AE (SYM-H)?

What is the relation between B_z and AE (SYM-H)?

Searching for a spectral dependency





[Silverman & Shapiro (1983), Tsurutani+ (1990)]

* A spectral break is observed at about 5 hours

* The magnetosphere responds as a low-pass filter

[Tsurutani+ (1990), Klimas+ (1996)]

* Is the spectral break the evidence of a nonlinear response of AE to B_z?

* Could only some scales be affected by the solar wind variability?

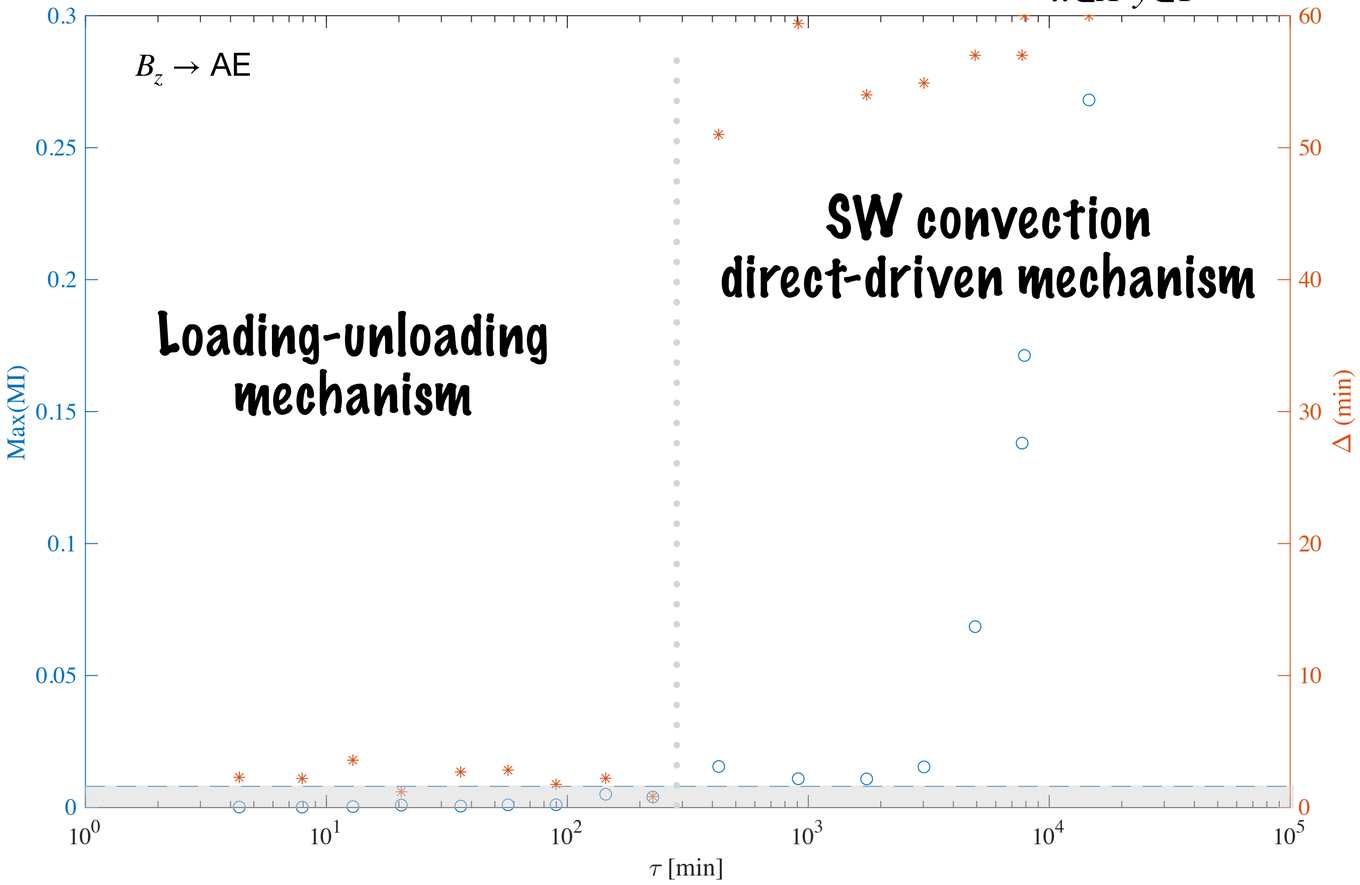
[Lyons+ (1997), Alberti+ (2017)]

What is the relation between B_z and AE (SYM-H)?

Searching for a nonlinear delayed scale dependency via mutual information

$$MI(\Delta) = \sum_{x \in X} \sum_{y \in Y} p(x, y) \log \frac{p(x, y)}{p(x)\, p(y)} \quad (1)$$

[Wing+ (2016), Alberti+ (2017)]



[Rostoker+ (1987), Kamide & Kokubun (1996), Consolini & De Michelis (2005)]

Detect intrinsic scales via a decomposition method (here the EMD)

[Huang+ (1998)]

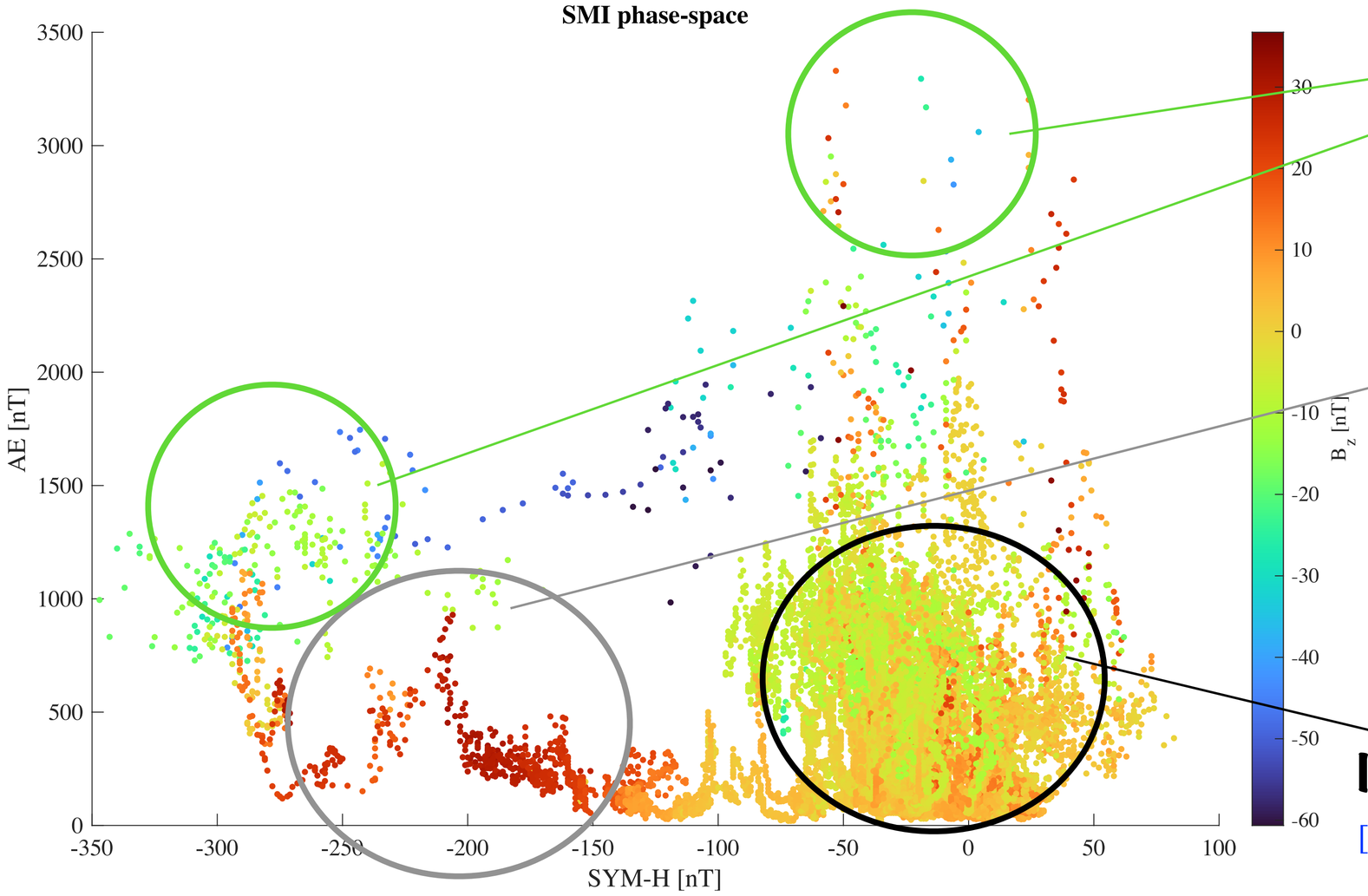
Evaluate mutual information (Eq. (1)) between the SW input and the geomagnetic output at each scale τ

A significant information transfer is found for $\tau > 200$ min with a delay of $\Delta \sim 60$ min

Similar relations between different couples of inputs/outputs

[Alberti+ (2017)]

How to interpret now the SMI reduced phase-space?



Substorm-storm relation?

[Daglis+ (1994), De Michelis+ (2011)]
[Runge+ (2018), Stumpo+ (2020)]

External feedback determines this "transient" region

[Tsurutani+ (1990), Alberti+ (2017)]

Internal dynamics dominates this region

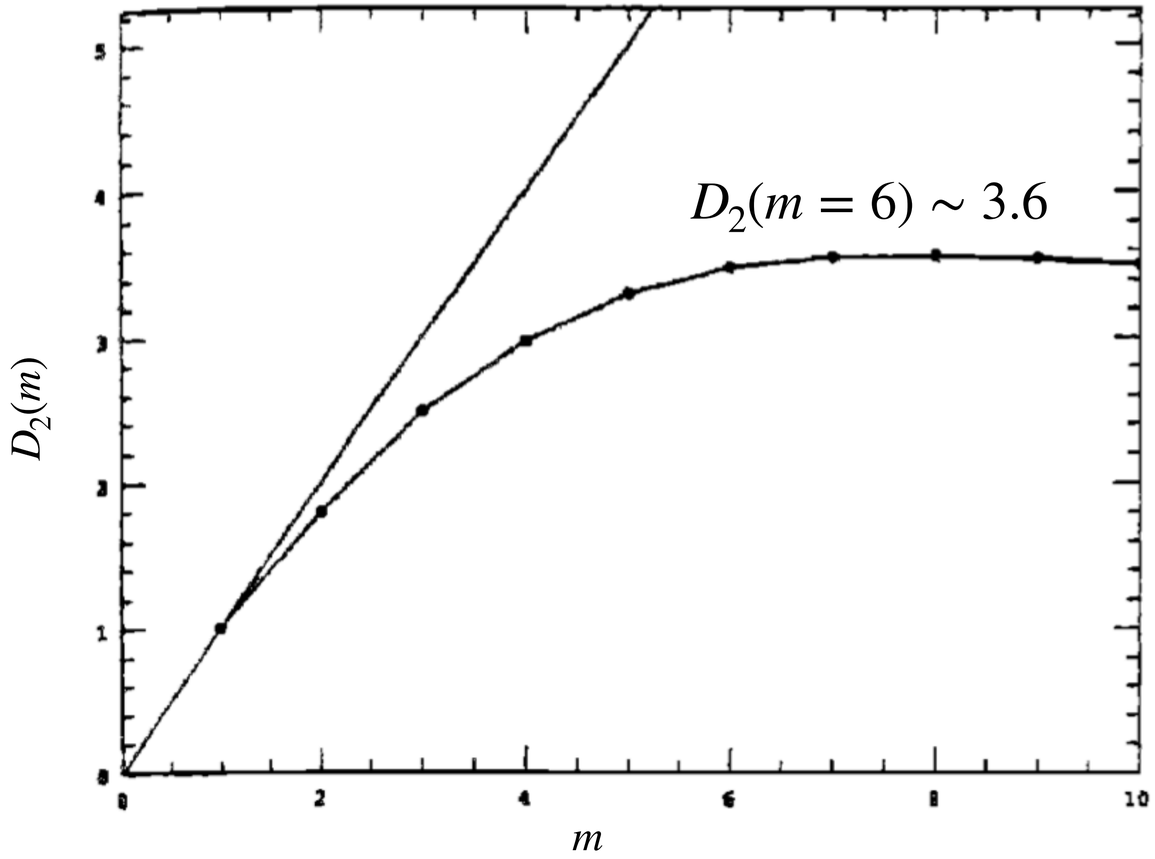
[Kamide & Kokubun (1996), Consolini & De Michelis (2005)]
[Sitnov+ (2001), Alberti+ (2020b)]

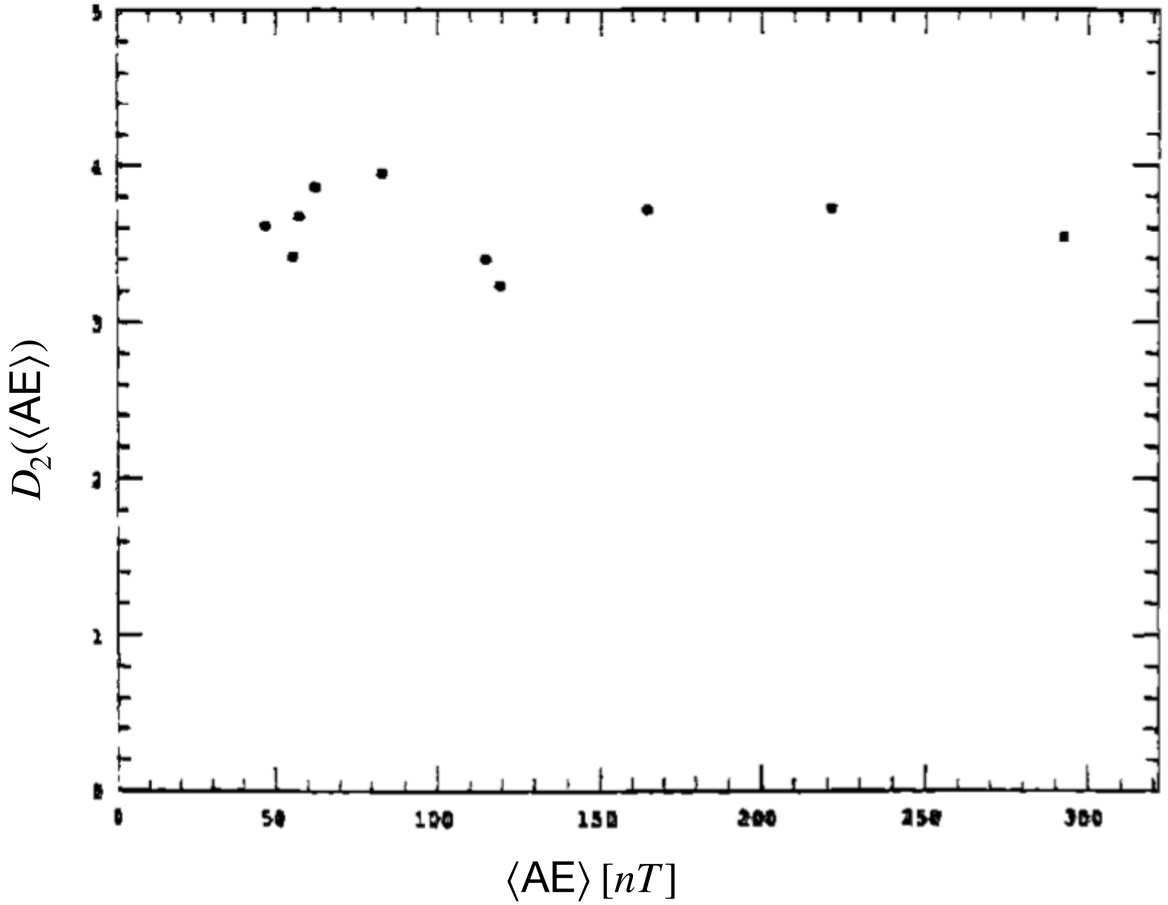
C. Complexity measures: a scale-to-scale approach

Measuring the SMI reduced phase-space

Looking for the correlation dimension

[Vassiliadis+ (1990)]





$D_2(m) < m$: not noise, low-dimensional system

D_2 almost independent on geomagnetic activity

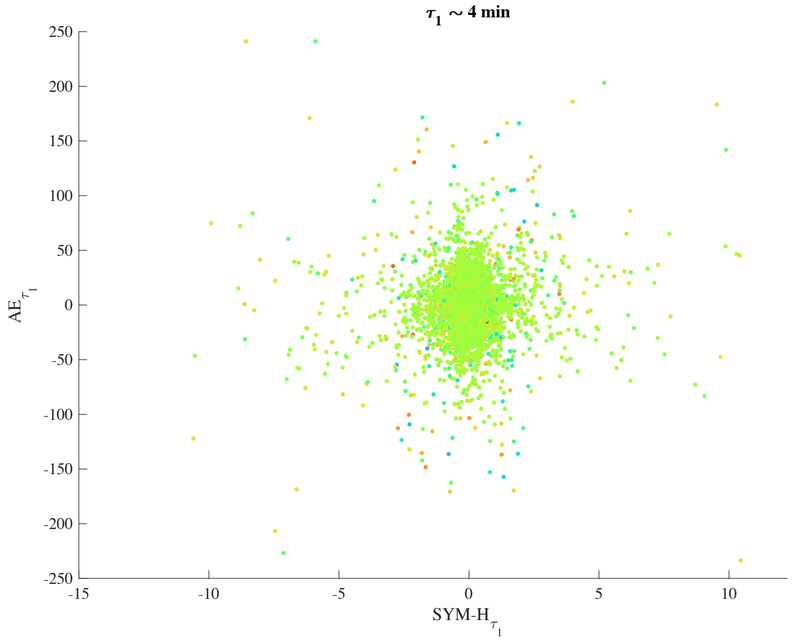
system: not the global magnetosphere, only its response represented by AE

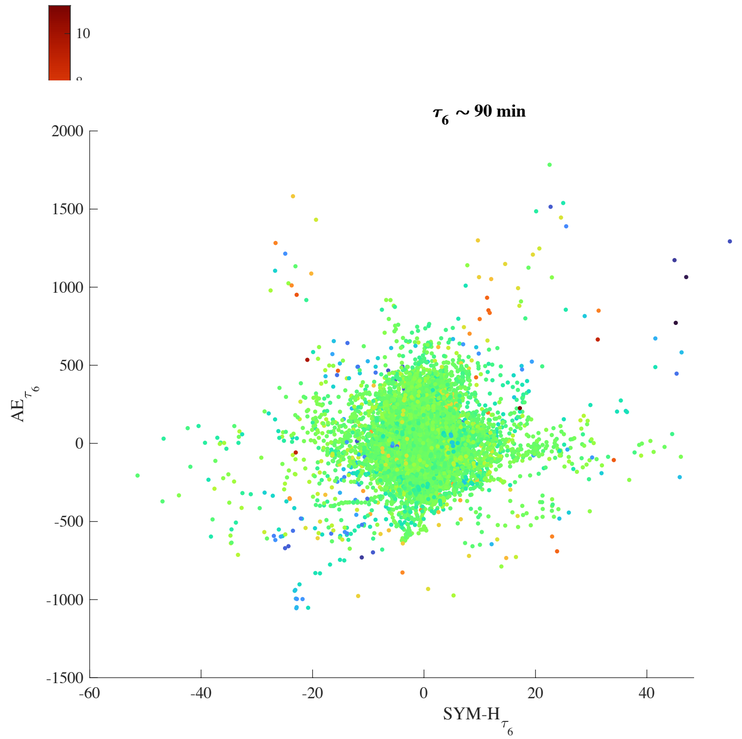
Measuring the SMI reduced phase-space

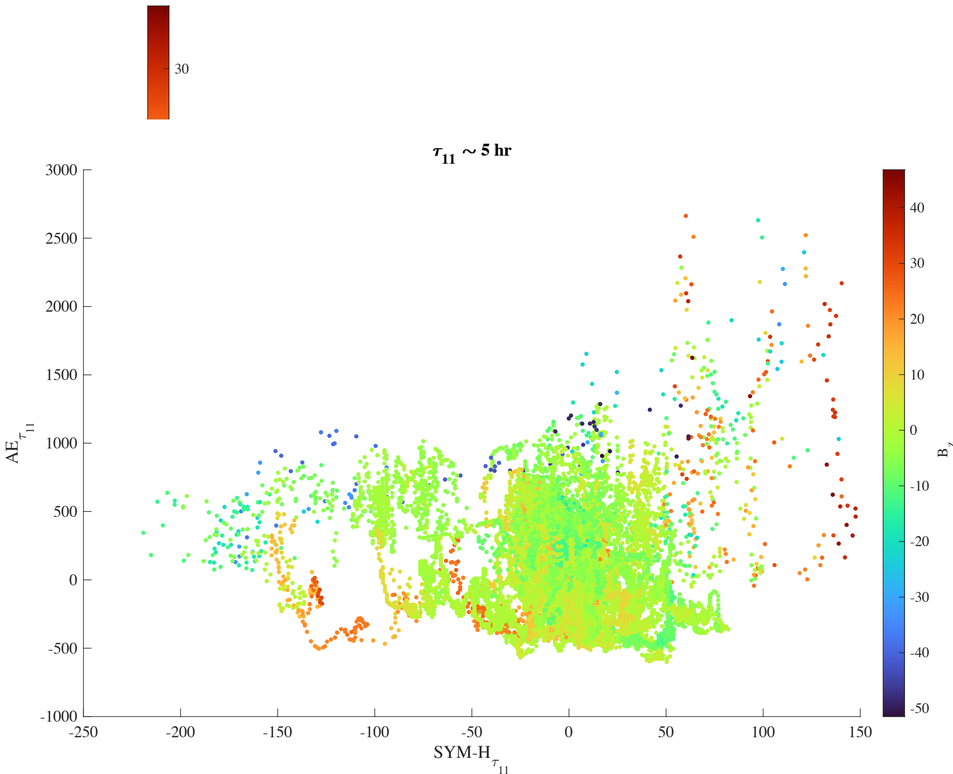
Looking for a scale-dependent phase-space

Detect intrinsic modes via a decomposition method (here the MEMD)

$$\mathbf{X}(t) = \left[B_z(t), \mathrm{AE}(t), \mathrm{SYM\text{-}H}(t)\right]^T = \sum_k \mathbf{C}_k(t) + \mathbf{R}(t)$$







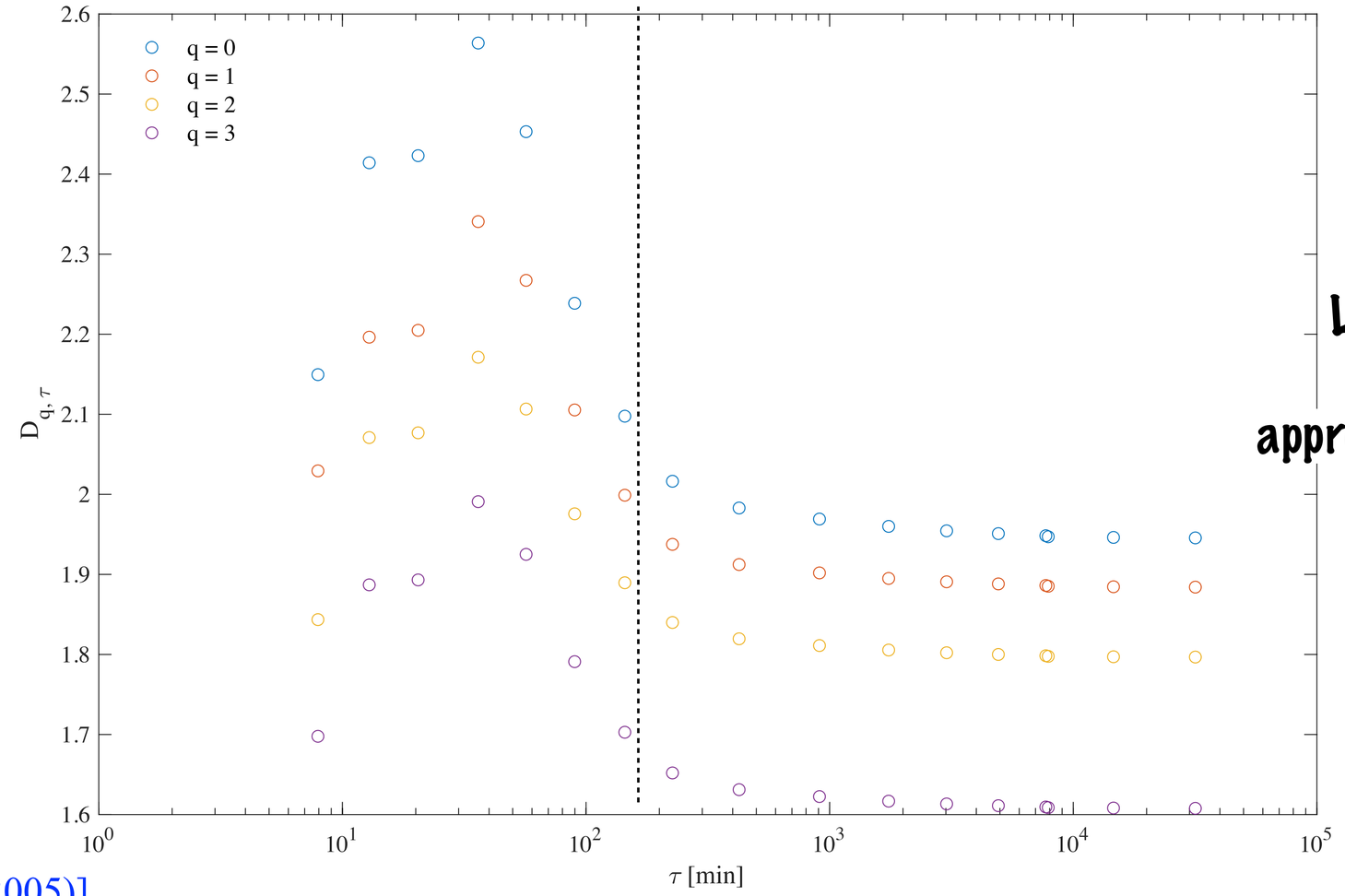
Measuring the SMI reduced phase-space

Looking for a scale-dependent fractal dimensions

[Alberti+ (2020a)]

$$D_{q,\tau} = \frac{1}{q-1} \lim_{r \to 0} \frac{\log \sum_i \mathscr{P}_{i,\tau}{}^q}{\log r}$$

Short-term variability has larger $D_{q,\tau}$



Long-term variability has lower $D_{q,\tau}$ approaching a constant value for $\tau \gtrsim 200$ min

Multifractal features emerge at all scales

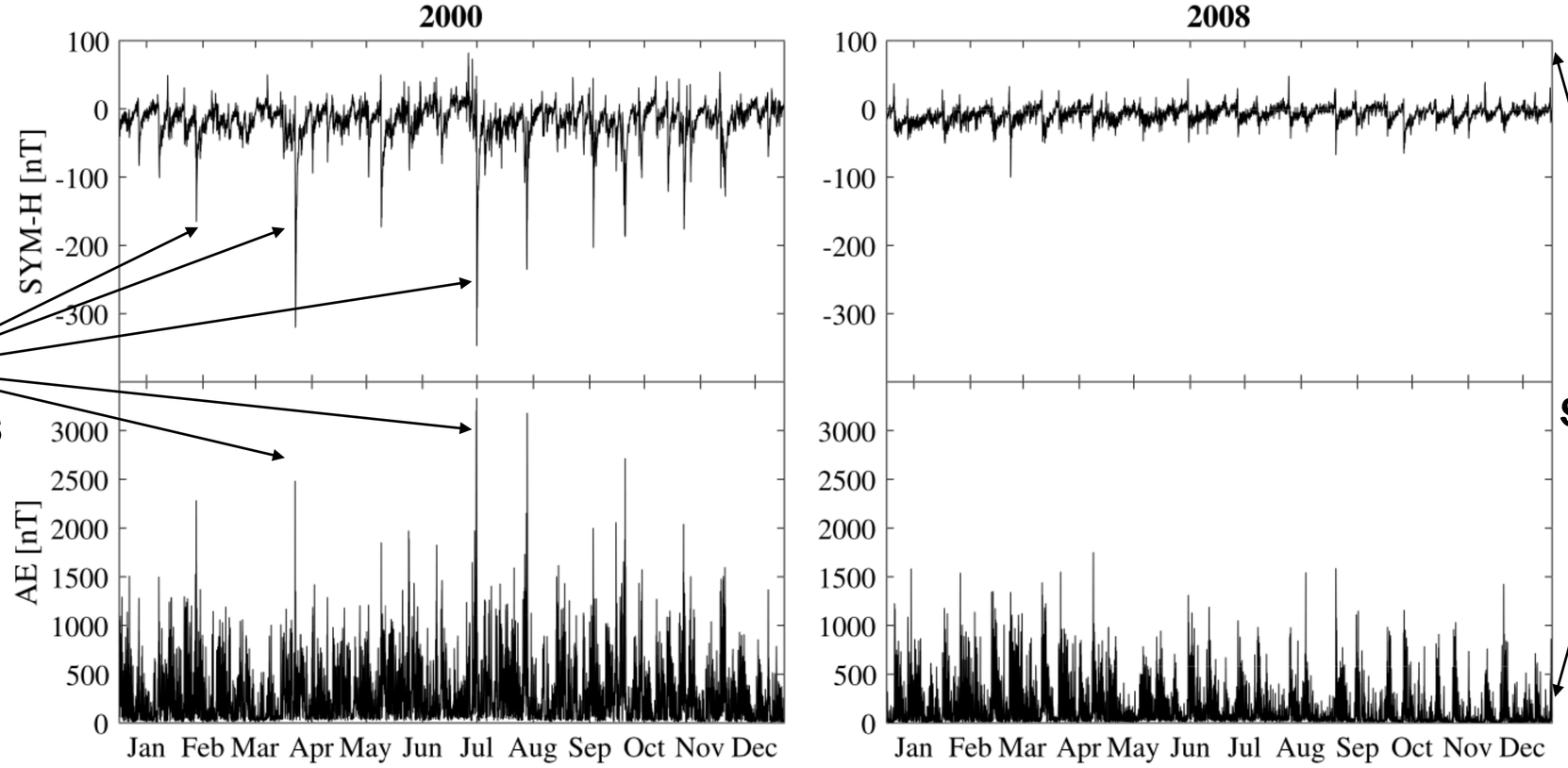
[Consolini+ (1996), Wanliss (2005)]

Is there any dependence on solar cycle phase?

Scale-dependent complexity measures during maxima and minima

Maximum phase of SC23

Minimum phase of SC23



A lot of storms and substorms

No or low-intense storms and substorms

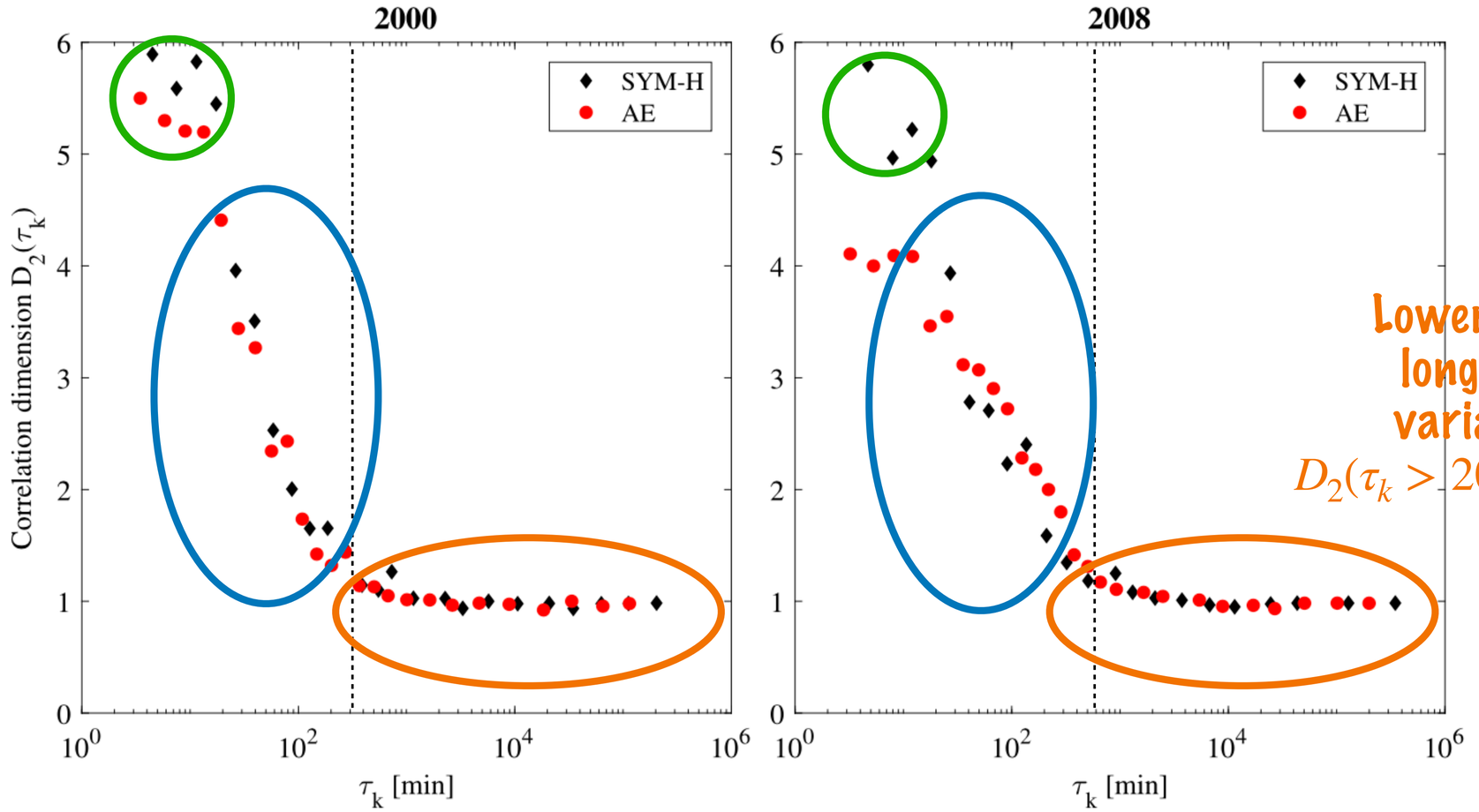
[Consolini+ (2018)]

Is there any dependence on solar cycle phase?

Scale-dependent $D_2(\tau_k)$ for each time series and $m = 6$ as in Vassiliadis+ (1990)

Noise?
$D_2 \sim m$

Larger D_2 for short-term variability



[Consolini+ (2018)]

Lower D_2 for long-term variability
$D_2(\tau_k > 200\,\text{min}) \sim 1$

[Rostoker+ (1987), Kamide & Kokubun (1996), Consolini & De Michelis (2005)]

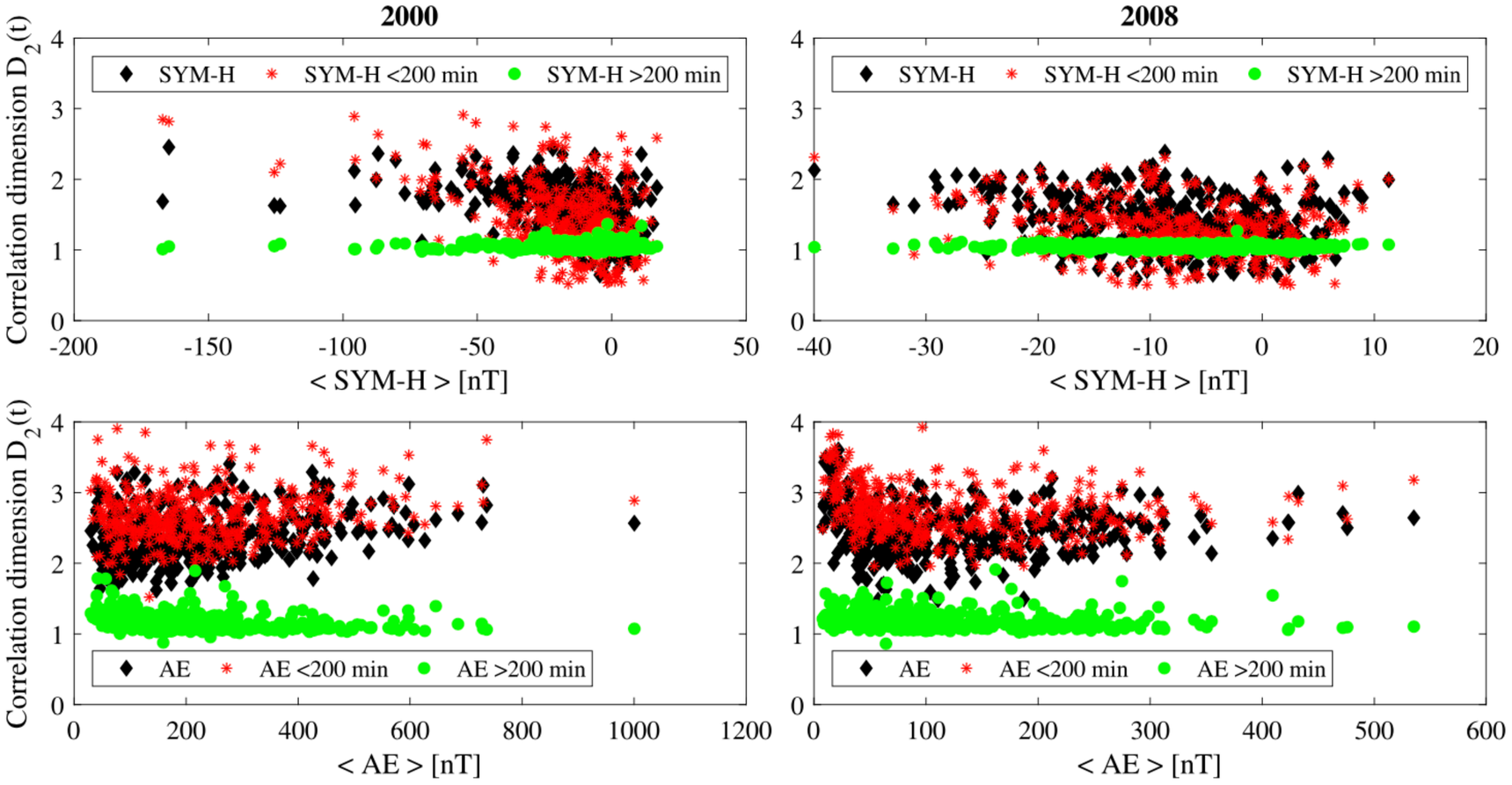
Is there any dependence on geomagnetic activity?

Short-/long-term variability vs. geomagnetic activity

SYM-H/AE < 200 min = $\sum_{k=1}^{k^*} C_k(t)$ s.t. $\tau_{k^*} = 200$ min

SYM-H/AE > 200 min = $\sum_{k=k^*+1}^{N_k} C_k(t)$ s.t. $\tau_{k^*} = 200$ min

$D_2 \gtrsim 1$ for short-term variability, increasing with geomagnetic activity



$D_2 \sim 1$ for long-term variability independent on geomagnetic activity

[Consolini+ (2018)]

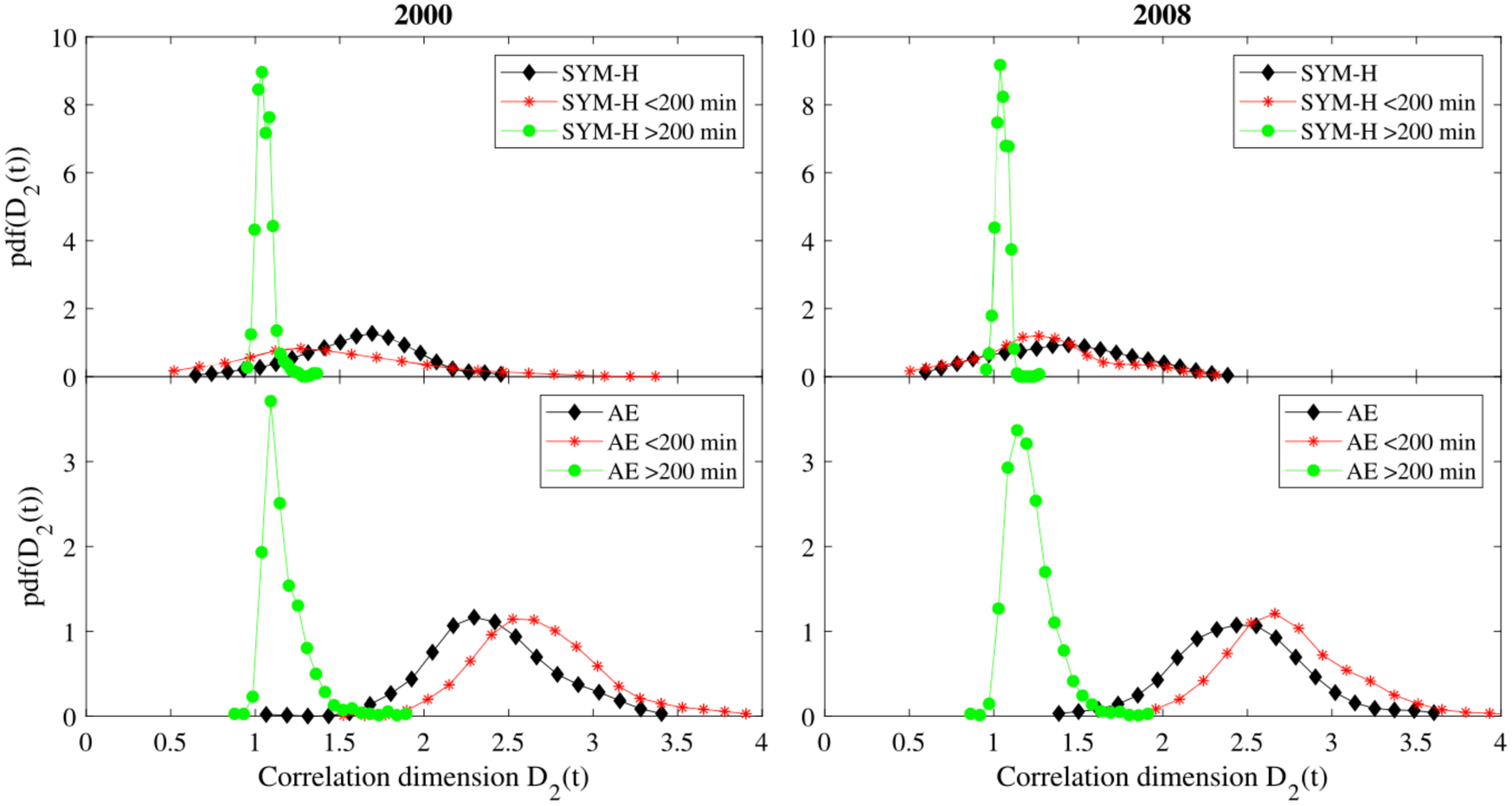
Is there any dependence on geomagnetic activity?

Short-/long-term variability vs. geomagnetic activity

SYM-H/AE < 200 min = $\sum_{k=1}^{k^*} C_k(t)$ s.t. $\tau_{k^*} = 200$ min

SYM-H/AE > 200 min = $\sum_{k=k^*+1}^{N_k} C_k(t)$ s.t. $\tau_{k^*} = 200$ min

Larger values are found for AE wrt SYM-H for short-term variability



Similar values are found for AE and SYM-H for long-term variability

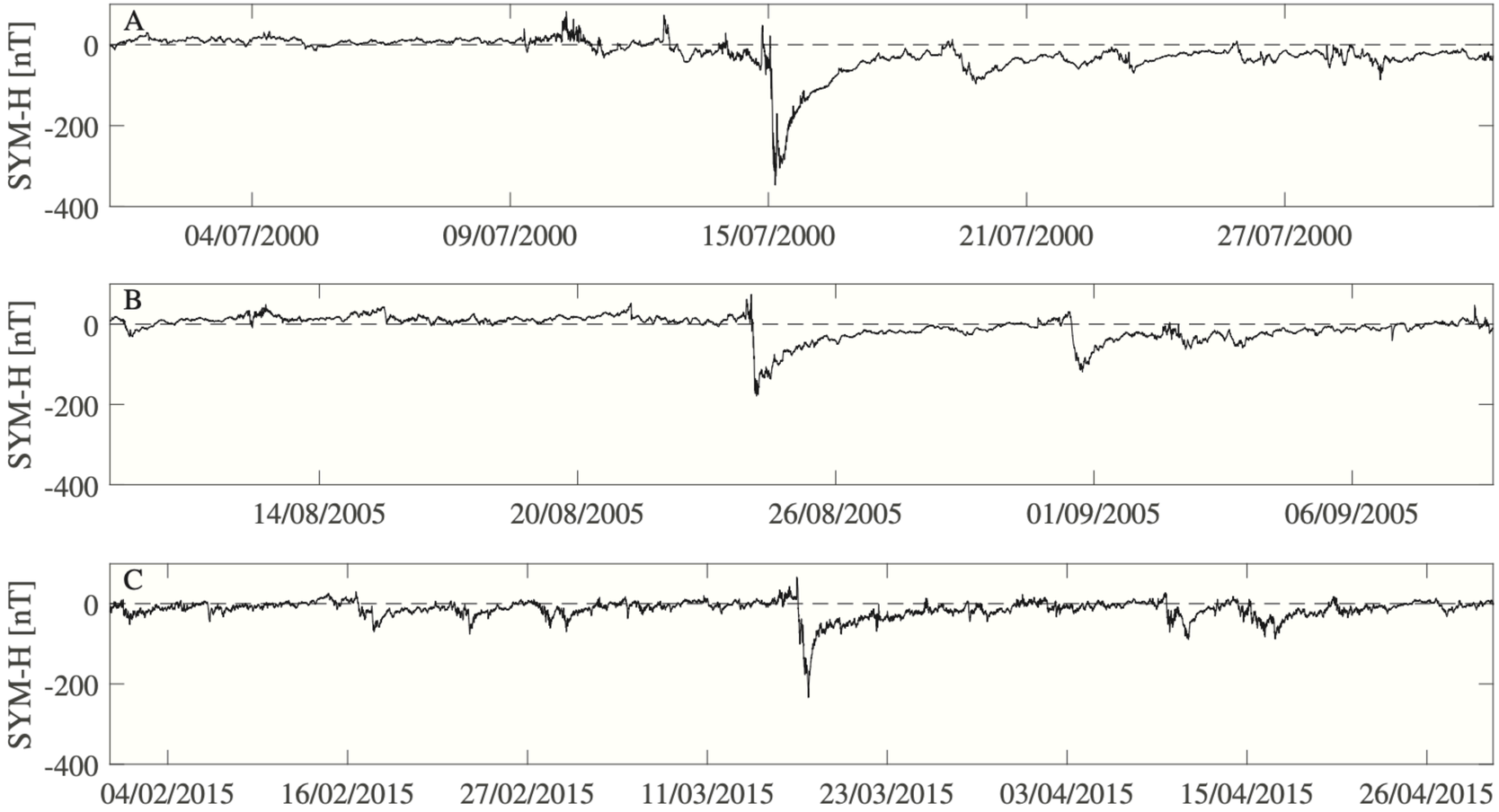
[Consolini+ (2018)]

Could the scale τ be considered a "control" parameter?

Are short- and long-term dynamics related to different fixed points?

[Alberti+ (2018)]

Three case studies of geomagnetic storms



Different intensities

Different morphology

Same length of time intervals

Could the scale τ be considered a "control" parameter?

Are short- and long-term dynamics related to different fixed points?

Use a simple Langevin model to describe the time evolution of SYM-H

$$\dot{x}(t) = -\frac{dU(x)}{dx} + \sigma\,\eta(t)$$

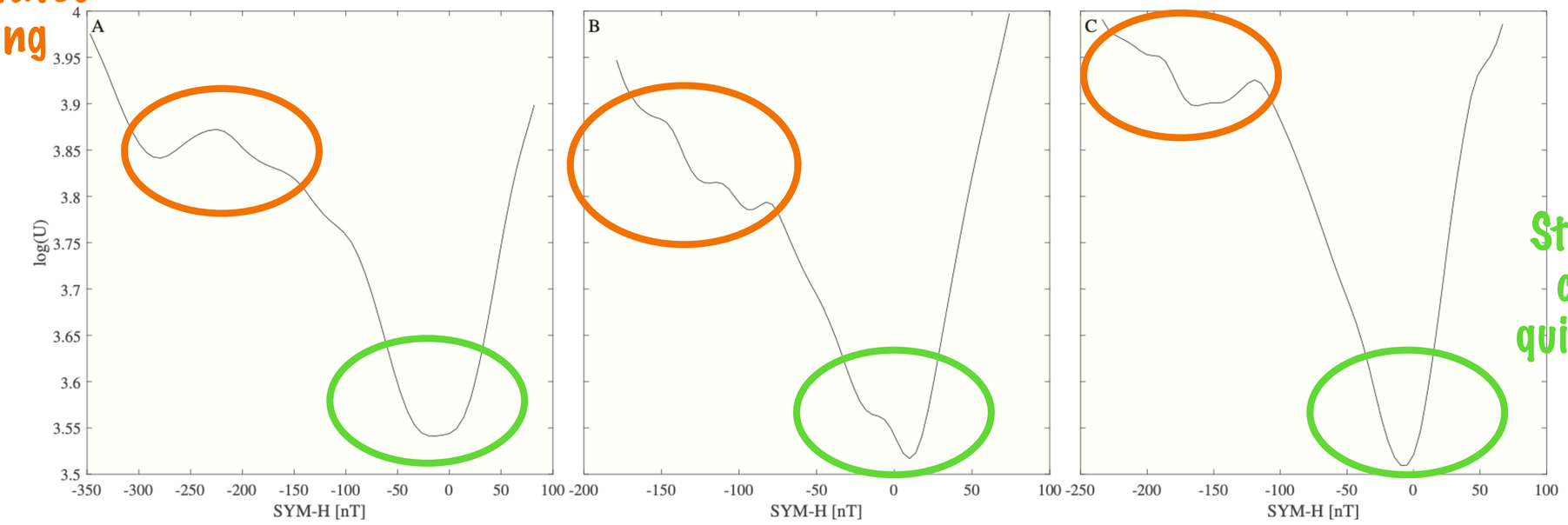
Derive the corresponding Fokker-Planck equation

$$\frac{d}{dt}\rho(x,t) = -\frac{\partial}{\partial x}\left[U(x)\rho(x,t)\right] + \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2}\left[\rho(x,t)\right]$$

Find its stationary solution

$$U(x) = -\frac{\sigma^2}{2}\log\left[\rho(x)\right]$$

Meta-stable states forming during the storm



Stable steady-state corresponding to quiet-time conditions

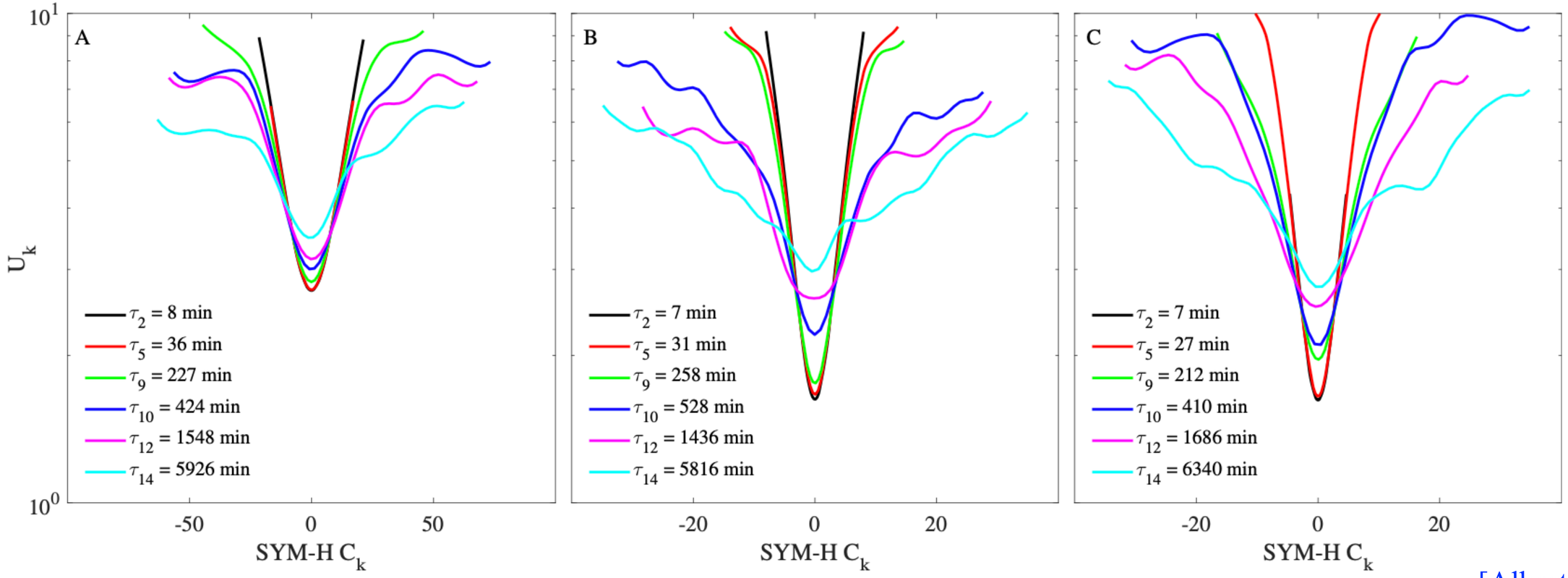
[Alberti+ (2018)]

Could the scale τ be considered a "control" parameter?

Are short- and long-term dynamics related to different fixed points?

Single-state function is found at short scales

Meta-stable states characterize state function at large scales



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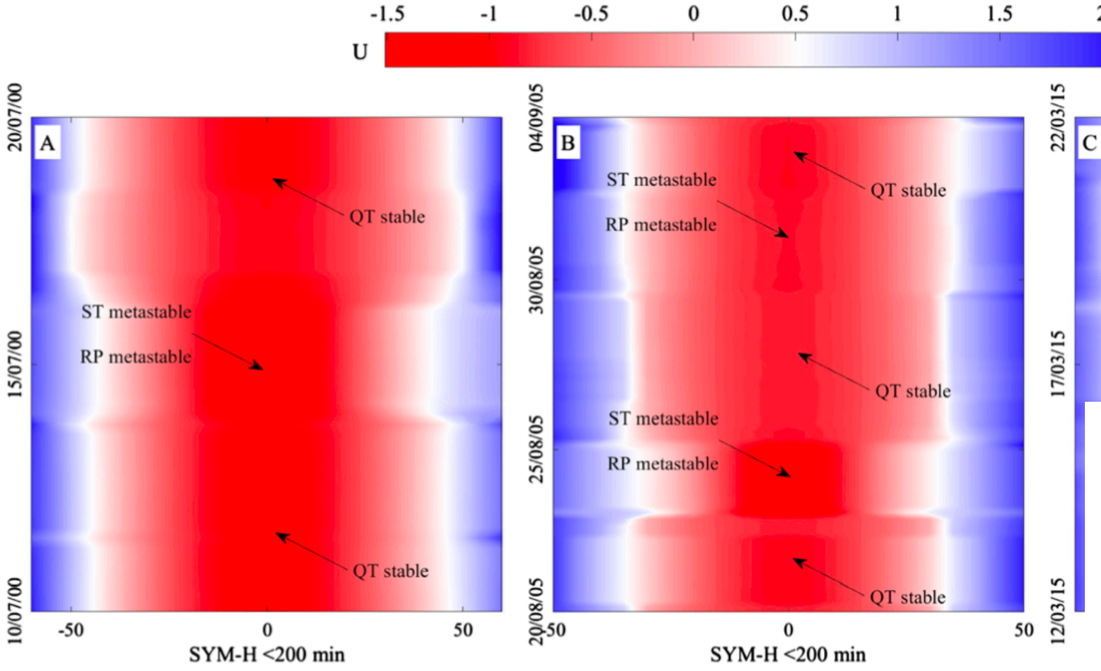
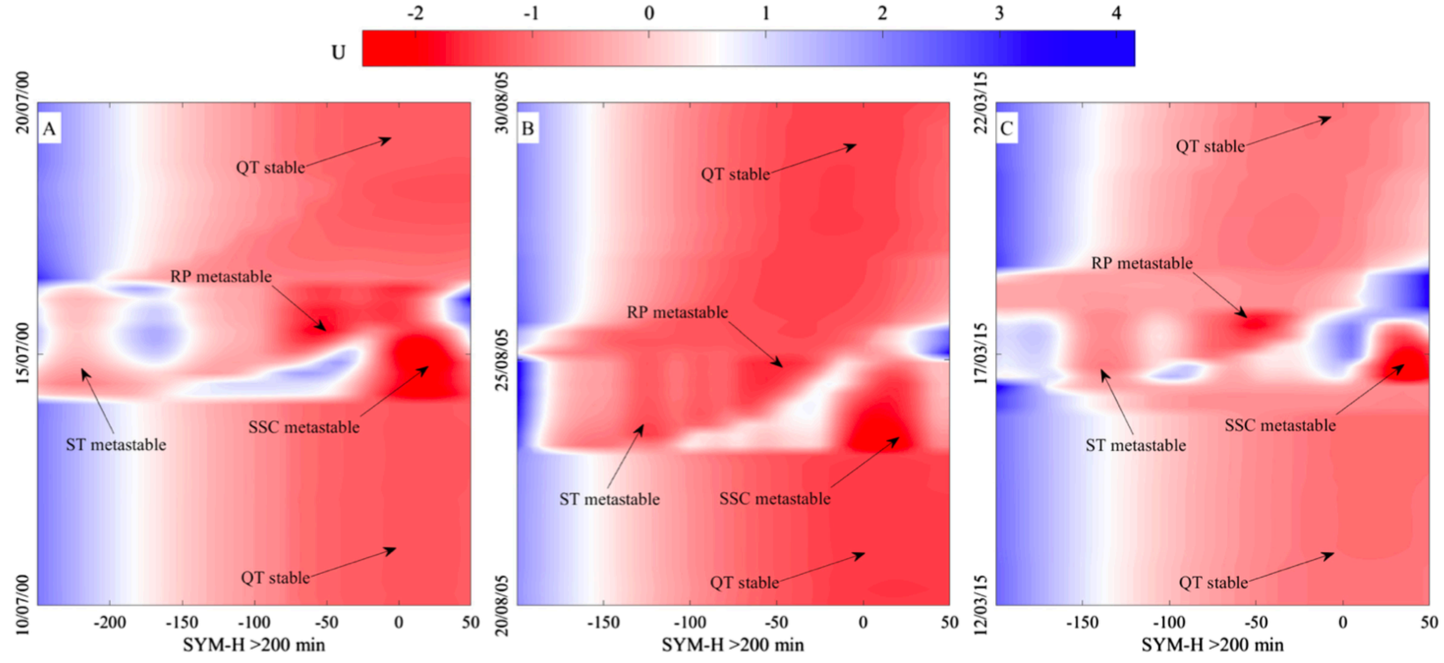


Single-stable states are not time-dependent characterize short-term variability also during a geomagnetic storm

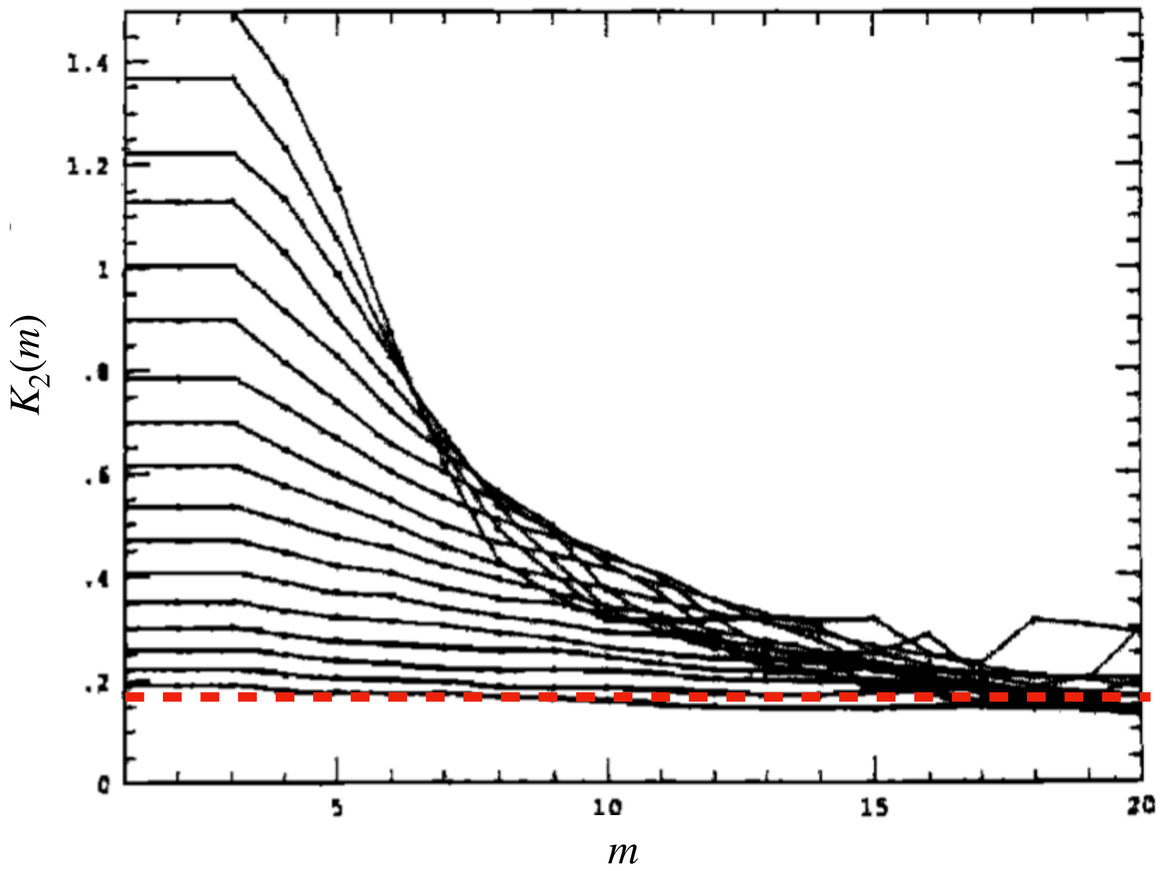


Meta-stable states are not only scale-dependent but also time-dependent and are related to the development of the geomagnetic storm

D. Chaotic measures: can we forecast the whole dynamics?

Forecasting geomagnetic indices variability

Why it is not so easy?



$$K_2 = \lim_{r \to 0} \frac{1}{\Delta t} \log \frac{C(r, m)}{C(r, m+1)}$$

* **Chaotic system:** $K_2 < \infty$
* **Non-deterministic system:** $K_2 \to \infty$
* **Forecast horizon:** $\tau_2 = K_2^{-1}$

[Vassiliadis+ (1990)]

$K_2(m) > 0.2\ \text{min}^{-1} \rightarrow \tau_2 < 5\ \text{min}$

We can accurately predict the behavior of the system with a forecast horizon of 5 min!

Remember: not the global magnetosphere, only its response represented by AE

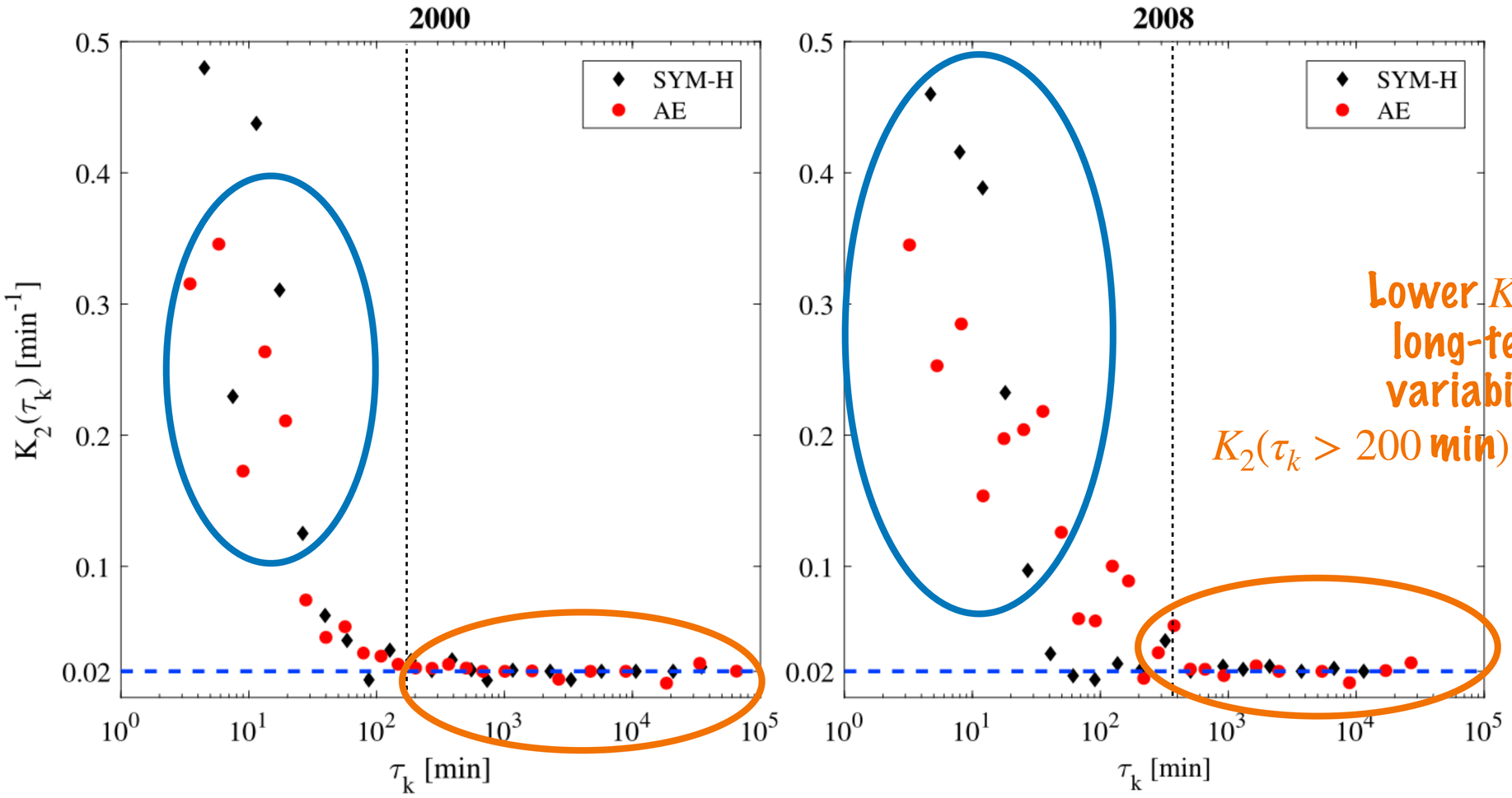
Forecasting geomagnetic indices variability

The role of chaos on short-term variability

[Consolini+ (2018)]

Larger K_2 for short-term variability

The forecast horizon is $\tau_2 \sim 2$ min!



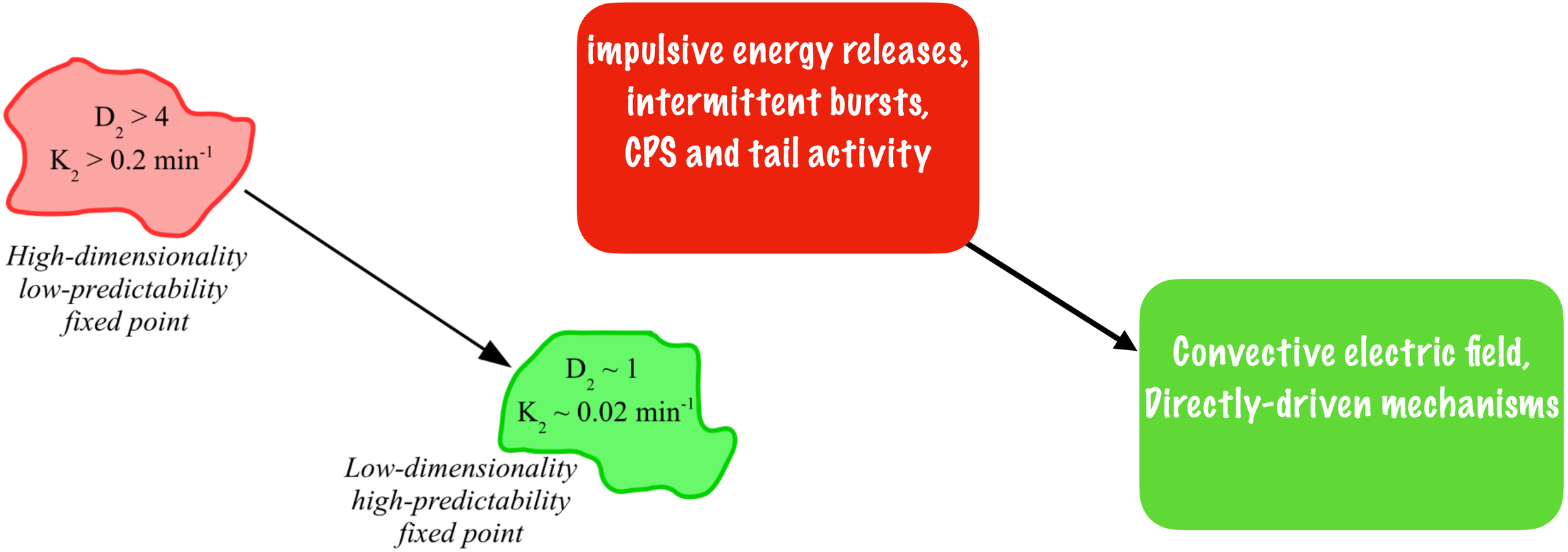
Lower K_2 for long-term variability

$K_2(\tau_k > 200\,\text{min}) \sim 0.02\,\text{min}^{-1}$

We can accurately predict the long-term behavior of the system with a forecast horizon of 50 min!

Forecasting geomagnetic indices variability

The role of chaos on short-term variability



more work is needed to determine some proxies for the tail dynamical state with a time resolution of seconds, necessary to overcome complications associated with the forecasting of short-timescale dynamics

[Kamide & Kokubun (1996), Consolini & De Michelis (2005), Alberti+ (2017), Consolini+ (2018), Alberti+ (2018, 2020b)]

E. Final remarks

Final remarks

1. Be careful: the overall Earth's magnetosphere is not a low-dimensional system
2. Its response to solar wind changes is scale-dependent and time-dependent
3. $1 < D_2(m) < m \rightarrow$ more than one independent variable to describe only one index
4. $2\,\text{min} < K_2^{-1} < 50\,\text{min} \rightarrow$ a scale-dependent forecast horizon
5. Suitable proxies are needed to understand and then forecast the internal dynamical state

The SMI system is a high-dimensional complex system that looks random but it is not random, it looks deterministic but it is not, it looks driven by but not only

Thanks for the attention

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