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Dynamical Systems and Machine Learning Approaches to Sun-Earth Relations

Dynamical Complexity in the Earth's Magnetospheric Dynamics

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L'Aquila, February 03/04, 2021

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Contents

Aims

1) To present an overview of past results on Earth's magnetosphere dynamical complexity
2) To underline the necessity to study and model *the Middle Earth* (mesoscopic domain) dynamics
3) To discuss the impact of complexity in forecasting magnetospheric dynamics

Part 1

Lexicon

Lexicon: Complexity

- What do we mean with the term *Complexity* and *Dynamical Complexity* ?
- When one considers a phenomenon or a thing that is *complex*, one generally associates it with something that is *hard to separate, analyze or to solve* (The Merriam-Webster Dictionary, 1997), i.e. as a synonym to *difficult*.
- Instead, we refer to a *complex system* as one whose phenomenological laws that describe the global behavior of the system are not necessarily directly related to the elemental laws that regulate the evolution of its elementary parts.
- Complexity is *the emergence of a non-trivial behavior due to the interactions of the subunits that form the system itself.*
- Complex systems may be described at different level.

Lexicon: Complexity

- What characterizes complex systems is:

  the presence of hierarchical multiscale structures,

  the emergence of long-range correlations,

  the cooperativeness,

  multiscale cross-coupling, symmetry breaking,

  scale-invariance, fractal topologies and criticality,

  universality features.

Lexicon: Complexity

An example of Complexity Emergence: the Wolfram Algorithmic Rules

Let us consider 3 simple algorithmic boolean rules [Wolfram, 2002] (here $f = 0,\ 1$ and $\bar{f}$ is the not operation):

- $\#\ 250\ f_{(i,j)} = f_{(i+1,j-1)} + \bar{f}_{(i+1,j-1)} f_{(i-1,j-1)}$
- $\#\ 90\ f_{(i,j)} = \bar{f}_{(i-1,j-1)} f_{(i+1,j-1)} + f_{(i-1,j-1)} \bar{f}_{(i+1,j-1)}$
- $\#\ 30\ f_{(i,j)} = \bar{f}_{(i-1,j-1)} f_{(i,j-1)} + \bar{f}_{(i,j-1)} (\bar{f}_{(i-1,j-1)} f_{(i+1,j-1)} + f_{(i-1,j-1)} \bar{f}_{(i+1,j-1)})$

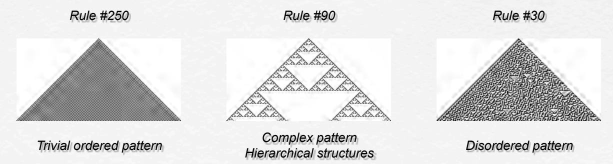


Figure 1: The three patterns of Rules # 250, # 90 and # 30.

Lexicon: Scale-Invariance

- What do we mean with the term *Scale-Invariance* ?
- Scale-invariance is special kind of symmetry feature, that can be observed in systems characterized by a huge range of scales (see e.g., fluctuations in proximity of a second-order phase transition) [Stauffer & Stanley, 1996; Sornette, 2000, Lesne & Laguës, 2012].
- Scale-invariance is the appearance of spatial and/or temporal structures without a characteristic length or time scale and refers to invariance over changes of scales, i.e., the system is reproducing itself under a coarse-graining transformation.
- Existence of a symmetry of scale invariance means a lack of a characteristic scale for the system.
- The concept of scale invariance can be expressed in mathematical terms by the simple following rationale.

Lexicon: Scale-Invariance

- Let us consider an observable quantity $O(x)$, which depends on a *control parameter* x.
- The quantity $O(x)$ is scale invariant if under an arbitrary change of the control parameter $x \rightarrow \lambda x$ there is a number $\mu(\lambda)$ such that

$$x \rightarrow \lambda x,\ O(x) = \mu O(\lambda x); \quad \mu(\lambda) = \lambda^{\alpha}$$

- The last equation is a first order homogeneous equation, defining homogeneous functions, and is generally encountered in the theory of critical phenomena, etc.
- The most simple solution of a first order homogeneous equation is a power law.

$$O(x) = cx^{\alpha} \text{ with } \alpha = -\frac{\ln \mu}{\ln \lambda}$$

- Thus, power-laws are the signature of a scale invariance property.
- Sometimes the scale invariance is applied in a statistical sense.

Lexicon: Scale-Invariance

- In this case, the invariance is related to changes with scale of the shape of PDF, i.e. to a sort of invariant function [Stauffer & Stanley, 1996; Hnat et al., 2002a, 2002b, 2003].
- For instance, consider the following very simple random walk:

$$x(n) = x(n-1) + \eta(n), \text{ where } \langle \eta(n)\eta(n') \rangle = \sigma^2 \delta(n-n')$$

- The statistics of the $x(n)$ for a large number of replica (ensemble of iterations) follows a scale invariant property (Quinconce de Galton).

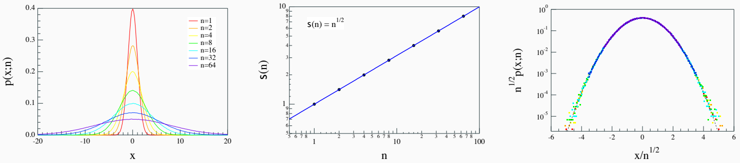


Figure 2: The statistics of $x(n)$ at different time step n (left panel). The scaling of the standard deviation $s(n)$ with time step n (central panel). The rescaled statistics and the corresponding *data collapsing* (right panel).

$$\Pi(x;n) = \frac{1}{\sqrt{2\pi n}} \exp\left[-\frac{x^2}{2n}\right], \ x \to x/\sqrt{n} \text{ then } \frac{1}{n^{-1/2}}\Pi(x;n) = \Pi\left(\frac{x}{\sqrt{n}};1\right) = \Xi(\tilde{x})$$

Lexicon: Fractals

- Several natural complex objects and systems show irregular features at all scales of observation, that make them different from classical Euclidean objects.
- The characterization of this natural complexity required the introduction of novel concepts.
- In 1982 Mandelbrot introduced the concepts of *fractal* object and *fractal dimension*.
- According to Mandelbrot, *a fractal (or set) is a rough or fragmented geometrical shape that can be subdivided into parts, each of which is a reduced-size copy of the whole* [in D. Sornette, 2000].
- Mathematically, a self-similar set $\mathcal{A}$ is said to be a fractal if its *Hausdorff dimension* D_0 exceeds the *ordinary topological dimension* d of its constituents and is less than the *embedding dimension* E

Lexicon: Fractals

- The quantity D_0 is named *fractal dimension* (also *capacity or self-similarity dimension*), and for simple self-similar geometrical objects is generally *a real number*.
- This generalization of the concept of dimension from integers to real numbers reflects the emergence of *continuous scale invariance* [Sornette, 2000].
- One of the most striking feature of a fractal set or objects is its *non-analytic* (singular) character at all scales.
- In order to clarify the above concepts, let us consider a simple example of fractal: the *Koch curve*.

Lexicon: Fractals

The Koch curve

- A simple fractal object is the famous *Koch snowflake*.
- Let's start with an equilateral triangle with sides of unit length. Then, we divide each side in 3 equal parts and add a small equilateral triangle of side 1/3 in the middle part. By iterating this process we get the *Koch snowflake*.

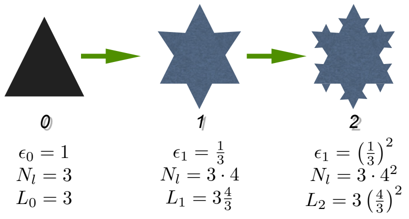
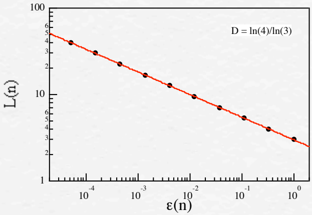


Figure 3: The Koch snowflake construction procedure.

$$\epsilon_n = \left(\frac{1}{3}\right)^n,\ N_l = 3 \cdot 4^n \text{ and } L_n = 3\left(\frac{4}{3}\right)^n$$

The Koch curve

- The length L diverges for increasing n, and can be expressed in as an inverse power-law of the resolution $\epsilon(n)$.
- This allows to evaluate its fractal dimension $D = 1.26186\ldots$



$$L_n \sim \epsilon_n^{1-D} \longrightarrow D = \frac{\ln 4}{\ln 3}$$

Figure 4: The Koch snowflake length $L(n)$ and its dimension D.

Lexicon: Geomagnetic Indices

- The Earth's magnetospheric dynamics can be investigated from different point of view; local dynamics and global evolution.
- While the analysis of the local dynamics is based on measurements at a local level via the observation of some plasma and magnetic field quantities, the global evolution of the Earth's magnetospheric dynamics is generally performed by means of some geomagnetic indices, which are proxies of dissipative processes.
- Among the various geomagnetic indices the most reliable and widely use in analysing the Earth's magnetospheric dynamics are *i)* the *Auroral Electrojet (AE) indices* and *ii)* the *Disturbance Storm Time indices (Dst)* and its variations (SYM-H, Asy-H, ...)

Lexicon: Geomagnetic Indices

- The Auroral Electrojet (AE) indices, introduced by Davis and Sugiura (1966), are a set of four geomagnetic indices (AE, AU, AL, AO) characterizing global auroral electrojet currents and computed from the horizontal component (H) of the magnetic field measured on the ground in the Northern auroral regions (geomagnetic latitudes $60^\circ - 70^\circ$).
- Among these four indices the AE-index (defined as $AE = AU - AL$ where AU and AL are the upper and lower envelopes of the ground based measurements of the H component) is representative of the energy dissipation rate in the auroral regions [Ahn et al., 1983].

$$E_i = \int_{\Omega_i} \delta AE(t)dt, \text{ where } \delta AE(t) = AE(t) - AE_{ref}$$
$$\Rightarrow \; E_i(J) = 1.38^{10} E_i(nT \cdot min)$$

Lexicon: Geomagnetic Indices

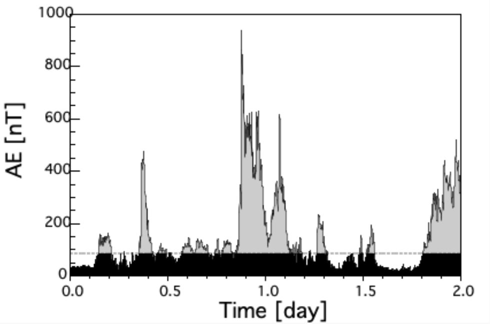


Figure 5: A sample of AE-index. The horizontal dashed line refers to the average value during quiet period (AE_{ref}).

Lexicon: Geomagnetic Indices

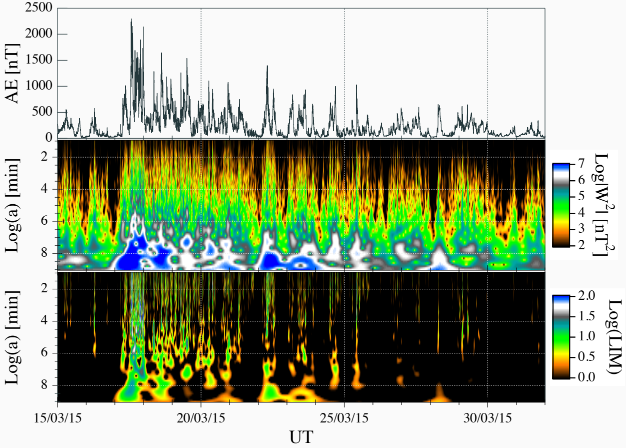


Figure 6: AE-index coherent bursts identified via Local Intermittency Measure (LIM).

Lexicon: Geomagnetic Indices

- The *Disturbance Storm Time (Dst)* index and its variants ($SYM - H$) aim to monitor variations of the equatorial magnetospheric ring current.
- It is a 1 hr (1 min) time resolution index derived from the horizontal component H of the geomagnetic field, as measured at ground observatories distant from the auroral and equatorial electrojets and approximately equally distributed in longitude.
- It was originally introduced by Sugiura (1969) and is produced by the World Data Center (WDC) for Geomagnetism, Kyoto, and is available from 1957 up until present (wdc.kugi.kyotou.ac.jp/dstdir/index.html).
- SYM-H is the Dst high resolution version (1 min) and represents the symmetric part of the equatorial disturbance as measure by a larger set of geomagnetic observatories.
- SYM-H index [Iyemori, 1990; Wanliss and Showalter, 2006] provides estimate of the longitudinally symmetric part of the disturbance field.

Lexicon: Geomagnetic Indices

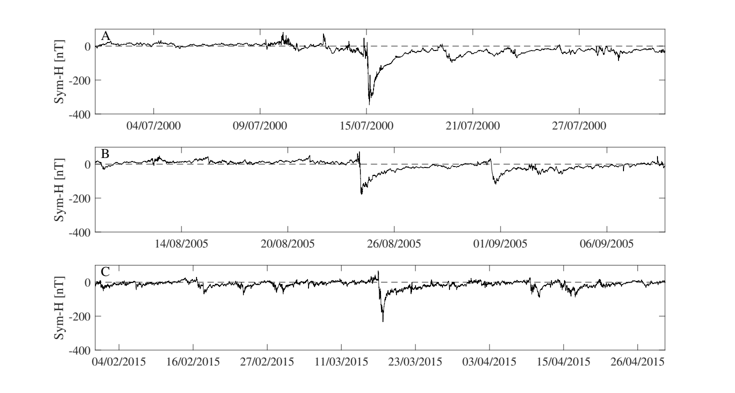


Figure 7: Behavior of SYM-H index for three different periods (from Alberti et al., J. Space Weather Space Clim., 8, A56, 2018).

The Earth's magnetospheric dynamics: A general view

The Earth's magnetospheric dynamics: A general view

- The Earth's magnetosphere is a structured dynamical system in a nonequilibrium configuration, continuously interacting with the interplanetary medium and the Earth's ionosphere-atmosphere system.
- In particular, as clearly shown by its comet-shape the Earth's magnetosphere can indeed be considered a far-from-equilibrium extended dissipative system, continuously driven by solar wind.
- Evidences of non-equilibrium are:

    non-symmetric shape (comet shaped);
    a complex system of continuously flowing currents (dissipative structures);
    the occurrence of fast energy releases.

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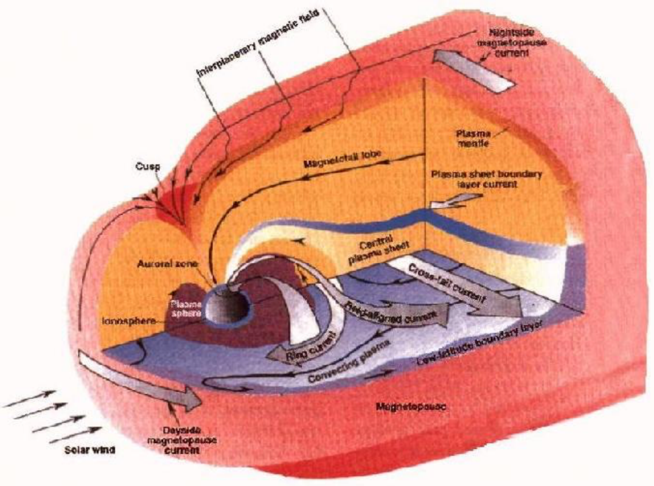

Figure 8:
A schematic view of the Earth's magnetosphere. (Credit NASA-ISTP web site `http://www-istp.gsfs.nasa.gov`)

The Earth's magnetospheric dynamics: A general view

- In the early years, magnetospheric studies primarily dealt with large scale phenomena and processes, achieving significant results.

The Earth's magnetospheric dynamics: A general view

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- Since the early 90s it was realized that the dynamics of the Earth's magnetosphere in response to solar wind changes is characterized by nonlinearity and chaotic features [Vassiliadis et al., 1990].

The Earth's magnetospheric dynamics: A general view

- In the early years, magnetospheric studies primarily dealt with large scale phenomena and processes, achieving significant results.
- Since the early 90s it was realized that the dynamics of the Earth's magnetosphere in response to solar wind changes is characterized by nonlinearity and chaotic features [Vassiliadis et al., 1990] .
- One of the first evidences of the nonlinear character of the Earth's magnetosphere was the seminal paper by *Tsurutani et al.* (1990).

The Earth's magnetospheric dynamics: A general view

GEOPHYSICAL RESEARCH LETTERS, VOL. 17, NO. 3, PAGES 279-282, MARCH 1990

THE NONLINEAR RESPONSE OF AE TO THE IMF B_S DRIVER: A SPECTRAL BREAK AT 5 HOURS

Bruce T. Tsurutani[1], Masahisa Sugiura[2,3], Toshihiko Iyemori[3], Bruce E. Goldstein[1],
Walter D. Gonzalez[4], Syun I. Akasofu[5] and Edward J. Smith[1]

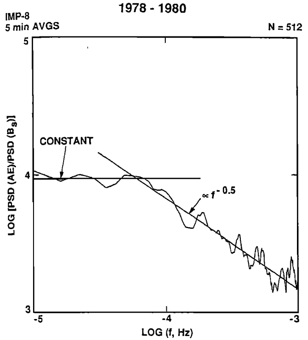


Fig. 5. The ratio of the power spectrum of AE to the power spectrum of the IMF B_S for 1978-1980.

The Earth's magnetospheric dynamics: A general view

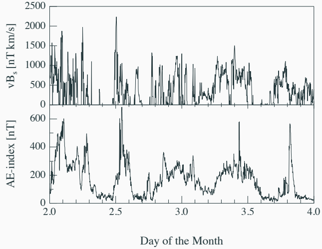


Fig. 5 The behavior of vB_s (upper panel) in comparison with the behavior of the AE index (lower panel) for a period of two days from December 02, 1994 to December 04, 1994. Notice the nonlinear response of the AE index to the solar wind driving.

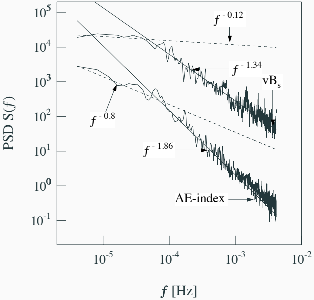


Fig. 6 Comparison between the spectra of vB_s and AE relative to the first 12 days of December, 1994. Solid and dashed lines are power-law best fits.

Figure 9: A view of the nonlinear response and not one-to-one coincidence of $vB_s - AE$ activity bursts (Consolini, Fractals, 2002).

The Earth's magnetospheric dynamics: A general view

- In the early years, magnetospheric studies primarily dealt with large scale phenomena and processes, achieving significant results.
- Since the early 90s it was realized that the dynamics of the Earth's magnetosphere in response to solar wind changes is characterized by nonlinearity and chaotic features [Vassiliadis et al., 1990
- One of the first evidences for the nonlinear character of the Earth's magnetosphere was the seminal paper by *Tsurutani et al.* (1990).
- This work opened a new view of the magnetospheric dynamics, providing the starting point to study the emergence of chaos and/or complexity.

The Earth's magnetospheric dynamics: A general view

- Anyway, the comprehension of the highly structured and dynamical features of magnetospheric processes during magnetic substorms and storms seems to require a different approach and novel concepts.
- This observation motivated a sequence of studies focused on the possible occurrence of low-dimensional chaos [Baker et al., 1990; Sharma, 1995; Klimas et al., 1996].

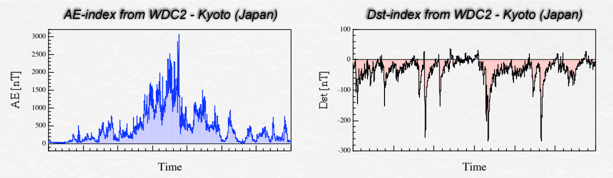


Figure 10: A view of AE and Dst geomagnetic indices.

The Earth's magnetospheric dynamics: A general view

- However, later it was realized that the Earth's magnetospheric dynamics dynamics cannot be discussed in terms of *autonomous systems* but more reasonably using analogue models displaying a *certain degree of organization*, i.e., in terms of driven nonlinear dynamical system rather than autonomous system [Klimas et al., 1996].

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 101, NO. A6, PAGES 13,089–13,113, JUNE 1, 1996

The organized nonlinear dynamics of the magnetosphere

A. J. Klimas,[1] D. Vassiliadis,[2] D. N. Baker,[3] and D. A. Roberts[1]

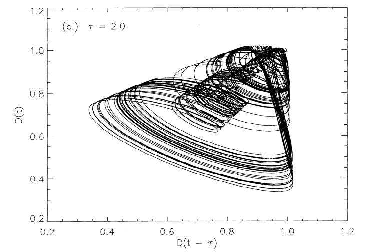


Figure 11: A view of Dst attractor reconstruction from Klimas et al. (1996).

The Earth's magnetospheric dynamics: A general view

- The main problem deals with the correct characterization of the fractal/multifractal character of the magnetospheric dynamics as monitored by geomagnetic indices (AE, Dst, SYM-H, etc).

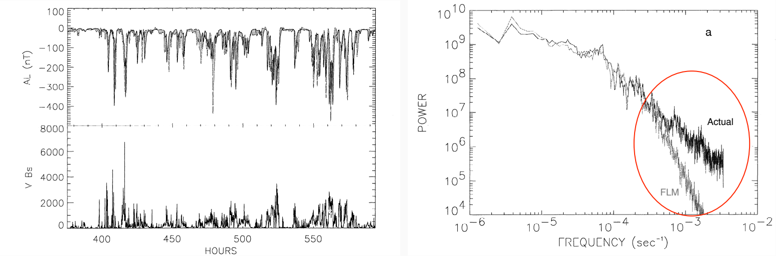


Figure 12: Comparison between measured AL and FLM reconstruction (Klimas et al., 1992, 1994) and the corresponding PSDs [from Klimas et al., 1996].

The Earth's magnetospheric dynamics: A general view

- Indeed, it was recognized that the nonlinear features of geomagnetic indices cannot be ascribed to a finite-dimensional dynamics [Prichard & Price, 1992; 1996] but conversely resemble more a stochastic signal than a chaotic one [Takalo et al., 1993, 1994; Takalo & Timonen, 1994].
- For instance, AE-index fluctuations shows large departures from the Gaussian statistics at several time-scales, along with a multifractal character [Consolini et al., 1996].

The Earth's magnetospheric dynamics: A general view

VOLUME 76, NUMBER 21 PHYSICAL REVIEW LETTERS 20 MAY 1996

Multifractal Structure of Auroral Electrojet Index Data

Giuseppe Consolini,* Maria F. Marcucci, and Maurizio Candidi

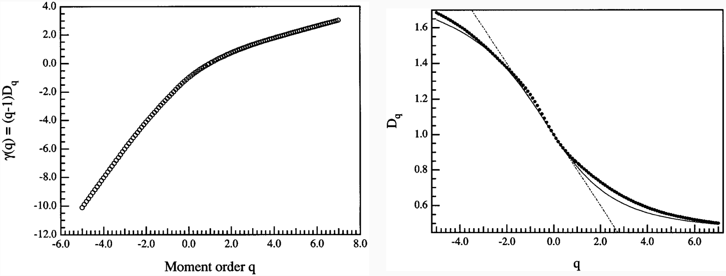


Figure 13: Left: Partition function scaling exponents, $\gamma(q)$; Right: The Renyi dimensions, D_q [from Consolini et al., 1996].

The Earth's magnetospheric dynamics: A general view

- The multifractal character of geomagnetic indices (AE) provides the evidence of a punctuated dynamics characterized by a many degree-multiscale process; i.e., the one of the first evidences of a complex dynamics characterized by many degree of freedom, as in turbulent systems.
- Indeed, one of the most striking feature of the magnetospheric dynamics is the *punctuated character*, i.e., the occurrence of large energy release sparsely distributed in time (period of stasis punctuated by crisis).

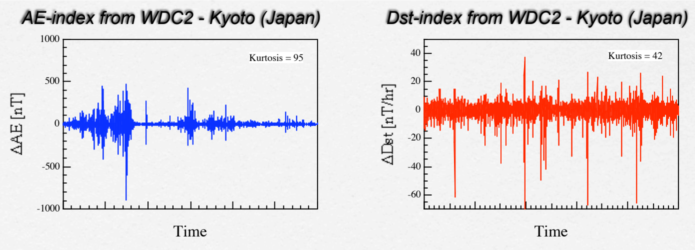


Figure 14: The AE-index and Dst-index small scale increments ($\Delta X(\tau) = X(t+\tau) - X(t)$).

Part 2

Complexity and Criticality

- Starting from different considerations, Chang [1992] suggested that magnetospheric dynamics may be that of an infinite-dimensional nonlinear system near criticality.
- This hypothesis was corroborated by successive analysis of the nonlinear character of the AE-index in terms of dissipation burst events and changes in the fractal (self-similarity) features during substorms [Consolini, 1997, 2002; Uritsky and Pudovkin, 1998].

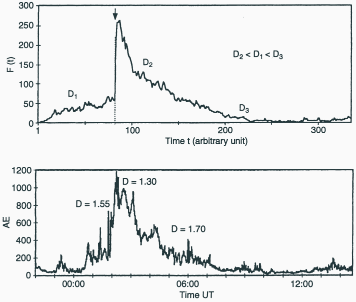


Fig. 6. The result of investigation of the model reaction to the single reversible decrease of Z_c using a 50×50 array. Time of Z_c change is marked by *arrow*

Fig. 7. Example of temporary evolution AE index during a real magnetospheric substorm. *Values* of fractal dimension D of AE trace corresponding to three stages of the substorm development are shown

Figure 15: From Uritsky and Pudovkin, Ann. Geophys., 1998.

Evidence of scale-invariant dissipation processes during substorms

$$s = \int_{\Omega} [AE(t) - L_{AE}(t)] dt$$

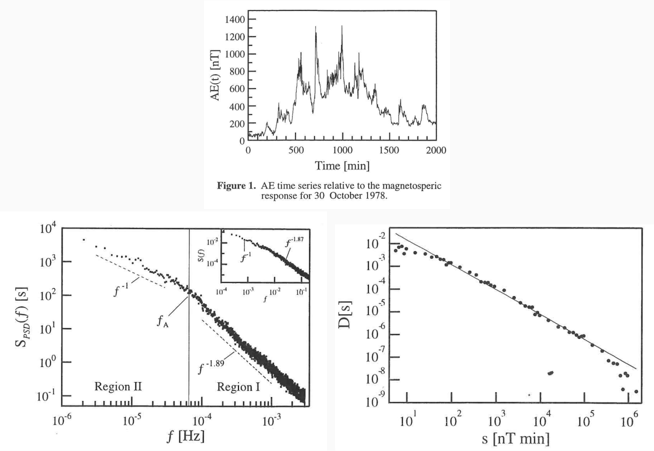
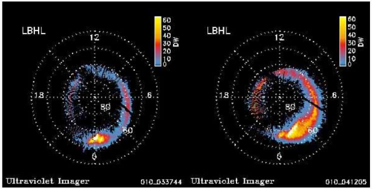


Figure 16: From Consolini, 1997.

Complexity and Criticality

- The evidence of scale invariance in the auroral indices led to the idea that the magnetosphere might behave as a *self-organized critical system* (SOC).
- Other evidences of scale-invariant energy dissipation, supporting the original proposal of Chang (1992), were:
    - Angelopoulos et al. (1999) - scale invariant PdF of BBF durations -
    - Lui et al. (2000), - spatial energy dissipation and size distribution of auroral luminosity -
    - Uritsky et al. (2002), - space-time features of auroral blobs -
- These features were interpreted as signatures of a dynamics similar to that of an out-of-equilibrium stochastic system near *forced and/or self-organized criticality* - FSOC [Chang, 1998, 1999; Chapman et al., 1998; Uritsky & Pudovkin, 1998; Takalo et al., 1999, 2001; Klimas et al., 2000; Consolini & De Michelis, 2001].

Complexity and Criticality



JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 107, NO. A12, 1426, doi:10.1029/2001JA000281, 2002

Scale-free statistics of spatiotemporal auroral emissions as depicted by POLAR UVI images: Dynamic magnetosphere is an avalanching system

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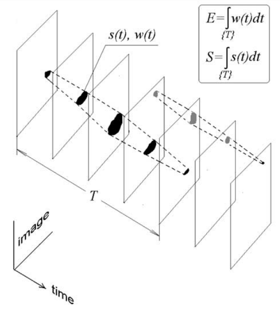
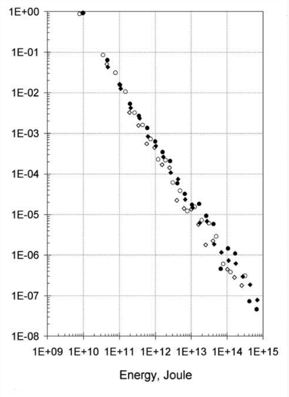
Received 29 August 2001; revised 21 March 2002; accepted 9 August 2002; published 5 December 2002.





Complexity and Criticality

- In spite of the large evidence of a scale-invariant dynamics in substorms features, Freeman et al. (2000) conjectured that the observed scale invariance could be due to similar features of the solar-wind coupling functions (vB_s and ϵ).
- However, different dynamical scaling features exist for time scales shorter than 3.5 hr [Uritsky et al., 2001].

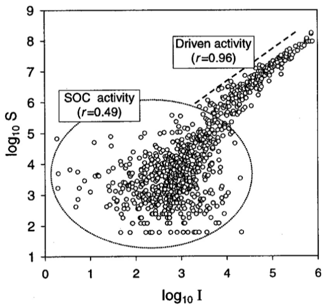


Figure 17: From Uritsky et al., 2001.

Complexity and Criticality

- This is a clear evidence of the double nature of the magnetospheric dynamics in response to solar wind changes:
  - scale-invariance related to internal processes due to loading-unloading
  - directly driven features (convection) on longer timescales.

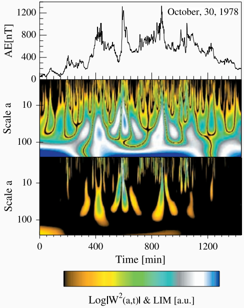


Figure 2. LIM analysis of AE index data for 30 October 1978. (top) Actual AE index. (middle) Scalogram $|W(a, t)|^2$. (bottom) Part of the scalogram satisfying $LIM_{a,t_0} > 1$. Colors in middle and bottom panels report the scalogram $|W(a,t)|^2$, and LIM intensity in arbitrary units (see color bar).

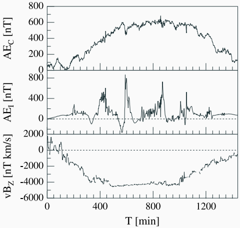


Figure 4. Comparison among the two components (AE_I and AE_C), and vB_z, evaluated using ISEE3 data.

Figure 18: From Consolini & De Michelis, 2005.

- On the other hand from a different approach, Sharma et al. (2000) and Sitnov et al. (2001), applying the singular spectrum analysis to a set of correlated data showed that:

  > the global low-dimensional manifestations of substorm dynamics and the observed scale invariance

  are well in agreement with the behavior of conventional first- and/or second-order phase transitions [Stanley, 1971].

SITNOV, SHARMA, PAPADOPOULOS, AND VASSILIADIS

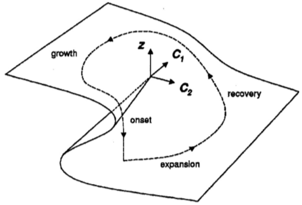


FIG. 6. Hypothetical cusp catastrophe manifold that was expected to approximate the substorm dynamics of the magnetosphere according to the model [62]. The evolution of an isolated substorm is shown by dashed arrows.

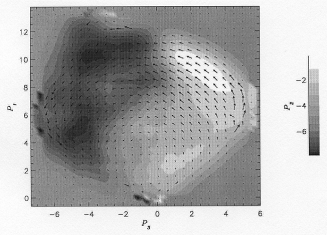


Figure 19: From Sitnov et al., 2001.

- To try to separate the scale-invariant character from the global low-dimensional behavior (perhaps directly driven by SW) in AE-index, we applied a moving window technique (local low-pass filter, $\tau = 120$ min).

$$\delta AE(t) = AE(t) - AE_B(t) \;\Rightarrow\; \Omega_i = \{t \in [t', t''] \mid \delta AE(t) \geq 0\}$$

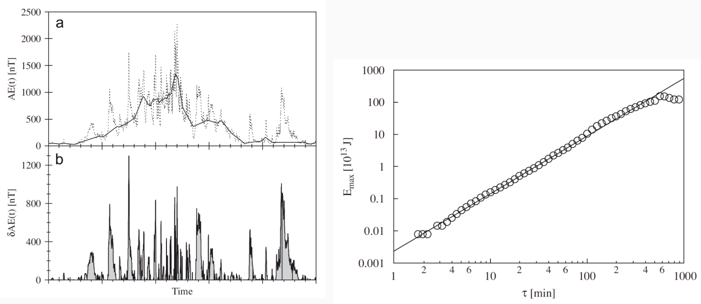


Figure 20: From Consolini and Kretzschmar, PSS, 2007.

$$E_{max}(\tau) \sim \tau^{\nu}$$

Complexity and Criticality

- Emergence of a characteristic scale for typical size of energy relaxation (~ 1 R_E)

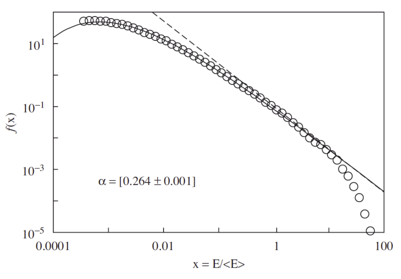


Figure 21: From Consolini and Kretzschmar, PSS, 2007.

$$f(x) = \left(\frac{\sigma^{\alpha}}{x}\right)^{1/(1-\alpha)} \exp\left[-\frac{1-\alpha}{\alpha}\left(\frac{\sigma}{x}\right)^{\alpha/(1-\alpha)}\right]$$

Complexity and Criticality

- Most of the studies focused on the energy/event size statistics limiting the discussion of the waiting times to a comparison between a power-law behavior and the Poisson statistics expected in the case of SOC systems.
- For instance, the observed waiting times statistics of AE index activity bursts follows a power-law plus some finite-size effects.

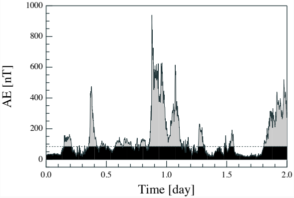


Fig. 2. A sample of a 2-day interval of AE-index records. The horizontal dashed line refers to the threshold used to discriminate quiet and active periods (grey in the figure).

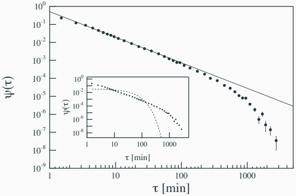


Fig. 3. The waiting times distribution function $\psi(\tau)$. The solid line refers to a power-law nonlinear best fit. The inset shows a comparison between the observed distribution and the expected Poissonian distribution characterized by the same average waiting time.

Figure 22: From Consolini & De Michelis, NPG, 9, 412 (2002).

Complexity and Criticality

- A different point of view to explain the observed waiting time statistics could be due to a complex energy landscape (phase space) for the nonequilibrium states (metastability & topological randomness).

$$f(\delta E) \simeq \exp\left[-\left(\frac{\delta E}{\delta E_0}\right)^{\alpha}\right] \;\rightarrow\; \tau \sim \tau_0 \exp\left(\beta \delta E\right) \;\rightarrow\; \psi(\tau) \simeq \frac{1}{\tau} \exp\left[-A \ln^{\alpha} \tau\right]$$

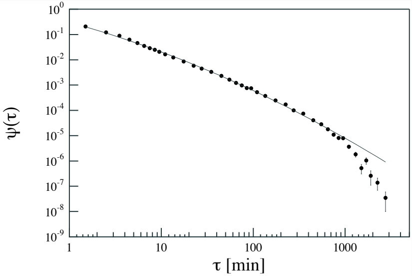


Fig. 4. The comparison between the observed waiting times distribution function $\psi(\tau)$ and the expression of Eq. (5). The solid line refers to a nonlinear best fit.

Figure 23: From Consolini & De Michelis, NPG, 9, 412 (2002).

- The analysis returned a value of $\alpha \sim 2$, suggesting that the distribution of the energy barriers δE follows a semi-Gaussian statistics.
- In the previous scenario β plays a role of an *inverse temperature* where can be related to the amplitude (variance) of the local internal fluctuation field.
- In practice, the fast energy releases (avalanche dynamics) related to AE index bursts could be the result of a walk in a complex free energy landscape with Gaussian statistics for the local minima.

Complexity and Criticality

- Thus, “neither SOC nor self-organization models taken separately can explain the whole variety of the magnetospheric activity on substorm scales” [Sitnov et al, 2001].
- Furthermore, “the behavior of the Earth’s magnetosphere resembles very close that of real sandpiles. Both systems reveal scale-invariant behavior for relatively small avalanches and first-order phase transition-like behavior for the largest avalanches” [Sitnov et al, 2001].
- The complexity of the magnetospheric dynamics during substorms is the results of competing processes such as fluctuating forces and dissipative ones.

Complexity and Criticality

A view of the concepts dealing with the Earth's magnetospheric dynamical complexity

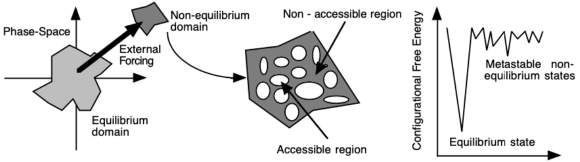


Figure 4. A schematic view of the phase space, dealing with the topological complexity, and of the self-similarity of the accessible states which reflects onto the scale-free behavior of the magnetospheric dynamics.

Figure 24: From Consolini & Chang, SSR, 9, 412 (2001).

Complexity and Forecasting

Complexity and Forecasting

- One of the main targets of Space Weather is the capability to predict/forecast geomagnetic disturbancies.
- This capability is of an extreme importance in mitigating the damaging effects of Space Weather events on anthropic technologies.
- In the past many attempts have been done using different approaches from linear/nonlinear differential equation approaches to linear prediction models or Nonlinear AutoRegressive Moving Average with eXogenous inputs (NARMAX).
- One of the first attempts was done by Burton et al. (1975) which applied a linear prediction model to forecast the evolution of 1hr Dst index;

$$\frac{d}{dt} Dst_0 = F(E) - a Dst_0 \text{ where } Dst_0 = Dst - b(P_d)^{1/2} + c$$

$$F(E) = d(E_y - 0.5) \quad \leftrightarrow \quad E_y > 0.5 \text{ mV/m and } P_d = nV^2$$

Complexity and Forecasting

VOL. 80, NO. 31 JOURNAL OF GEOPHYSICAL RESEARCH NOVEMBER 1, 1975

An Empirical Relationship Between Interplanetary Conditions and *Dst*

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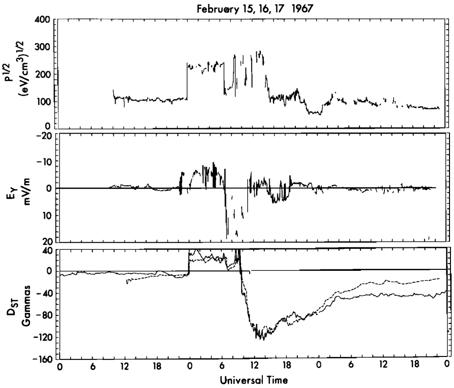

Fig. 6. Observations and predictions for February 15–17, 1967.

Boynton et al., **Data derived NARMAX Dst Model**, Ann. Geophys. 29, 965 (2011)

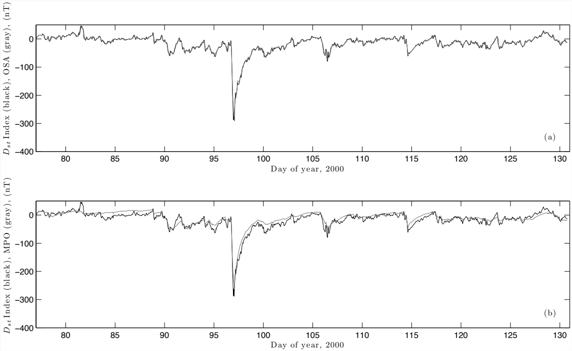

Fig. 3. (a) Measured Dst index in black and OSA Dst in grey. **(b)** Measured Dst index in black and MPO Dst in grey. Both for the same time period starting from 01:00 UT 17 March 2000 until 18:00 UT 9 May 2000.

$$y(t+1) = F\left[y(t), y(t-1), y(t-2), \ldots u(t), u(t-1), u(t-2), \ldots \eta(t), \eta(t-1), \eta(t-2), \ldots\right]$$

Here, $F(\cdot)$ is a nonlinear function.

- In the framework of Machine Learning methods and, more specifically, *Artificial Neural Networks* (ANN), the first attempts to make geomagnetic indices forecasting dates to the early 90's.
- For instance, we mention the works by Gleisner et al., (1996), Lundstedt and Wintoft (1994) and Wu and Lundstedt (1997) that attempted a prediction of Dst index based on interplanetary features, i.e., magnetic field and plasma parameters.
- Pallocchia et al. (2006) demonstrated that it is possible to get a reliable forecasting of Dst index using only the information contained in the interplanetary magnetic field (IMF) by an Elmann's ANN architecture.

Pallocchia et al., **Geomagnetic Dst index forecast based on IMF data only**, Ann. Geophys. 24, 989 (2006)

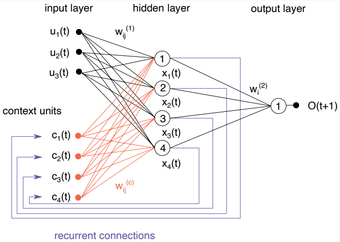

Fig. 2. The Elman scheme used for EDDA. u_1, u_2 and u_3 are the inputs (normalized B_z, B^2 and B_y^2); $w's$ are the network weights. The blue lines indicate copying of each hidden layer output x into the corresponding context unit c, so that $c_k(t) = x_k(t-1)$ (see details in text).

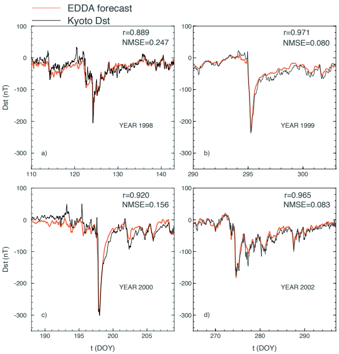

Fig. 4. Comparison of the EDDA D_{st} (red lines) with the Kyoto D_{st} (black lines) for geomagnetically disturbed periods. r is the linear correlation coefficient and NMSE the normalised mean square error (see text for details).

Inputs: B_z, B_y^2 and B^2.

- In spite of the good results in forecasting long time scales variations of geomagnetic indices (Dst and Kp) using ANNs and/or other approaches, the correct evaluation and prediction of shot time scale variations of geomagnetic indices is far from to be achieved.
- This is particularly true when we look for 1 min AE-index.

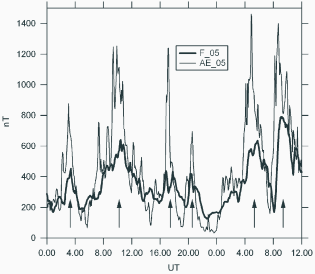

Fig. 1. AE_{05} (thin line) and F_{05} (thick line) plotted against time from 00 UT on 07 September 1995 to 12 UT on 08 September 1995 (see text for details).

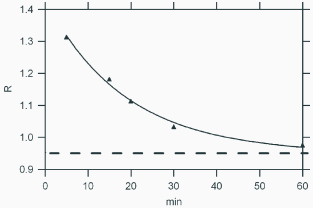

Fig. 4. Normalised standard deviation (R) as a function of the algorithm time resolution (triangles). The solid line represents the exponential fitting the triangles. The horizontal dashed line is the exponential asymptote.

Figure 25: 5 min-AE index forecasting. From Pallocchia et al., JASTP, 70, 663 (2008).

- The main issue in forecasting small time scale variations is related to the internal magnetospheric dynamics, which depends on the instantaneous state of the plasma in the CPS regions of the magnetospheric tail.

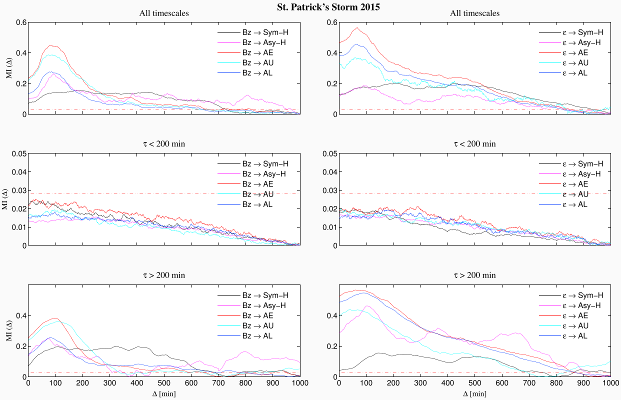

Figure 26: Mutual information between driver and driven quantities. From Alberti et al., JGR:SP, 122, 4266 (2017).

Complexity and Forecasting

- The observed character of the magnetospheric dynamics which is thus a comprise between directly driven (long time scales - slow fluctuations) and internally triggered phenomena (short time scales - fast fluctuations) which depend on the internal dynamical state.
- An attempt to solve this problem was recently done by including some proxies of the geomagnetic internal dynamics: some delayed geomagnetic indices (SYM-H, AE,...).

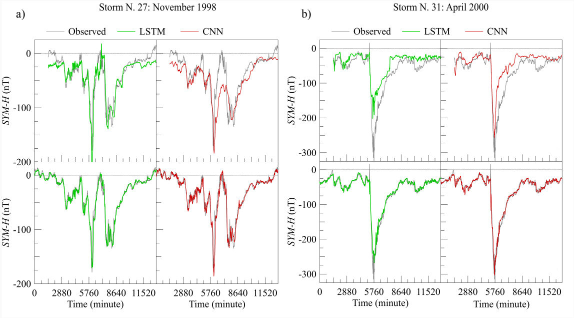

Figure 27: From Siciliano et al., Space Weather, in press (2021).

- The inclusion of some geomagnetic indices as proxies of the internal magnetospheric state significantly increases the forecasting performances.

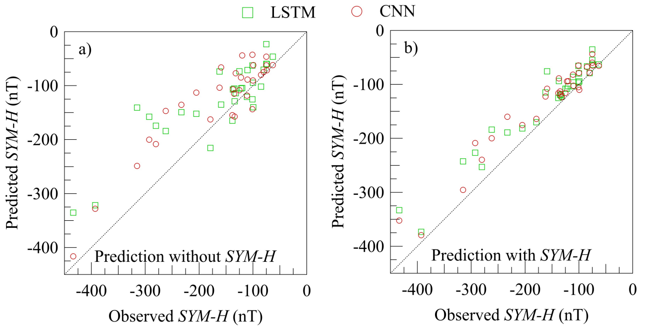

Figure 28: From Siciliano et al., Space Weather, in press (2021).

Conclusions and Open Questions

Conclusions and Open questions

- Complexity is clearly a feature of the magnetospheric dynamics and, in particular, of the substorm dynamics.
- Indeed, the dynamics is characterized by both scale-invariance and low dimensional behavior.
- We have to keep in mind that we deal with an *open system* in which the mechanisms giving rise to the fluctuating forces are distinct from and independent of those producing dissipative forces.
- Indeed, instabilities generated by fluctuations are not uncommon in open systems, whose dramatic effects are the so-called fluctuation-induced phase transitions [Horsthemke & Lefever, 1984; Chang et al., 1990].
- A possible scenario could be that of an extended system characterized by disordered and highly dynamical complex topology, involving the interplay of kinetic, intermediate, and MHD scale fluctuations [see e.g., Chang, 1999, 2001].

Conclusions and Open questions

- Most of the studies (only partially shown here) deals with the use of geomagnetic indices and/or local observations.
- What could be the best set of variables to characterize the complex dynamics of the Earth's magnetosphere ?
- How to construct a multiscale fitness landscape for this systems ? or at least for the substorm dynamics ?
- What are the relevant control parameters of the observed scale-invariance features ?

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