UNIVERSITÀ DELLA CALABRIA
Dipartimento di FISICA

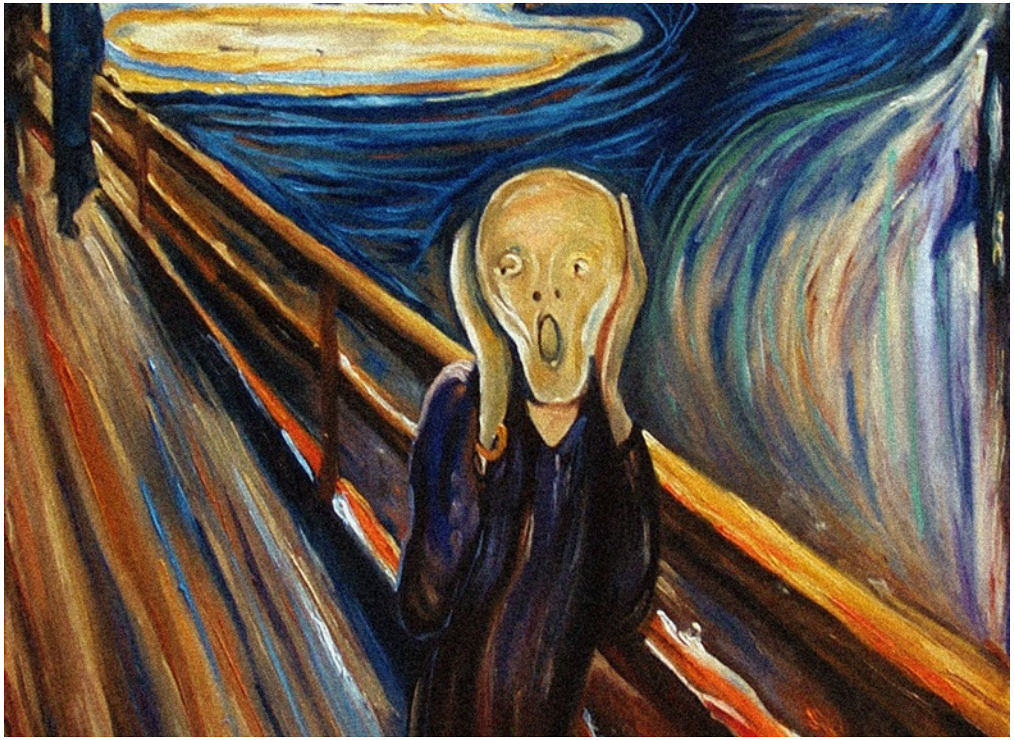
Turbulence in Space Plasmas: From MHD scales to kinetic domain

Vincenzo Carbone

Dipartimento di Fisica, Università della Calabria

ISSS course:
Dynamical Systems and Machine Learning Approaches to Sun-Earth Relations
L'Aquila, February 2020

SUMMARY: I would like to describe how fluctuations, usually observed in the interplanetary space, can reach small scales thus dissipating energy in a collissionless plasma

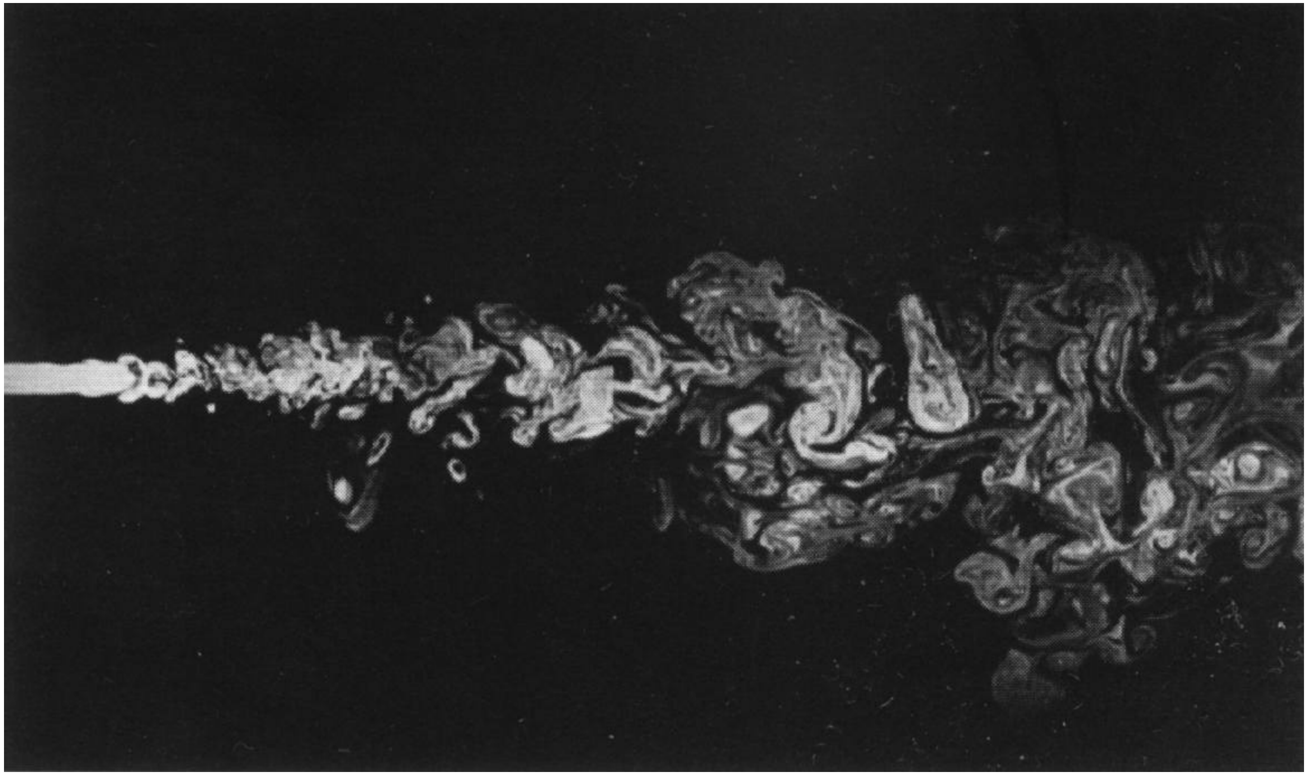


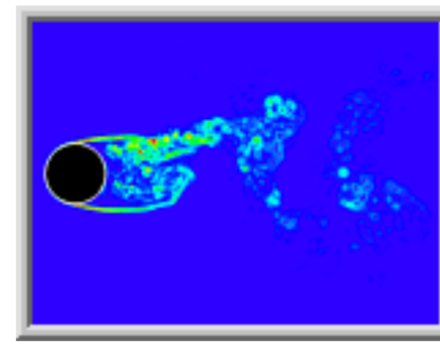
1. Fluid turbulence presents some problem;
2. Magnetic field add some annoyance;
3. In Space Plasmas, both 1. and 2. add a lot of contradiction and confusion.

I was sitting at my desk looking at the topic of my lectures, at the wide, sometimes confusing and contradictory literature … and I sensed an infinite scream passing through the universe …

paraphrase by E. Munch, 1893

A peculiar stochastic process: strange mixing of order and chaos



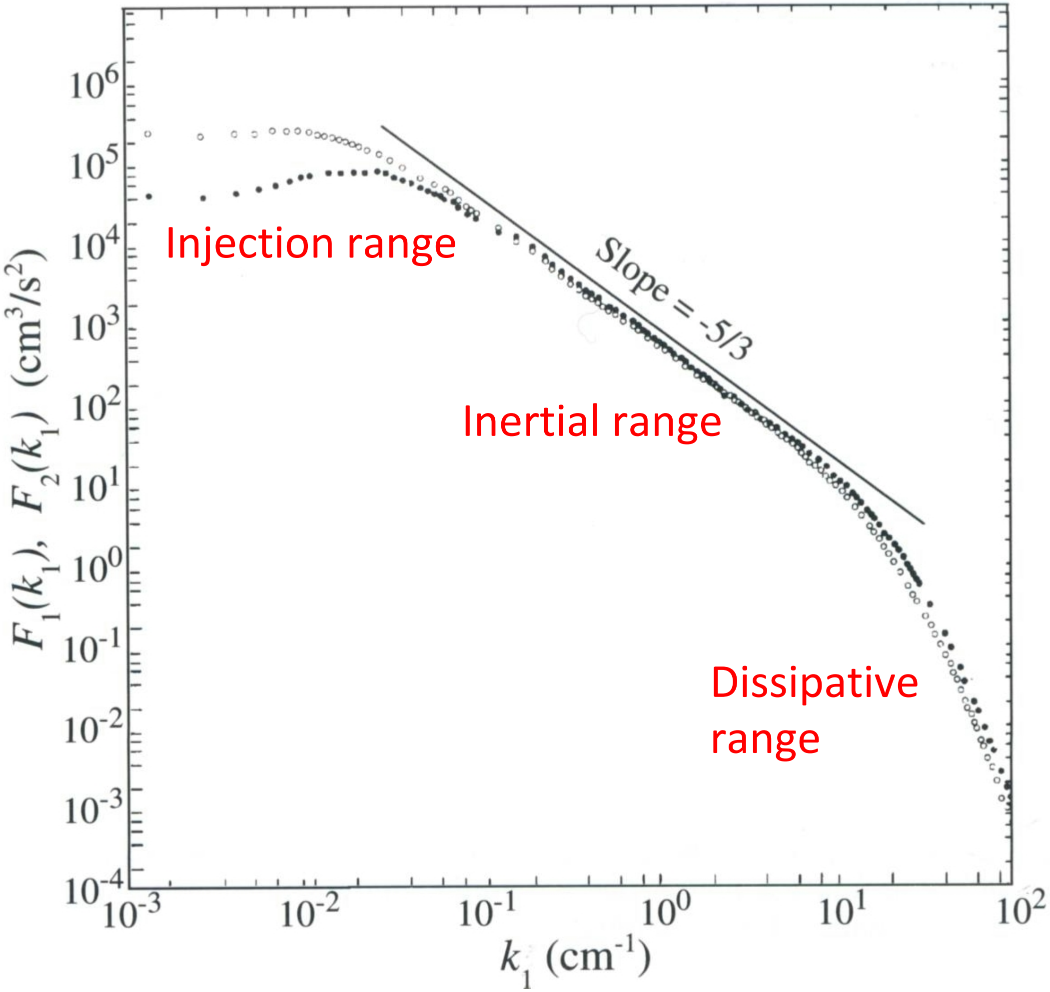


Turbulence is far from a sequence of random numbers with a well defined spectrum and uncorrelated phases. You cannot reproduce a "turbulent field" putting at random sand on a table and collecting snapshots!

Main features:
1) Randomness both in space and time
2) Turbulent "structures" (eddies) on all scales
3) Unpredictability and instability to very small perturbations

Details of the turbulent motion are unpredictable, but statistical behaviours are reproducible

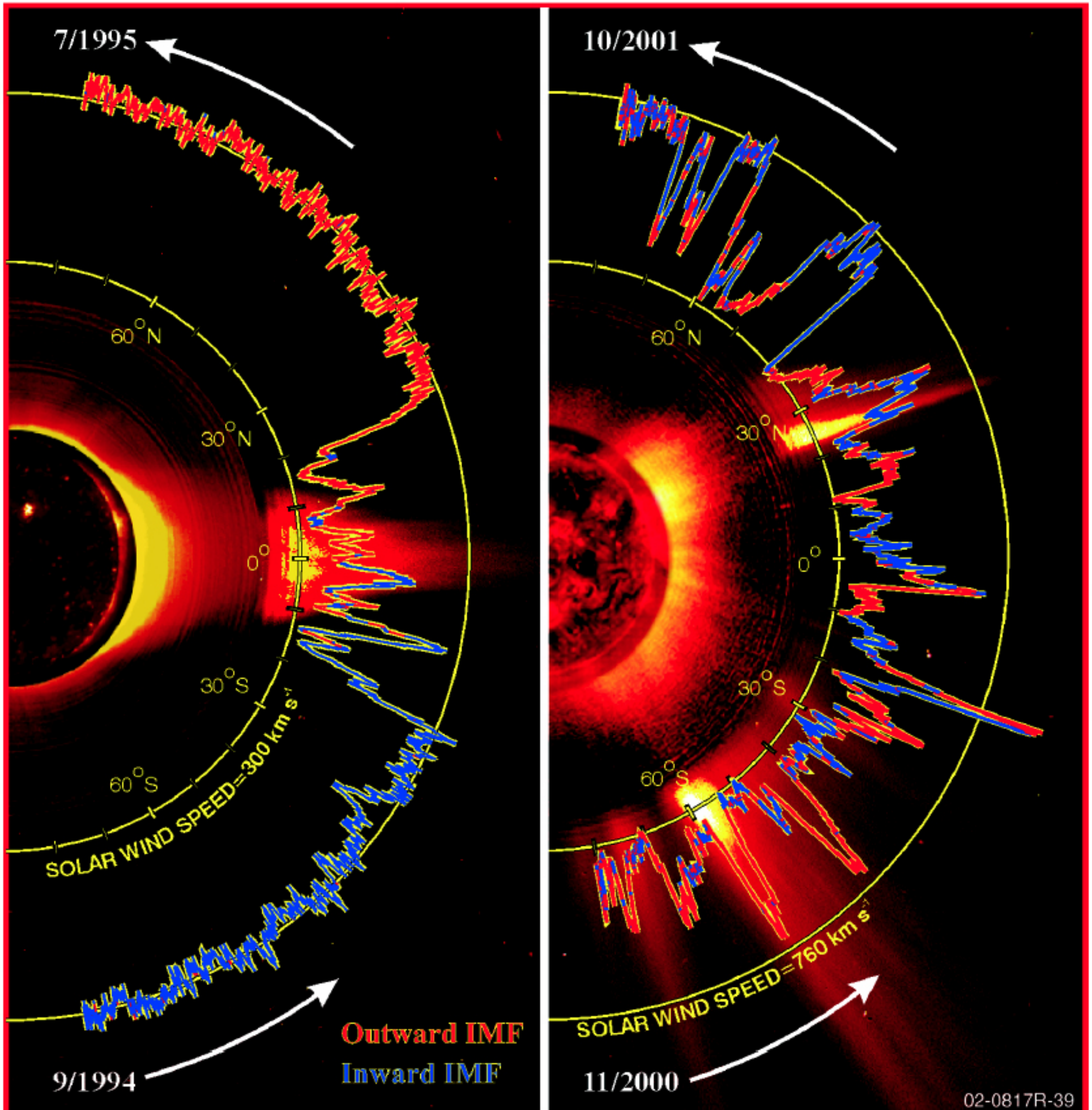
$$\Delta u_r = u(x+r) - u(x) \qquad \left\langle \Delta u_r^2 \right\rangle = 2\int_0^\infty E(k)g(kr)\,dk$$



The Kolmogorov energy spectrum can be observed almost everywhere in turbulent flows

$$E(k) \approx k^{-5/3}$$

The -5/3 scaling law within the inertial range, rapidly becomes a distinctive feature of turbulence.

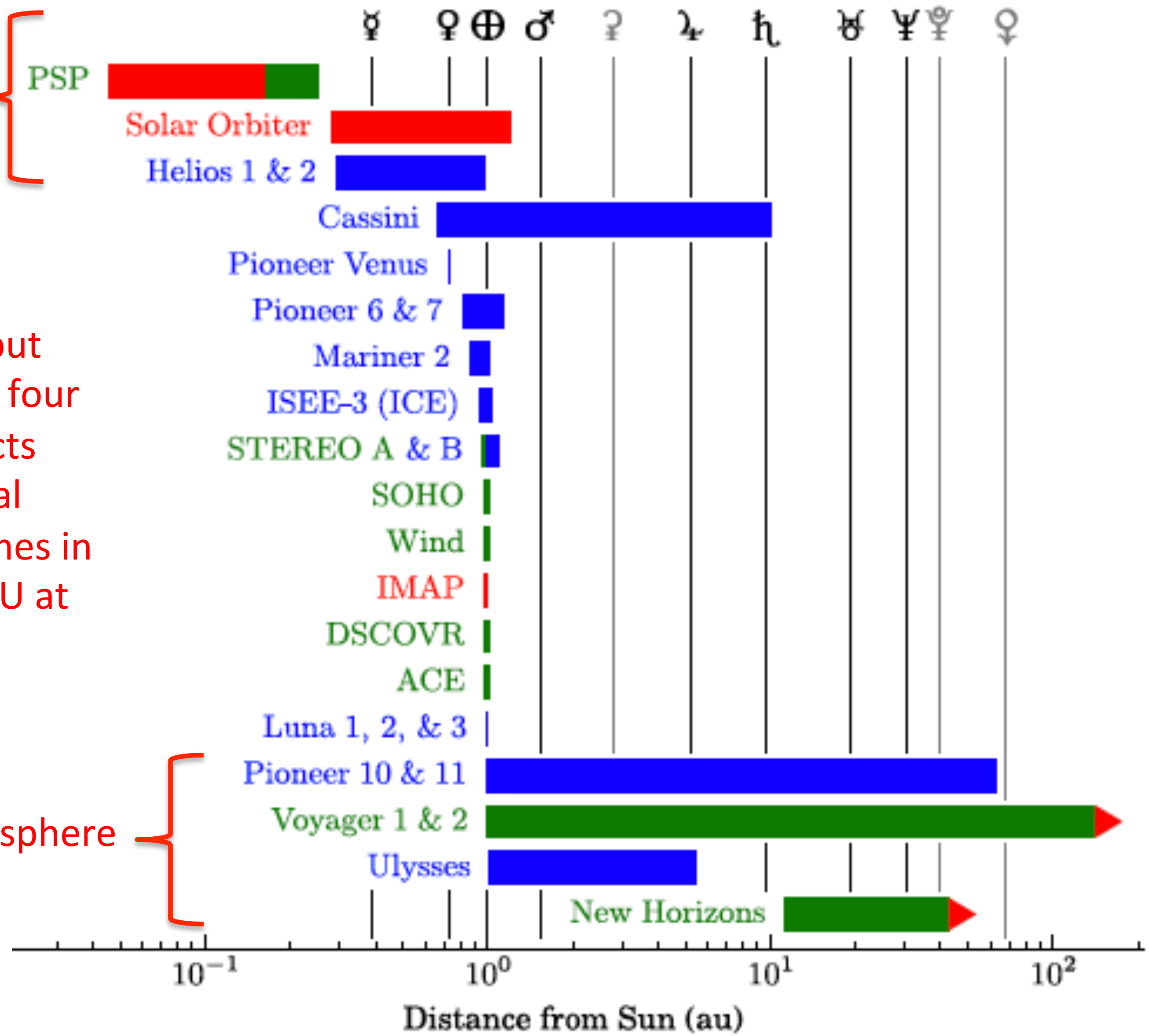


The interplanetary space is permeated by the solar wind, a supersonic plasma flow coming from the exterior of the Sun (solar corona).

Spacecrafts represent local probes in the solar wind. They detected high amplitude fluctuations of plasma parameters within solar wind.

Radial coverage of spacecrafts

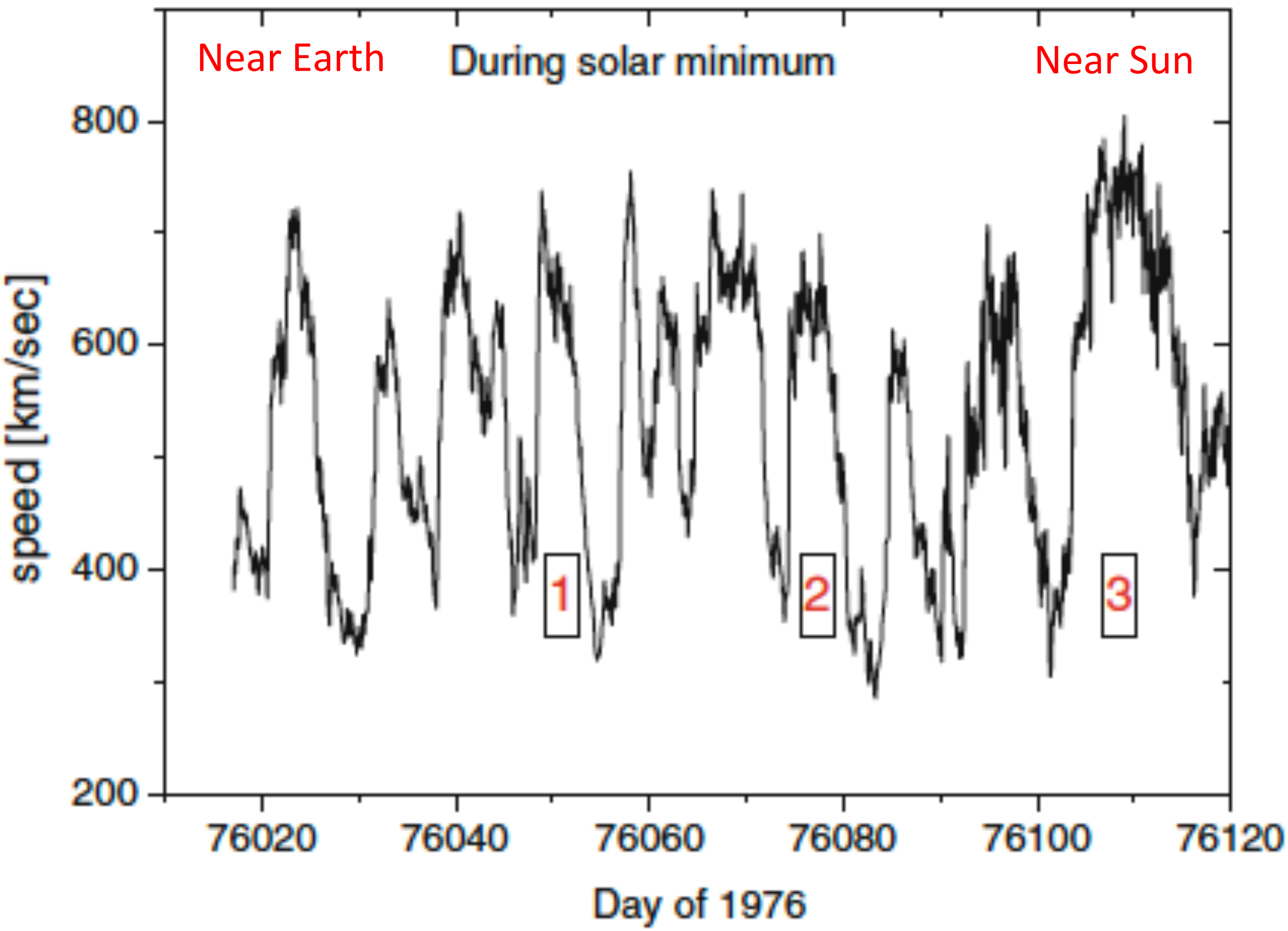
Inner heliosphere



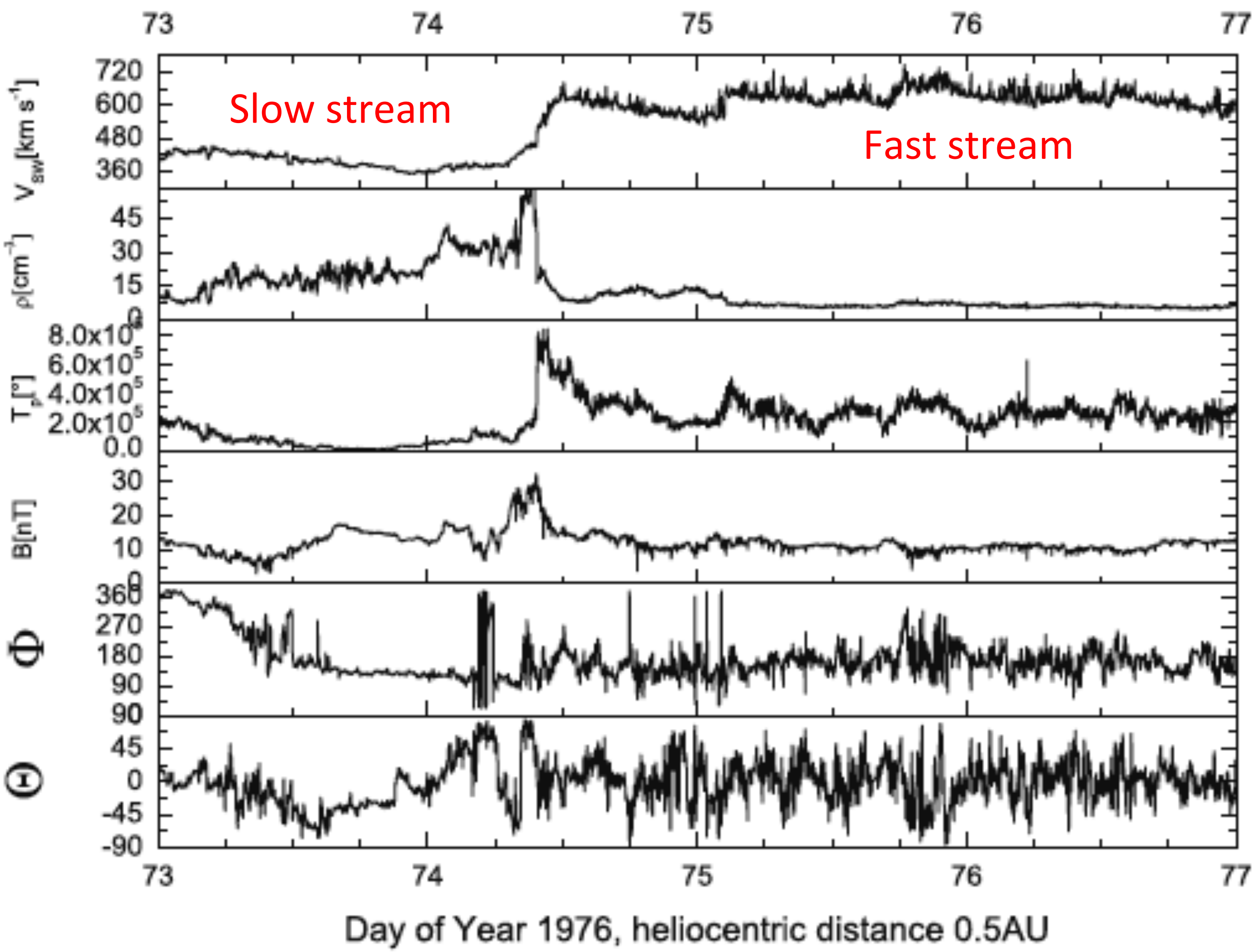
Single spacecrafts but CLUSTER: a suite of four identical spacecfracts flying in a tetrahedal formation (sometimes in solar wind near 1 AU at Earth's orbit).

Outer heliosphere

Actually two kind of wind streams are observed, **FAST streams** and **SLOW streams** (coming from different sources) superimposed to fluctuations with different characteristics.



Some differences between fluctuations in fast and slow streams

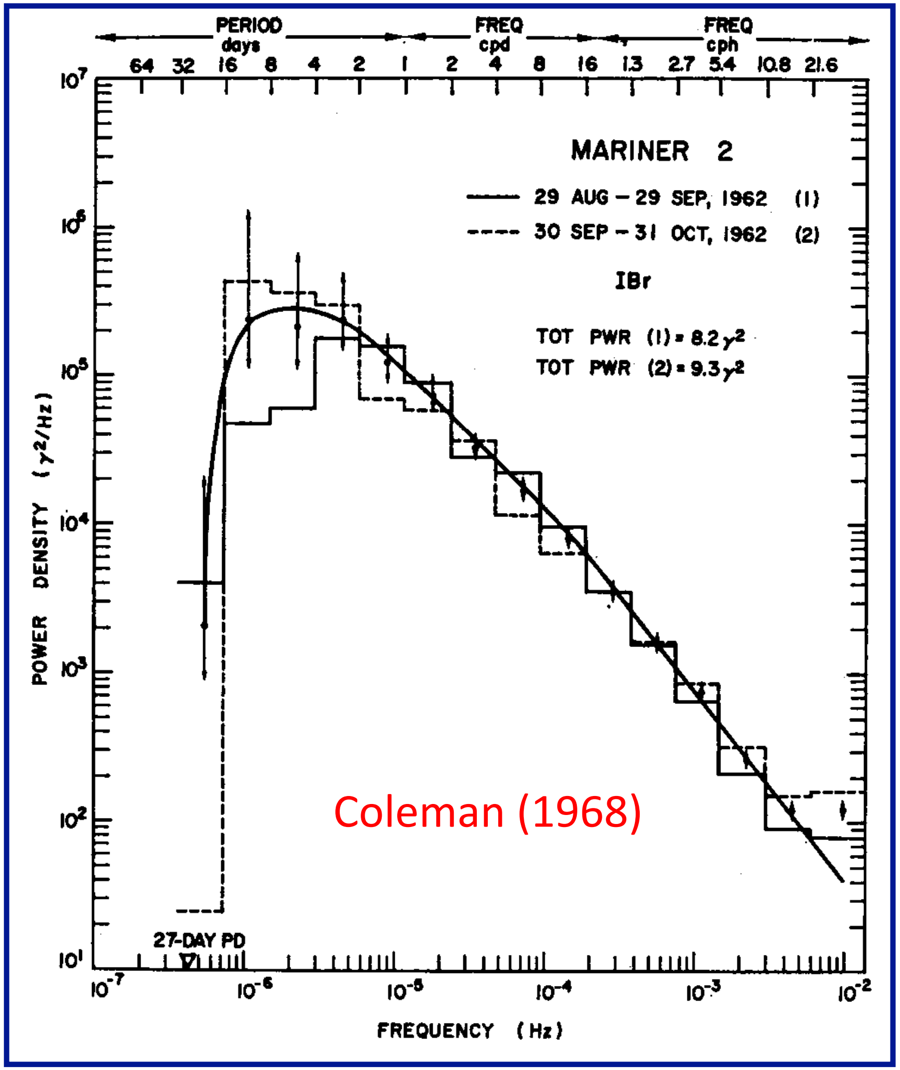


SLOW STREAMS: Enhanced density fluctuations

FAST STREAMS: Enhanced temperature fluctuations

Fig. 3.12 High velocity streams and slow wind as seen in the ecliptic during solar minimum

First observations of spectral properties of magnetic fluctuations indicated the existence of a Kolmogorov-like energy spectrum spanning almost four decades.



Spectral properties are detected by time series from the Mariner 2 spacecraft

$$\Delta B_\tau = B(t+\tau) - B(t)$$

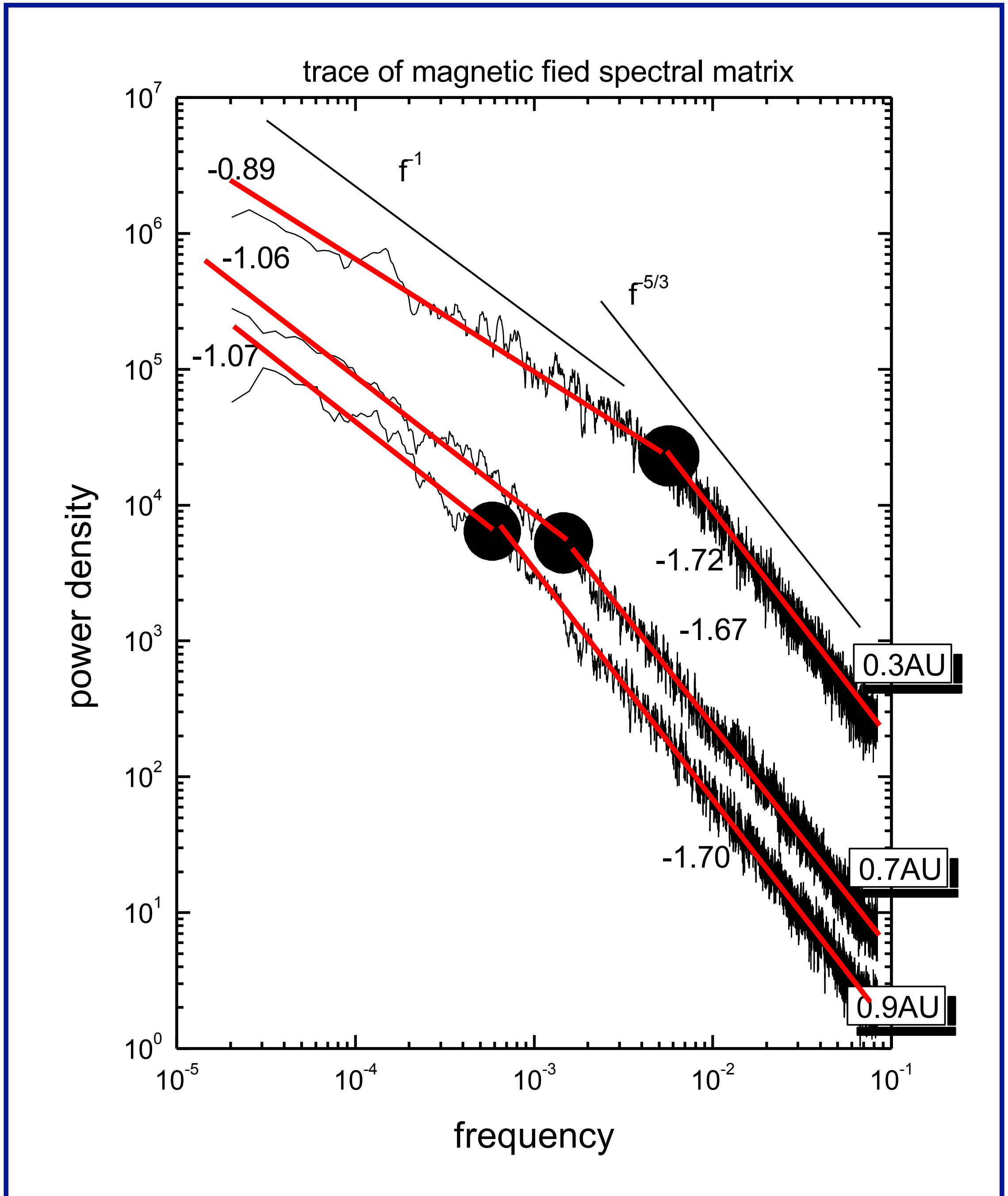
$$\langle \Delta B_\tau \rangle = 2\int_0^\infty P(f) g(f\tau)\, df$$

$$P(f) \approx f^{-5/3}$$

Using the Taylor hypothesis the spectra are interpreted in terms of wavevectors scaling law

$$E(k) \approx k^{-5/3}$$

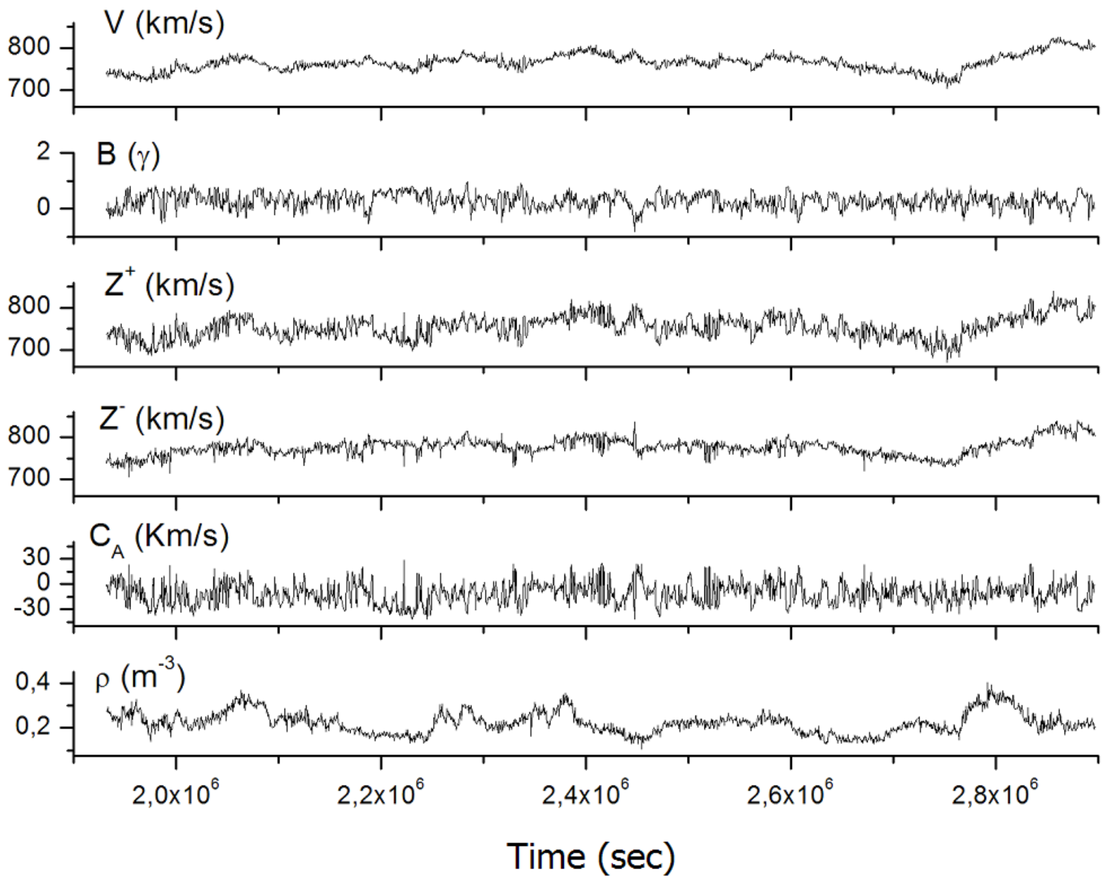
This was enough to frame fluctuations in SW as an example of fully developed turbulence



Despite some theoretical complications (inhomogeneities, compressibility, anisotropy) we have a lot of convinging evidences of Kolmogorov-like power spectrum for magnetic fluctuations:

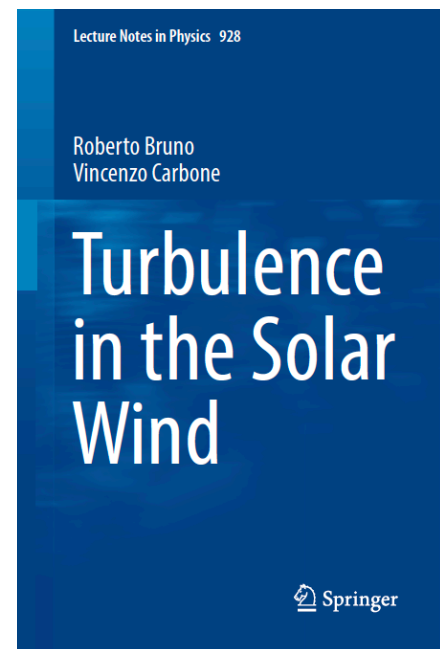
Low-frequency fluctuations in solar wind can be described in the framework of classical turbulence

The solar wind as a wind tunnel



$$z_i^{\pm} = v_i \pm b_i = v_i \pm \frac{B_i}{\sqrt{4\pi\rho}}$$

In situ measurements of high amplitude fluctuations for all fields (velocity, magnetic field, temperature…). A unique possibility to measure low-frequency turbulence in plasmas over a wide range of scales.



For a review:
R. Bruno & V. Carbone, Turbulence in the Solar Wind
Lecture Notes in Physics 928 (Springer, 2016)

Low-frequency plasma fluctuations are described by MHD equations. Turbulence is the result of nonlinear dynamics

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{u} = -\nabla P_{\mathrm{tot}} + \nu\nabla^2\mathbf{u} + (\mathbf{b}\cdot\nabla)\,\mathbf{b}$$

$$\frac{\partial \mathbf{b}}{\partial t} + (\mathbf{u}\cdot\nabla)\,\mathbf{b} = -(\mathbf{b}\cdot\nabla)\,\mathbf{u} + \eta\nabla^2\mathbf{b}.$$

$$\mathbf{b} = \mathbf{B}/\sqrt{4\pi\rho}$$

Incompressible MHD

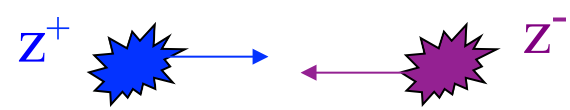
MHD equations couple velocity field fluctuations and magnetic fluctuations, share the same "structure" with Navier-Stokes equations → quadratic nonlinearities vs dissipation.

$$\frac{\partial z_i^\pm}{\partial t} + \left(z_j^\mp \frac{\partial z_i^\pm}{\partial x_j}\right) = -\nabla P + \nu\nabla^2 z_i^\pm$$

$$z_i^\pm = v_i \pm \frac{B_i}{\sqrt{4\pi\rho}}$$

Elsasser variables define pseudo-energies

$$\left\langle \left(\Delta z^\pm\right)^2 \right\rangle = E^\pm$$



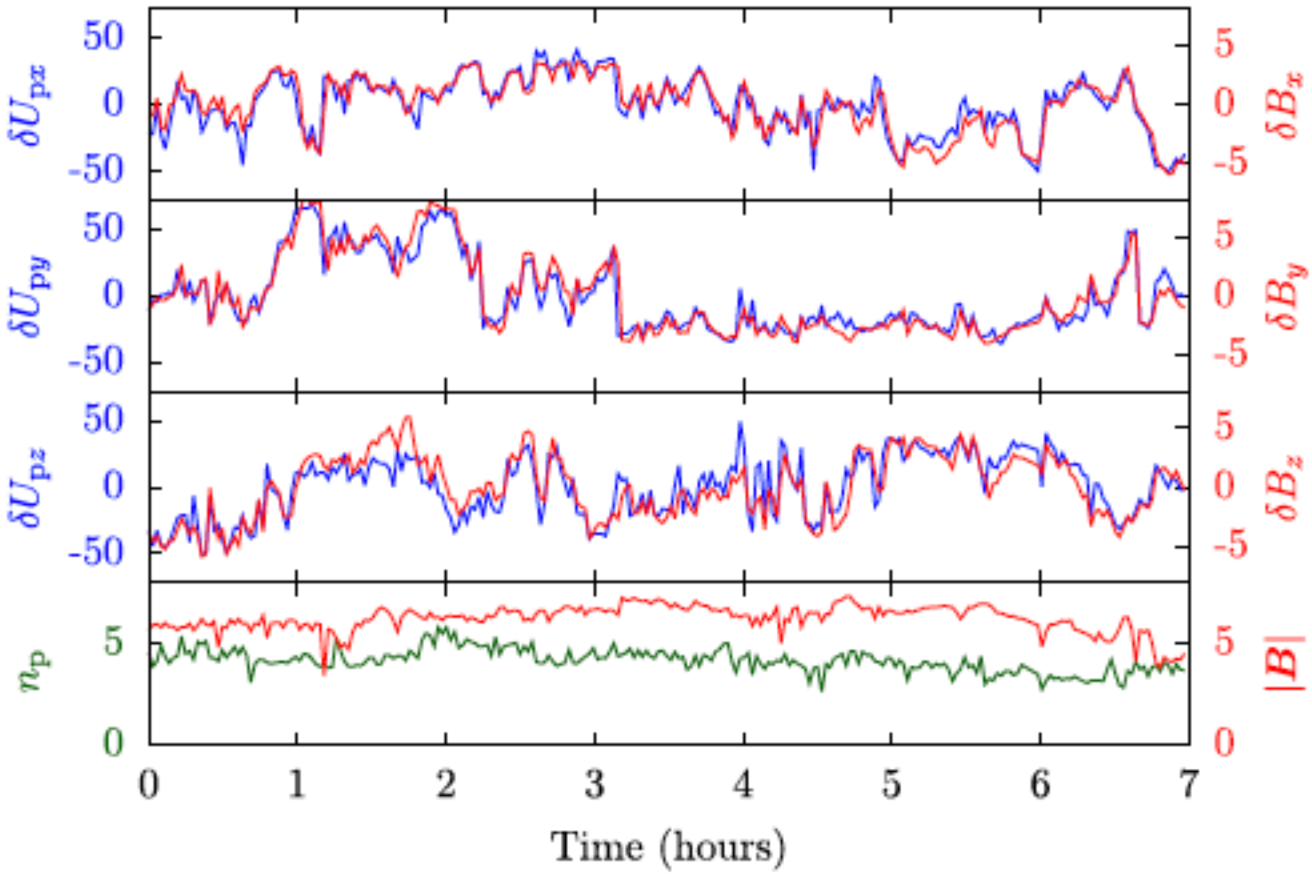
Nonlinear interactions in MHD happens only between fluctuations propagating in opposite direction with respect to the large-scale magnetic field
→ slow down of nonlinear interactions with respect to fluids: eddies move apart.

A further complication: u-b correlations

Alfvènic fluctuations at 0.3 AU within the high-speed stream

$$u \approx \pm \frac{b}{\sqrt{4\pi\rho}}$$

$$z^{\mp} \approx 0$$



Nonlinear interactions might be damped during these periods.

Does this means no turbulent cascade in fast wind?

Phenomenological arguments leading to Kolmogorov's spectrum (from MHD)

$$\partial_t z_i^{\pm} + z_\alpha^{\mp} \partial_\alpha z_i^{\pm} = -\partial_i P + \nu \partial_\alpha^2 z_i^{\pm}$$

In the limit of zero viscosity equations are **SCALING INVARIANT**, say they remains the same (for any value of h) for the following scaling transformations

$$r \Rightarrow r' \lambda$$

$$z^{\pm} \Rightarrow (z^{\pm})' \lambda^{h}$$

$$\Rightarrow \quad \frac{\Delta z^{\pm}}{r^h} \approx \frac{z^{\pm}(x+r) - z^{\pm}(x)}{r^h}$$

Are scale-invariant

$\Delta z_r^{\pm}$ Characteristic turbulent fluctuations across eddies at the scale r.

We expect scaling solutions where

$$\Delta z_r^{\pm} \approx r^h$$

Let us consider the dissipation rate for both pseudo-energies

$$\varepsilon^{\pm} \approx \frac{\left(\Delta z_r^{\pm}\right)^2}{\tau_{NL}^{\pm}}$$

The characteristic time (eddy-turnover time) represents the lifetime of turbulent eddies

$$\tau_{NL}^{\pm} \approx \frac{r}{\Delta z_r^{\mp}} \quad \Rightarrow \quad \varepsilon^{\pm} \approx \frac{\left(\Delta z_r^{\pm}\right)^2 \Delta z_r^{\mp}}{r}$$

Scaling invariance and statistics: fluid-like

$$\varepsilon^{\pm} \Rightarrow \varepsilon'^{\pm} \lambda^{3h-1}$$

The energy transfer rate is constant (Kolmogorov's hypothesis) only when

h = 1/3

$$\Delta z_r^{\pm} \approx \Delta z_r^{\mp} \approx \Delta u_r$$

This leads to the Kolmogorov scaling law

$$\Delta u_r \approx r^{1/3}$$

Second-order moment of fluctuations are related to the usual spectral energy density

$$\left\langle \left[u(x+r) - u(x) \right]^2 \right\rangle = 2 \int_0^{\infty} E(k) \left[1 - \frac{\sin kr}{kr} \right] dk$$

$$\left\langle \left(\Delta u_r \right)^p \right\rangle = C_p \varepsilon^{p/3} r^{p/3}$$

r ≈1/k

$$E(k) \approx \varepsilon^{2/3} k^{-5/3}$$

Kolmogorov scaling

Phenomenological arguments for magnetically dominated turbulence

When the flow is dominated by a (large-scale) magnetic field, there exists a new physical time, the Alfvèn time, related to the sweeping of Alfvenic fluctuations due to the large-scale magnetic field

$$\varepsilon^{\pm} \approx \frac{\left(\Delta z_r^{\pm}\right)^2}{T_r^{\pm}}$$

T is the time to effectively realize the cascade

Since the Alfvèn time in some case is LESSER than the eddy-turnover time, nonlinear interactions are reduced because the cascade is effectively realized in a time T:

$$T_r^{\pm} \approx \tau_{NL}^{\pm}\left(\frac{\tau_{NL}^{\pm}}{\tau_A}\right)$$

$$\tau_A \approx \frac{r}{c_A}$$

$$\tau_{NL}^{\pm} \approx \frac{r}{\Delta z_r^{\mp}}$$

A different expression for the pseudo-energies transfer rates

$$\varepsilon^{\pm} \approx \frac{\left(\Delta z_r^{\pm}\right)^2\left(\Delta z_r^{\mp}\right)^2}{r}$$

Kraichnan spectrum

$$\varepsilon^{\pm} \Rightarrow \varepsilon'^{\pm} \lambda^{4h-1}$$

The energy transfer rate is scaling invariant only when

h = 1/4

$$\Delta u_r \approx r^{1/4}$$

A different scaling

Iroshnikov-Kraichnan scaling

Linear scaling for the p-th order moments

$$\left\langle \left(\Delta z_r^{\pm}\right)^p \right\rangle = C'_p \left(c_A \varepsilon^{\pm}\right)^{p/4} r^{p/4}$$

$$\left\langle \left(\Delta z_r^{\pm}\right)^2 \right\rangle = C'_2 \left(c_A \varepsilon^{\pm}\right)^{1/2} r^{1/2} \Rightarrow E^{\pm}(k) \approx k^{-3/2}$$

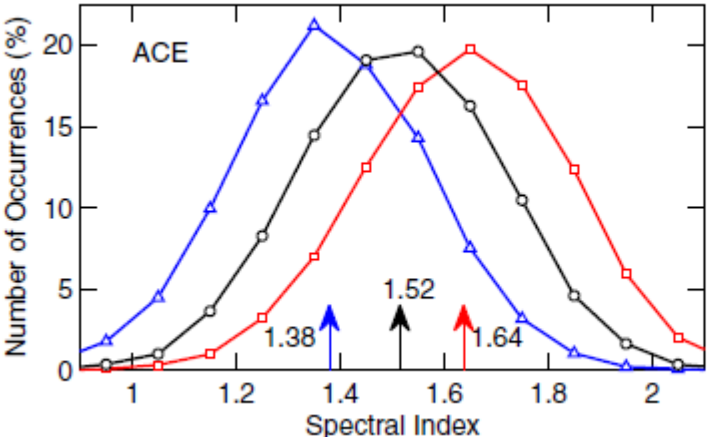
A flatter spectrum in the inertial range

At variance with the fluid-like case, here both pseudo-energies are transferred at "the same" rate

$$\varepsilon^{+} \approx \varepsilon^{-}$$

DMV-80: An initial unbalance between both pseudo-energies is maintained by the energy cascade. This should be enough to explain BOTH the existence of a power spectrum and the presence of one single Alfvènic "mode" fluctuation.

Data analysis and numerical simulations: Perhaps a Kolmogorov spectrum for magnetic energy, a flatter spectrum for kinetic energy



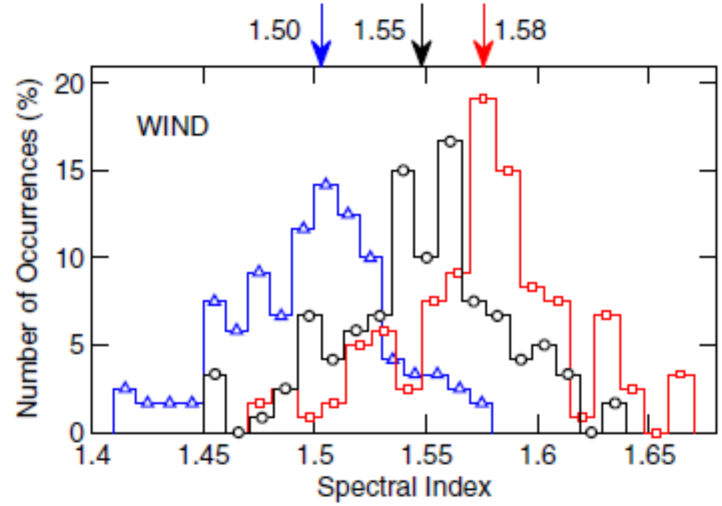


Figure 3. Histograms of measured spectral indices for the velocity spectrum (blue triangles), magnetic field spectrum (red squares), and total energy spectrum (black circles) in the solar wind using data from the *ACE* and *Wind* spacecraft. The average spectral indices are indicated by the arrows. Note the different horizontal scales in the two plots.

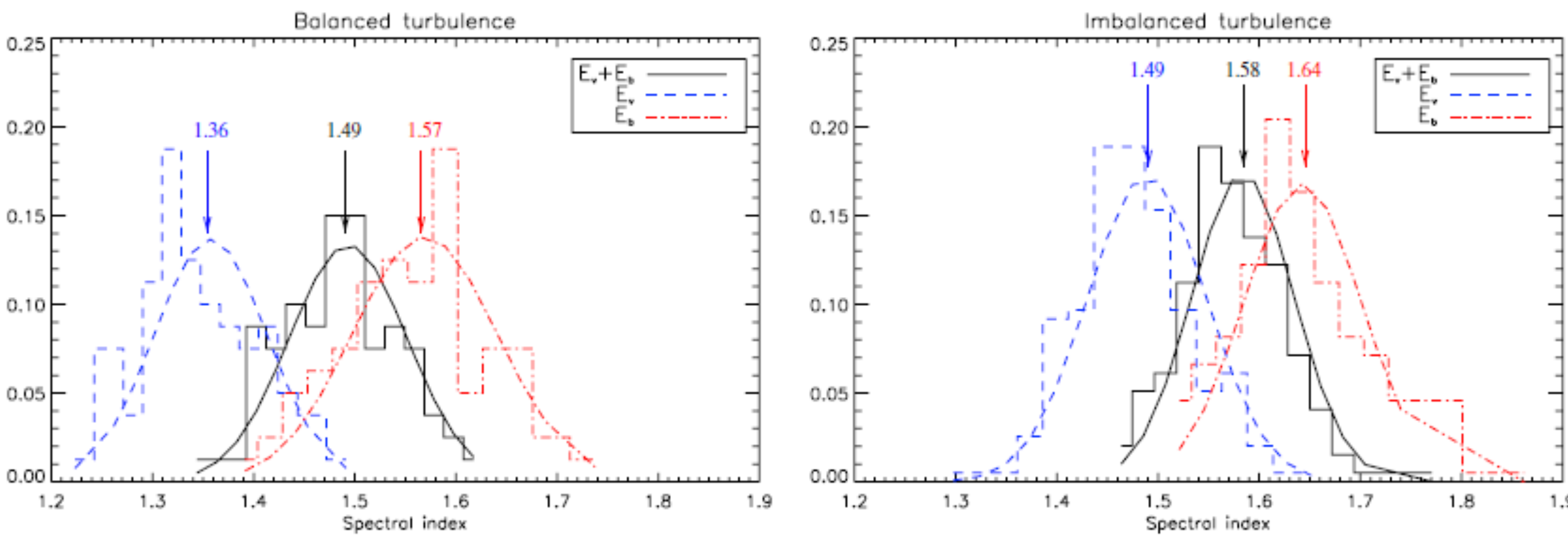


Figure 2. Distributions of spectral indices for kinetic, magnetic, and total energies for individual snapshots in numerical simulations of MHD turbulence. Left plot: balanced turbulence, 80 snapshots; right plot: imbalanced turbulence, 196 snapshots. The average spectral indices are indicated by arrows. Normal distributions with the mean values and variances matching those of the data are also shown.

Yaglom's law

An exact relation from MHD equations similar to Kolmogorov's 4/5-law

An exact Yaglom's statistical relation can be derived from MHD equations

A statistical relation obtained from fluid equations (NS or MHD) for the third-order moment of fluctuations, in the stationary state.

$$\Delta Z_i^{\pm} = Z_i^{\pm}(x_i') - Z_i^{\pm}(x_i) \qquad x_i' = x_i + \ell_i$$

Two-points differences of Elsässer variables

Large-scale inhomogeneity

Pressure anisotropy

$$\frac{\partial}{\partial \ell_\alpha}\langle \Delta Z_\alpha^{\mp}\left(\Delta Z_i^{\pm}\Delta Z_j^{\pm}\right)\rangle = -\langle Z_\alpha^{\mp}\left(\partial_\alpha' + \partial_\alpha\right)\left(\Delta Z_i^{\pm}\Delta Z_j^{\pm}\right)\rangle - \Pi_{ij} + \\ + 2\nu \frac{\partial^2}{\partial_\alpha^2}\langle \Delta Z_i^{\pm}\Delta Z_j^{\pm}\rangle - \frac{4}{3}\frac{\partial}{\partial \ell_\alpha}\left(\epsilon_{ij}^{\pm}\ell_\alpha\right)$$

Dissipation

$$\epsilon_{ij}^{\pm} = \langle\left(\partial_i Z_j^{\pm}\right)\left(\partial_j Z_i^{\pm}\right)\rangle$$

Pseudo-energy dissipation rate tensor

Assuming local isotropy and global homogeneity, finte transfer rate in the limit of vanishing viscosity, after longitudinal integration, the equation reduces to a Yaglom-like relation → a linear relation between the third-order mixed moment calculated through separations along the longitudinal (streamwise) direction, and the separation itself.

$$\langle \Delta Z_\ell^{\mp} | \Delta Z_i^{\pm} |^2 \rangle = -\frac{4}{3} \epsilon^{\pm} \ell$$

MHD turbulence

$$\langle \Delta u_\ell^3 \rangle = -\frac{4}{5} \epsilon \ell$$

Fluid turbulence (Kolmogorov's 4/5-law)

Note that the Yaglom law for MHD looks similar to the Yaglom law for a passive scalar in fluid turbulence

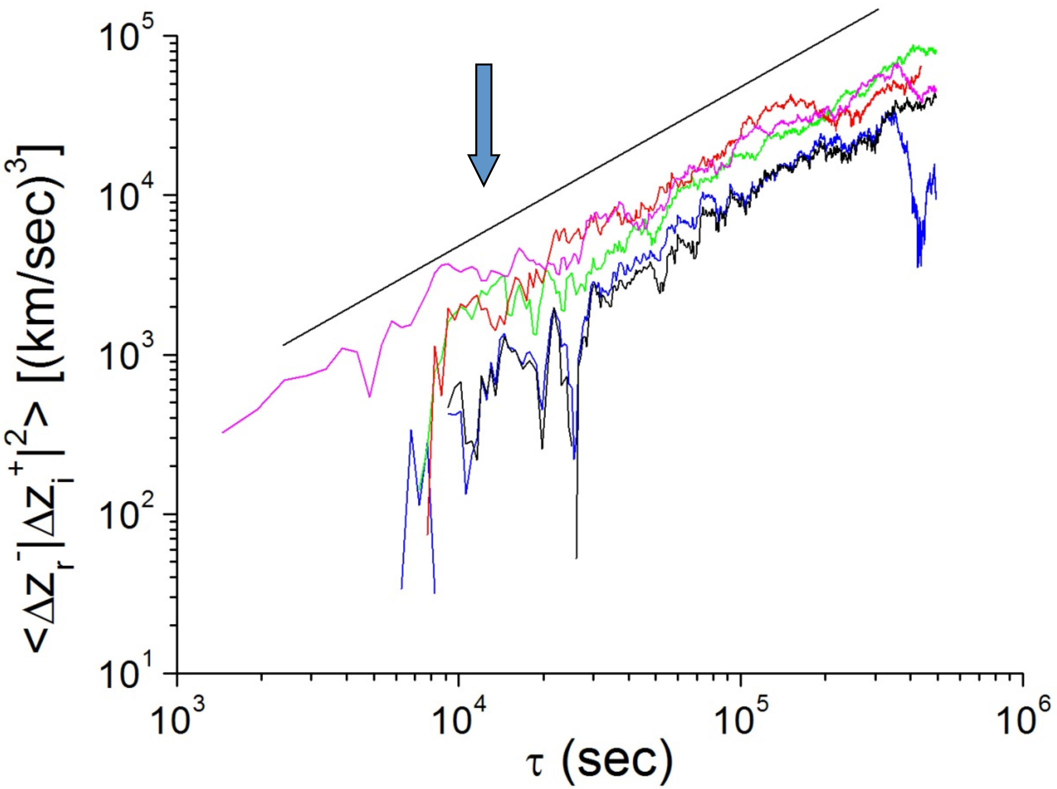
$$\partial_t \phi(\mathbf{r},t) - \mathbf{u}(\mathbf{r},t) \cdot \nabla \phi(\mathbf{r},t) = \kappa \Delta \phi(\mathbf{r},t) + f(\mathbf{r},t)$$

$$\langle [\delta_r \mathbf{u} \cdot \hat{\mathbf{r}}][\delta_r \phi]^2 \rangle = -\frac{4}{3} \epsilon_\phi r,$$

The passive scalar is advected by the velocity field

The Yaglom relation is satisfied by most datasets of Ulysses spacecraft during fast streams

$$\langle \Delta Z_\tau^\mp |\Delta Z_i^\pm|^2 \rangle = \frac{4}{3} U_{rms} \epsilon^\pm \tau$$

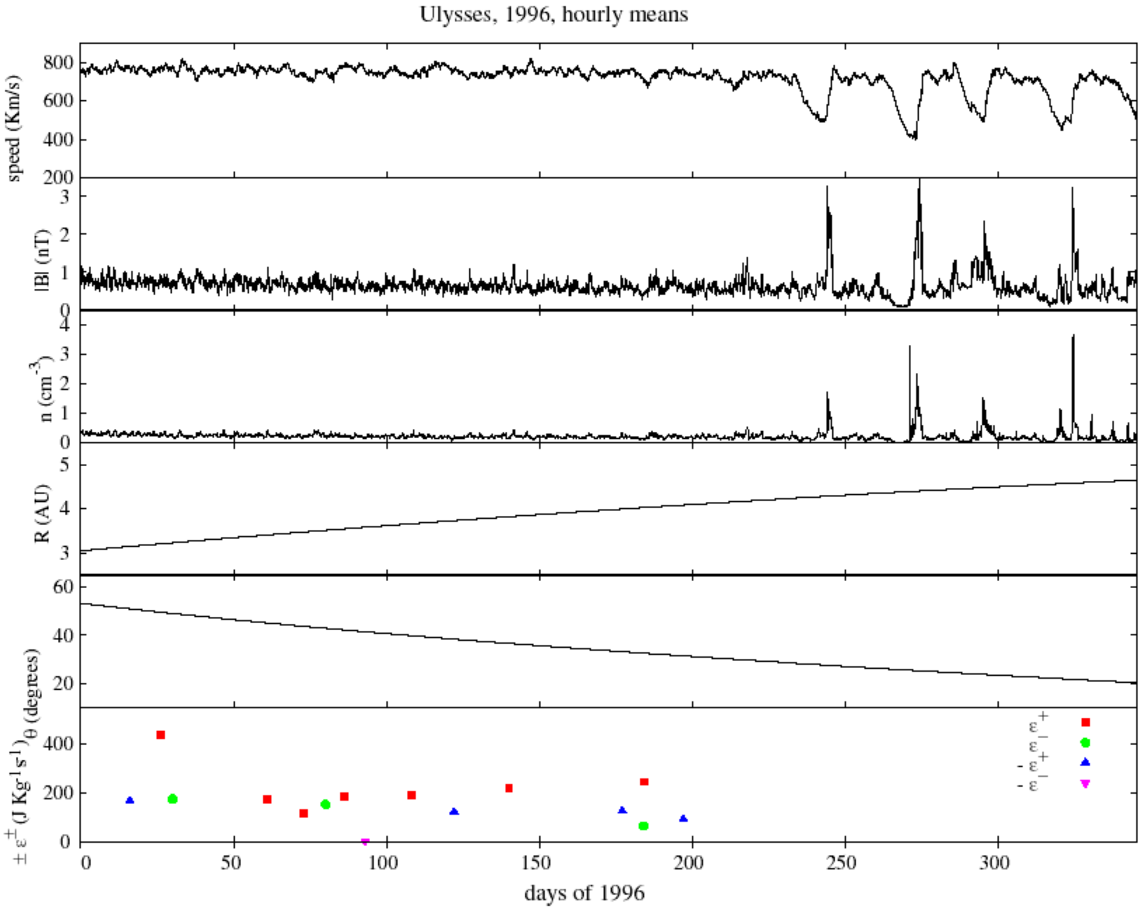


Although the data are somewhat contaminated by the inhomogeneity and local anisotropy, the observed scale collapse onto the Yaglom law appears very robust in most periods of Ulysses dataset.

The first REAL evidence that (low frequency) solar wind can be described in the framework of MHD turbulence

L. Sorriso-Valvo et al., PRL (2007)

From Yaglom's law we can estimate the values for the energies transfer rates at different heliocentric distances



Roughly of the same order of magnitude, about few hundred J/Kg sec.

FIG. 2 (color online). Hourly averaged quantities are represented as a function of the flight time of Ulysses. The top panels represent, respectively, the solar wind speed, the magnitude of the magnetic field, the particle density, the distance from the Sun and the heliolatitude angle. In the bottom panel are plotted the values of $\epsilon^{\pm}$, calculated through a fit using the relation (3) during the periods where a clear linear scaling exists.

As a comparison, energy transfer rate per unit mass in usual fluid flows is about 1 ÷ 50 J/Kg s

$$\epsilon \simeq (1.5 \pm 0.5) \times 10^6$$

cgs units

The problem of solar wind heating

Solar wind model → Adiabatic expansion, temperature should decrease with helioscentric distance (radial cooling) with a typical scaling with dfistance

$$T(r) \approx r^{-4/3}$$

Spacecraft measurements → actually temperature decay is slower than expected. Conjecture: **Turbulence should heat solar wind.**

$$T(r) \approx r^{-\xi}$$

$$\xi \in [0.7;1]$$

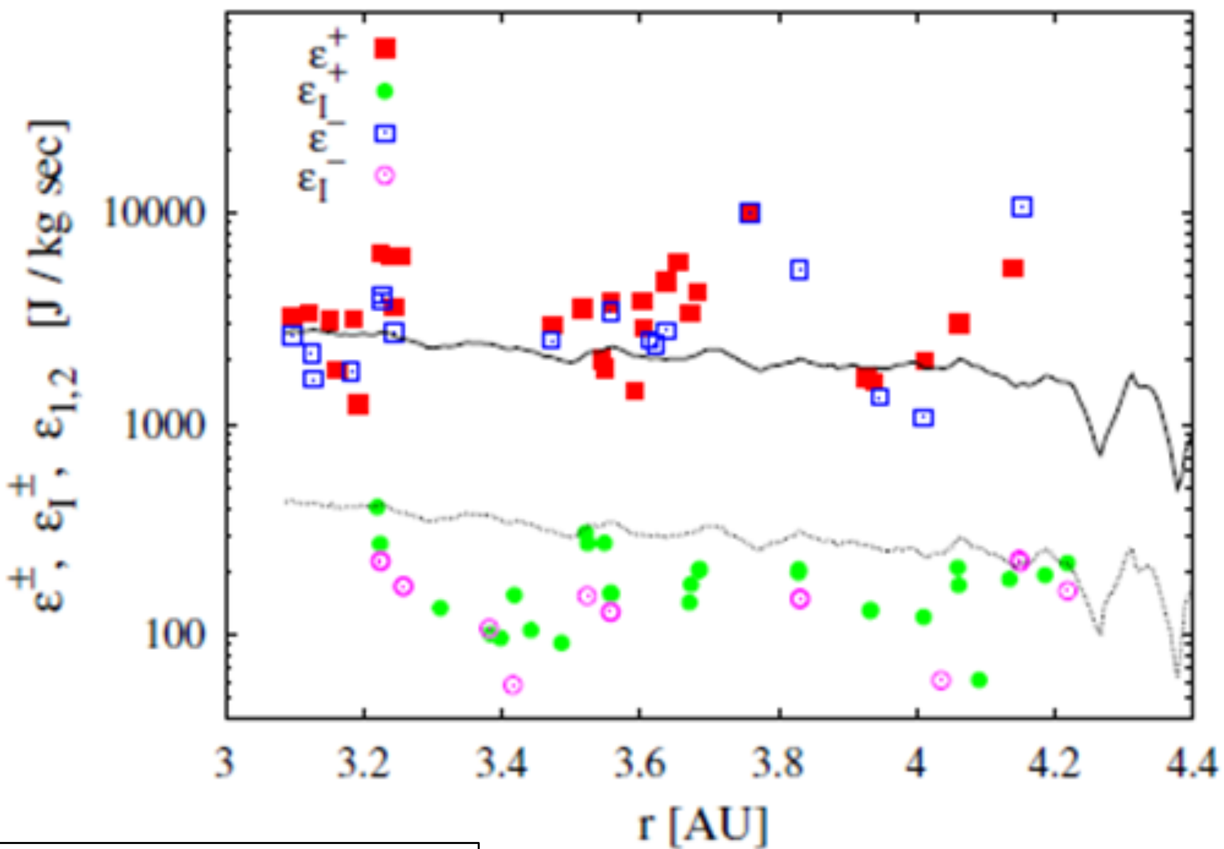
Estimate of the heating rate needed to heat the solar wind (say to obtain the observed small radial cooling)

$$\epsilon_{heat}(r) = \frac{3}{2}\left(\frac{4}{3} - \xi\right)\frac{V_{SW}(r) k_B T(r)}{r m_p}$$

Vasquez et al., JGR 2007

We can ask whether the estimated turbulent energy flux towards dissipative scales, from Yaglom's law, is enough for solar wind heating

A good agreement of the radial evolution of dissipation energy rate measured from Yaglom's law, with the model of heating rate needed to explain the slower cooling of solar wind, with respect to adiabatic cooling.



Heating by turbulent energy means that energy MUST be dissipated in some way at small scales.

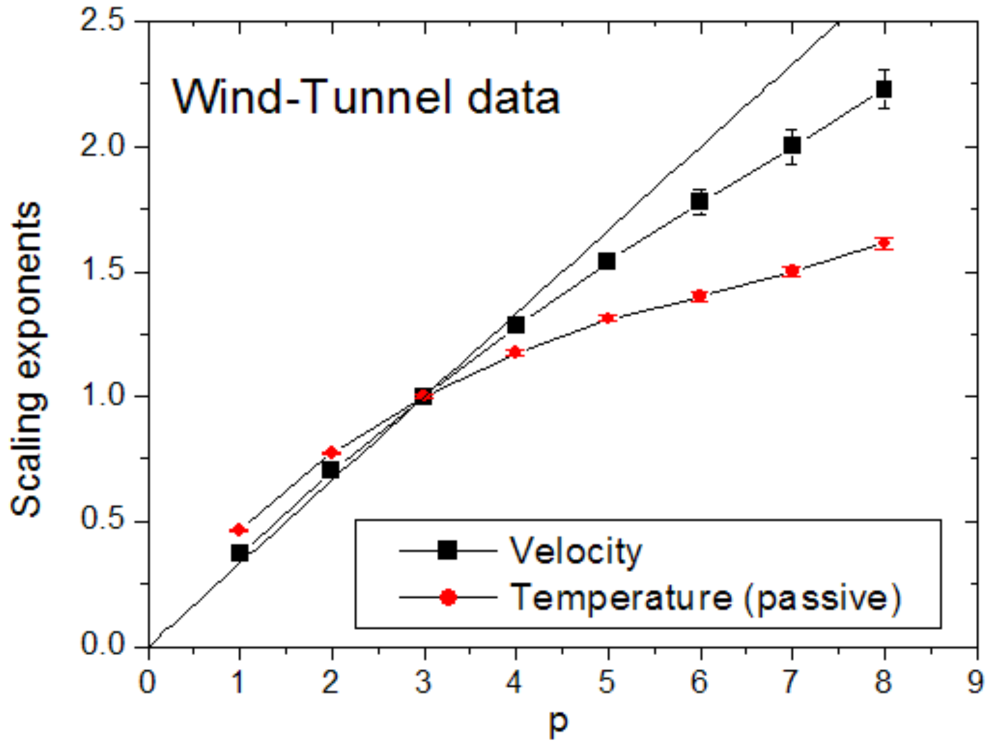
Intermittency

The second-order moment (power spectrum) does not play any privileged role. Turbulence in Solar Wind shares anomalous scaling laws with usual fluid flows for high-order moments of fluctuations

Despite the Yaglom-law and a 5/3-spectrum are observed, experiments show a strong departure from the Kolmogorov's conjecture for high-order moments

$$S_n(\tau) = \langle [u(t+\tau) - u(t)]^n \rangle \sim \tau^{\zeta_n}$$

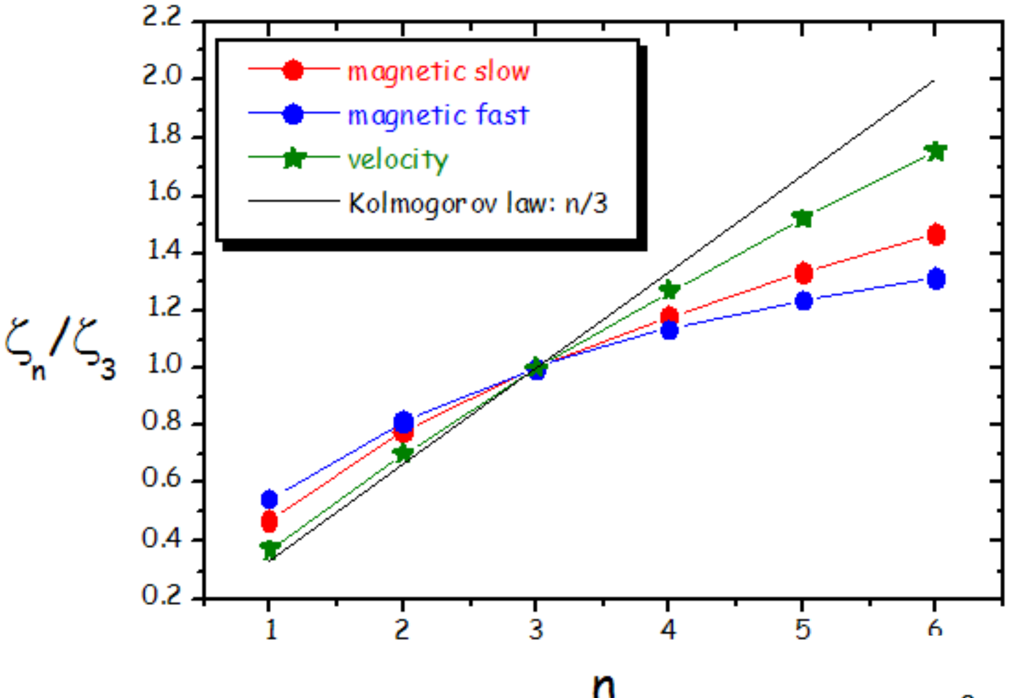
1) u along the main flow;
2) Taylor hypothesis to transform length scales in time scales



The departure has been attributed to **INTERMITTENCY** in fully developed turbulence

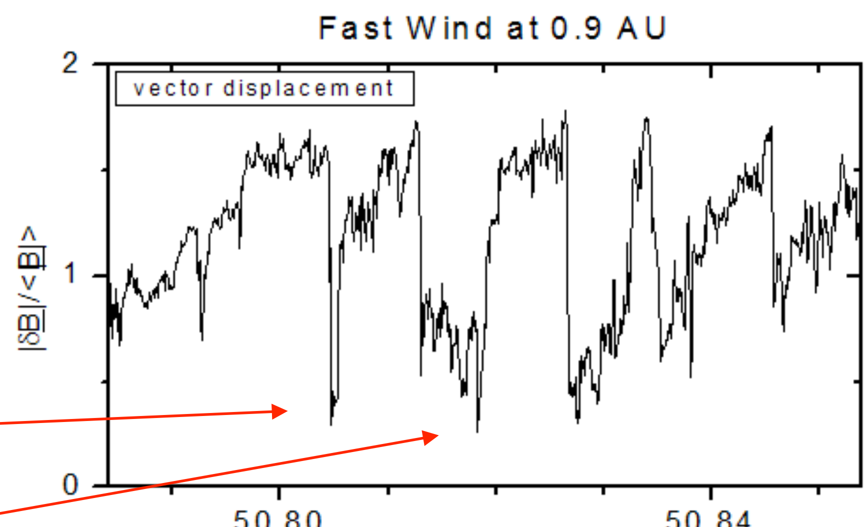
Fluid flows: Intermittency, measured as the distance from the Kolmogorov's linear law, is stronger for passive scalar

The same behaviour in Solar Wind turbulence



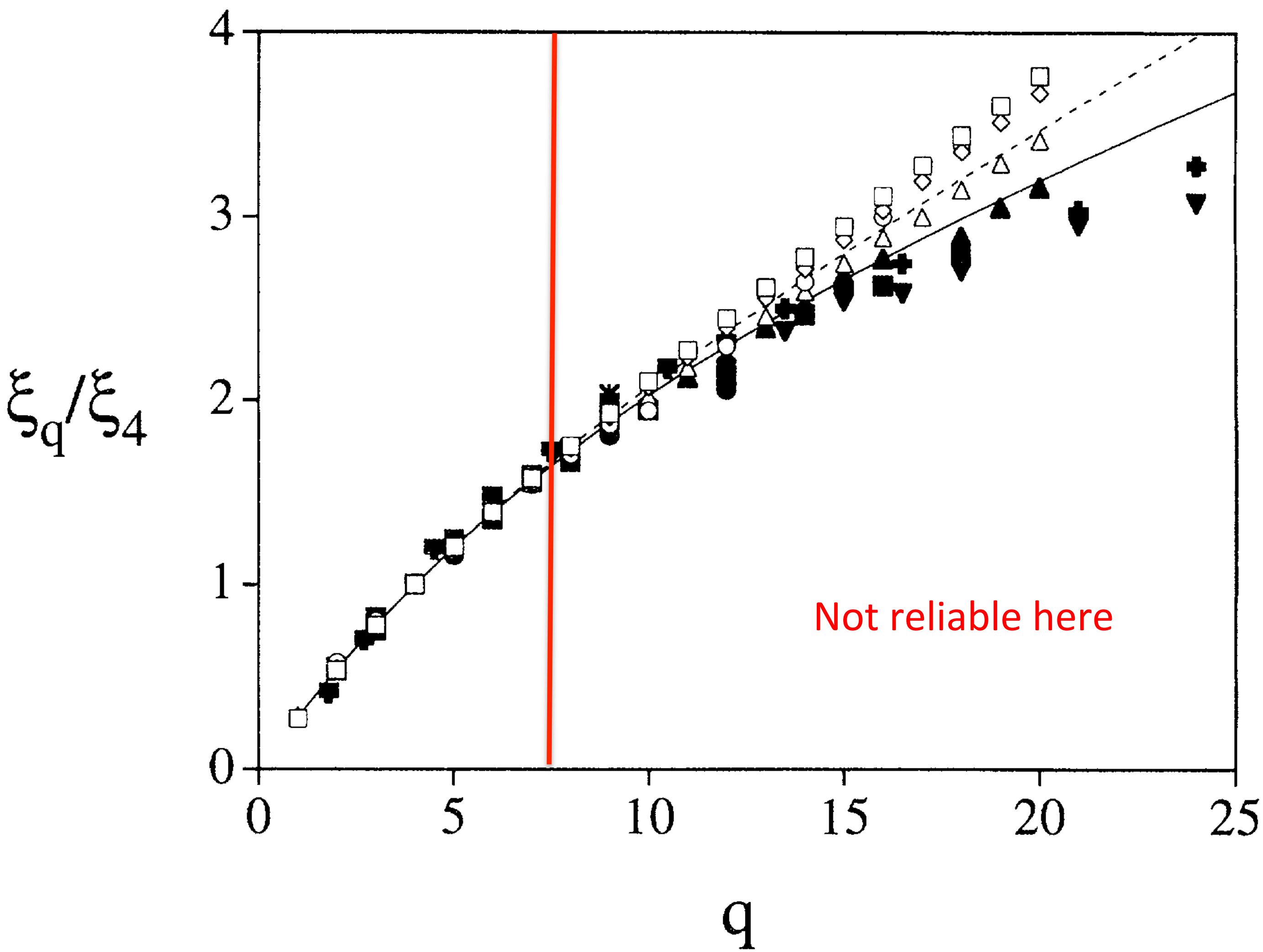
Solar wind: Intermittency is stronger for magnetic field than for velocity field. Scaling laws for velocity field in the solar wind coincide with that observed in fluid flows (through extended self-similarity)

THIS DOES NOT IMPLY THAT THE MAGNETIC FIELD IS A "PASSIVE VECTOR": statistics cannot prove, just disprove



Strong jumps of magnetic orientation are responsible for the strongest intermittency

Comparison with velocity fluctuations in fluid flows



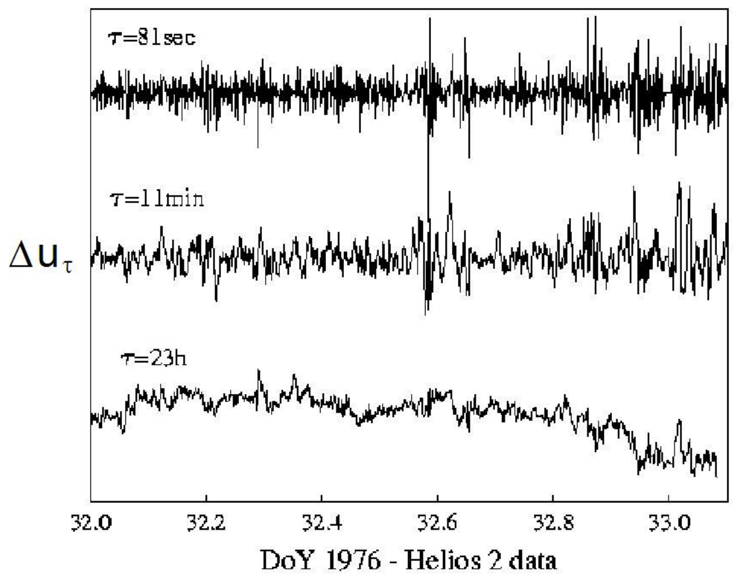
A collection of data from laboratory fluid flows (black symbols) and solar wind velocity (white symbols).

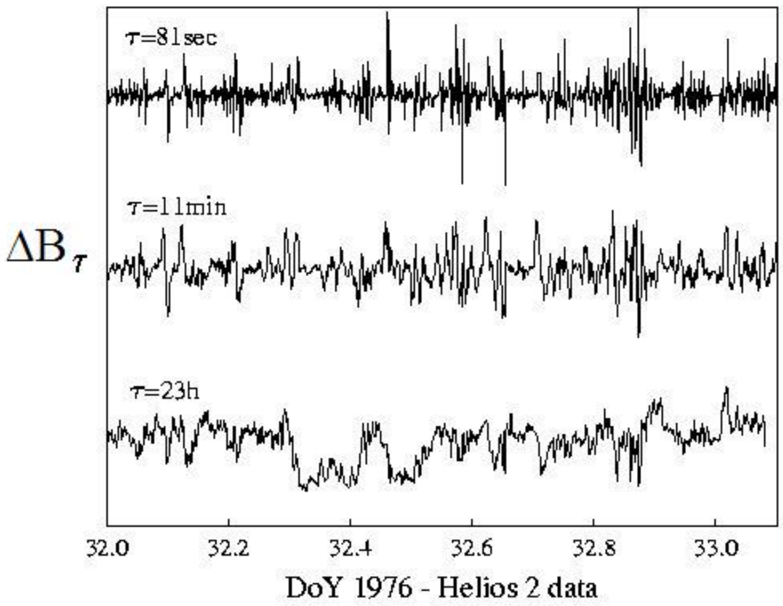
Differences only for unreliable high order moments, perhaps due to different geometry of dissipative structures.

What is “intermittent” in turbulence

Natural variables:
Fluctuations at a given scale
(separation time)

$$\Delta u = u(t+\tau) - u(t)$$
$$\Delta B = B(t+\tau) - B(t)$$



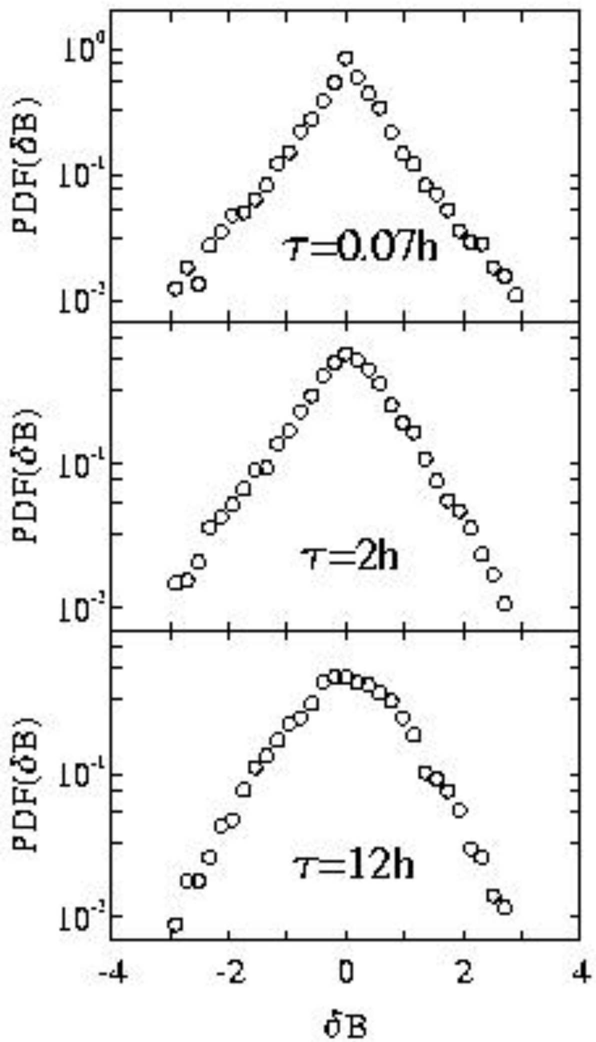


1) A random signal at large separations;
2) Bursts of activity at smaller separations

Intermittency implies a departure from global self-similarity (multifractals)

Stretched exponential PDF at small scales

Gaussian PDF at large scales



PDFs of normalized variables changes with scale

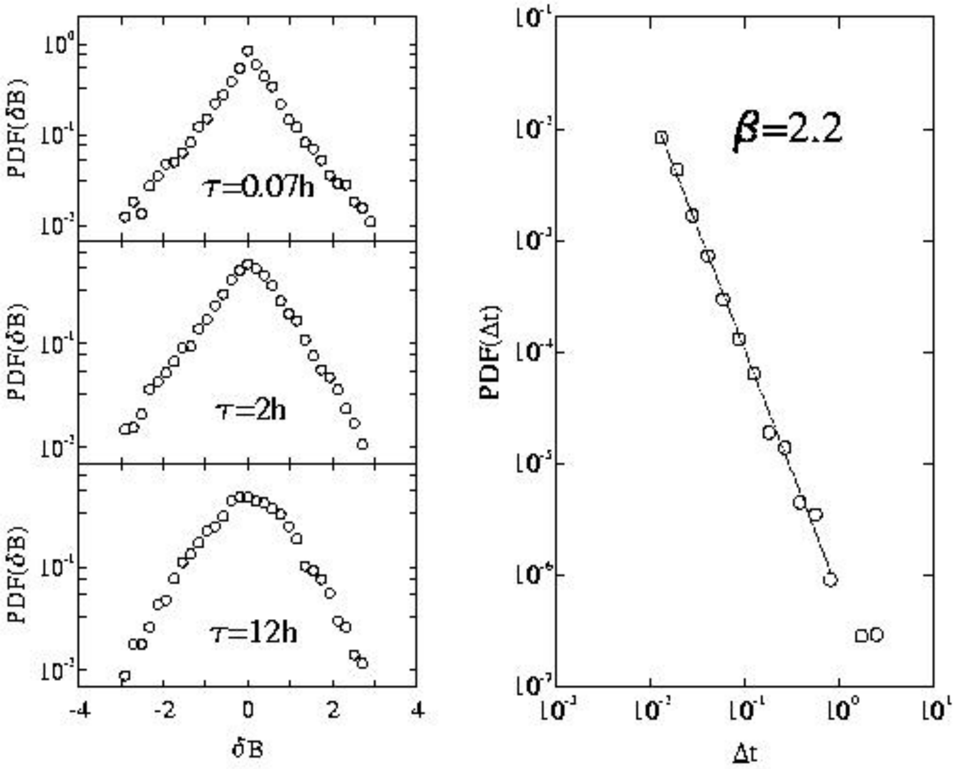
$$\frac{\Delta B_\tau}{\sqrt{\langle \Delta B^2 \rangle}}$$

Probability of occurrence of strongest events are higher than a Gaussian

→ Random fluctuations, with highly correlated phases, are present, they are an unavoidable characteristic of real turbulence.

Turbulence CANNOT be described by a random phase process

Times between bursts



The times between events (waiting times) at the smallest scale, are distributed according to a power law

$$P(\Delta t) \approx \Delta t^{-\beta}$$

The turbulent energy cascade generates intermittent "coherent" events at all scales.

Interesting! the underlying cascade process is NOT POISSONIAN, that is the intermittent (more energetic) bursts are NOT INDEPENDENT

Wavelets and Local Intermittency Measure: disentangle “structures”

Intermittency: the energy content, at each scale, is not homogeneously distributed

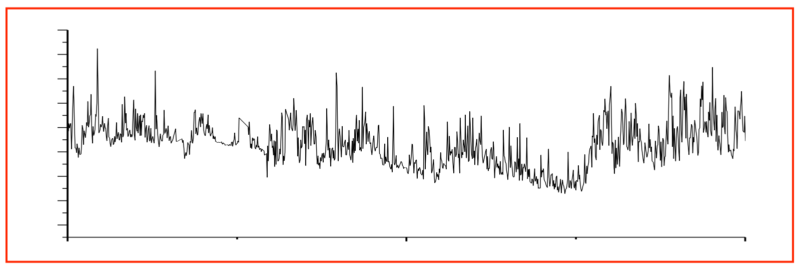
$$f(x) = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} w_{ij} \psi_{ij}(x)$$

$$l.i.m. = \frac{|w_{ij}|^2}{\left\langle |w_{ij}|^2 \right\rangle_i}$$

L.i.m. greater than a threshold means that at a given scale and position the energy content is greate than the average at that scale

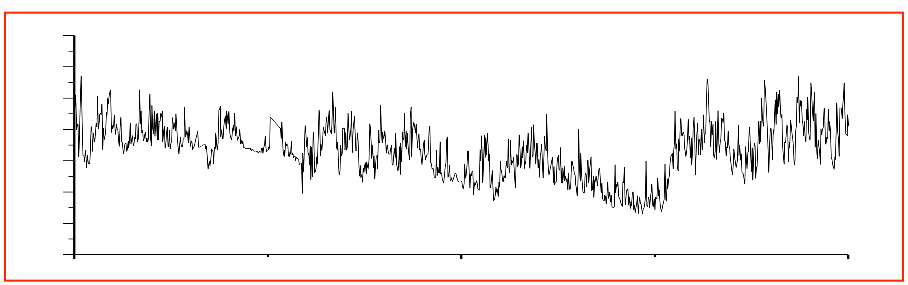
$$w_{ij} = \int_{-\infty}^{\infty} f(x) \psi_{ij}(x) dx$$

l.i.m. smaller than threshold

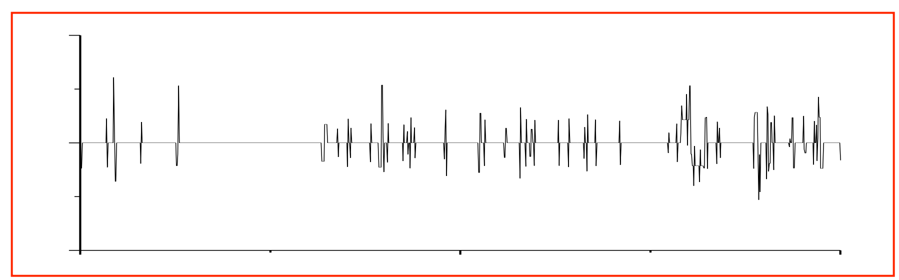


Complete signal

l.i.m. larger than a gaussian threshold



Gaussian background

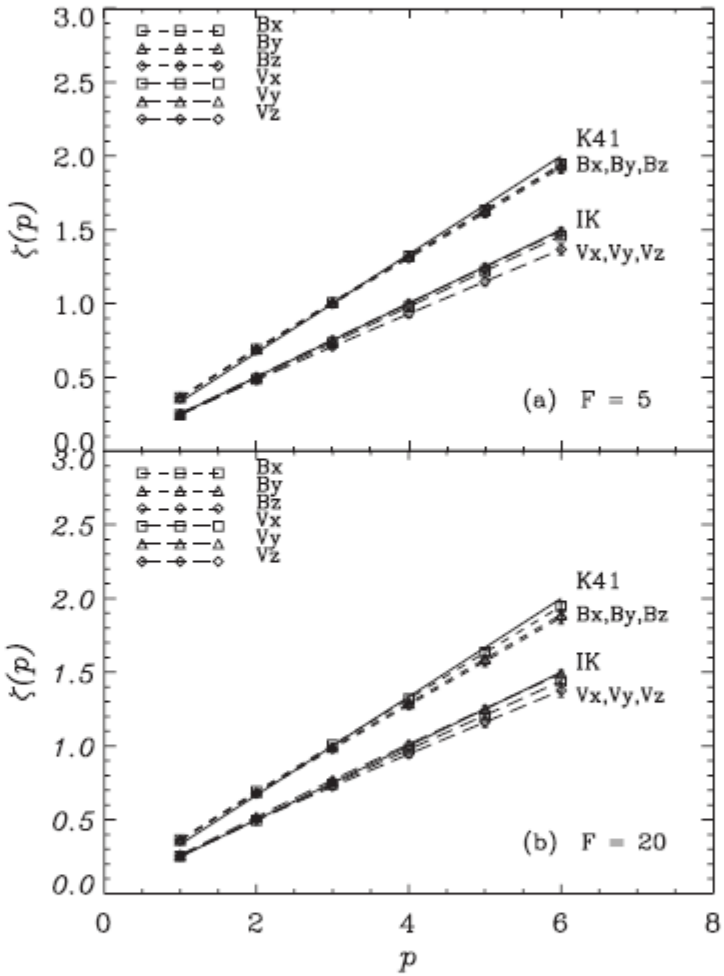


Structures

THE ASTROPHYSICAL JOURNAL, 702:537–553, 2009 September 1
doi:10.1088/0004-637X/702/1/537

SOLAR WIND MAGNETOHYDRODYNAMICS TURBULENCE: ANOMALOUS SCALING AND ROLE OF INTERMITTENCY

C. SALEM[1], A. MANGENEY[2], S. D. BALE[1], AND P. VELTRI[3]
[1] Space Sciences Laboratory, University of California, Berkeley, CA 94720, USA; salem@ssl.berkeley.edu
[2] LESIA, Observatoire de Paris-Meudon, F-92195 Meudon, France
[3] Dipartimento di Fisica, Università della Calabria, Rende (Cs), Italy
Received 2009 January 9; accepted 2009 June 25; published 2009 August 13

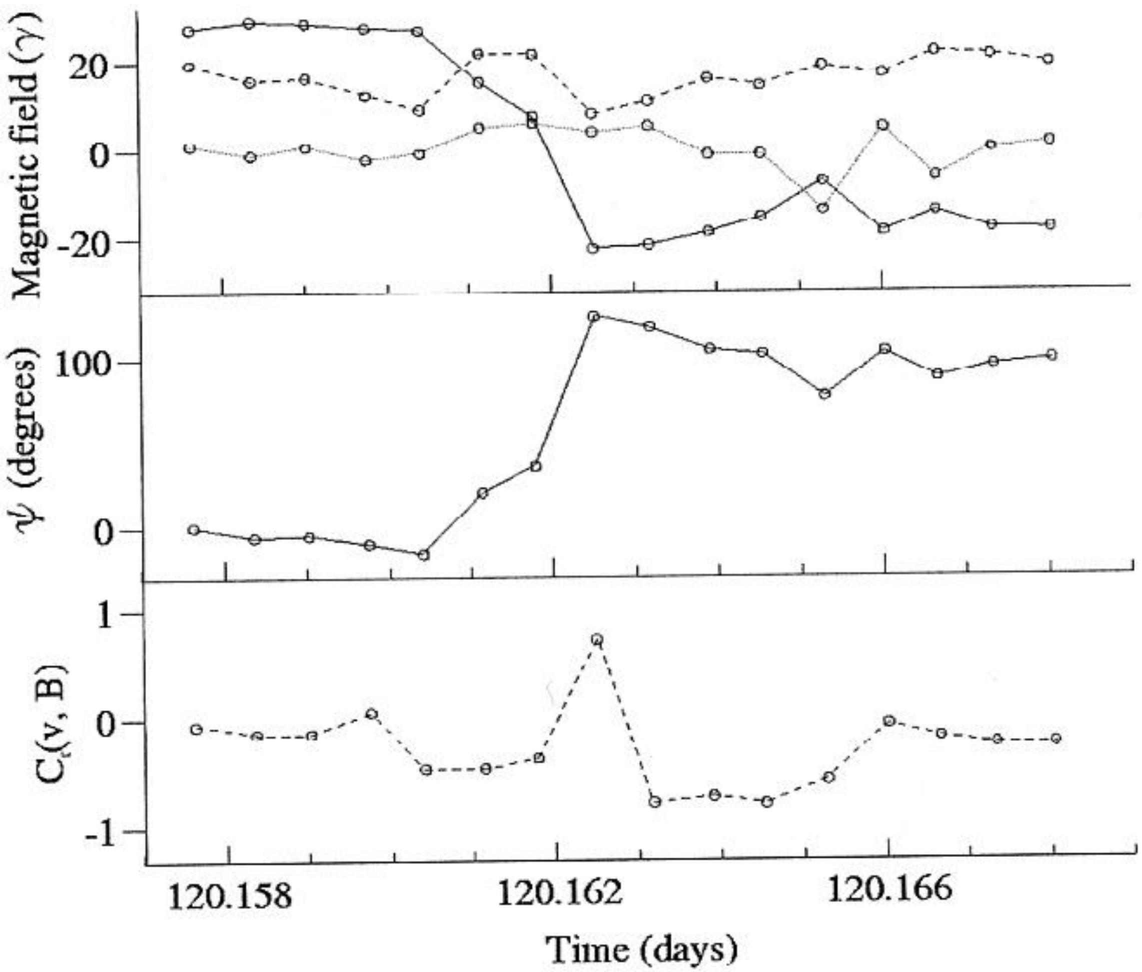


Conditioned structure functions (using only the background) do not show anomalous scalings!

Localized structures with an high-energy content, represent the main ingredient for intermittency

Analysis of the magnetic field fluctuations around isolated structure allows to identify them.

What kind of structures in MHD (1)

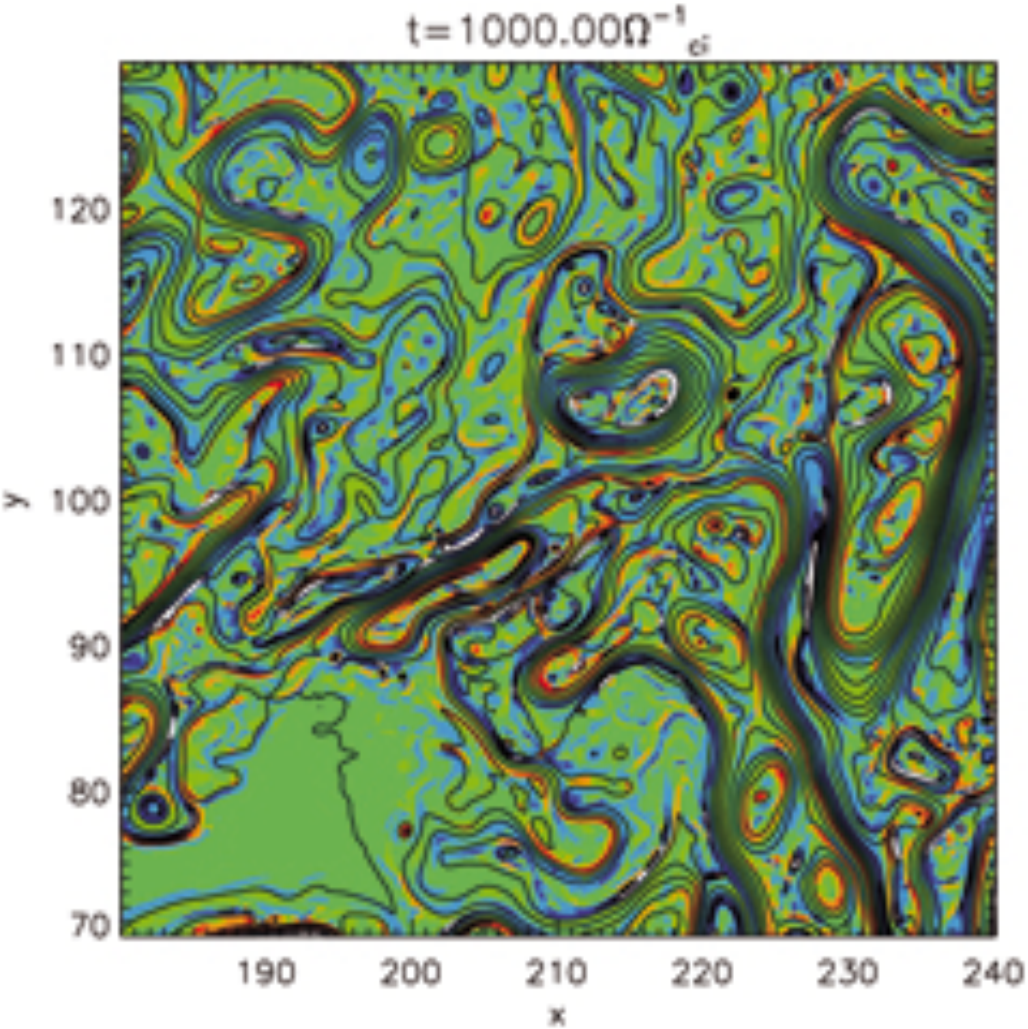


Veltri & Mangeney, 1999
Bruno et al., 1999
Veltri et al., 2005

The component of the magnetic field which varies most changes sign, and is almost perpendicular to the average magnetic field.

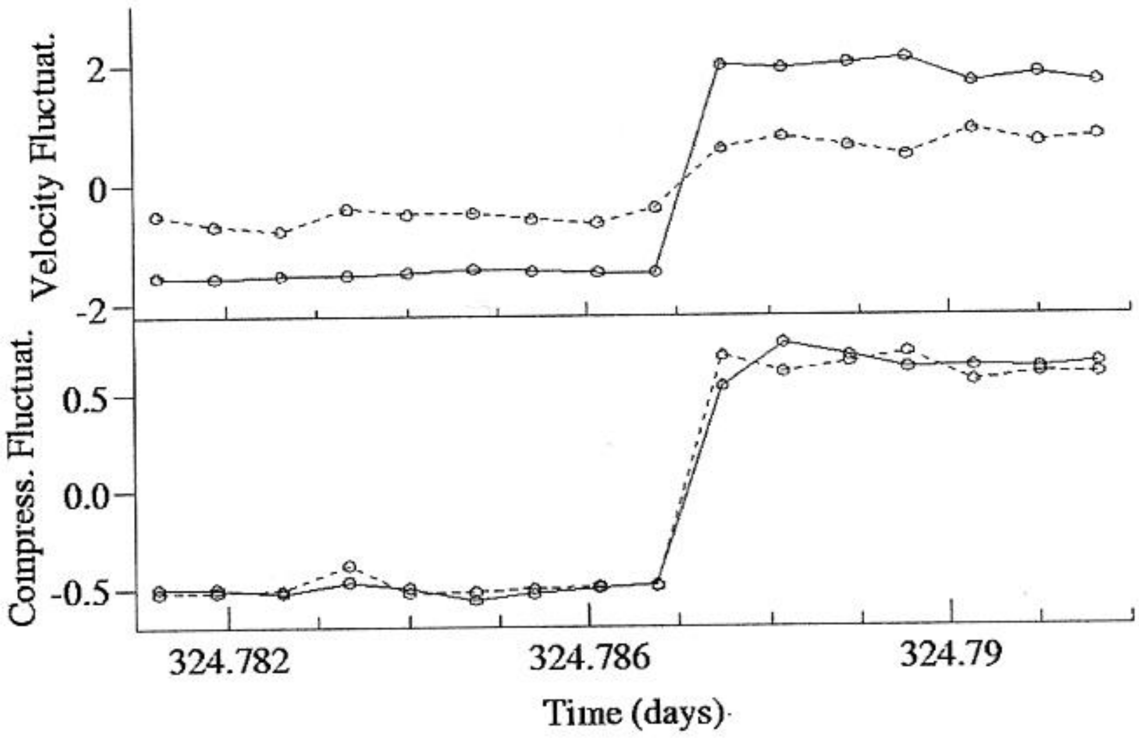
Identified as tangential discontinuities (current sheets). They are spontaneously generated at all scales inside MHD turbulence by the nonlinear dynamics.

In MHD isolated structures are recognized as current sheets, where a lot of physical processes occur: reconnection, particle acceleration, magnetic annihilation, etc.



Intermittent current sheets are generated by the nonlinear energy cascade and observed both in space, or better, in numerical simulations at very small scales.

What kind of structures in MHD (2)



Compressive discontinuities are sometimes observed. These structures can be either parallel shocks or MHD slow-mode (like) wave trains.

Veltri & Mangeney, 1999
Bruno et al., 1999, 2001, 2003, 2004
Veltri et al., 2005

Anisotropy

Large scales in fluid flows are anisotropic, Kolmogorov's hypothesis K41 requires a return-to-isotropy at intermediate (inertial range) scales.

The solar wind fluctuations are intrinsically anisotropic, a large scale magnetic field cannot be eliminated through a galilean transformation.

1) Polarizations anisotropy: the three components of fluctuations have different amplitudes along different directions;
2) Wavevectors anisotropy: fluctuations in Fourier space depend differently on the wavevector directions parallel and perpendicular to the mean field.

Polarization anisotropy

$$S_{ij} = \langle B_i B_j \rangle - \langle B_i \rangle \langle B_j \rangle$$

Determine the eigenvalues and eigenvectors of the one-point variance matrix

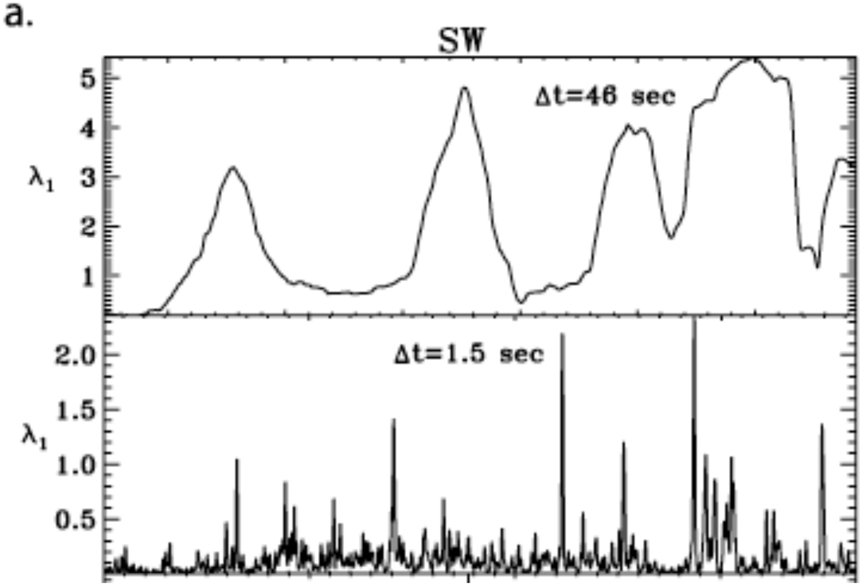
1) Ratios of eigenvalues → statistical properties of anisotropy of magnetic fluctuations;
2) Eigenvectors → three unitary vectors forming a (minimum variance) reference system where one of the axis is aligned along the direction of minimum fluctuations.

Globally

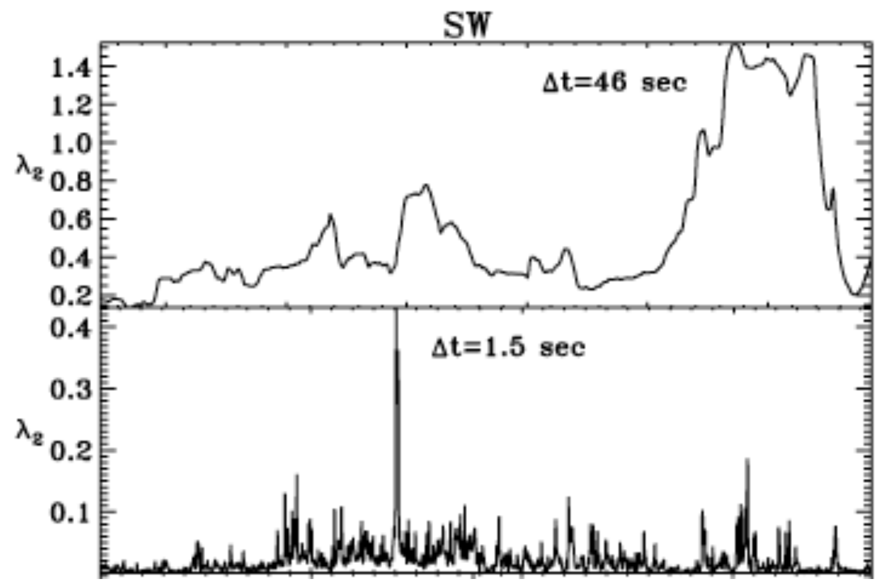
$$\lambda_3 \ll \lambda_2 \leq \lambda_1$$

$$S_{ij}^{(s)}(\Delta t_n, t_l) = \langle B_i^{(s)}(t) B_j^{(s)}(t) \rangle - \langle B_i^{(s)}(t) \rangle \langle B_j^{(s)}(t) \rangle$$

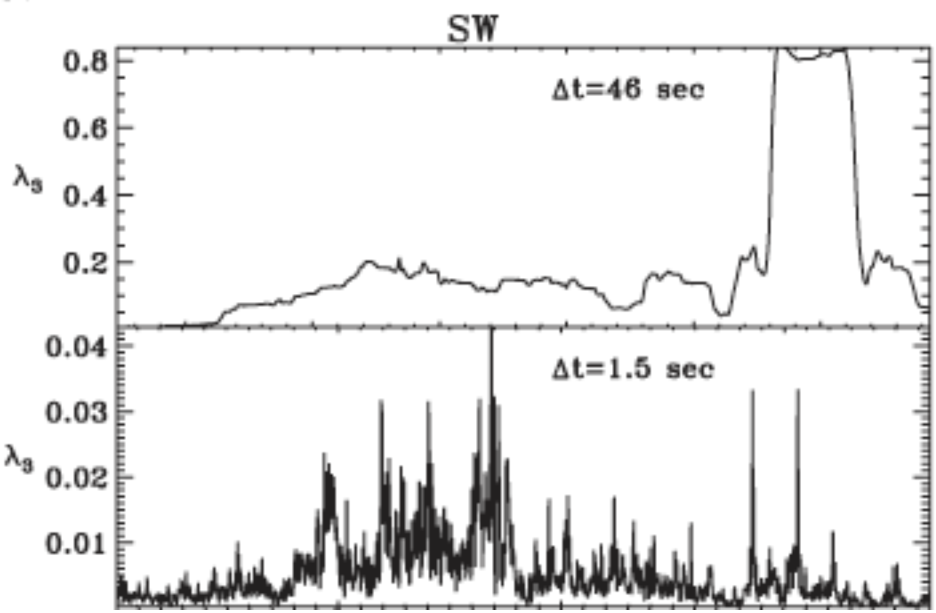
Variance matrix computed on running averages of different amplitudes, gives information both on the time evolution and on the scaling properties of anisotropy a.



a.



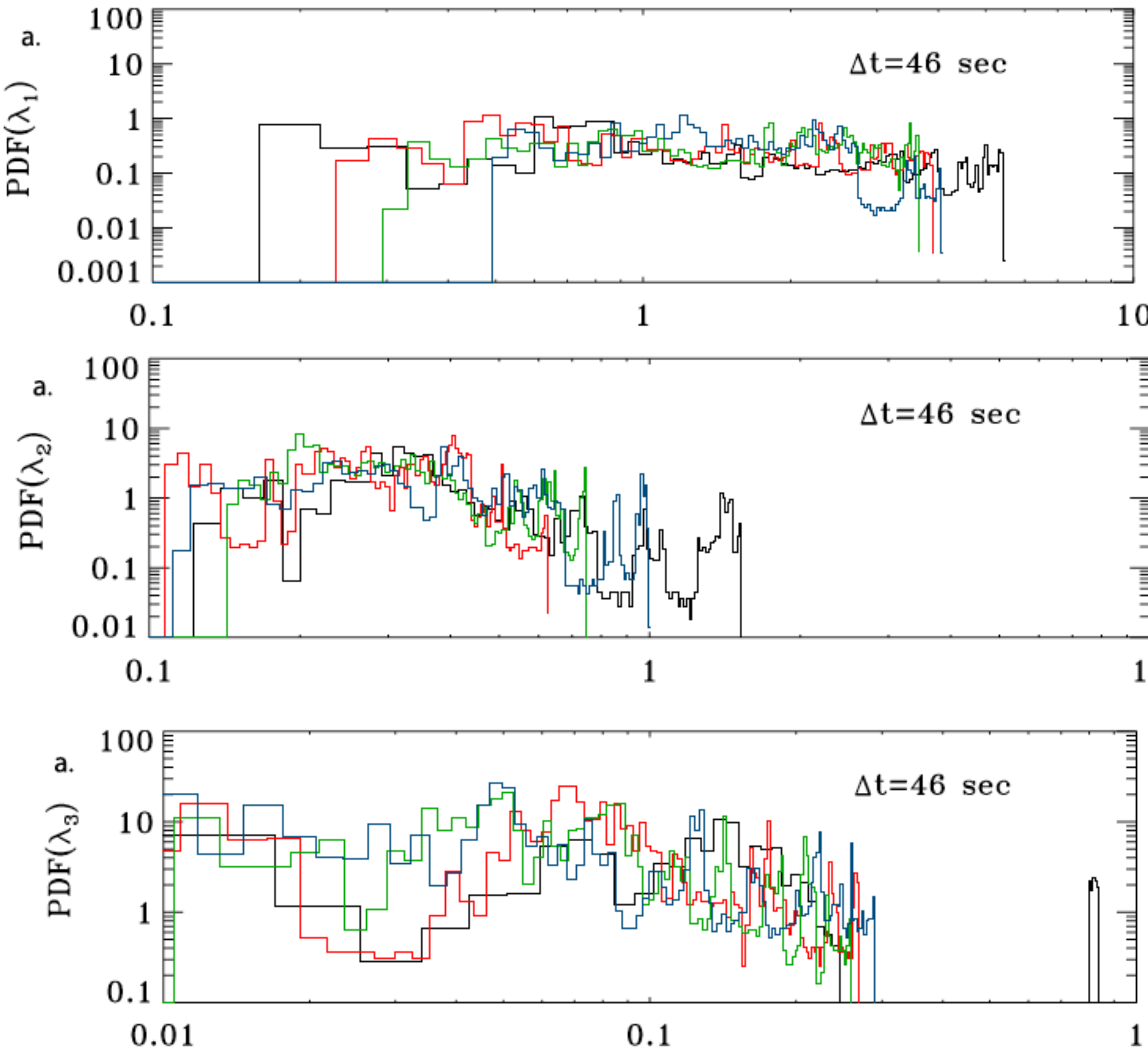
a.



Eigenvalues of variance matrix, as a function of time, at two different scales

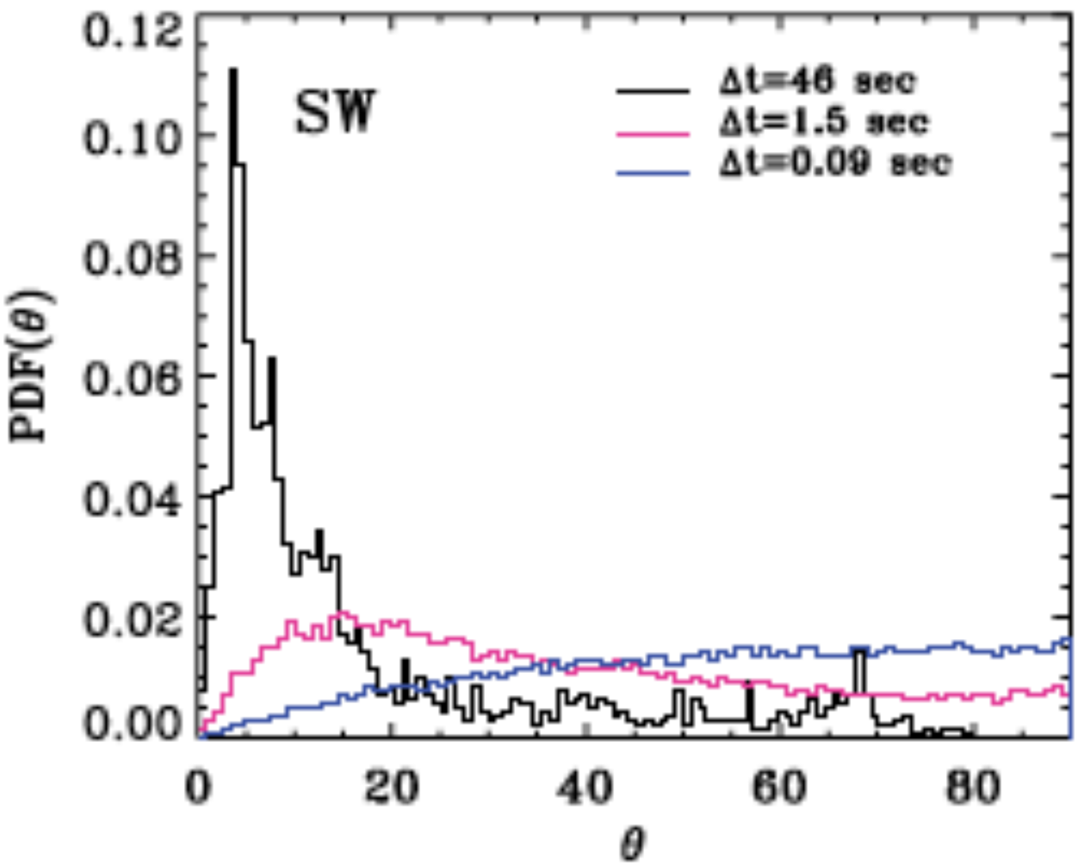
The scaling properties of the eigenvalues are evident. Burst-like behaviour of anistropy at small scales.

Pdfs of eigenvalues at a given scale



One of the eigenvalues sistematically smaller than the other two:

Fluctuatons lye on a plane → Magnetic turbulence approximately two-dimensional, at variance with usual fluid flows.



PDF's of the angle between the minimum variance eigenvector and the direction of the large-scale magnetic field, at three scales.

Minimum variance nearly aligned to the background magnetic field at large scale, broadening at small scales.

At large scales fluctuations lye in a plane almost perpendicular to the background magnetic field. At smaller scales the plane changes direction continuously

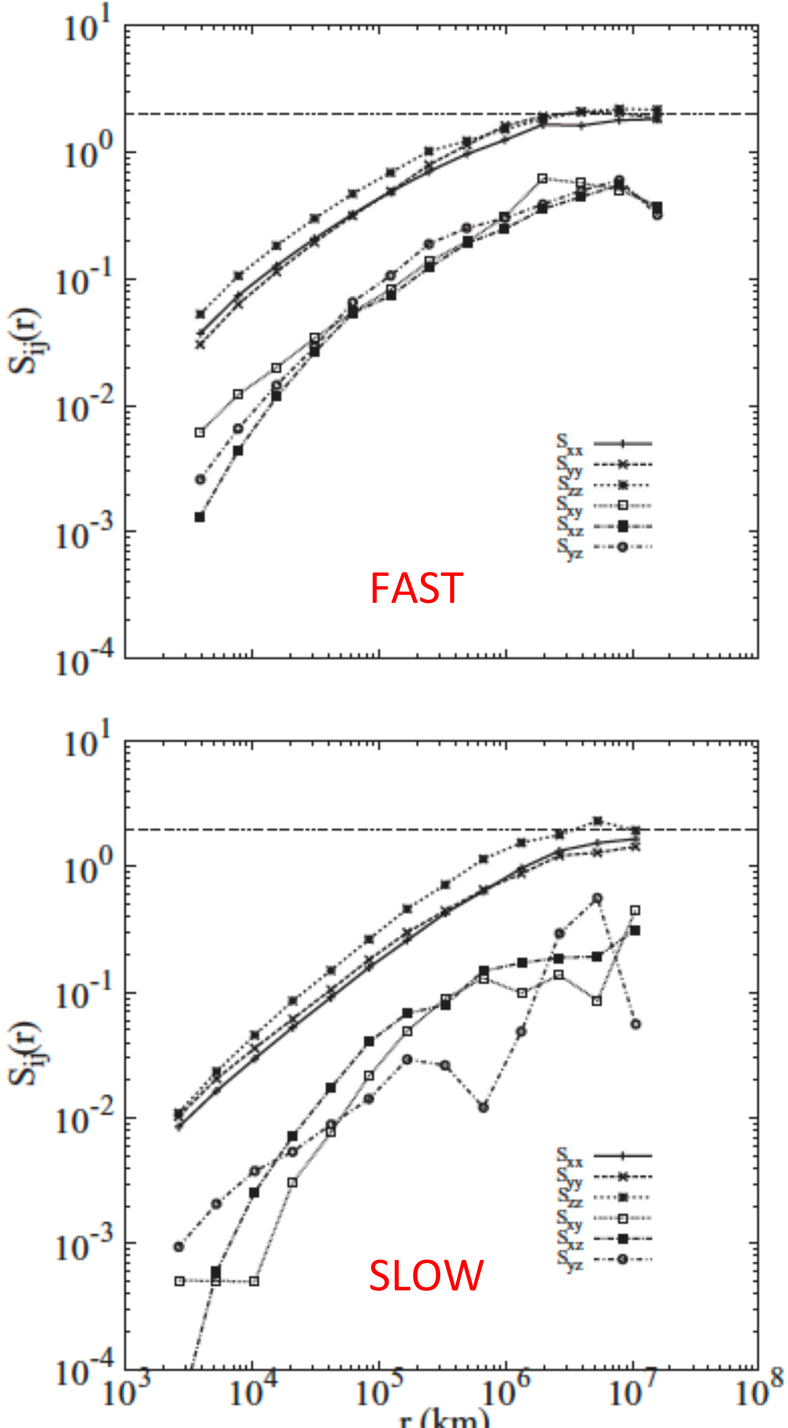
High-order polarization anisotropy

$$S_{\alpha_1,\alpha_2,\cdots,\alpha_n}(l) = \langle [B_{\alpha_1}(r+l) - B_{\alpha_1}(r)] \times [B_{\alpha_2}(r+l) - B_{\alpha_2}(r)] \cdots \times [B_{\alpha_n}(r+l) - B_{\alpha_n}(r)] \rangle .$$

Compute the n-th order variance matrix. Some out-of-diagonal elements contain only anisotropic contributions, and can be compared to diagonal elements where isotropic and anisotropic contributions cohexist.

$$S_{\alpha_1,\cdots,\alpha_n}(l) = \frac{A_{\alpha_1,\cdots,\alpha_n}\eta^n (l/\eta)^n}{\left[1 + B_{\alpha_1,\cdots,\alpha_n}(l/\eta)^2\right]^{C_{\alpha_1,\cdots,\alpha_n}}} \times [1 + D_{\alpha_1,\cdots,\alpha_n}(l/L_0)]^{2C_{\alpha_1,\cdots,\alpha_n} - n} .$$

Tensors can be fitted by a suitable analytic functional shape, where the anisotropic scaling exponents can be recovered



Second-order structure tensor

The contribution of anisotropic off-diagonal elements is not negligible for all scales, that is **the return-to-isotropy** invoked by K41 **fails in solar wind turbulence**

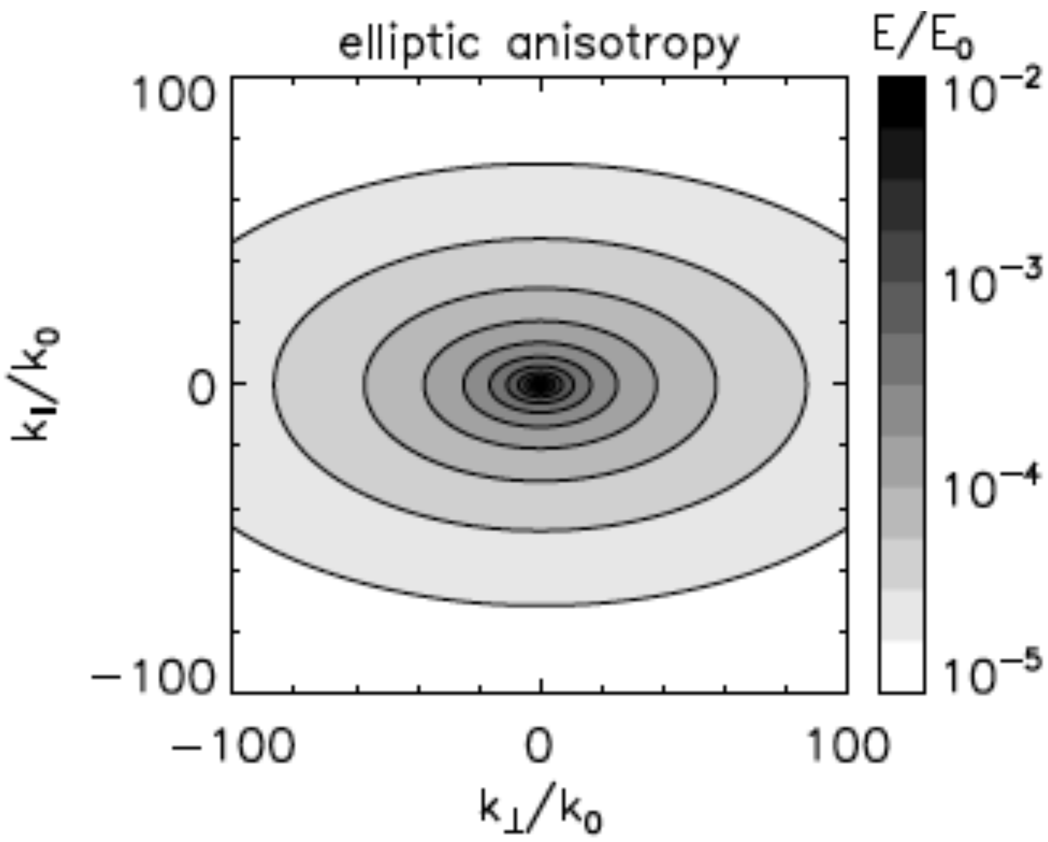
	Fast	Slow
$\zeta_{\alpha_1 \cdots \alpha_n}$	$R = 0.9$ AU	$R = 0.9$ AU
ζ_{xx}	0.66 ± 0.03	0.60 ± 0.02
ζ_{yy}	0.62 ± 0.03	0.62 ± 0.02
ζ_{zz}	0.66 ± 0.04	0.57 ± 0.02
$\zeta_{xy}(*)$	0.66 ± 0.03	0.60 ± 0.02
$\zeta_{xz}(*)$	0.67 ± 0.03	0.59 ± 0.02
$\zeta_{yz}(*)$	0.59 ± 0.02	0.62 ± 0.02
ζ_{xxxx}	1.19 ± 0.03	1.18 ± 0.04
$\zeta_{xzzz}(*)$	1.54 ± 0.03	1.33 ± 0.03
$\zeta_{xxxz}(*)$	1.55 ± 0.03	1.37 ± 0.02

* → fully anisotropic components

Differences among scaling exponents are small, the anisotropic contribution does not vanishes

Wavevector anisotropy

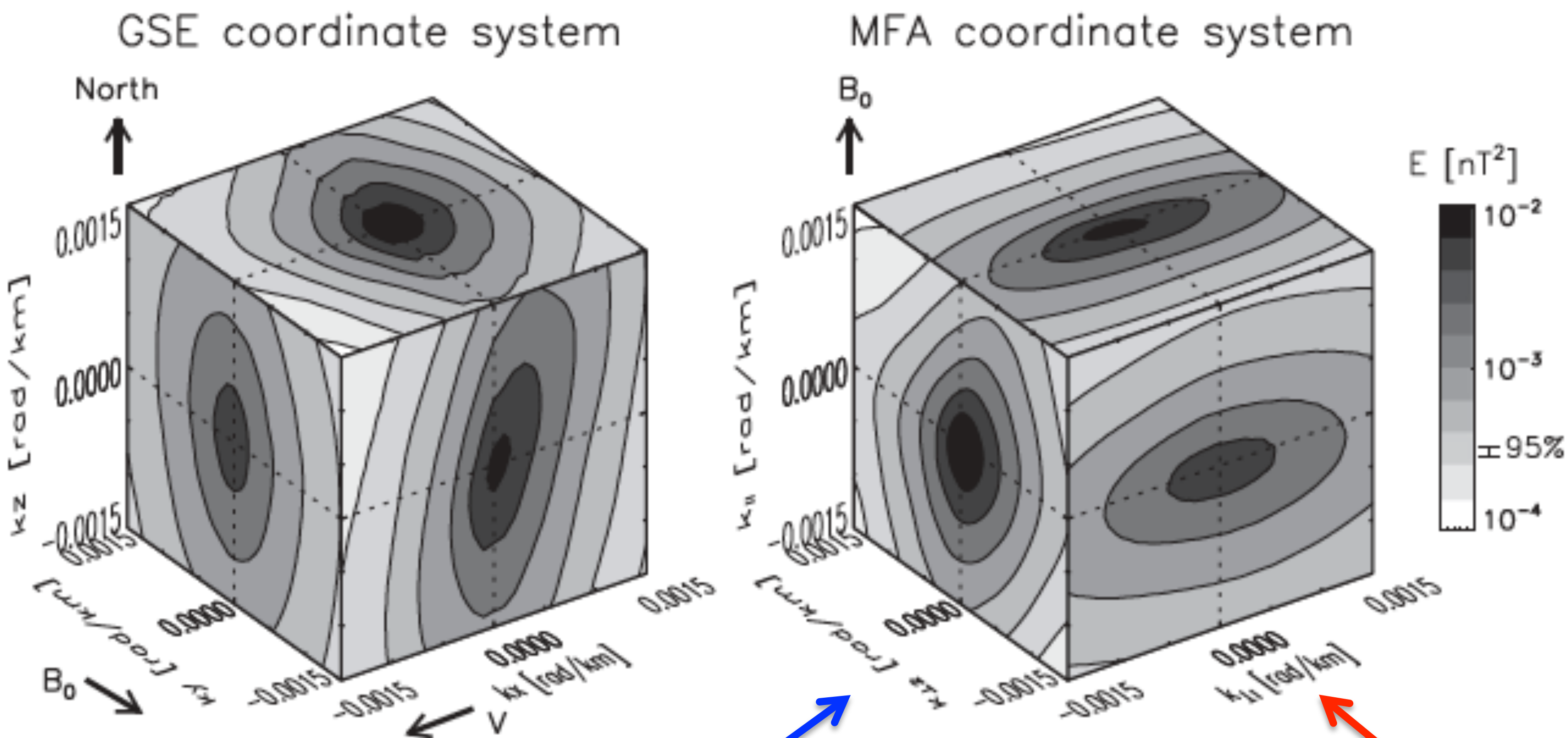
Single spacecrafts cannot be used to compute wavevector anisotropy. The four Cluster spacecrafts have been recently used (k_filtering). Just some wavevector scales have been investigated, depending on the relative distances of the four satellites.



$$\frac{\partial}{\partial t} - i\mathbf{k}\cdot\mathbf{c}_A$$

Decorrelation time of fluctuations, when they move apart, depends on the angle between the wavevector of fluctuations and the "background" magnetic field. This time is shorter for perpendicular wavevectors, that is the turbulent energy cascade is roughly realized mainly in the perpendicular direction.

Elliptic anisotropy is observed



Minimum power direction in the perpendicular plane

Maximum power direction in the perpendicular plane

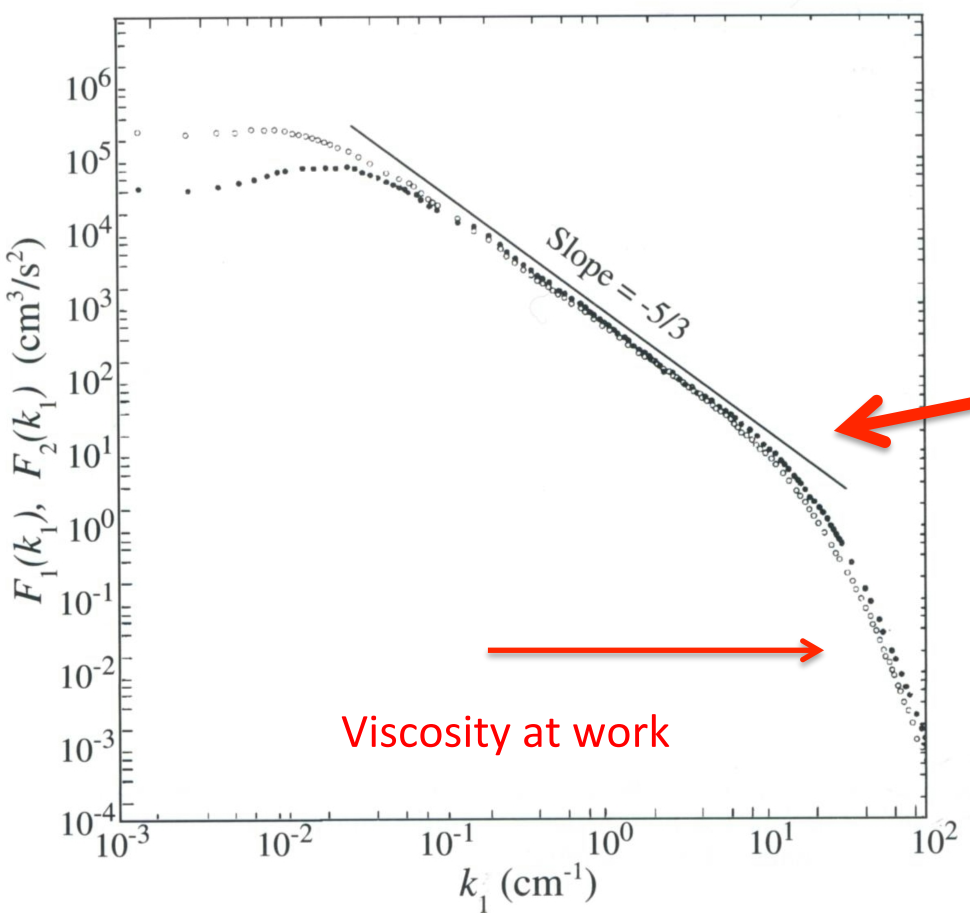
Narita et al. PRL (2010)

The dissipation of turbulent energy

Once the energy is transferred to small scales, it must be dissipated.
In usual fluid flows the dissipative term is at work at small scales. In MHD turbulence (numerical simulations) the situation is quite similar.

Dissipation of energy in classical turbulence

When the dissipative time becomes of the order of the nonlinear eddy turnover time, the energy cannot be transferred efficiently to small scales



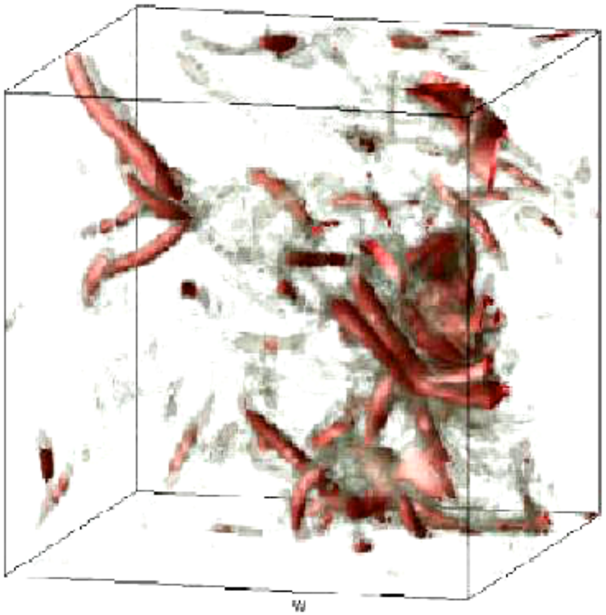
We observe a depletion in the energy spectrum starting at the Kolmogorov's characteristic scale

$$\eta \sim \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

$$\frac{\eta}{L} \sim R^{-3/4}$$

The larger the Reynolds number the smaller the dissipation scale

Dissipation of energy in classical turbulence through isolated bursts: finite-time singularities



Numerical simulation

Dissipative structures are very localized both in space and time (intermittency in the dissipative domain). Energy is dissipated through isolated bursts.
This process can be viewed as the generation of finite-time singularites:

$$\tau_\ell \sim \frac{\ell}{u_\ell} \sim \epsilon^{-1/3} \ell^{2/3}$$

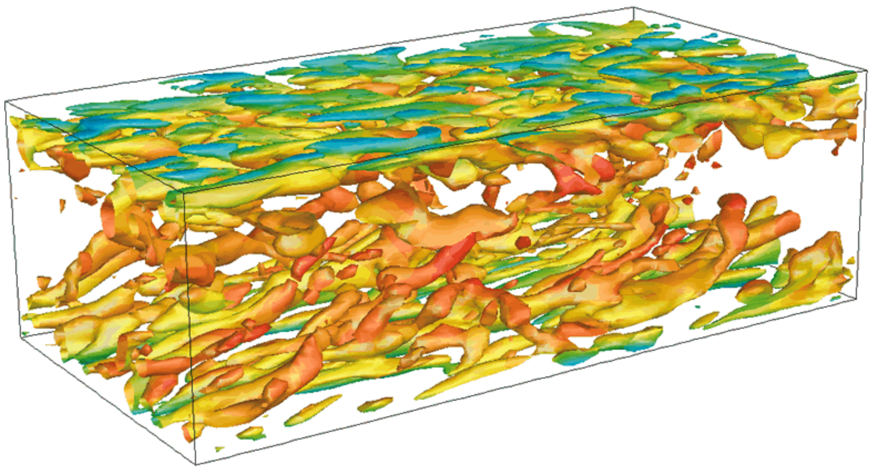
$$T_{tot} \sim \sum_\ell \ell^{2/3}$$

The sum of eddy-turnover times CONVERGES as the scale length tends to zero.

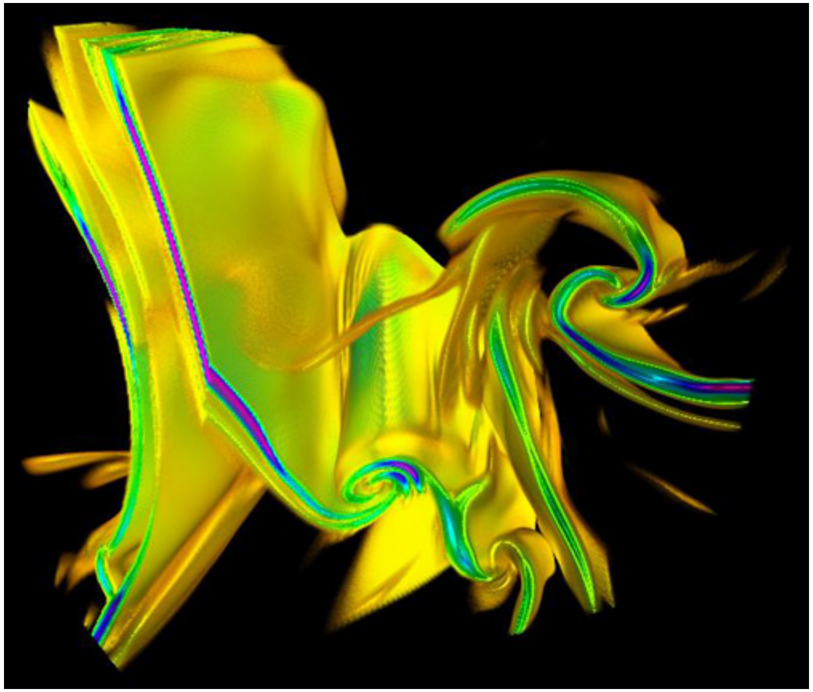
The energy is transferred towards structures of ZERO length in a FINITE time, this should generates a **singularity**.

Geometry of dissipative bursts

Intermittent dissipative structures:
Filaments in usual fluid flows, sheets in MHD flows

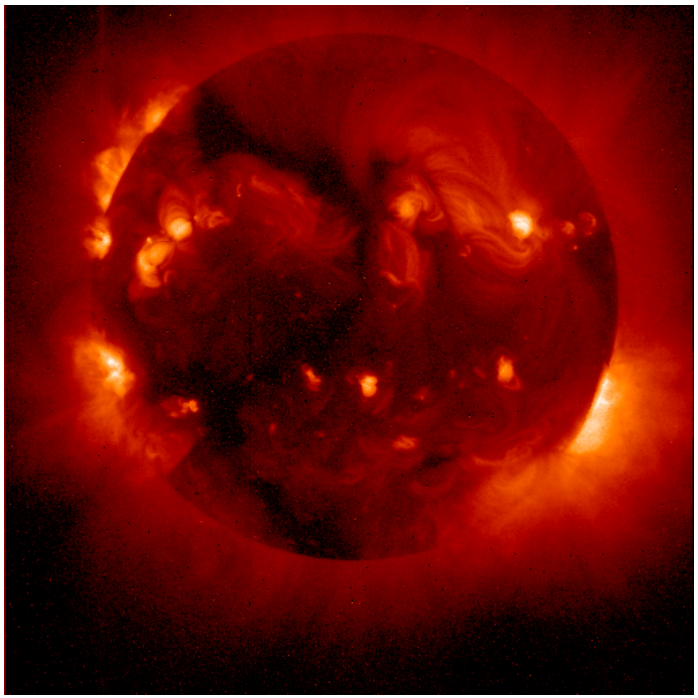


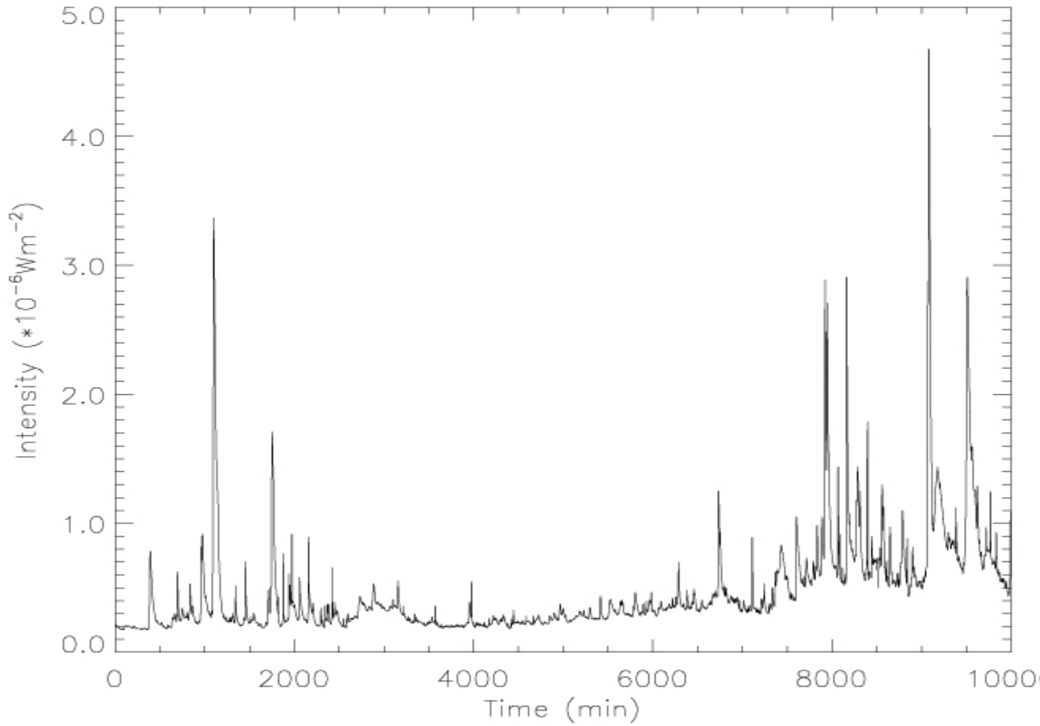
Dissipative structures near the wall



Current sheet

Example of Solar flares: impulsive annihilation of magnetic energy at spontaneously generated current sheets in a turbulence inside the solar corona

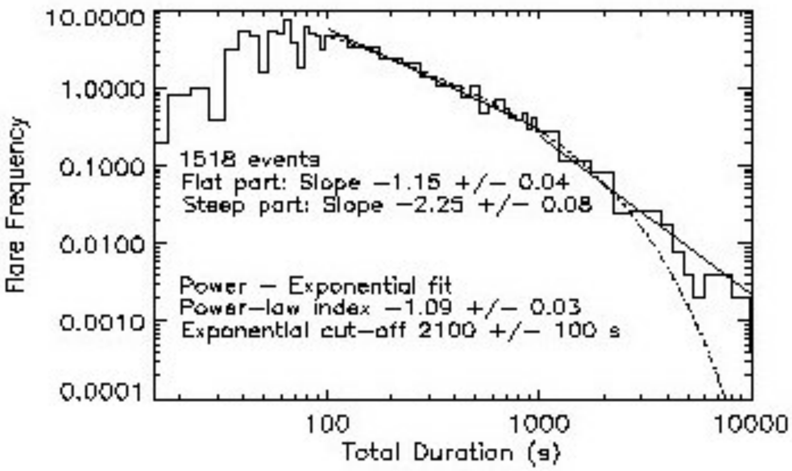


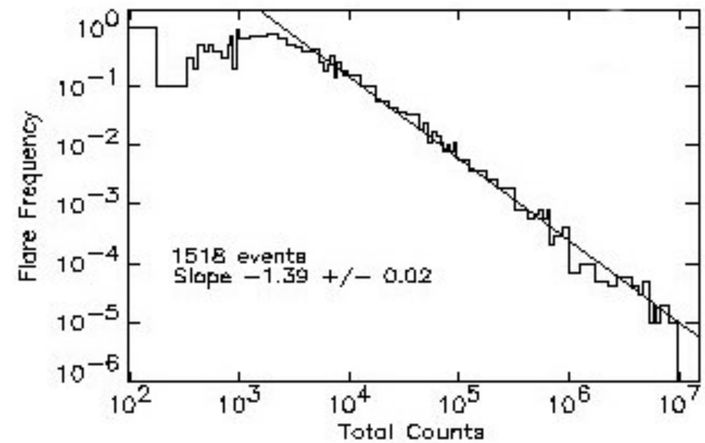


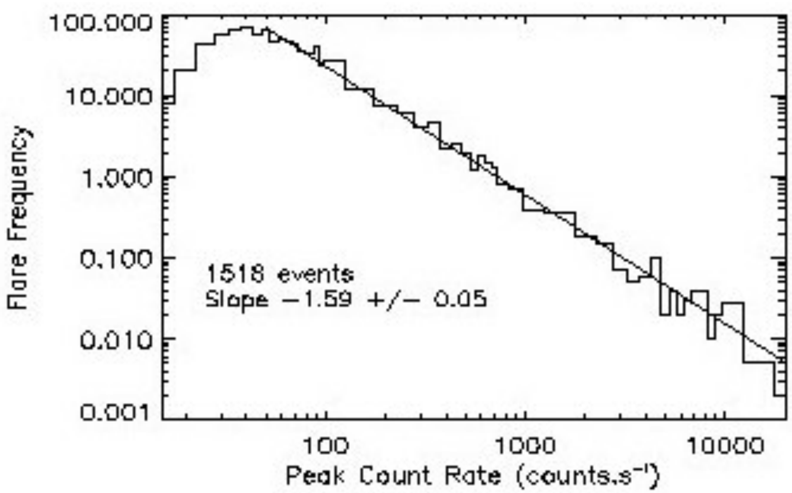
Time series of flare events

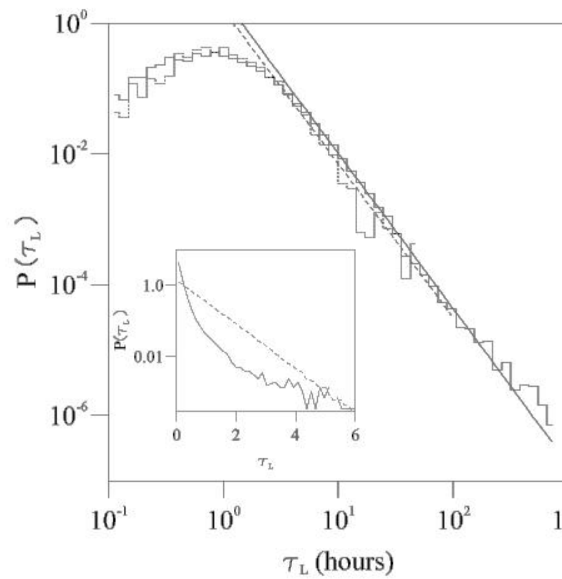
Hard X-ray (> 20 keV):
Intermittent spikes, duration 1-2 s,
$E_{max} \sim 10^{27}$ erg
Numerous smaller spikes down to 10^{24} erg (detection limit)

Power law statistics of flares





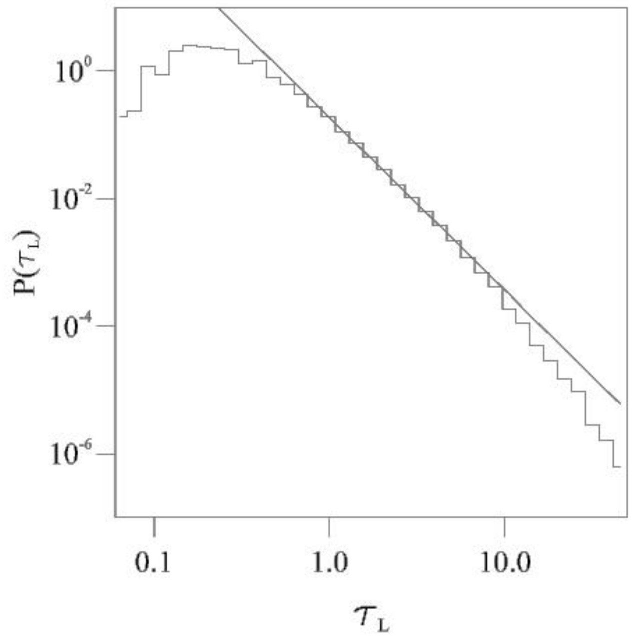


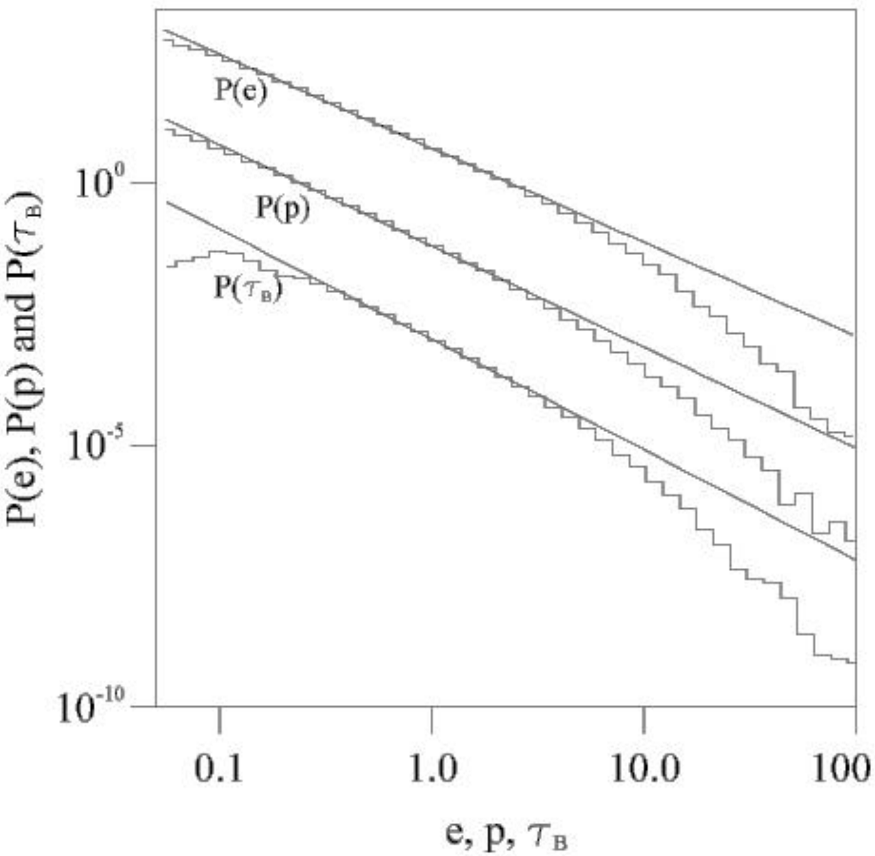


Total energy, separation times, peak energy and (more or less!) lifetime of individual bursts seems to be distributed according to power laws.

Same statistics for MHD simulations and shell models

1) Total energy of bursts
2) Time duration
3) Energy of peak
4) Waiting times





What happens in Solar Wind

1. Where does cascading turbulent energy go on small scales in solar wind?

2. How is turbulent energy dissipated at small scales, thus heating the medium?

At least four characteristic scales in plasmas (lengths and frequencies): ion-inertial length (shielding of protons to electromagnetic waves) and ion-gyration radius (size of gyration of ions). The same for electrons but at smaller scales (higher frequencies)

At scales lesser than the characteristic ion scales the MHD approximation fails, solar wind fluctuations become kinetic and cannot be described within the MHD framework: small scales fall in the realm of plasma physics.

Mean-free-path of the order of the Sun-Earth distance
λ = 1 AU

→

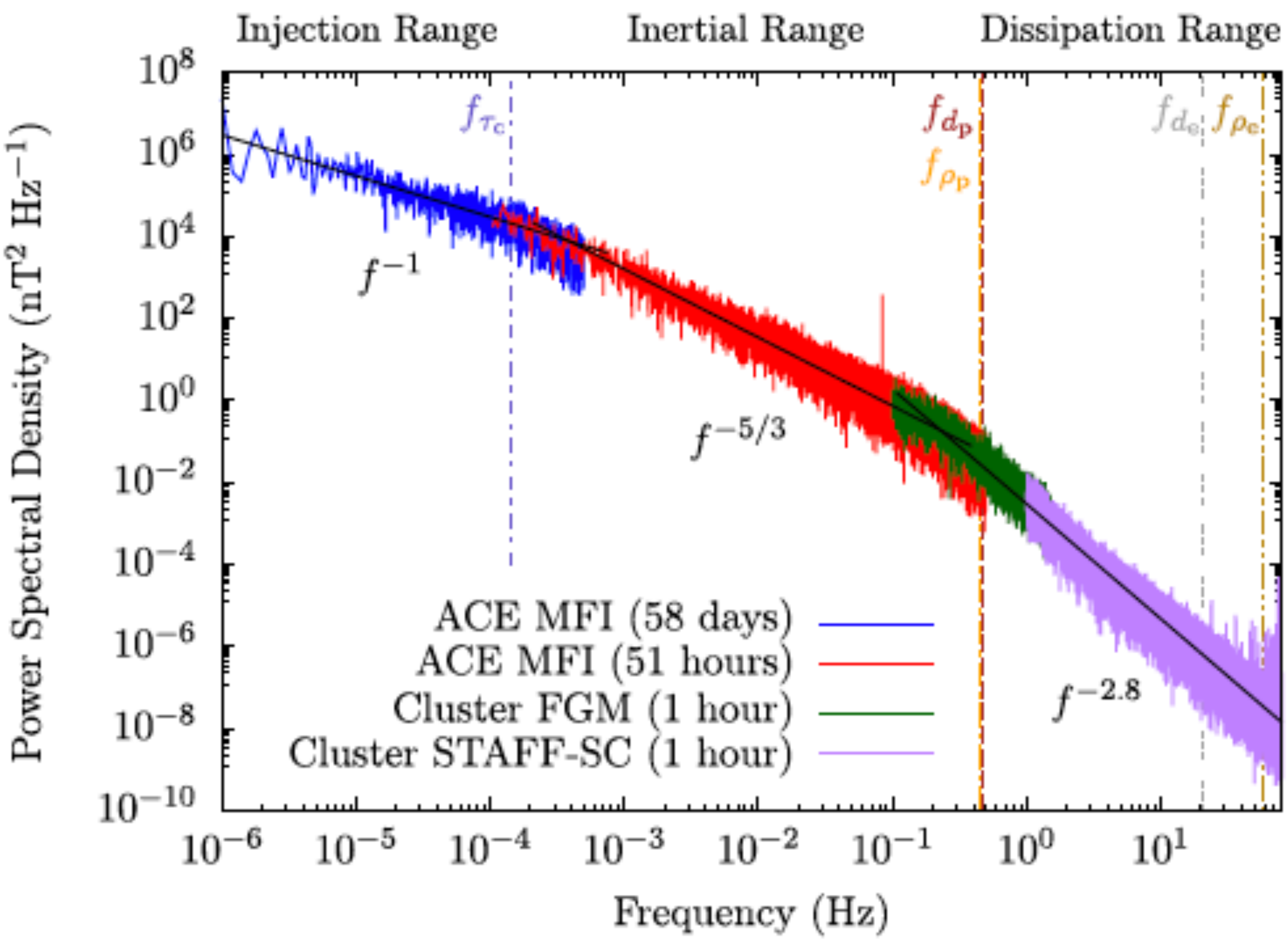
Spacecrafts probe a collisionless (or maybe a weakly collisional) medium. Viscosity cannot be a real physical quantity and a ∇^2-like dissipative term does not actually exist in space plasmas.

Three big questions rise

1. What we actually observe as "dissipation range" of solar wind turbulence, at scales lesser than the inertial range scales?
2. What is the framework to describe fluctuations at frequencies beyond the ion-cyclotron frequency?
3. What kind of process "replaces" viscous dissipation at small scales to dissipate energy?

A range of scales with a steeper power law energy spectrum is observed

All is present here, but viscosity

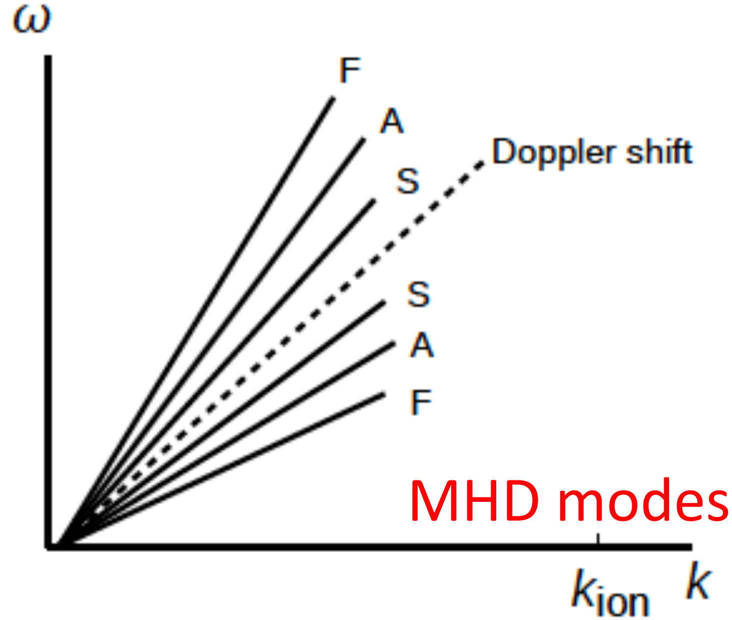


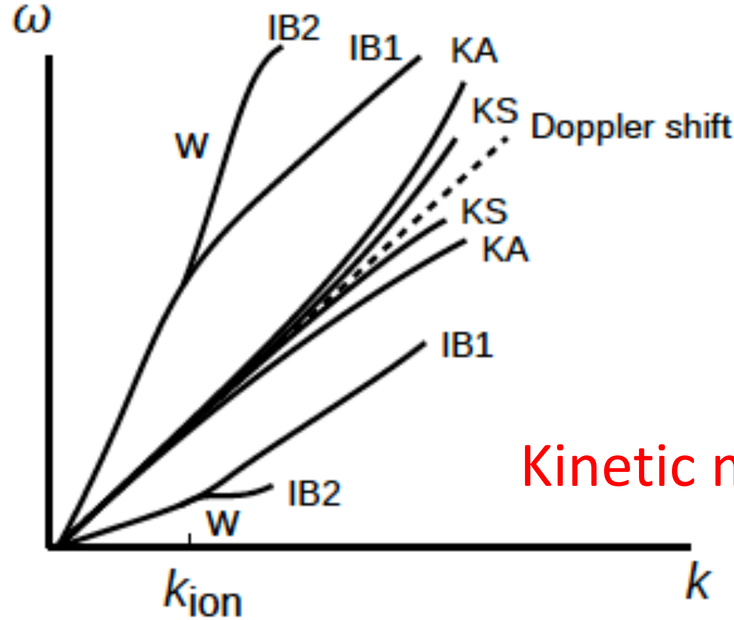
break at the proton inertial frequency, or at the proton gyroradius, both approximately near 1 Hz

How observations can be interpreted in terms of a "dissipation range" of turbulence?

Linear plasma theory. The usual MHD branches (Alfven waves, magnetosonic fast and slow waves) split into various branches of quasi-perpendicular (anisotropic) propagating modes (Ion-Bernstein, Kinetic Alfvèn waves, Whistler, kinetic slow modes) .

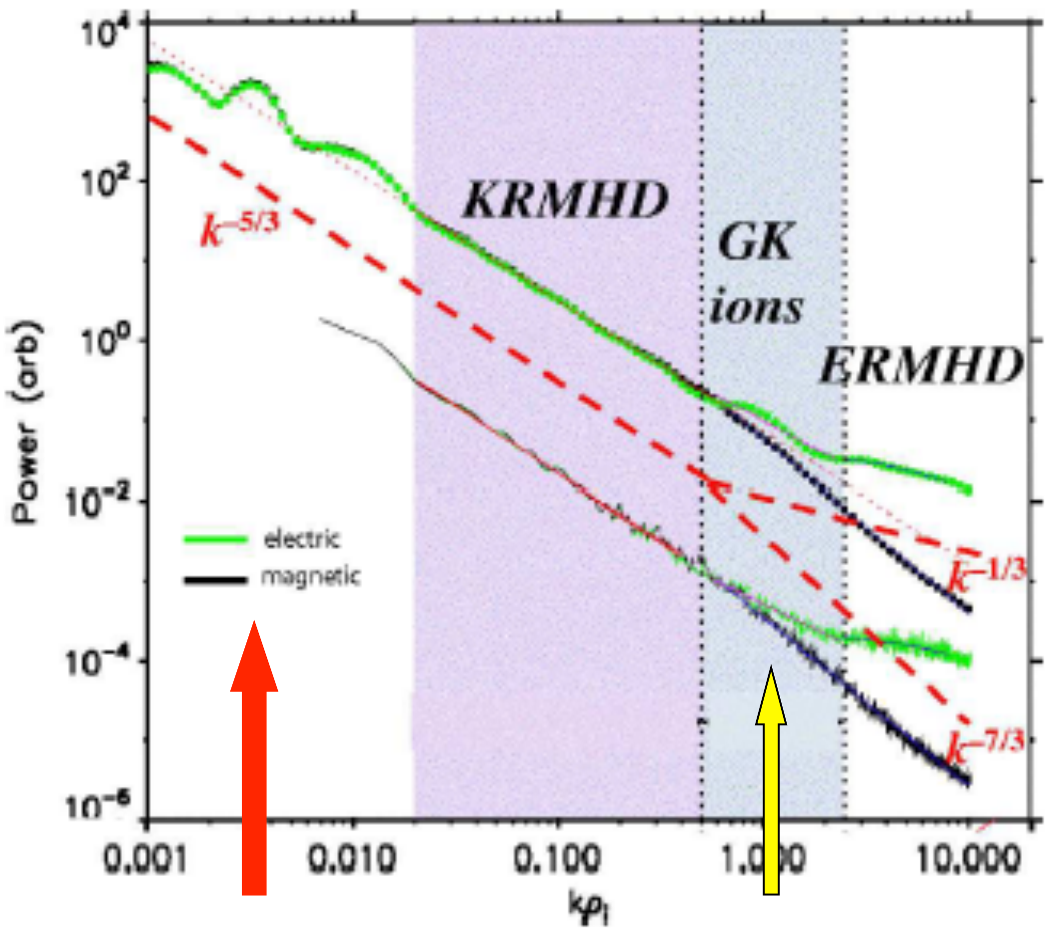
Theoretical dispersion relations of modes





Nonlinear dispersive effects on wave-wave couplings come into play: Small scales should represent a dispersive/dissipative range due to nonlinear wave couplings and collisionless dissipation.

Dispersive effects: Electric field fluctuations dominate at small scales with respect to magnetic fluctuations



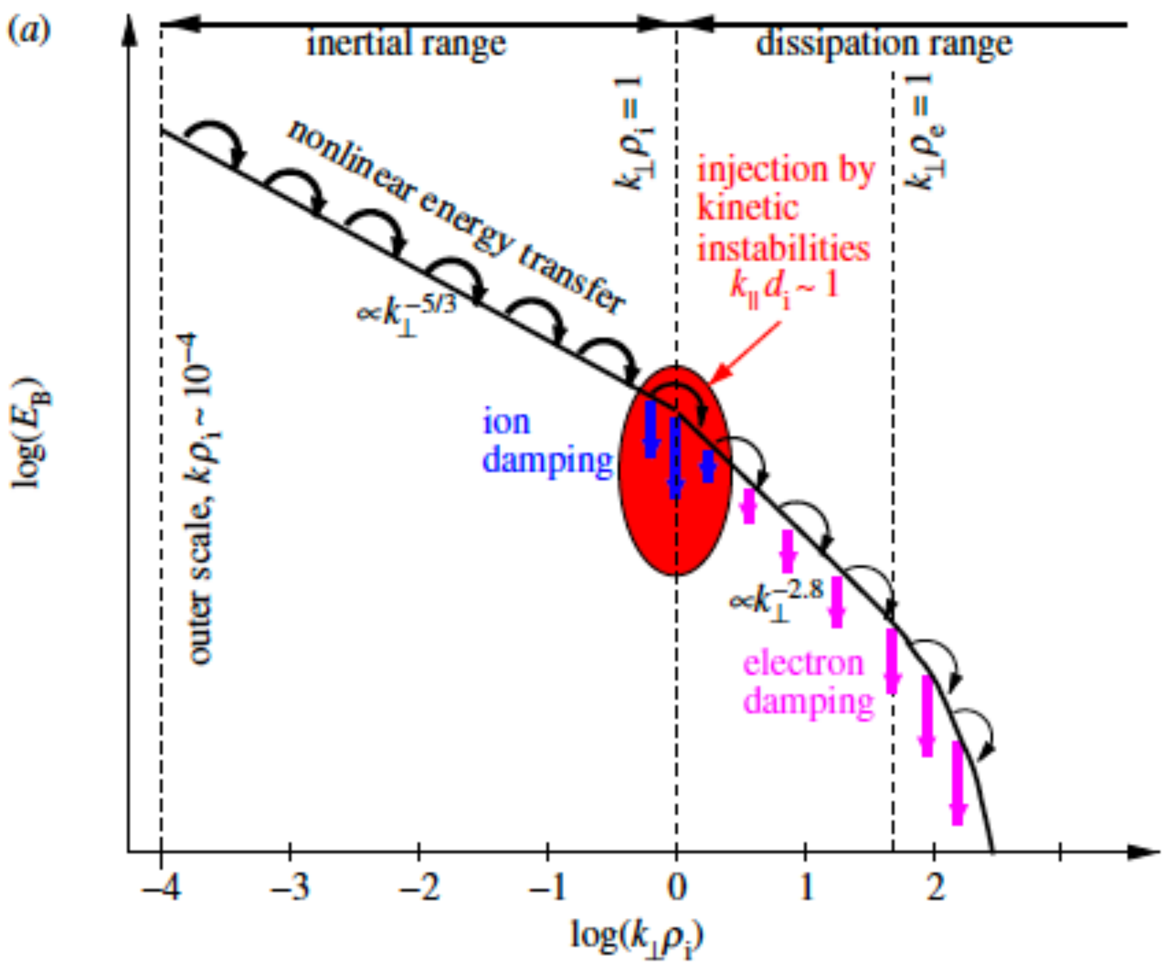
S.D. Bale et al., PRL (2005)

Evidence for electrostatic turbulence beyond the spectral break?

Magnetic fluctuations are residual → weak wave turbulence?

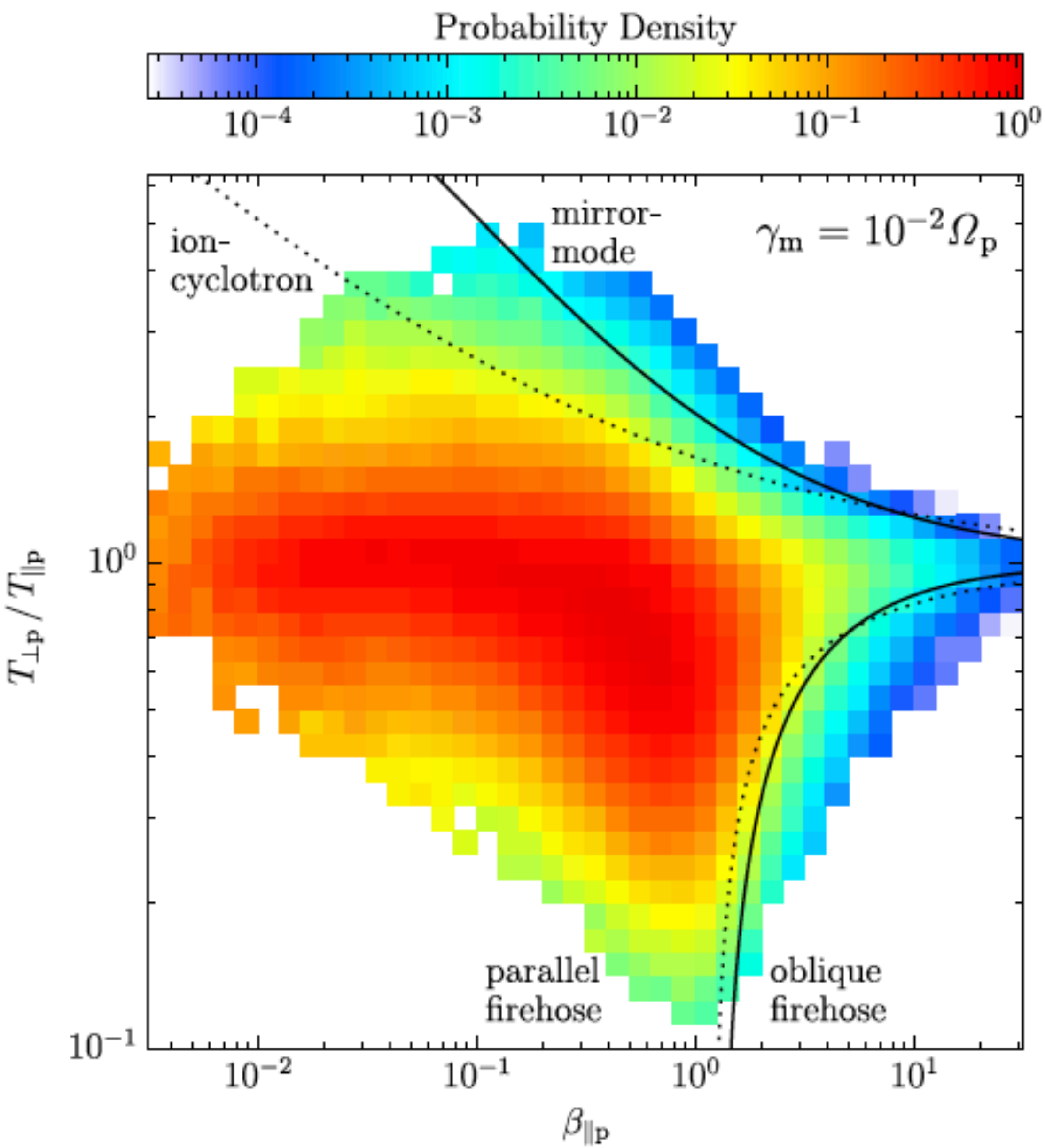
inertial range   dissipative/dispersive range

Overall picture of fluctuations: The energy cascade continues through nonlinear coupling of kinetic wave-modes which open several channels of energy transfer to small scales.



Assume that energy damping is localized on two main scales for ions and electrons.

Kinetic instabilities can inject further energy into the system at ion scale



Departure from temperature isotropy constrained by kinetic instabilities furnishes evidences for residual Coulomb collisions at ion scale, for low plasma beta

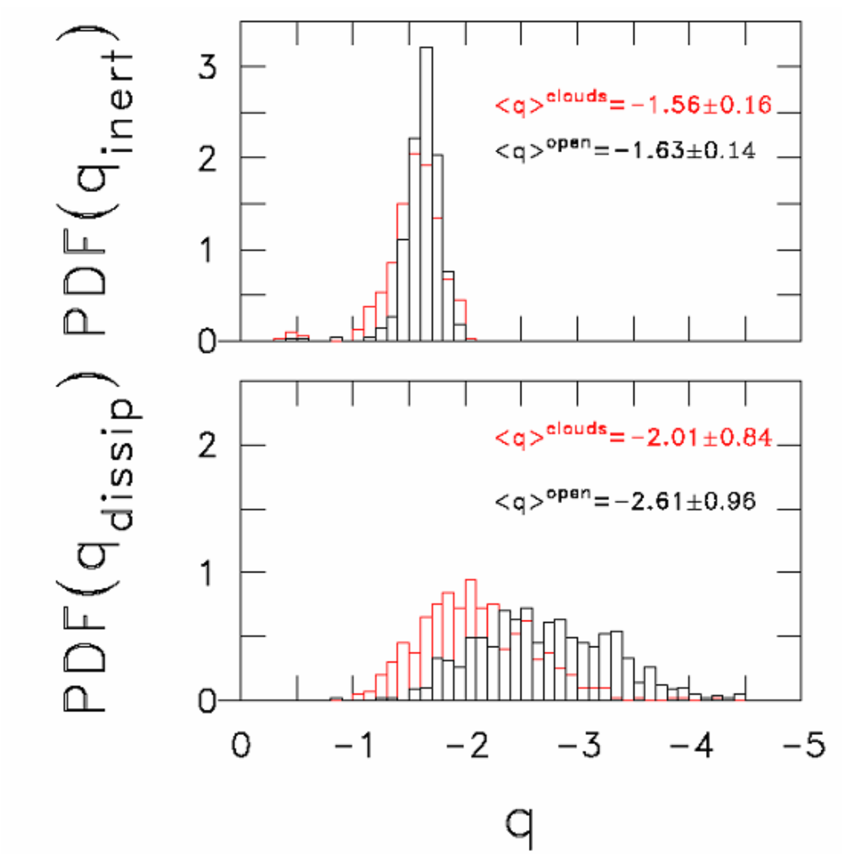
Kinetic instabilities at ion scales are generated by ion temperature anisotropy, observed at low plasma-beta.

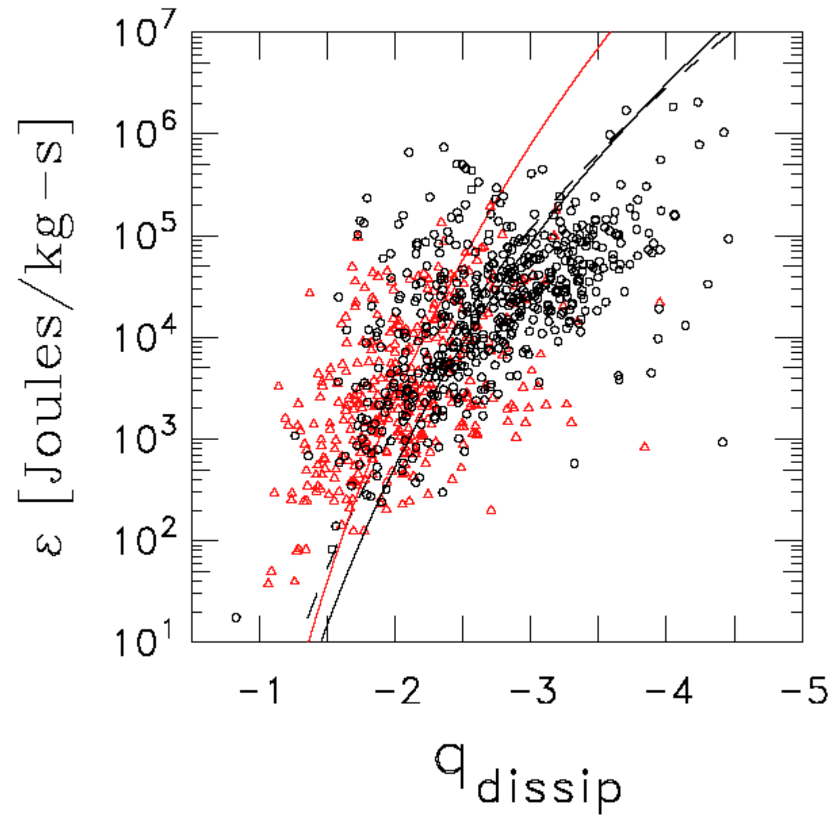
Plasma beta represents the ratio between kinetic and magnetic pressure.

Lack of statistical universality in the high-frequency range, spectral properties are related to the MHD energy cascade

ACE spacecraft

C. Smith et al., ApJ Letters (2006)

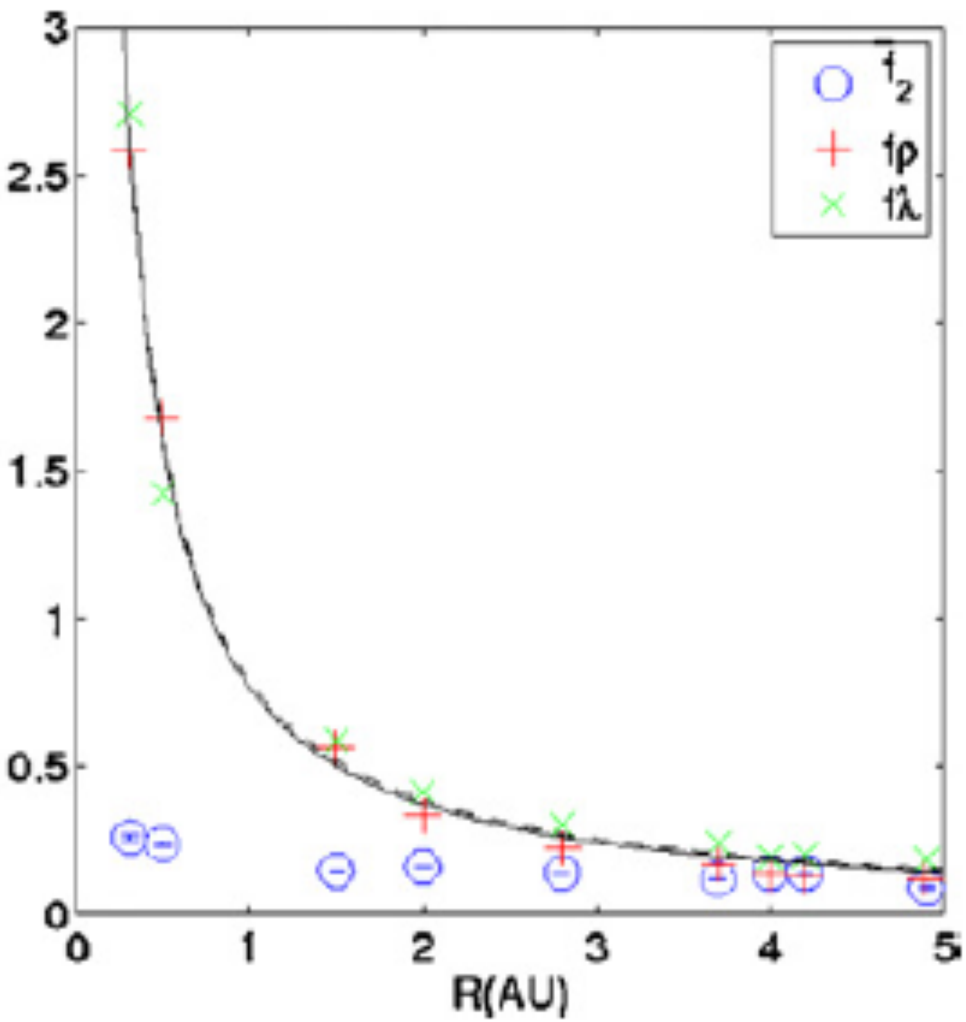




Broader distribution of observed spectral indices in the high-frequency range.

A rough estimate of the energy cascade rate of inertial range is directly related to the steepening of the high-frequency range: The higher ε, the steeper the spectrum.

Where does MHD turbulence breaks down in solar wind turbulence?

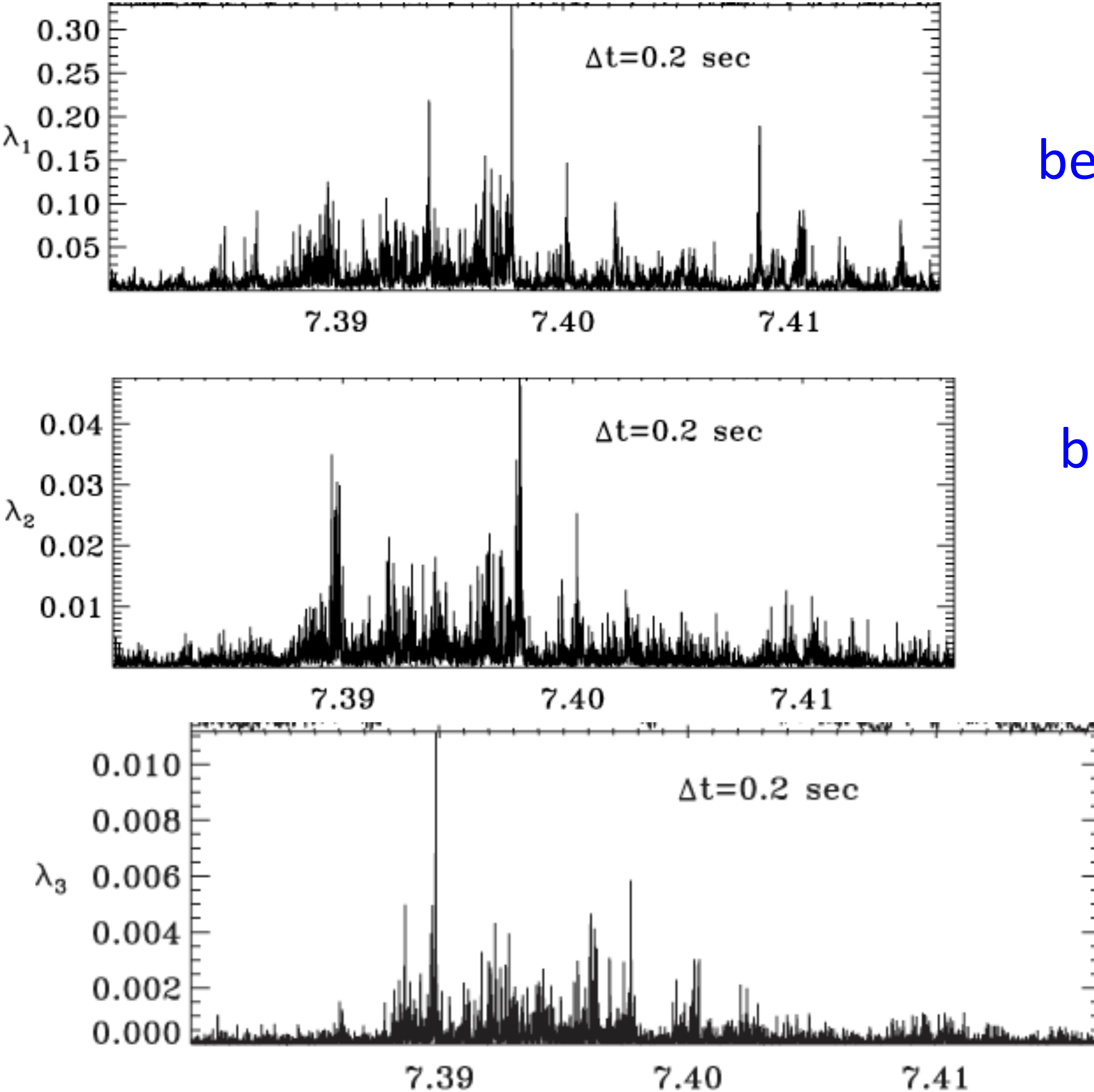


The characteristic frequency break of the Kolmogorov spectrum depends on the model assumed to describe small scale plasma fluctuations.

While the characteristic plasma frequencies evolve with distance from the Sun, the spectral break frequency looks to be constant.

→ Break neither directly related to proton inertial length nor to proton gyroradius. Perhaps affected by anisotropy.

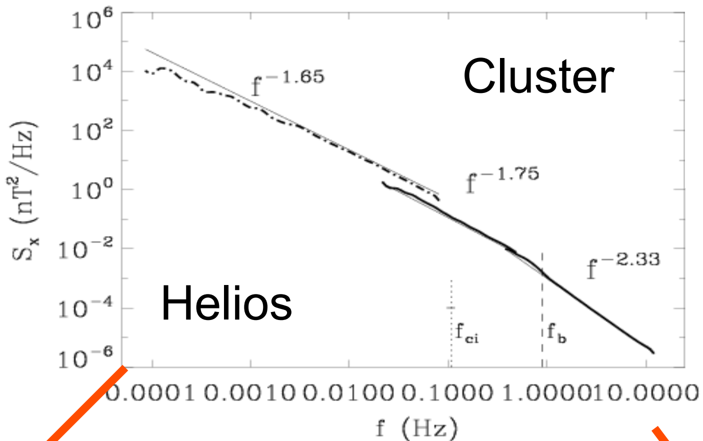
S. Perri et al., ApJL 2010

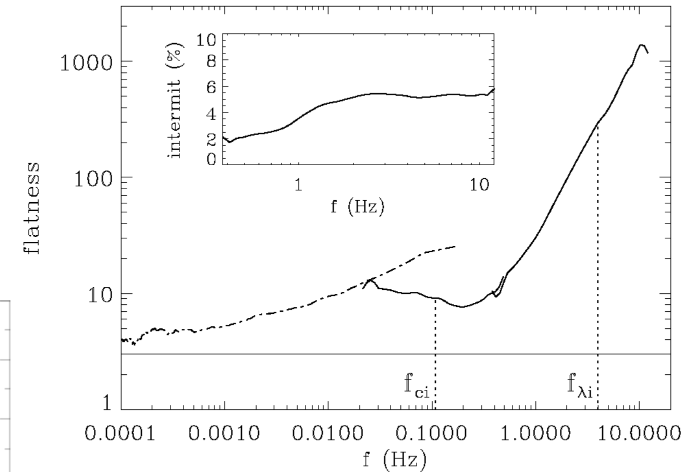


Anisotropy beyond the ion-cyclotron frequency much more bursty than at large scales

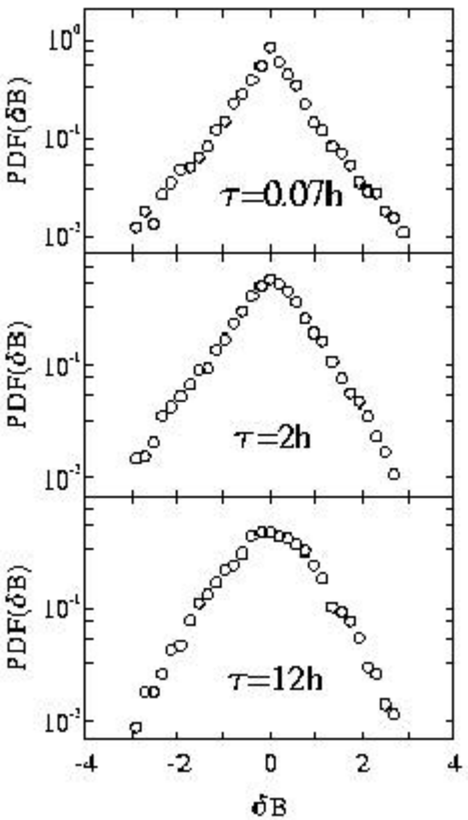
Observations of strong intermittency beyond the ion scale

O. Alexandrova et al., ApJ (2008)



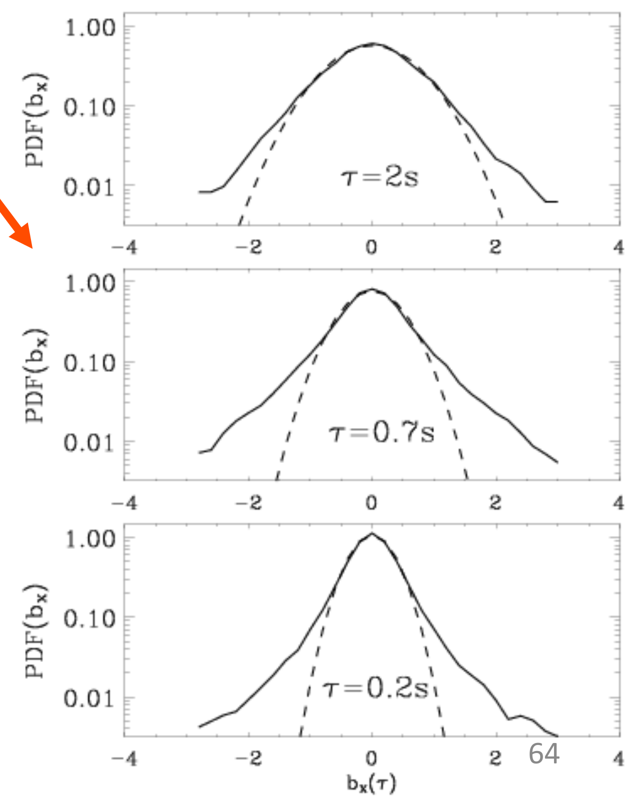


Kurtosis



Probability distribution functions (PDF) of normalized wavelets coefficients on different time scales depends on scale:

$$\frac{W_x}{\sqrt{\langle W_x^2 \rangle}}$$



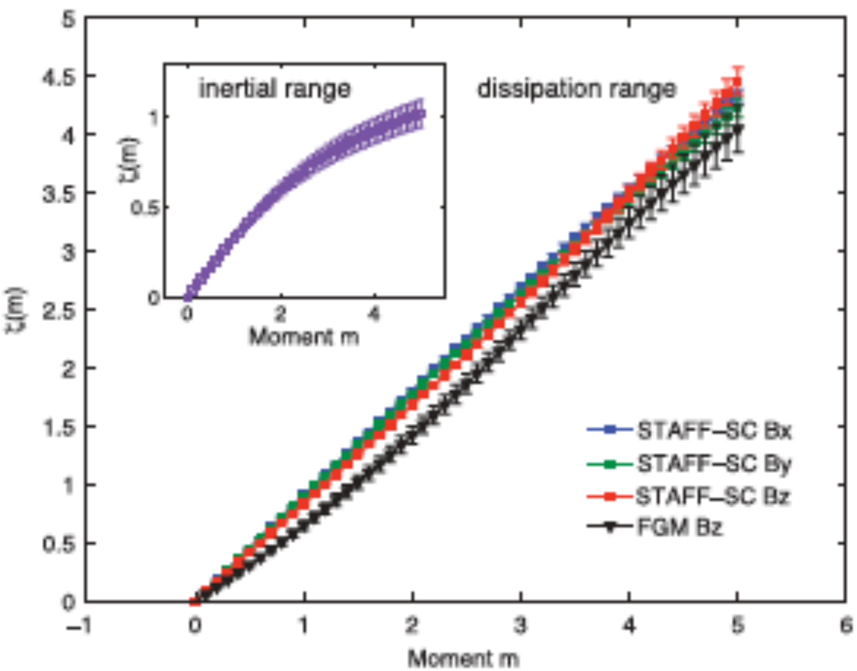
PRL 103, 075006 (2009) PHYSICAL REVIEW LETTERS week ending 14 AUGUST 2009

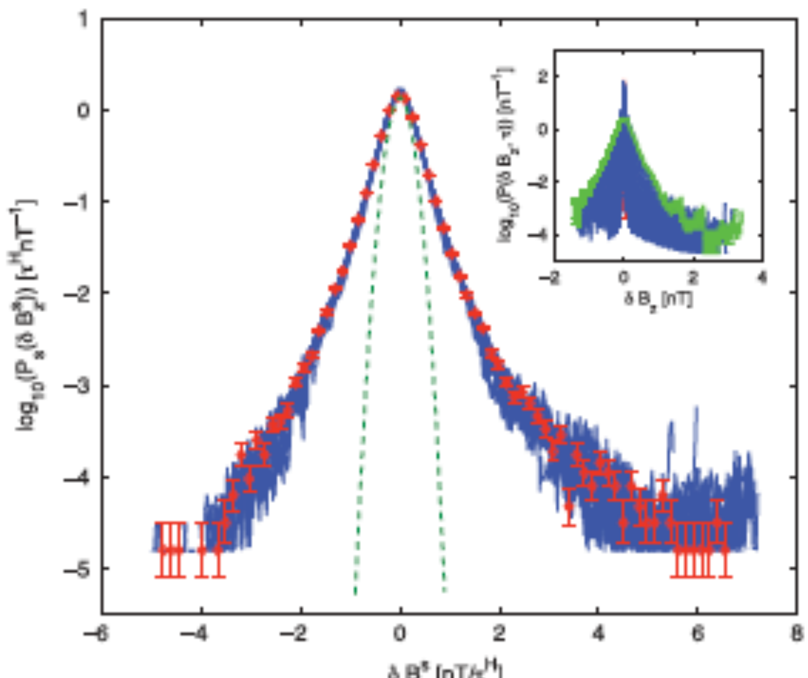
Global Scale-Invariant Dissipation in Collisionless Plasma Turbulence

K. H. Kiyani,[1,*] S. C. Chapman,[1] Yu. V. Khotyaintsev,[2] M. W. Dunlop,[3] and F. Sahraoui[4,5]

Sometime, no intermittency is observed at small scales. Scaling exponents look regular

At variance with inertial range, intermittency observed at small-scales seems to be not a universal feature of turbulence.





All kind of universality is lost at small scales, different samples give almost different results. What kind of plasma process generates magnetic fluctuations beyond the ion scale?

How the turbulent MHD cascade continues beyond the ion scale?

1) Wave-wave coupling channels;
2) A different kind of strong MHD-like turbulence.

Hall MHD turbulence: the simplest way to reproduce dispersive effects

A conjecture: At the proton cyclotron frequency there is a breakdown of the usual "alfvenic" turbulence, and a new kind of turbulence appears

$$\frac{\partial B}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \boxed{\frac{1}{R_i}\nabla \times (\frac{1}{\rho}(\nabla \times \mathbf{B}) \times \mathbf{B})} \qquad R_i = \frac{L\Omega_i}{V_A}$$

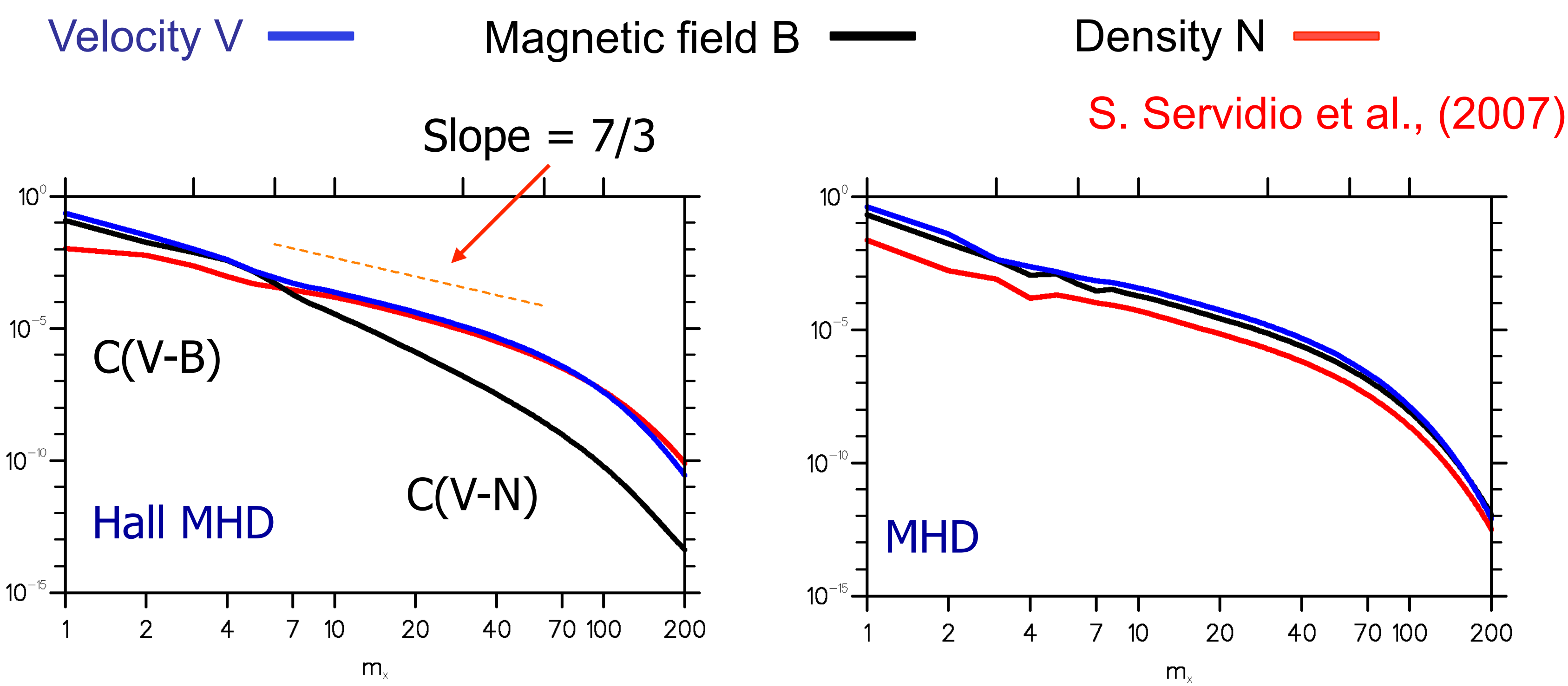
Hall term

A breakdown of the scale-free features, the Hall term introduces a characteristic scale. Two competing non-linear terms: energy is transferred on times of the order of the eddy-turnover time up to the Hall scale. At this scale the energy cannot be transferred on the same time, but on a new characteristic time.

$$\frac{B}{\tau_H} \sim \frac{B^2}{L^2\rho} \longrightarrow \tau_H \sim \frac{L^2\rho}{B}$$

$$\boxed{E(k) \approx k^{-7/3}}$$

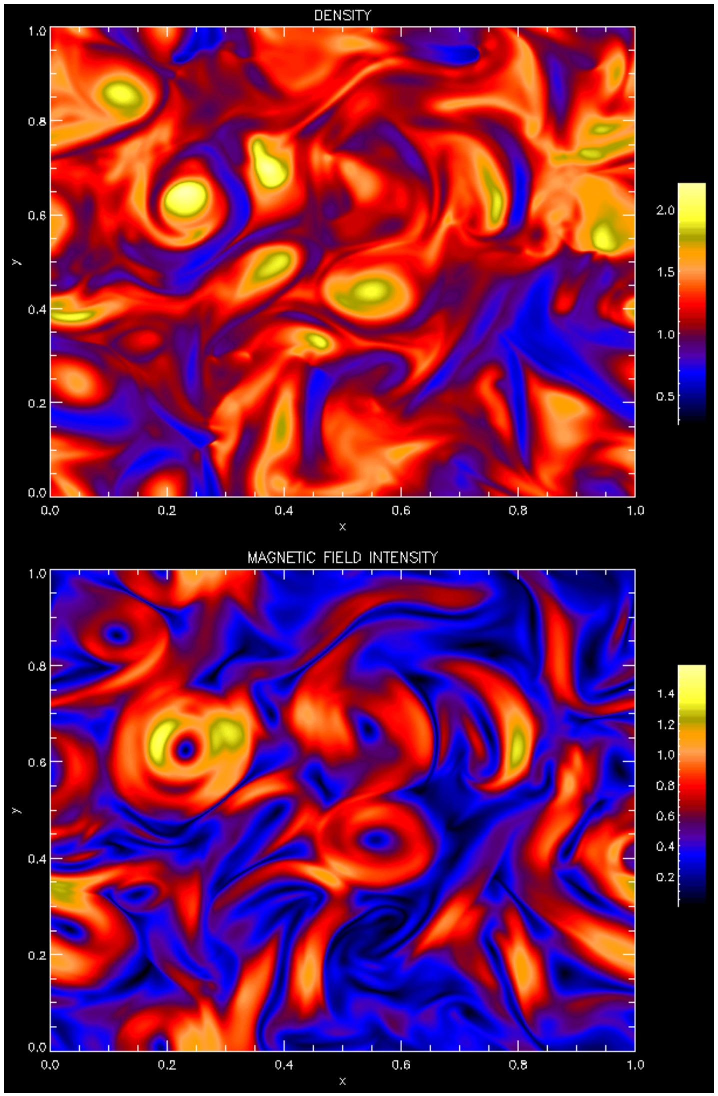
Hall MHD vs. MHD: energy spectra from numerical simulations



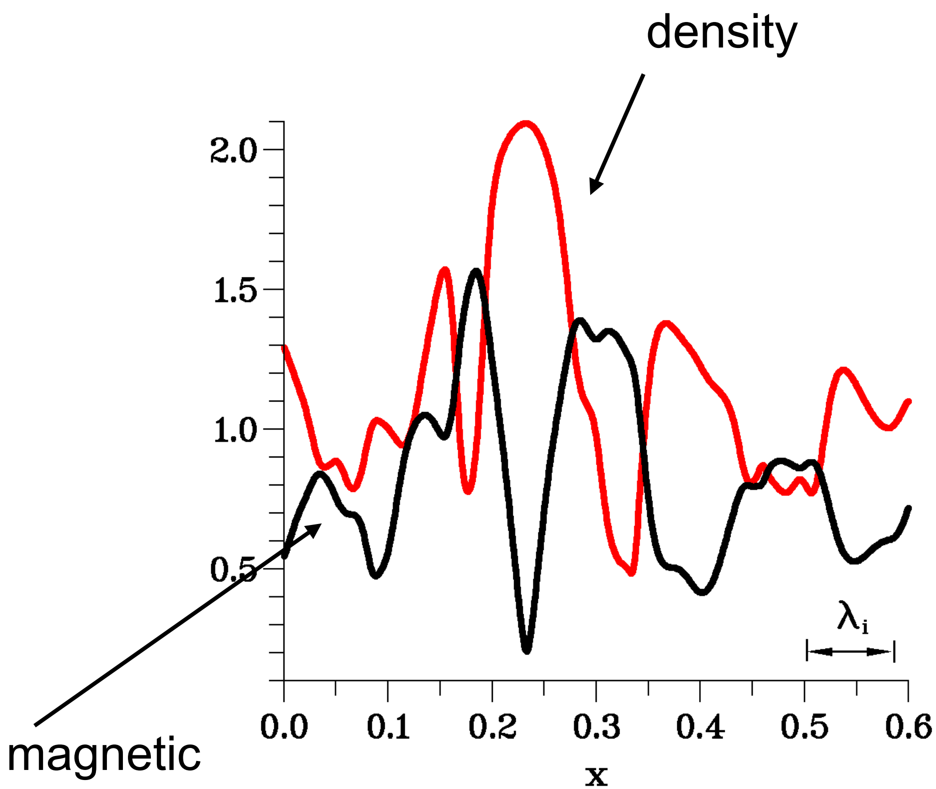
Alfvenic turbulence becomes dispersive. The spectral dynamical alignment between V and B is lost in favor of a spectral correlation between N and V.

The Hall effect causes a breakdown of Alfvènic turbulence to a "Magnetosonic Turbulence" → enhanced role of density

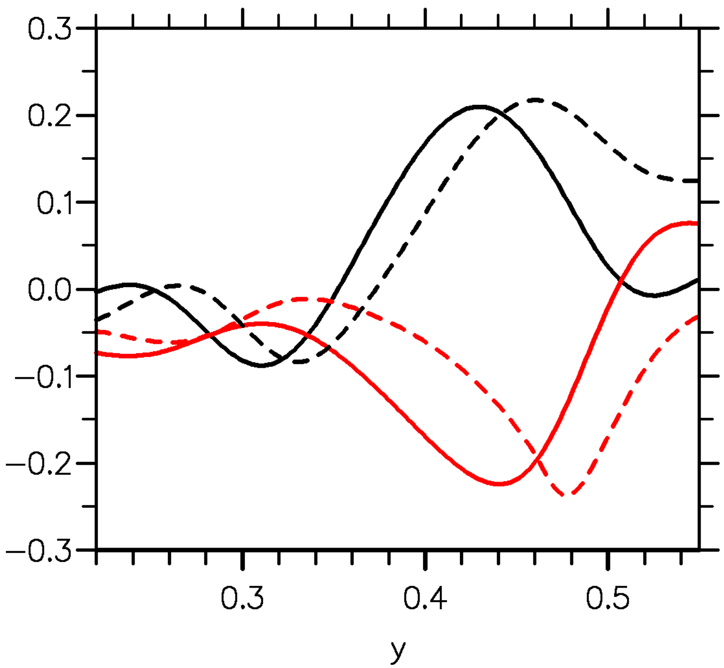
Hall-MHD turbulence generates anti-correlated fluctuations between density and magnetic fields



A cut along x
at a fixed y



“Travelling eddies” (soliton-like) within magnetosonic turbulence sometimes observed by Cluster spacecrafts



Density and magnetic field: a “travelling” structure at two different times

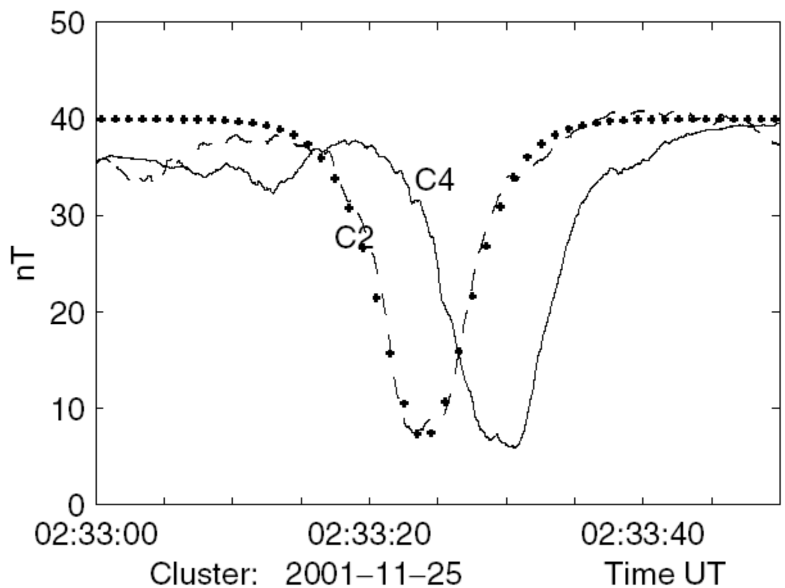
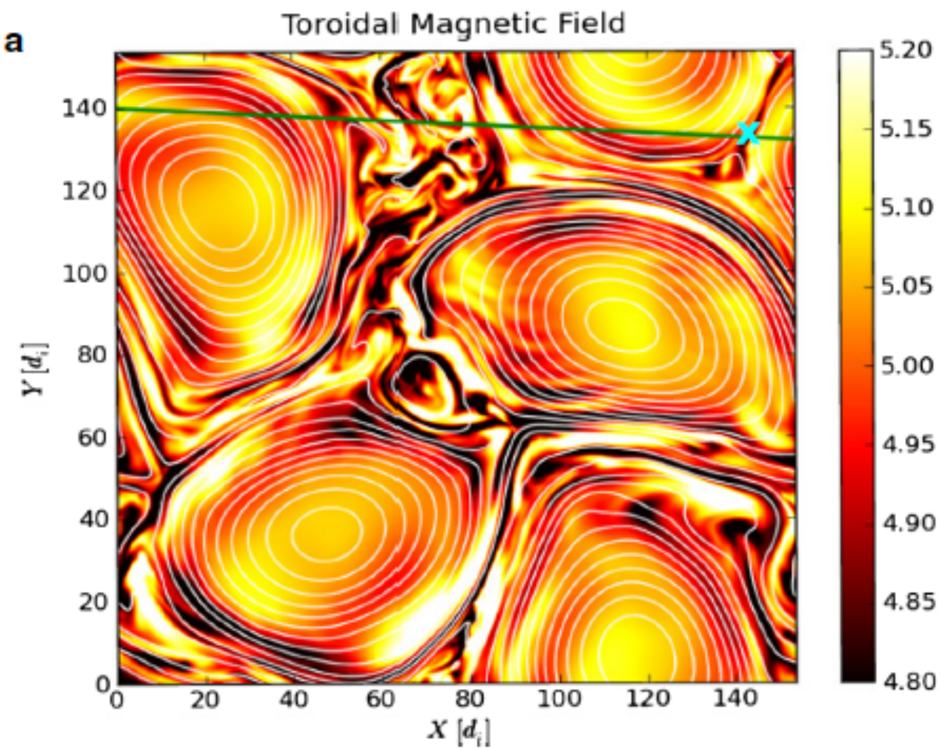


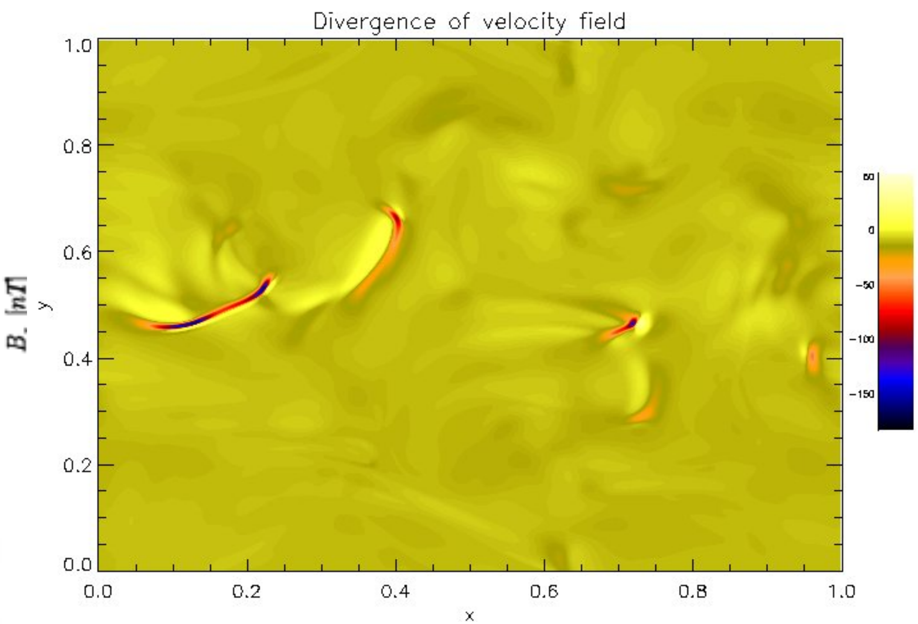
FIG. 1. A large scale soliton observed by Cluster spacecraft C2 (dashed) and C4 (solid) in the total magnetic field. Marked curve shows fit of $b_0 \,\mathrm{sech}^2[(t - t_0)/\delta t]$ with $b_0 = -33$ nT and $\delta t = 4.4$ s. The soliton moves with velocity $u_0 \approx 250$ km/s and has a width of 2000 km. The position of Cluster satellites was $(-4, 17, 5)\ R_E$ GSE.

Small scale structures appears in Hall-MHD as current sheets or compressive structures. A new kind of turbulence?

Current sheets



Strong divergence of velocity field



Gyrokinetic model a framework for wave-wave coupling, no fluid-like

Expansion of the distribution function of particles at the ion scale (ordering through a small parameter)

$$f_s(t, \mathbf{r}, \mathbf{v}) = F_{0s}(v) - \frac{q_s \varphi(t, \mathbf{r})}{T_{0s}} F_{0s}(v) + h_s(t, \mathbf{R}_s, v_\perp, v_\parallel).$$

Maxwellian

Response to maxwellian (dissipation)

Gyrocentric distribution function in terms of average gyroradius for particles

- The equation is supplied by equations for electric and magnetic fields
- Fokker-Planck collisional operator at the ionic scale to reproduce residual collisions

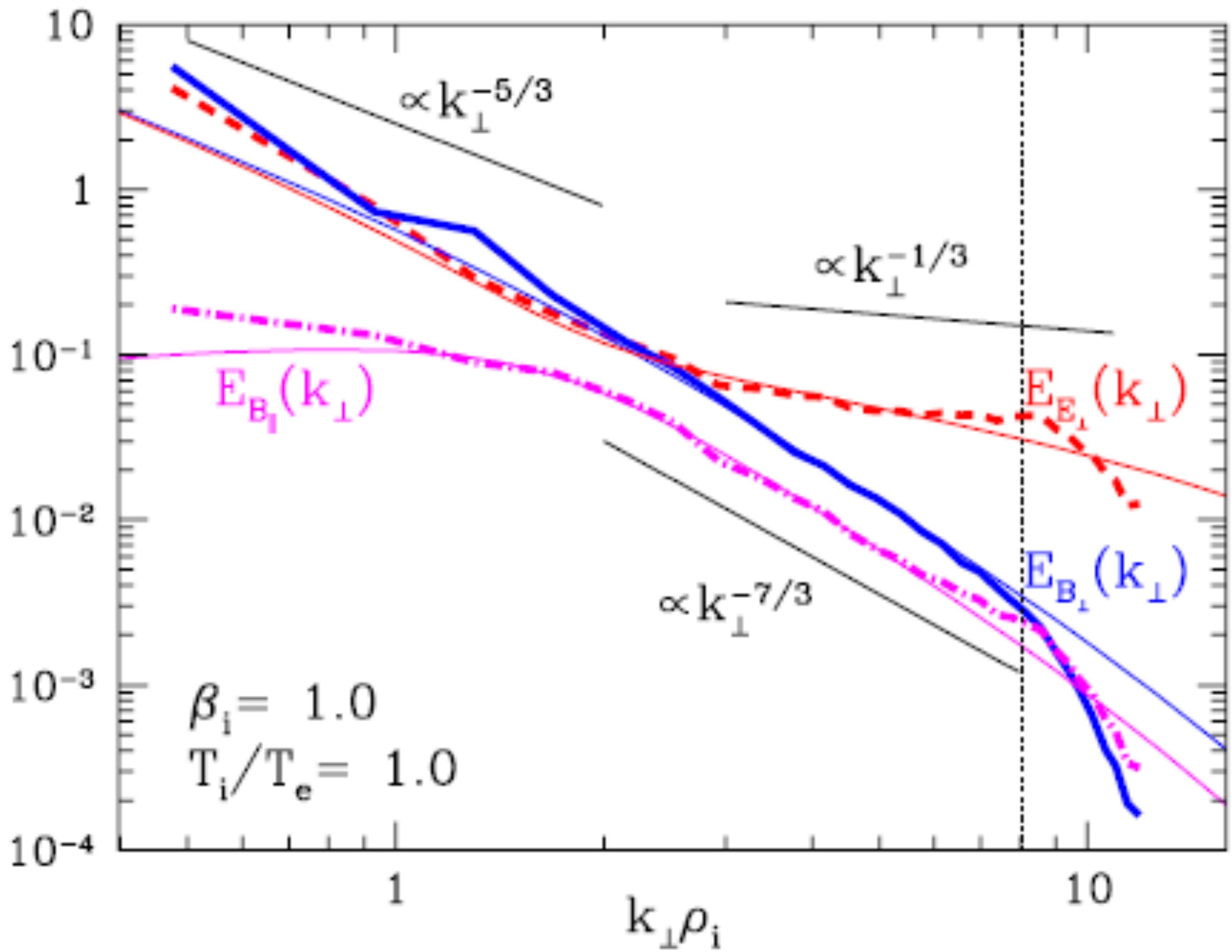
$$W = \int d^3\mathbf{r} \left(\sum_s \int d^3\mathbf{v} \frac{T_{0s} \delta f_s^2}{2F_{0s}} + \frac{|\delta \mathbf{B}|^2}{8\pi} \right).$$

A generalized energy is conserved, thus generating cascades in the phase space
Entropy and magnetic energy

Gyro-kinetic simulations

Couplings of Kinetic Alfven Waves (KAW) or Whistlers could generate the dispersive (anisotropic) spectrum

$$k_\perp^{-7/3}$$



Enhanced electric field at small scales (red line), enhanced role of compressibility (purple line)

Different scenarios for this range: weak/strong turbulence depending on the dispersion relation of kinetic modes used to describe fluctuations

1) Fast/Whistler mode turbulence or strong whistler turbulence starts at the proton inertial length.

Leith (1967), Zhou & Matthaeus, (1990), Biskamp et al. (1996, 1999) Stawicki, Gary & Li, (2001), Gary et al. (2008), Gary and Smith (2009).

2) Compressible or incompressible Hall MHD turbulence naturally results in a steepening of the magnetic turbulent spectrum at the Hall frequency, and enhanced compressibility

Gosh et al. (1996), Servidio et al., (2007), Galtier, (2006), Galtier & Buchlin, (2007), Alexandrova et al., (2008).

3) Nonlinear coupling of Kinetic Alfven Waves. Essentially transverse to the ambient field. At the ion Larmor radius, the Alfvènic cascade continues through the KAWs branch.

Howes et al. (2006, 2008), Schekochihin et al. (2009)

Observations dont help us to clarify the origin of the dispersive region → the description of the dispersive region remains somewhat confusing!

"At our present level of understanding, the best we can say is that quasi-parallel whistlers, quasi perpendicular whistlers, and Kinetic Alfvèn Waves all probably contribute to dispersion range turbulence in the solar wind.
Thus, the critical question is not which mode is present, but rather, what are the conditions which favor one mode over the other?"

Conclusions by P. Gary & C. Smith, JGR 114, A12105 (2009)

Looking for "modes" in solar wind turbulence (at kinetic scales)? Dispersion relation requires more than one satellite, then it can be investigated using Cluster satellites and k-filtering method of analysis.

Dispersion relation using 4 Cluster satellites

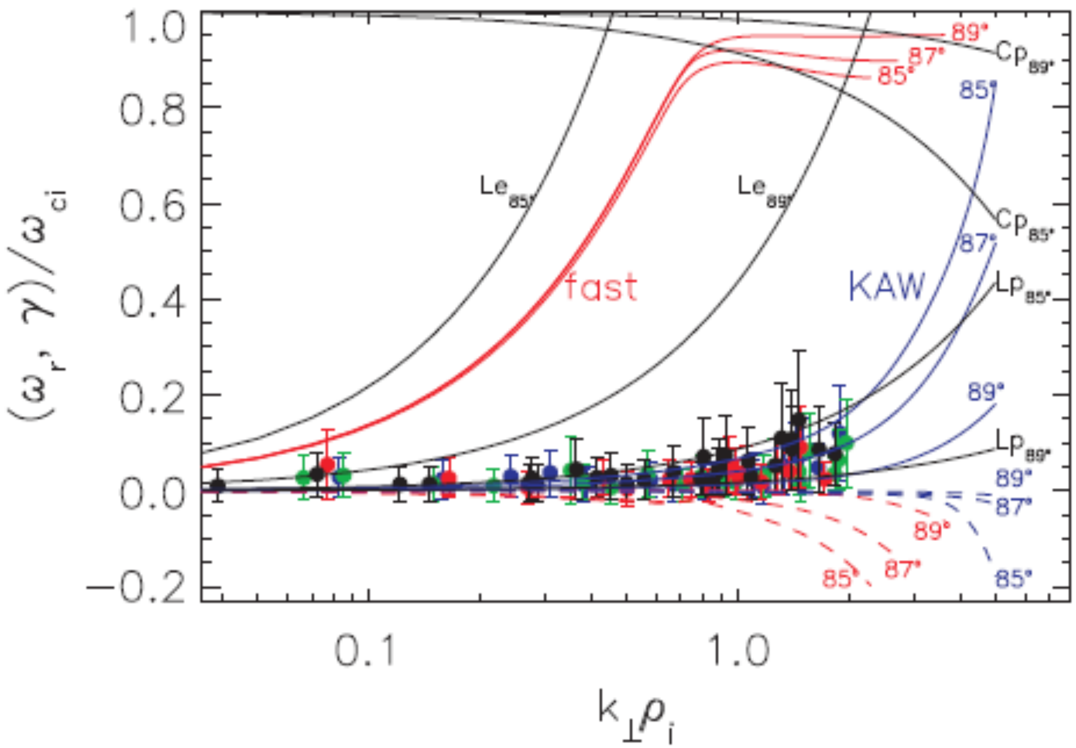


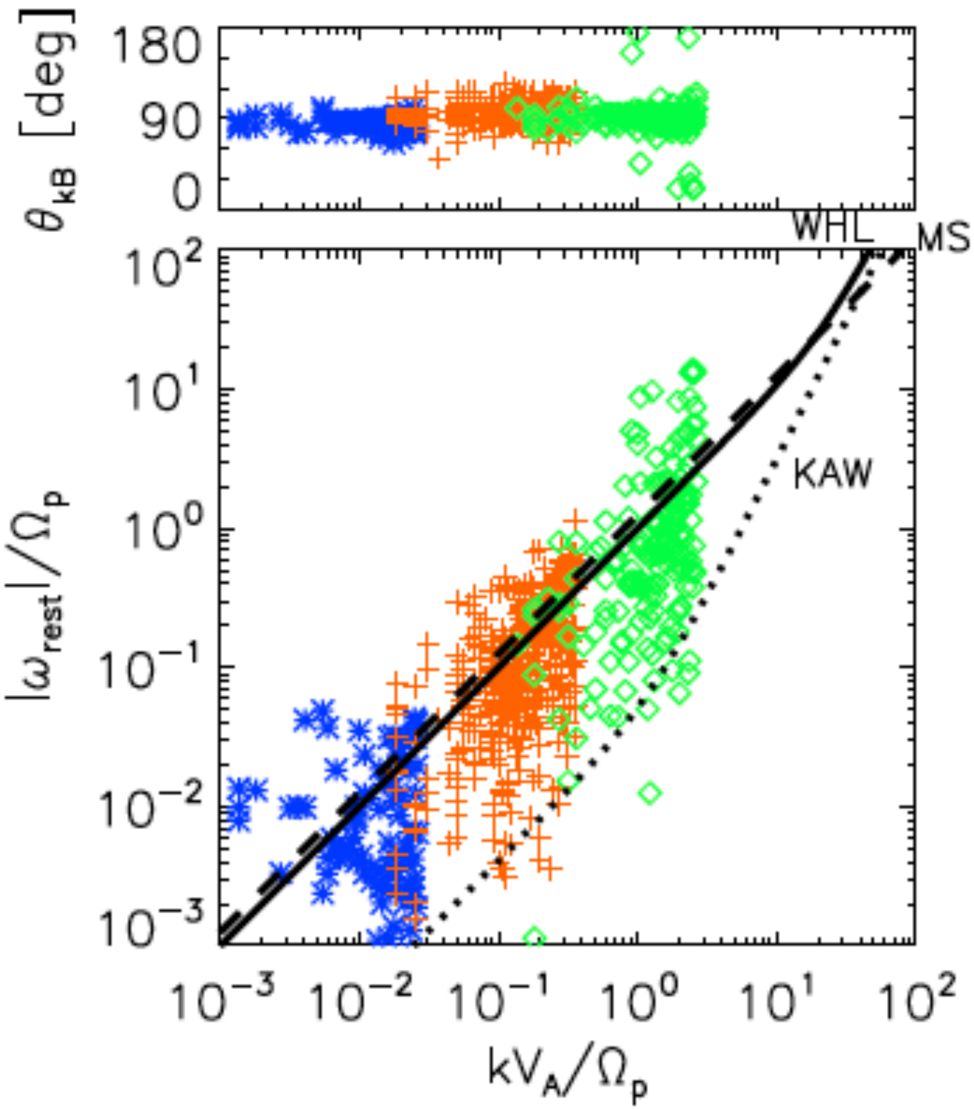
FIG. 5: Observed dispersion relations (dots), with estimated error bars, compared to linear solutions of the Maxwell-Vlasov equations for three observed angles $\Theta_{\mathbf{kB}}$ (the dashed lines are the damping rates). The black curves ($L_{p,e}$) are the proton and electron Landau resonances $\omega = k_\parallel V_{th_{i,e}}$, the curves C_p are the proton cyclotron resonance $\omega = \omega_{ci} - k_\parallel V_{th_i}$ (the electron cyclotron resonance is also plotted but it lies expectedly out of the plotted frequency range).

F. Sahraoui et al. (PRL, 2010) compared the observed dispersion relation with linear solutions of Vlasov equations.

The best fit seems to be obtained for KAW dispersion relation.

On data: Wave-telescope (k-filtering) technique, based on the assumption of plane-waves

Same data, same periods, same technique



All wave modes should eventually be present, wavevectors perpendicular to background magnetic feld

A k(ω) relation is not observed, that means a one-to-one relation between frequency and wavevector is not observed.

Results **STRONGLY** depend on the k-filtering assumptions

Y. Narita et al. 2011, GRL

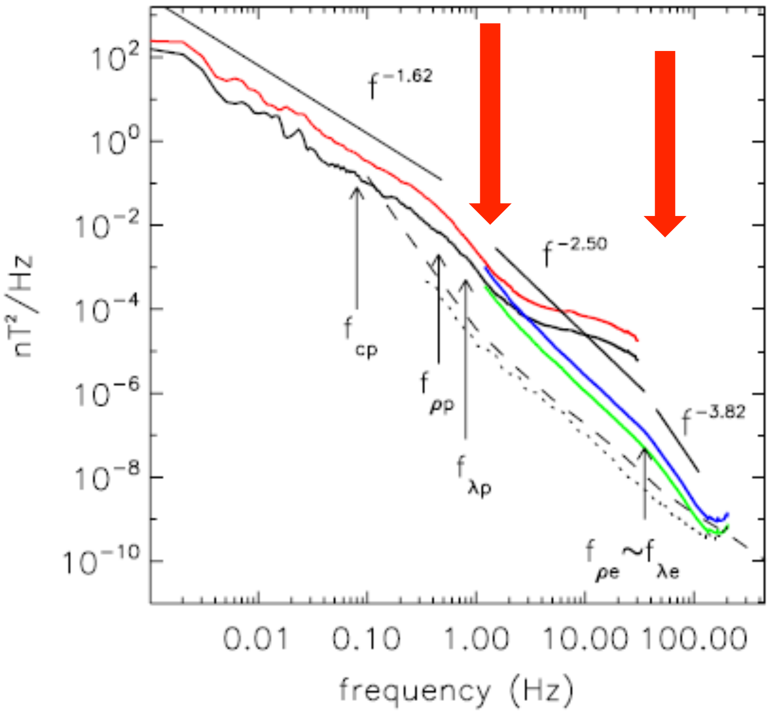
Don’t forget dissipation! What kind of process replaces viscosity in solar wind collissionless medium?

In the dispersive region, dissipation must be also at work, at the same scales, along with dispersive wave-wave couplings.

1. Wave-particle interactions: resonant collisionless heating;
2. Stochastic heating: non-resonant;
3. Something else …

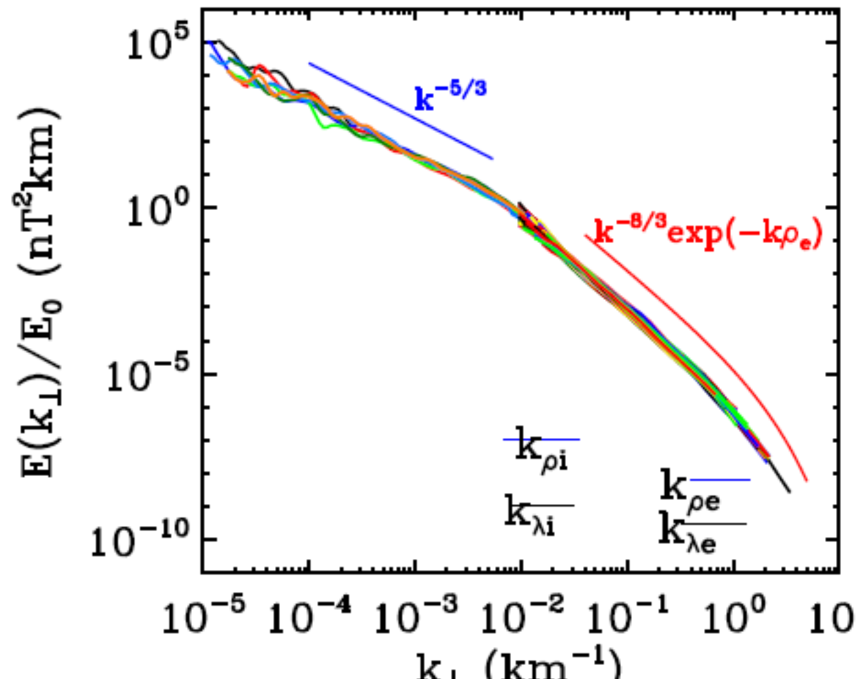
Cluster data up to 100 Hz seems to show two breakpoints: at the Doppler-shifted proton and electron gyroscales:

The “true” starting point of “dissipation”?



Double power laws

F. Sahraoui et al., PRL (2009)



$$E(k_\perp) = A k_\perp^{-8/3} \exp(-k_\perp \rho_e).$$

O. Alexandrova et al., PRL (2012)

Two different models but the same “interpretation”: Kinetic Alfven Waves turbulence followed by a “dissipative range” maybe generated by collisionless Landau damping

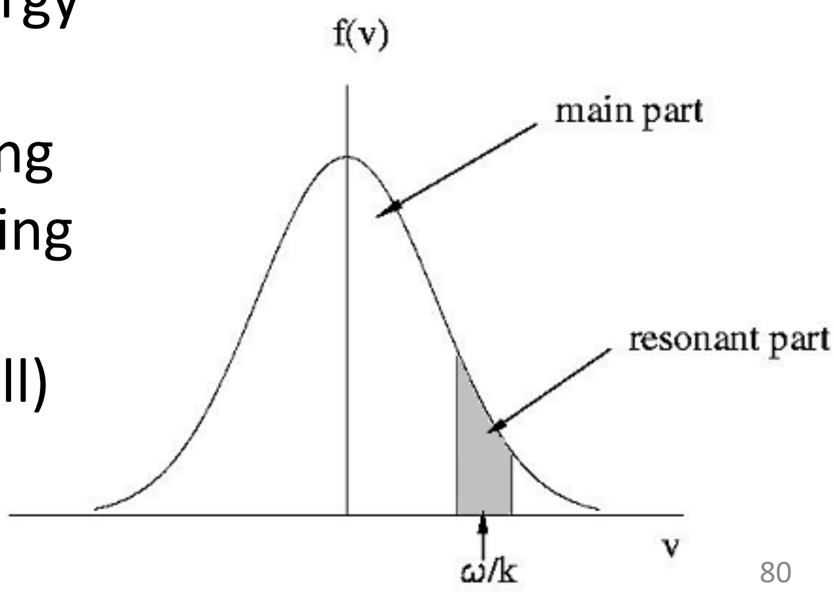
Wave energy can be dissipated through collisionless wave-particle interactions (Landau damping)

The Vlasov-Poisson system describe the self-consistent approach to collisionless wave-particle interaction (Boltzmann without collisions).

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} - \frac{eE}{m}\frac{\partial f}{\partial v} = 0$$

$$\frac{\partial E}{\partial x} = 4\pi e\left(n_0 - \int_{-\infty}^{+\infty} f dv\right)$$

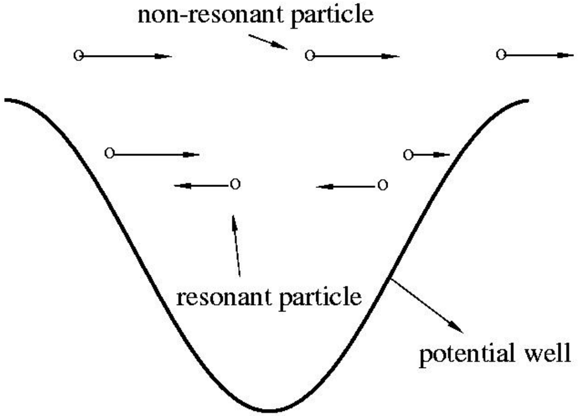
The physical mechanism (Landau, 1946): Resonant particles can exchange energy with the wave. In a decreasing f(x,v) there are always more electrons taking energy from the wave than those giving energy to it (head-on and tail-on collisions with the wave potential wall)



→ The wave damps

Landau damping is a linear process: nonlinear saturation

Nonlinear effects occur when the resonant particles, after the first energy exchange, reach the opposite side of the potential well.
O’Neil (1965): the particles which gain energy in the first interaction with the wave lose it after a time equal to the “trapping time”



$$\tau_p = \sqrt{\frac{m}{eEk}}$$

The resonant particles are trapped in the potential well: closed trajectories in the phase space, while the untrapped electrons travel through open trajectories.

Saturation of Landau damping by a kind of “phase-mixing”

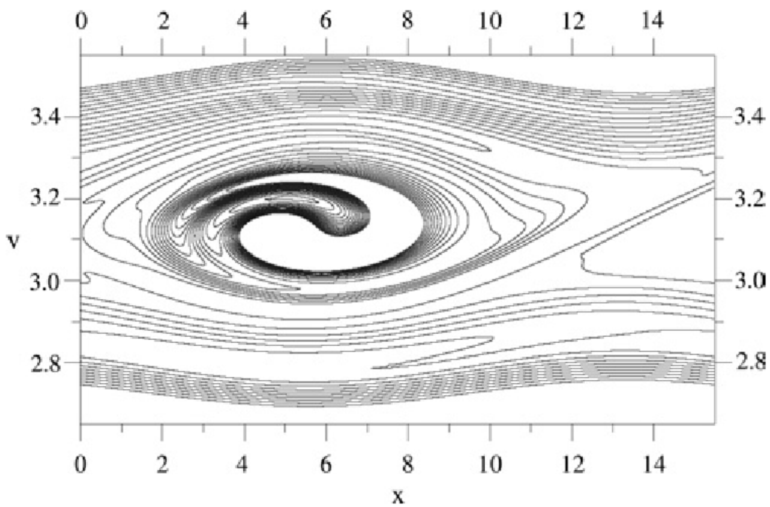


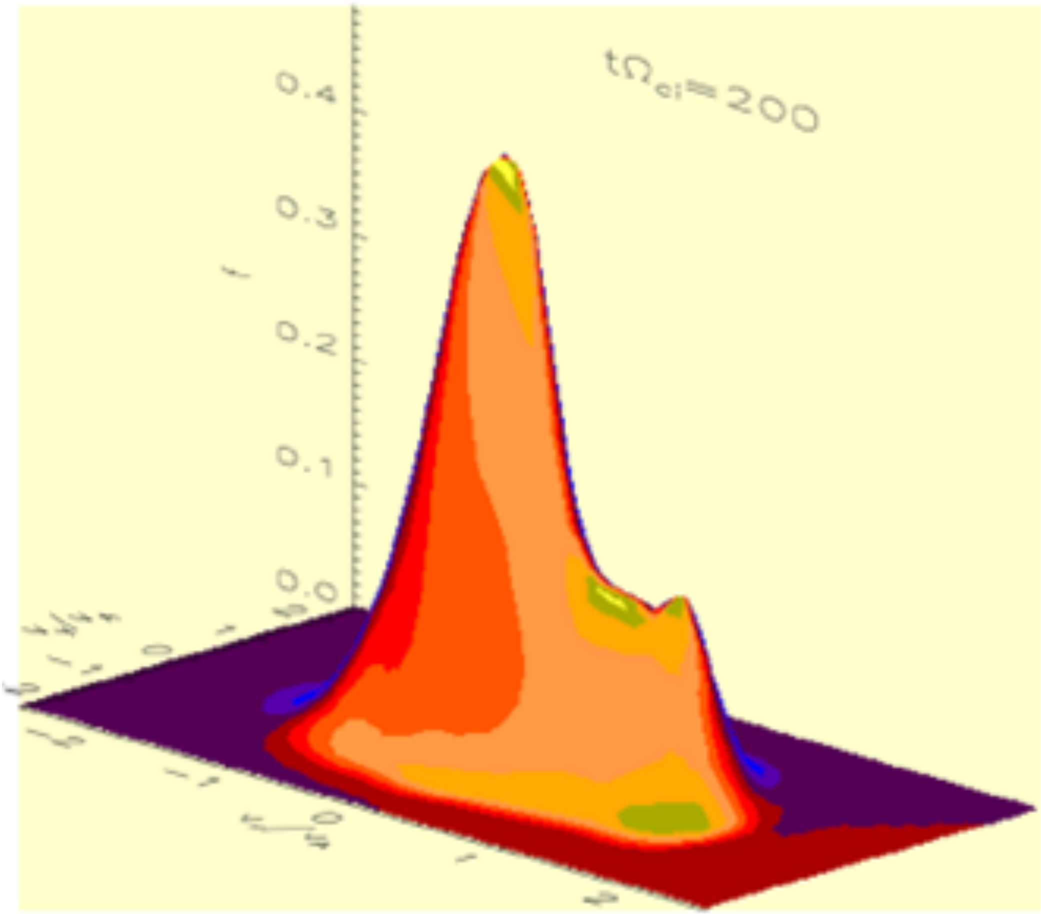
Figure 6. Contour plot of the electron distribution function in the phase space.

Vlasov-Maxwell simulations

Hybrid Vlasov-Maxwell simulations (ions → particle, electrons → fluid) 1d in space 3d in velocity

Trigger by circularly left-hand polarized Alfvèn waves in the perpendicular plane and in parallel propagation

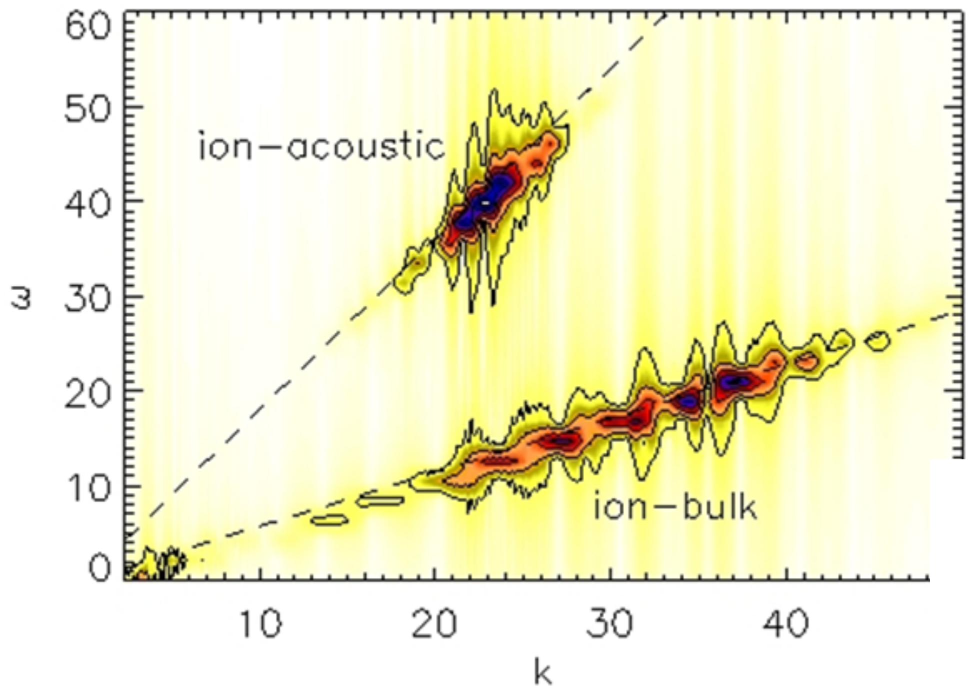
Energy is carried to small scales in longitudinal electrostatic fluctuations of acoustic form



A plateau is generated in the distribution function, due to the trapping of ions (nonlinear saturation of Landau damping)

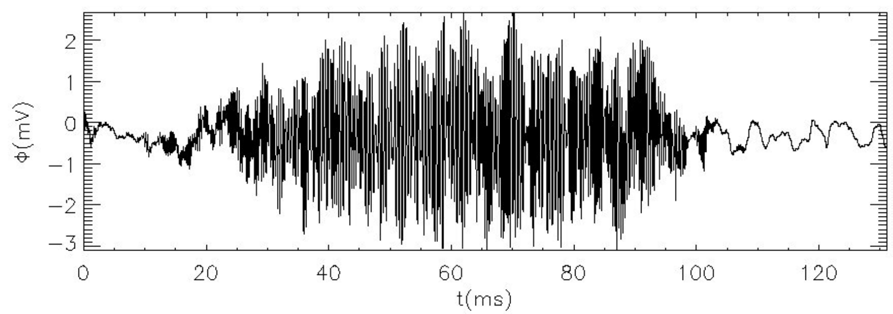
F. Valentini et al., 2008, 2009

Due to trapping, dispersion relation shows two branches of electrostatic waves at small scales

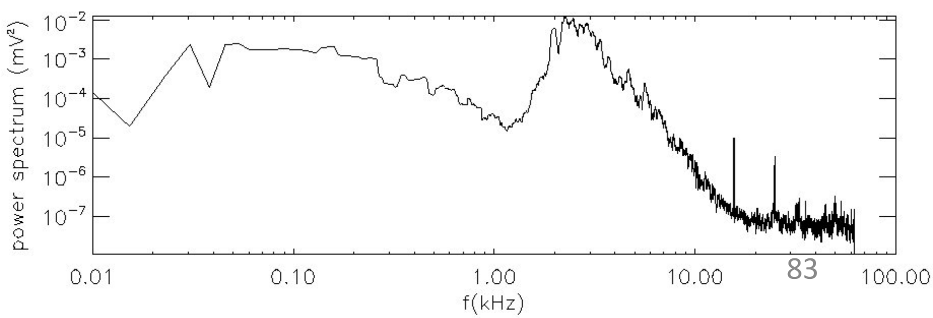


A new branch of waves (ion-bulk) in the numerical dispersion relation

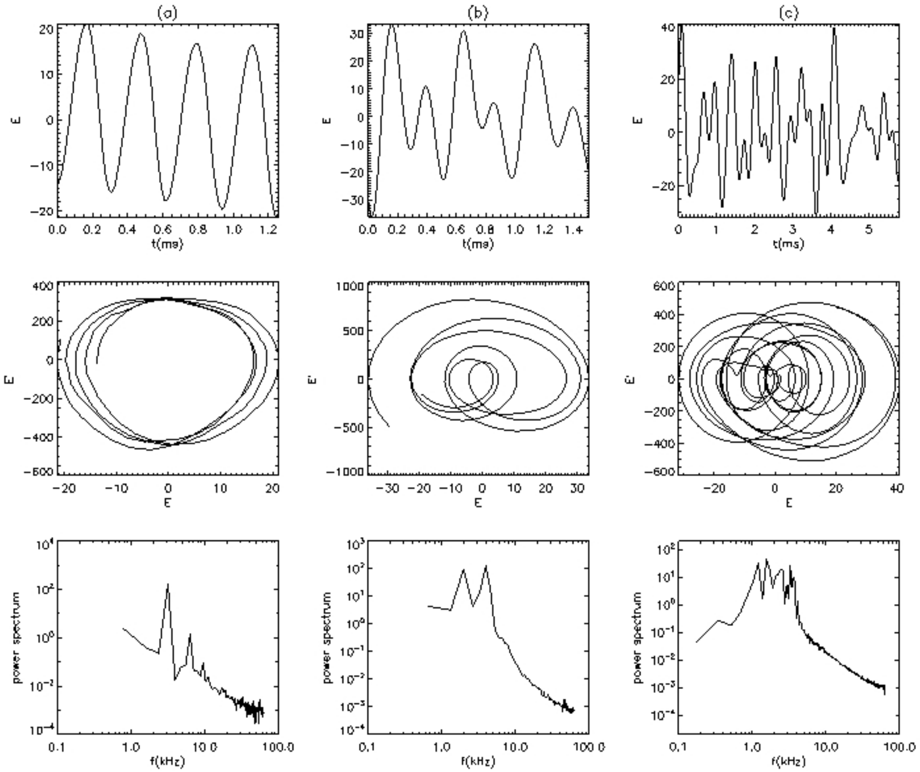
Bursts of electrostatic activity



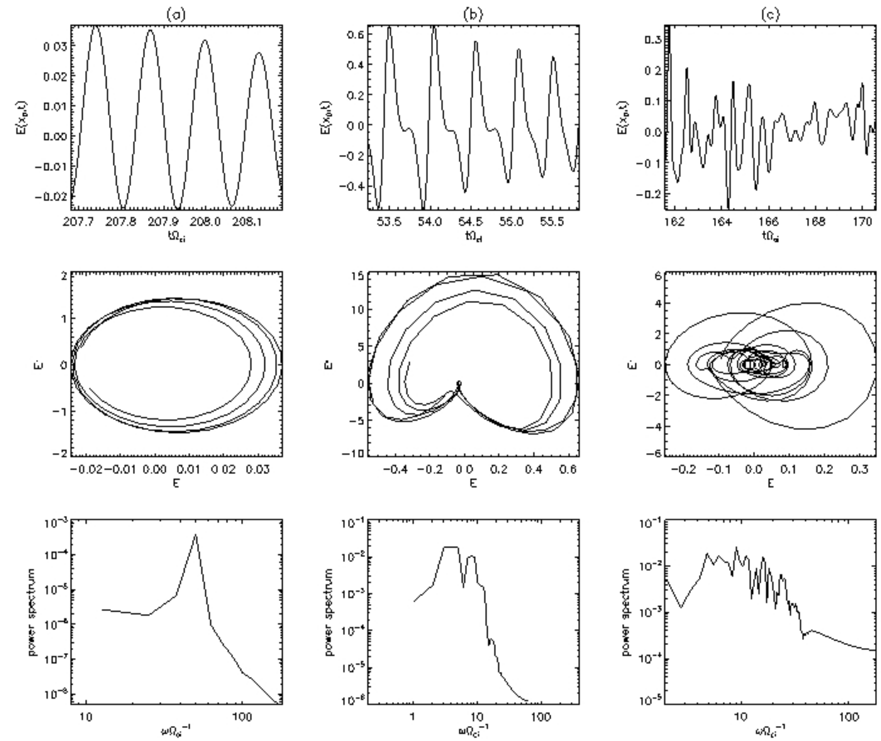
S/WAVES (onboard STEREO sc) → Wavepackets dominated by occasional electrostatic fluctuations observed in the range 0.7 - 4 KHz.



Typical observed waveforms look similar to simulations



Stereo sc observations in solar wind



Vlasov-Maxwell simulations

During the saturation phase of wave-particle interactions a field-aligned beam is generated.

→ coherent beam-wave instability:

Particle In Cell simulations: Heating due to field-aligned proton beams generated by parametric instability of Alfvèn-cyclotron waves

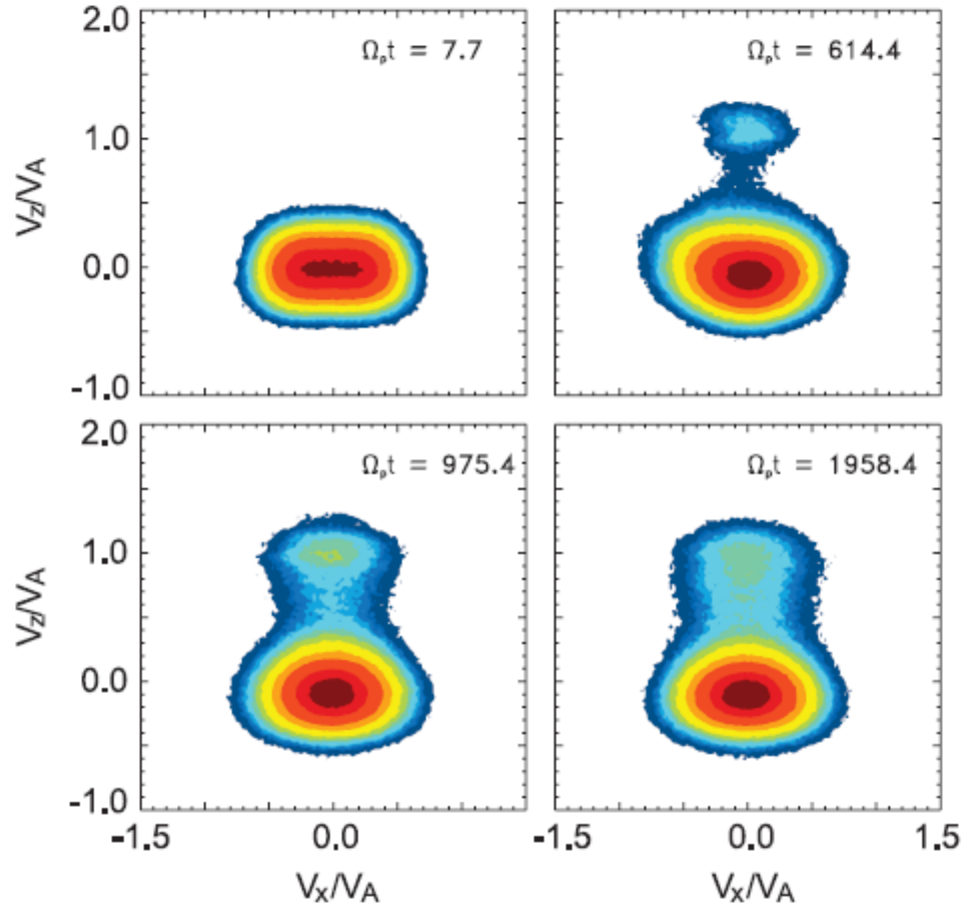


FIG. 5 (color). Contour plots of the proton VDF in the (v_x, v_z) plane for the dispersive-wave case at four instants of time. The color coding of the contour lines corresponds, respectively, to 75 (dark red), 50 (red), 10 (yellow) percent of the maximum, with a final beam density of about 7%.

Araneda et al. PRL (2008)

Del Zanna, Velli, Londrillo, SW10 (2003)

PIC code simulations: Signature for electron heating (mainly in the parallel direction), due to wave-particle interactions detected by elongation of the velocity distribution function

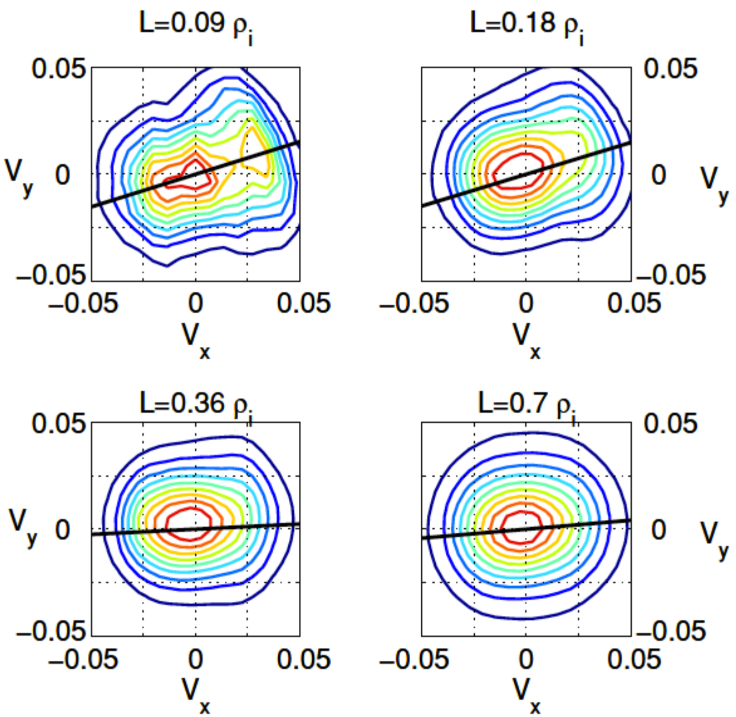
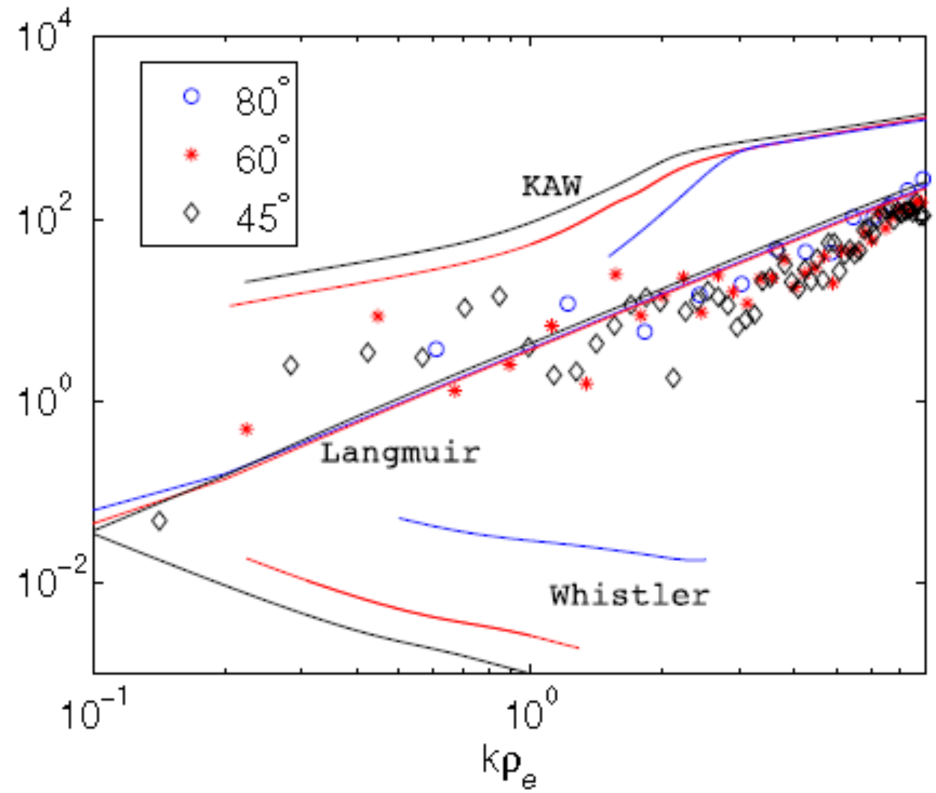


Figure 11. Electron distribution functions in the (x, y)-plane, collected in four nested boxes of increasing size L. The solid line shows the direction of the mean magnetic field within each box. Velocities are normalized to the speed of light.

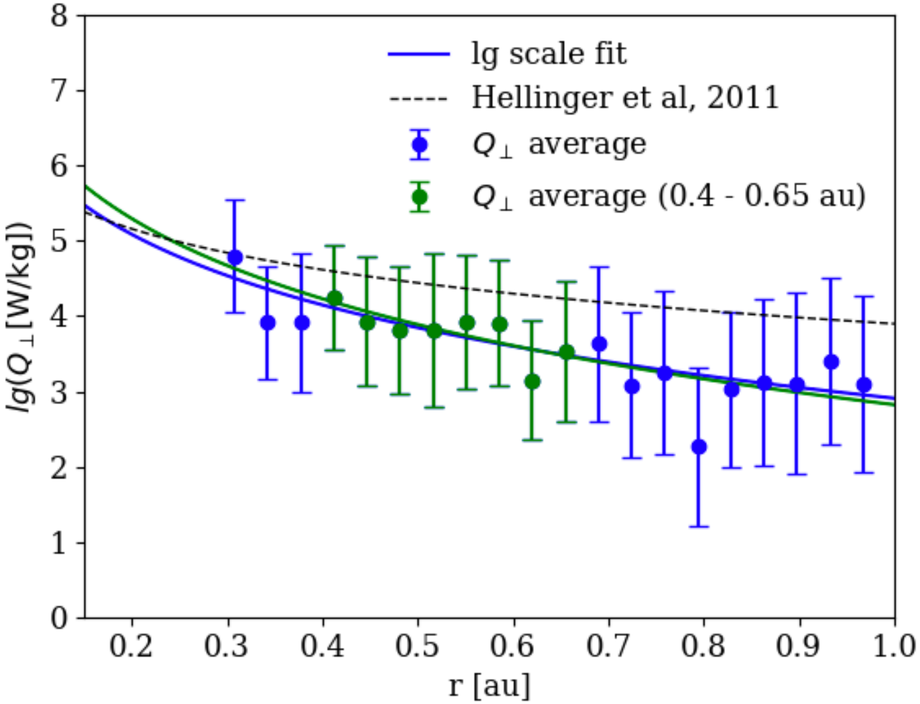


Comparison with linear Vlasov theory show that, although all modes are present, the Langmuir mode seems to be the one whose electron compressibility is close with numerical simulations.

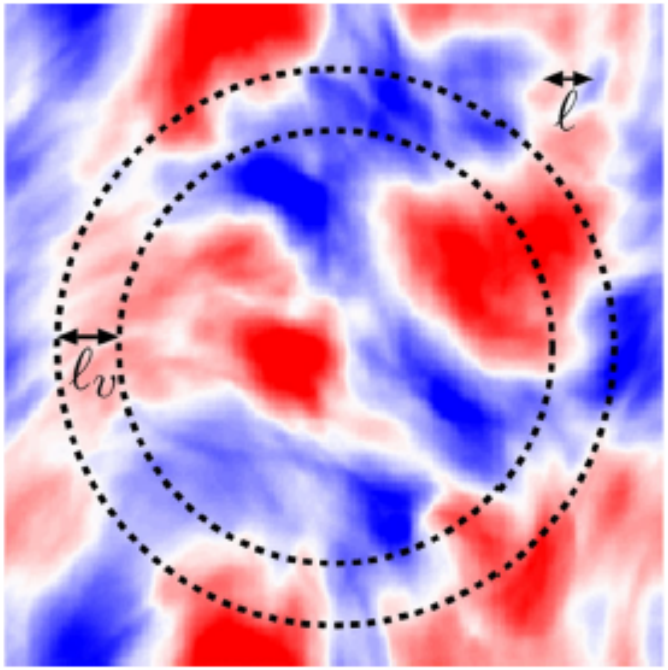
Camporeale & Burgess, ApJ (2010)

Stochastic heating

Non-resonant energy diffusion process due to strong fluctuations at the ion gyro-radius. In a constant background magnetic field particles diffuse in the perpendicular plane thus experiencing stochastic increasing of kinetic energy corresponding to a perpendicular heating. Perhaps enough for solar wind heating.



Gyrokinetic models present stochastic heating as phase mixing of entropy cascade in phase space, when a Fokker-Planck collisional operator is used in numerical simulations.



A different (almost provocative) approach to describe magnetic fluctuations at small scales

1) At small scales a lot of different characteristic plasma micro-scales appear, thus breaking the scale-free behaviour;
2) Kinetic wave-modes exhibit at the same time, and on the same scales, both a dispersive and dissipative character, due to wave-wave and wave-particle interactions;
3) Measurements at relatively large scales, just beyond the ion gyroscale, are interpreted in terms of "turbulent cascade" invoking specific wave-wave interactions, not really theoretically supported.

Let us consider a Ito stochastic equation for magnetic fluctuations with two competing contributions

$$db(t) = -\gamma b(t)dt + F_0 \xi(t)dt \ .$$

a) A linear term due to "dissipation" (wave-particle, stochastic heating,…)
b) A stochastic term due to wave-wave couplings, without bring into question any mode

$$\langle b_\omega b_\omega^\star \rangle = \frac{F_0^2 \langle \xi_\omega \xi_\omega^\star \rangle}{(\gamma - i\omega)(\gamma + i\omega)} ,$$

Spectrum of magnetic fluctuations depends on two-point correlations of stochastic forcing term.

The stochastic term is due to wave-wave couplings, dispersion, etc. Let us conjecture an exponential decay for the two-point correlations with some decay rates distributed according to a probability of occurrence

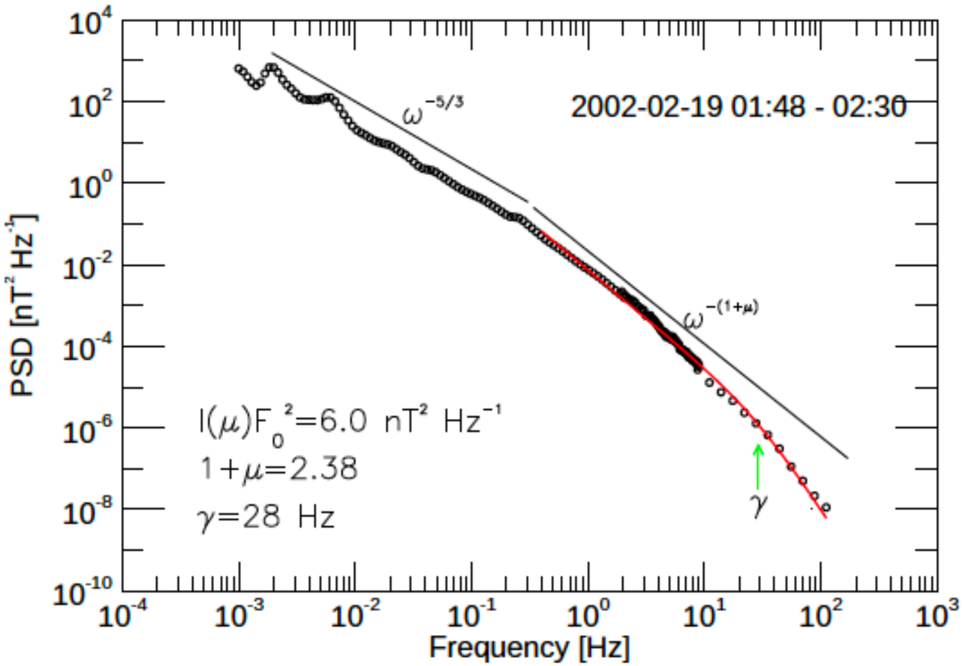
$$\langle \xi(t) \cdot \xi(t') \rangle = \exp[-\lambda(t' - t)]$$

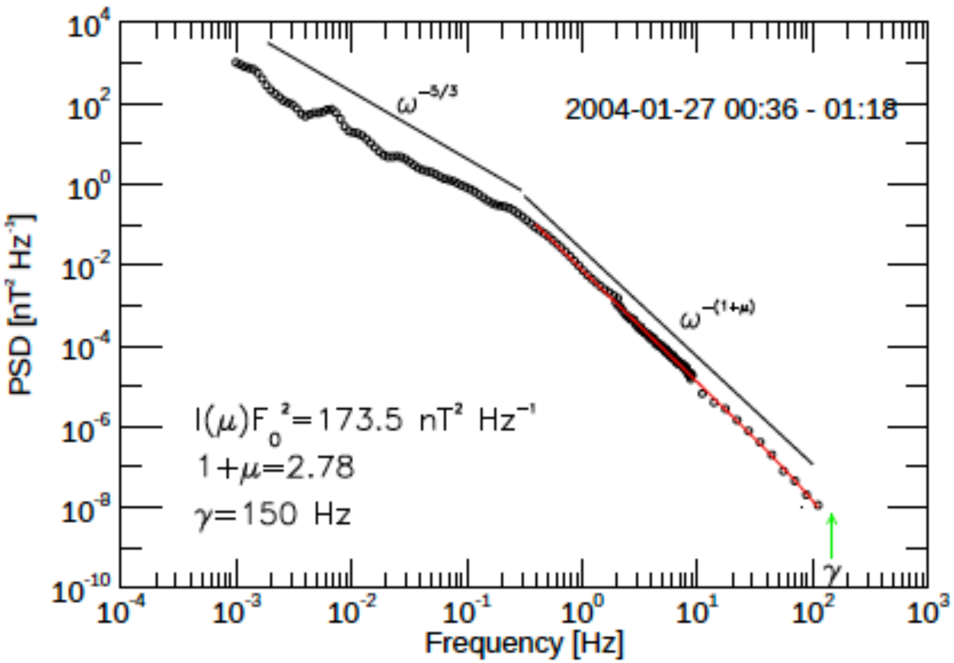
$$dP(\lambda) = (\lambda/\lambda_0)^{-\mu} d\lambda/\lambda_0$$

$$E(\omega) \simeq A(\mu) F_0^2 \omega^{-(1+\mu)} \left(\omega^2 + \gamma^2\right)^{-1} .$$

The power spectrum depends on the scaling exponent and on the dissipation rate. A continuous spectrum, also compatible with a double power law, as often invoked to interpret observations

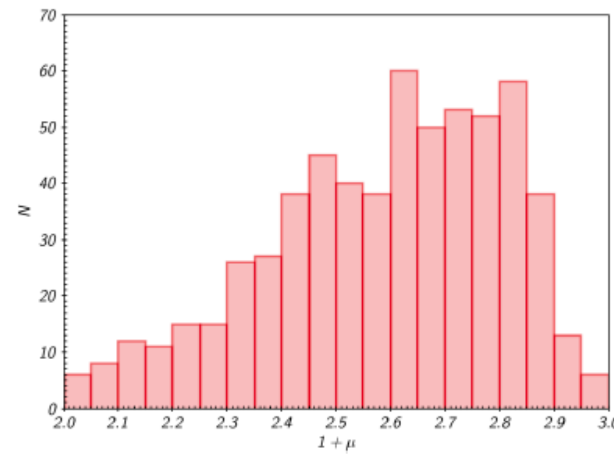
The relation is able to reproduce the observed spectra

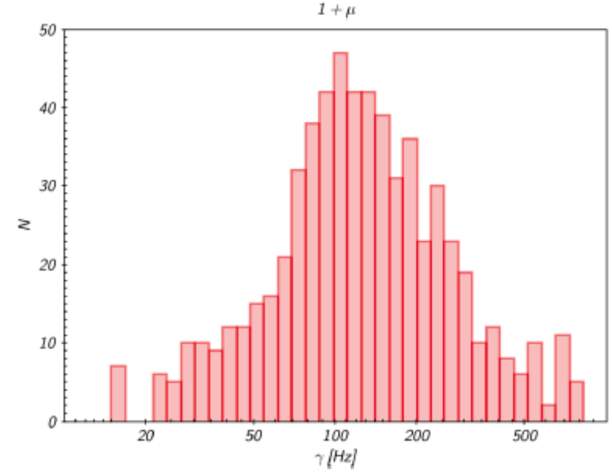




Statistical analysis using Cluster datasets allows us to recover scaling exponents and dissipation rates (often beyond the instrumental range).

$$1 + \mu \simeq 8/3$$





The steeper the scaling exponent, the larger the dissipation frequency.

$$\epsilon(t) = \langle b_j^2/2\mu_0\rangle$$

$$\frac{d\epsilon}{dt} + \gamma\epsilon = F_0\langle b(t)\xi(t)\rangle \; .$$

The long time evolution tends to a non-equilibrium statistically stationary state ρ(A,F,Ω)

Fluctuation-Dissipation Relation relates the average dissipation rate with the statistically stationary state

$$\left(\frac{\gamma}{\lambda_0}\right) \simeq \left[h(\mu)\left(\frac{B_0^2}{2\mu_0}\right)\right]^{1/(\mu-1)} [\rho(A,F,\Omega)]^{-1/(\mu-1)}$$

Let us identify ρ(A,F,Ω) as a non-equilibrium electron "temperature" $k_B T$ (even far from thermodynamic equilibrium neither maximal entropy nor maxwellian distributions are required), and using the typical spectrum of fluctuations

$$1 + \mu = 8/3$$

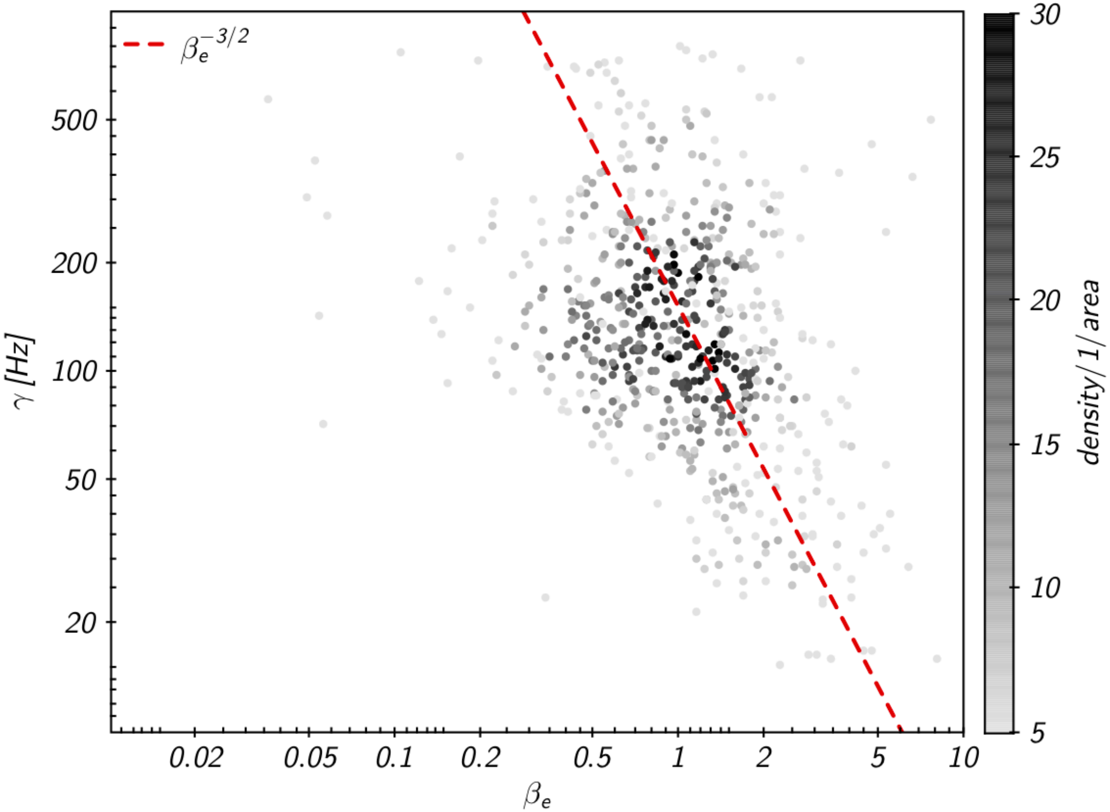
$$\gamma \approx \left(k_B T\right)^{-3/2}$$

The classical scaling of the damping rate of Landau damping

FDR in terms of electron plasma-beta

$$\left(\frac{\gamma}{\lambda_0}\right) \simeq [n_e h(\mu)]^{1/(\mu-1)} \beta_e^{-1/(\mu-1)}$$

A Landau-damping scaling is fully compatible with Cluster data within solar wind.



The spectral properties are not necessarily due to a cascade process, rather they are a consequence of FDR which governes both fluctuations and dissipation. Fluctuations and dissipation adjust themself to an out-of-equilibrium statistically stationary situation.

FDR identifies collisionless Landau damping as the main dissipation mechanism, and is able to open a “window” even on scales which cannot be directly observed, using measurements at ion scale.

Conclusions

1. Although some hypotheses do not exactly work, large-scale fluctuations in solar wind, allows us to think these fluctuations can be (robustly) described in the framework of MHD turbulence;
2. On smaller scales, at about ion gyro-radius or inertial length, linear MHD modes become kinetic, exhibiting **a dispersive and dissipative character** due to various kind of wave-wave and wave-particle interactions.
3. Observations of small scales fluctuations do not indicate so clearly a theoretical framework, apart for the fact that they represent some kind of “plasma fluctuations” which must be dissipated, perhaps through Landau damping. A “pandora’s box” of possible interpretation of the various phenomenon are then possible.