# Inflation
# &
# Primordial non-Gaussianity

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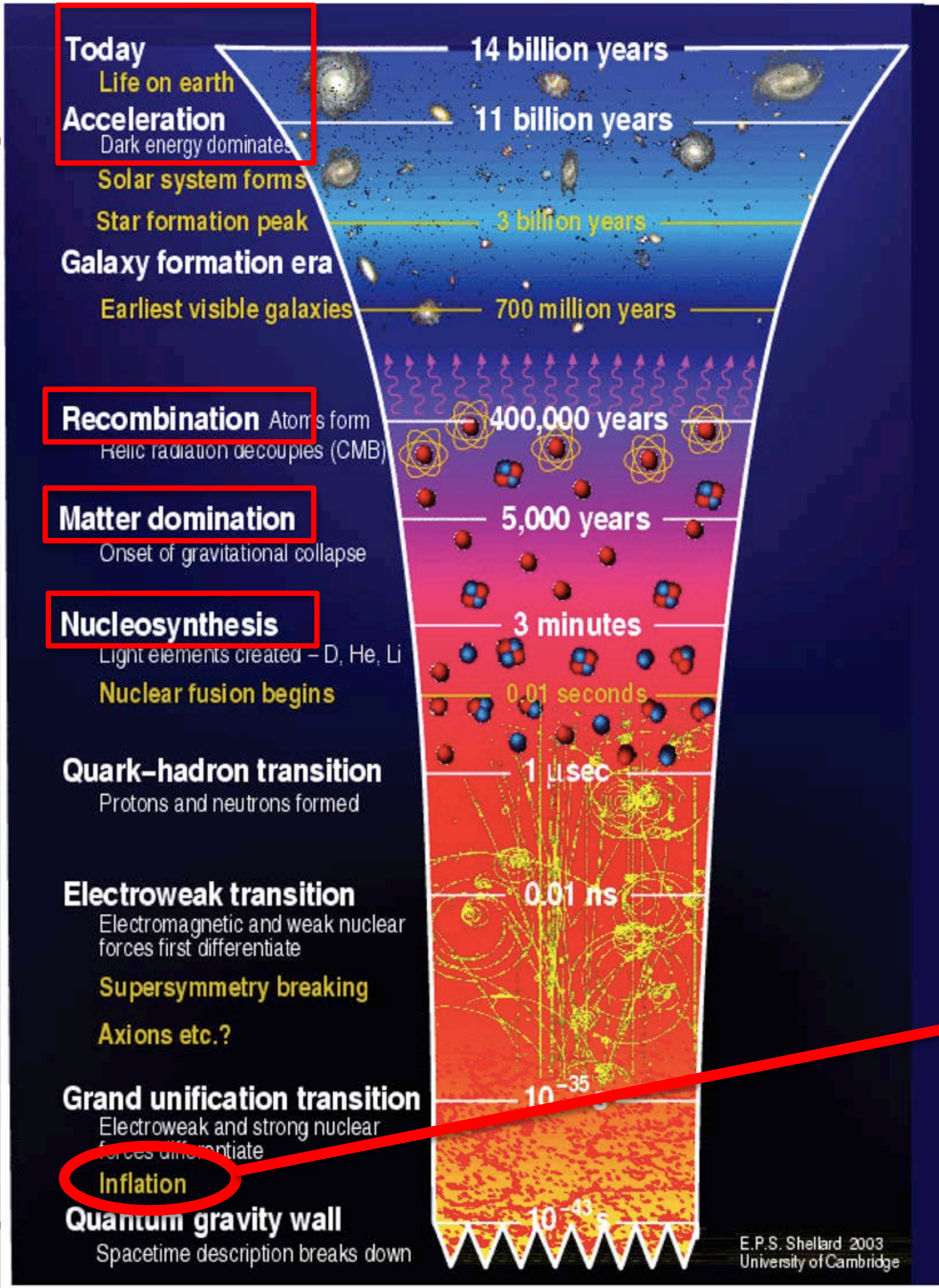
# The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada

*Planck* is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.




**We are here**

$Z_{rec}$~1100

$Z_{eq}$~3500

T~1 MeV

**We seek information about very early times and very high energies E~$10^{16}$ GeV**

**... and we got it!**

# The Big Bang “crisis”

i. Horizon problem: does our Universe belong to … a set of measure zero?

ii. Flatness problem: do we need to fine-tune the initial conditions of our Universe?

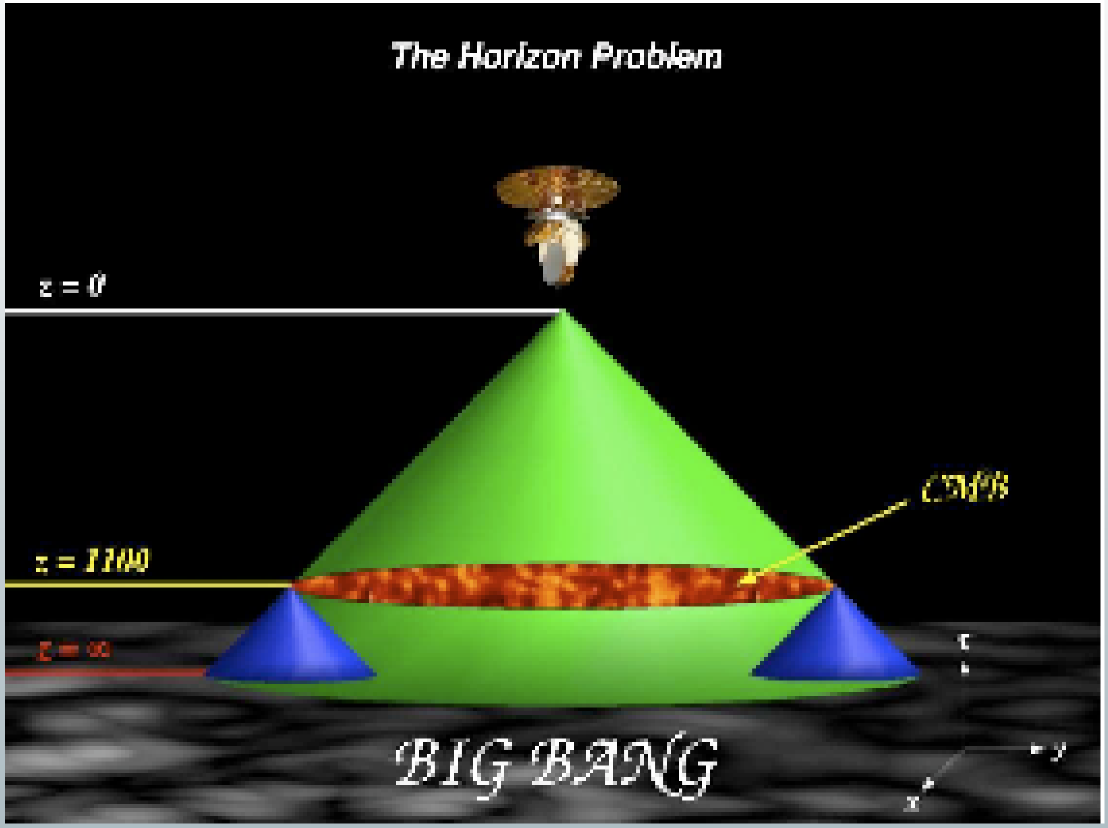
iii. Cosmic fluctuation problem: how did perturbations come from?

# The horizon problem

The comoving scale
of causal correlation

$$r_H(t) = 1 / a(t)H(t)$$

grows with time




*Kinney 2003*

# The rise and fall ... of the comoving Hubble horizon

(late-time dark energy dominance neglected for simplicity)

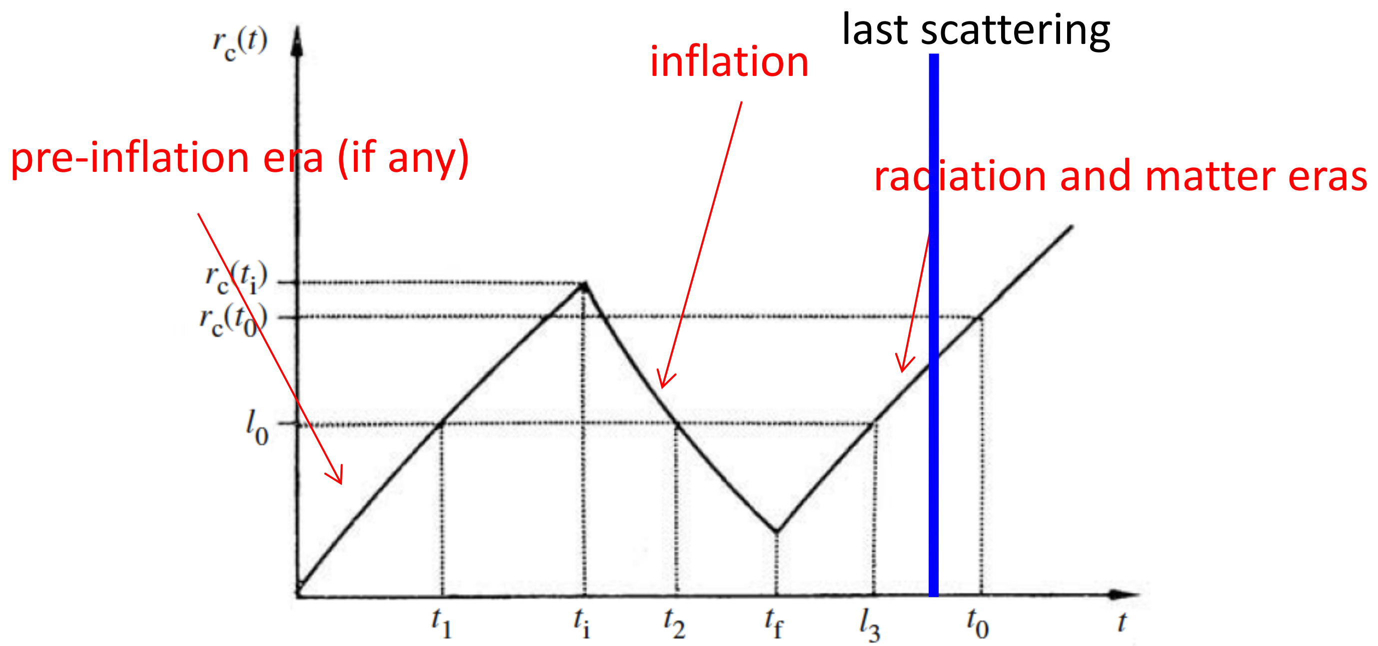
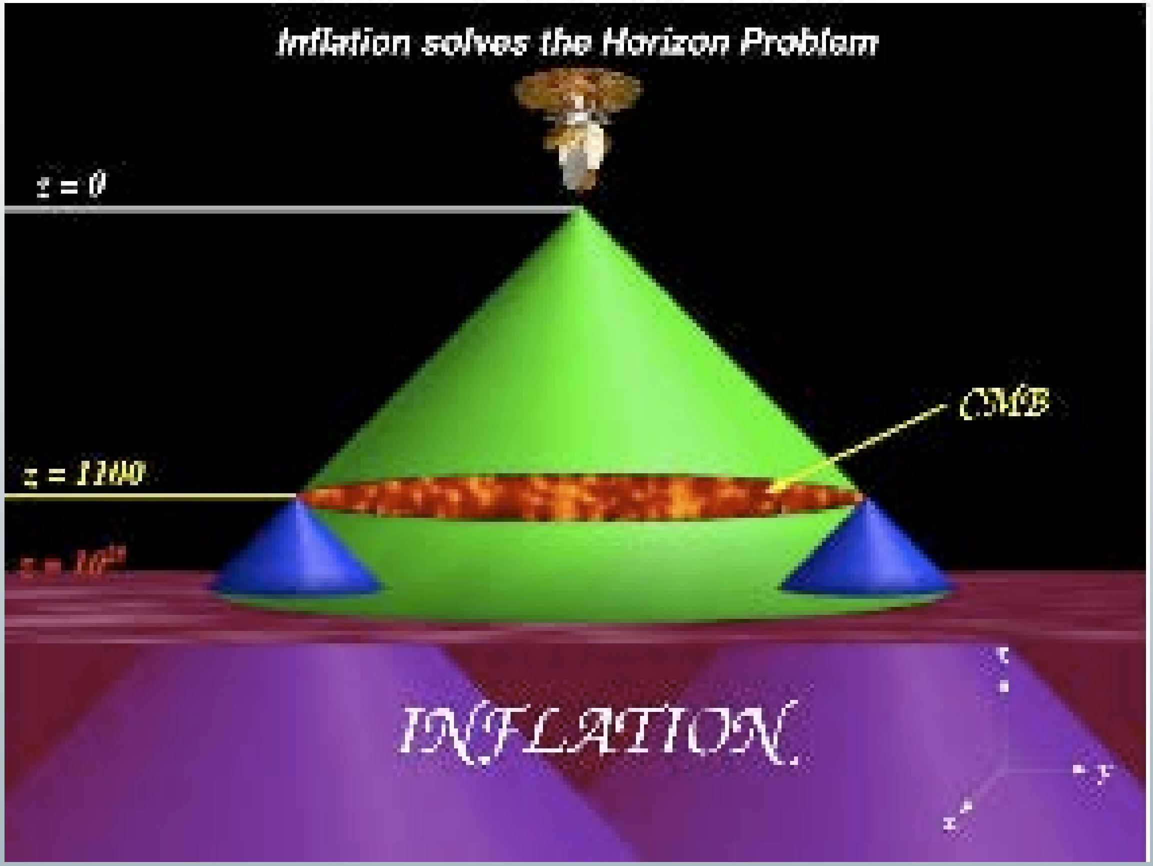



**Figure 7.4** Evolution of the comoving cosmological horizon $r_c(t)$ in a universe characterised by a phase with an accelerated expansion (inflation) from $t_i$ to $t_f$. The scale $l_0$ enters the horizon at $t_1$, leaves at $t_2$ and re-enters at $t_3$. In a model without inflation the horizon scale would never decrease so scales entering at $t_0$ could never have been in causal contact before. The horizon problem is resolved if $r_c(t_0) \leqslant r_c(t_i)$.

credits: Coles & Lucchin 2002

# Solution of the horizon problem

About ***60 e-folds*** of inflation suffice to solve the horizon and flatness problems. Inflation usually lasts much much longer.




*Kinney 2003*

# Solution of the horizon problem

The horizon problem is solved if a region that was causally connected at the beginning of inflation, $t_i$, whose typical size is $d_H(t_i) = a(t_i)\, r_H(t_i)$ after inflating by a factor

$$Z \equiv a(t_f)/a(t_i) = \exp \int_{t_i}^{t_f} dt H(t) \equiv \exp N_{\inf}$$

is able to contain the present Hubble radius scaled back to the end of inflation $t_f$:

$$r_H(t_i) \geq r_H(t_0)$$

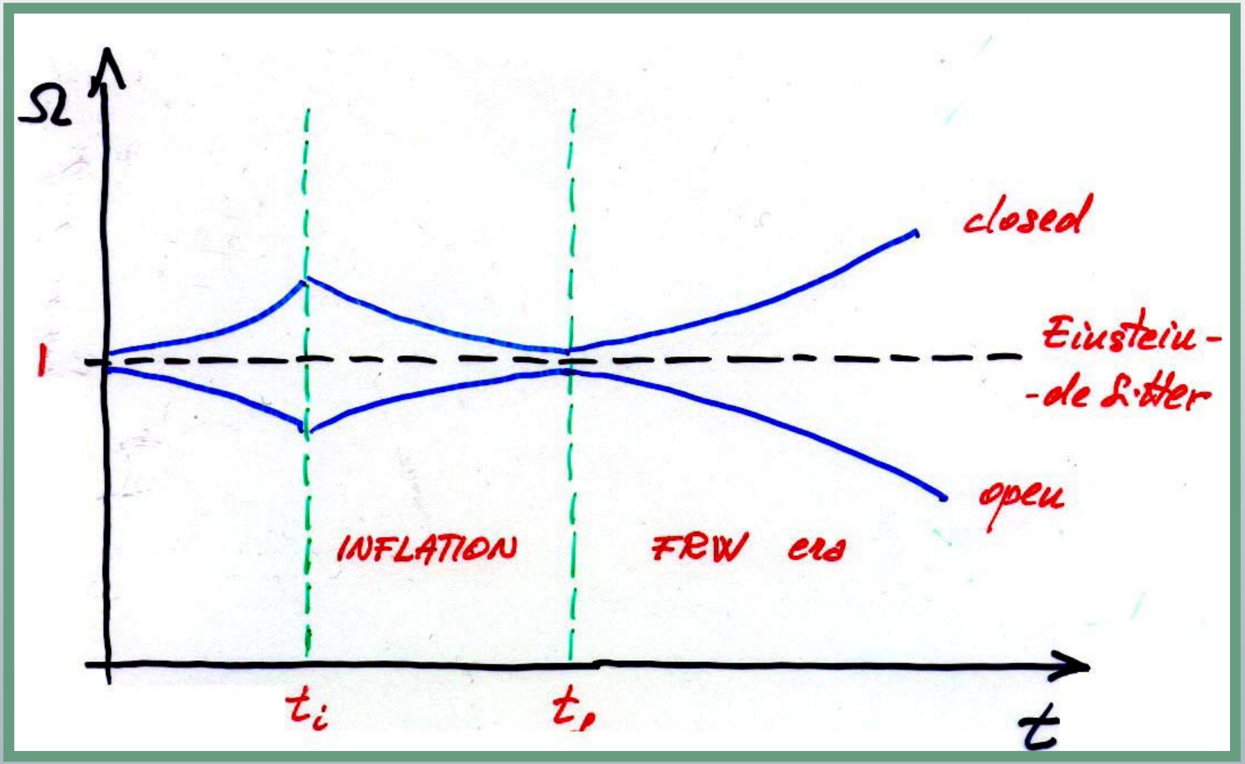
This is possible only if $r_H(t)$ decreases with time during inflation:

$$\dot{r}_H(t) < 0 \Leftrightarrow \ddot{a}(t) > 0$$

for a suitable time-interval

# Evolution of the density parameter: the flatness problem

The density parameter decreases with time if the Universe expansion is decelerated. One needs a fine-tuning of ~ 60 orders of magnitude (!) at the Planck time in order to allow for a density parameter of order unity today! A period of ***accelerated expansion*** automatically solves the problem.



# “Minimal inflation”: $Z=Z_{min}$

Take $Z=Z_{min}$ such that $r_H(t_0)=r_H(t_i)$. From the definition of Z one finds

$$Z_{\min} \approx \left(10^{30} T_f / T_{Planck}\right)^{2/|1+3w_{\inf}|} \qquad w_{\inf} = p_{\inf} / \rho_{\inf}$$

which, for inflation final temperature not far from the Planck energy ($T_{Planck}$~$10^{19}$GeV), and for a nearly de Sitter equation of state $w_{inf}$=-1, leads to a minimum number of inflation e-folds

$$\boxed{N_{inf} \sim 60}$$

→ With such a choice $\Omega_0=\Omega_i$ which automatically solves the flatness problem. More in general

$$(\Omega_0^{-1}-1)/(\Omega_i^{-1}-1)=(Z/Z_{min})^{-|1+3w|}$$

# Inflation in the early Universe

- Inflation is an epoch of accelerated expansion in the early Universe ( ~ $10^{-34}$ s after the "Big Bang") which allows to solve two inconsistencies of the standard Big Bang model (horizon: why is the Universe so homogeneous and isotropic on average + flatness: why is the Universe spatial curvature so small even ~ 14 billion years after the Big Bang?).

- Inflation (Brout et al. 1978; Starobinski 1980; Kazanas 1980; Sato 1981; Guth 1981; Linde 1982, Albrecht & Steinhardt 1982; etc. ...) is based upon the idea that the vacuum energy of a scalar quantum field, dubbed the "inflaton", dominates over other forms of energy, hence giving rise to a quasi-exponential (de Sitter) expansion, with scale-factor

$$\mathbf{a(t) \approx \exp(Ht)}$$

# Inflation predictions

- Quantum vacuum oscillations of the inflaton (or other scalar fields, such as the "curvaton") give rise to classical fluctuations in the energy density, which provide the seeds for Cosmic Microwave Background (CMB) radiation temperature anisotropies and polarization, as well as for the formation of Large Scale Structures (LSS) in the present Universe.

- All the matter and radiation which we see today must have been generated after inflation (during "reheating"), since all previous forms of matter and radiation have been tremendously diluted by the accelerated expansion ("Cosmic no-hair conjecture").

# Inflation dynamics I

- The vacuum expectation value of the *inflaton* scalar field behaves like a perfect fluid, but, unlike standard fluids, it can have negative pressure, thus driving a suitable period of accelerated expansion in the early Universe, dubbed “inflation”. If a long enough period of inflation (more than 60 e-folds) occurred it solves the horizon and flatness problem of the Big Bang model and generates the seeds of cosmic structure formation and CMB anisotropies by quantum oscillations of the vacuum state

## INFLATON DYNAMICS

equation of motion in a FRW background

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi}$$

$$\varphi(t) = \langle \Phi(\vec{x}, t) \rangle \,.$$

inflaton as a perfect fluid (on average)

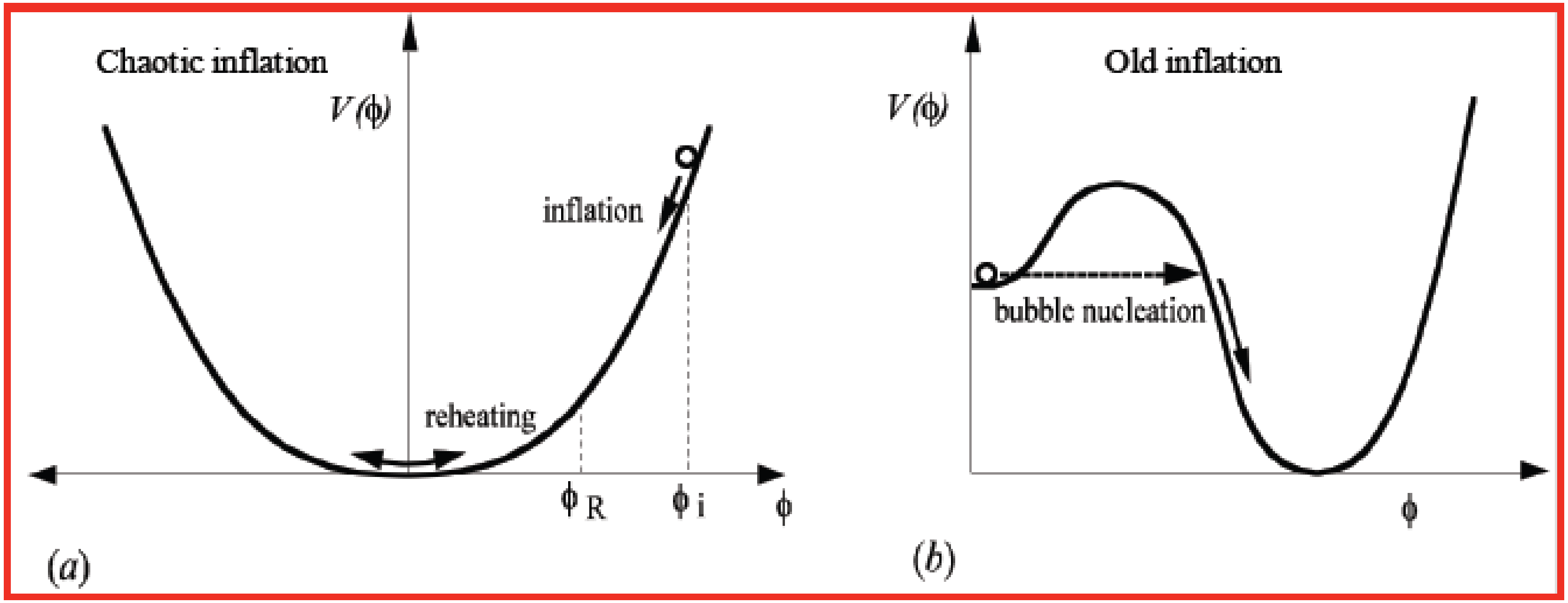
$$\begin{cases} P = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) & \text{pressure} \\ \rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) & \text{energy density} \end{cases}$$

equation of state

$$w = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\varphi}^2 - V}{\frac{1}{2}\dot{\varphi}^2 + V} < -\frac{1}{3}$$

# Inflation dynamics II

Different models of inflation derive from different potential and different initial conditions. Old inflation (*Guth 1981) assumes thermal initial conditions* (which are very difficult to achieve). Chaotic inflation (*Linde 1983) is based* on the application of the uncertainty principle at Planck energies.



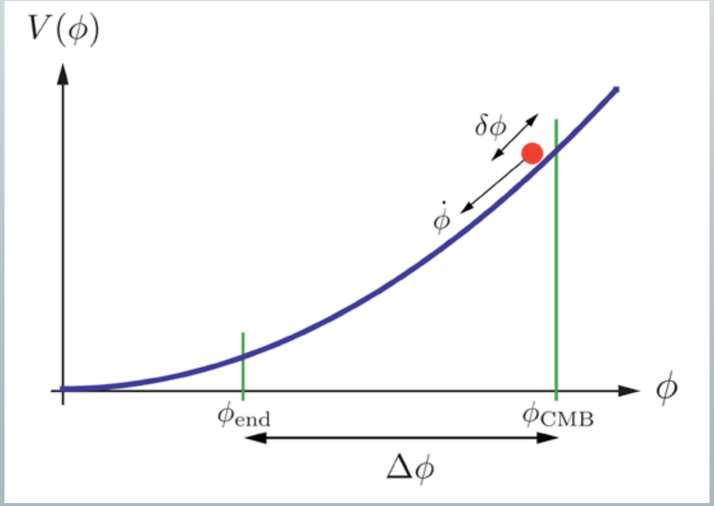
# Inflation and the Inflaton

$$\mathcal{L}_\phi[\phi, g_{\mu\nu}] = \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi)$$

***Standard kinetic term***

***Inflaton potential:*** describes the self-interactions of the inflaton field and its interactions with the rest of the world

Think the inflaton mean field as a particle moving under a force induced by the potential V




Ex: $V(\phi) = \frac{m^2}{2}\phi^2$

# Slow-roll Inflation

$$\begin{cases} H^2 \simeq \dfrac{8\pi G}{3} V(\phi) \\ \dot{\phi} \simeq -\dfrac{V_{,\phi}}{3H} \end{cases} \longrightarrow$$

$$V(\phi) \gg \frac{1}{2}\dot{\phi}^2 \Rightarrow \frac{(V_{,\phi})^2}{V} \ll H^2$$

AND

$$\ddot{\phi} << 3H\dot{\phi} \Rightarrow V_{,\phi\phi} \ll H^2$$

So the slow-roll conditions, as expected, means that the inflaton potential is very flat

It is then customary to parametrize inflationary models (i.e. the form of the inflaton potential ) in a sort of model-independent way by introducing the slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2} = 4\pi G \frac{\dot{\phi}^2}{H^2} \simeq \frac{1}{16\pi G}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1$$ : the Hubble rate changes slowly

$$\eta = \frac{1}{3}\frac{V_{,\phi\phi}}{H^2} = \frac{1}{8\pi G}\left(\frac{V_{,\phi\phi}}{V}\right) \ll 1$$ : attractor solution

# Observational predictions of inflation

- **Primordial density (scalar) perturbations**

$$\mathcal{P}_\zeta(k) = \frac{16}{9}\frac{V^2}{M_{\rm Pl}^4\dot{\phi}^2}\left(\frac{k}{k_0}\right)^{n-1}$$

*amplitude*

*spectral index: (or ``tilt'')* $n - 1 = 2\eta - 6\epsilon$

$$\epsilon = \frac{M_{\rm Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \ll 1; \; \eta = \frac{M_{\rm Pl}^2}{8\pi}\left(\frac{V''}{V}\right) \ll 1$$

- **Primordial (tensor) gravitational waves**

$$\mathcal{P}_{\rm T}(k) = \frac{128}{3}\frac{V}{M_{\rm Pl}^4}\left(\frac{k}{k_0}\right)^{n_{\rm T}}$$

Tensor spectral index: $n_{\rm T} = -2\epsilon$

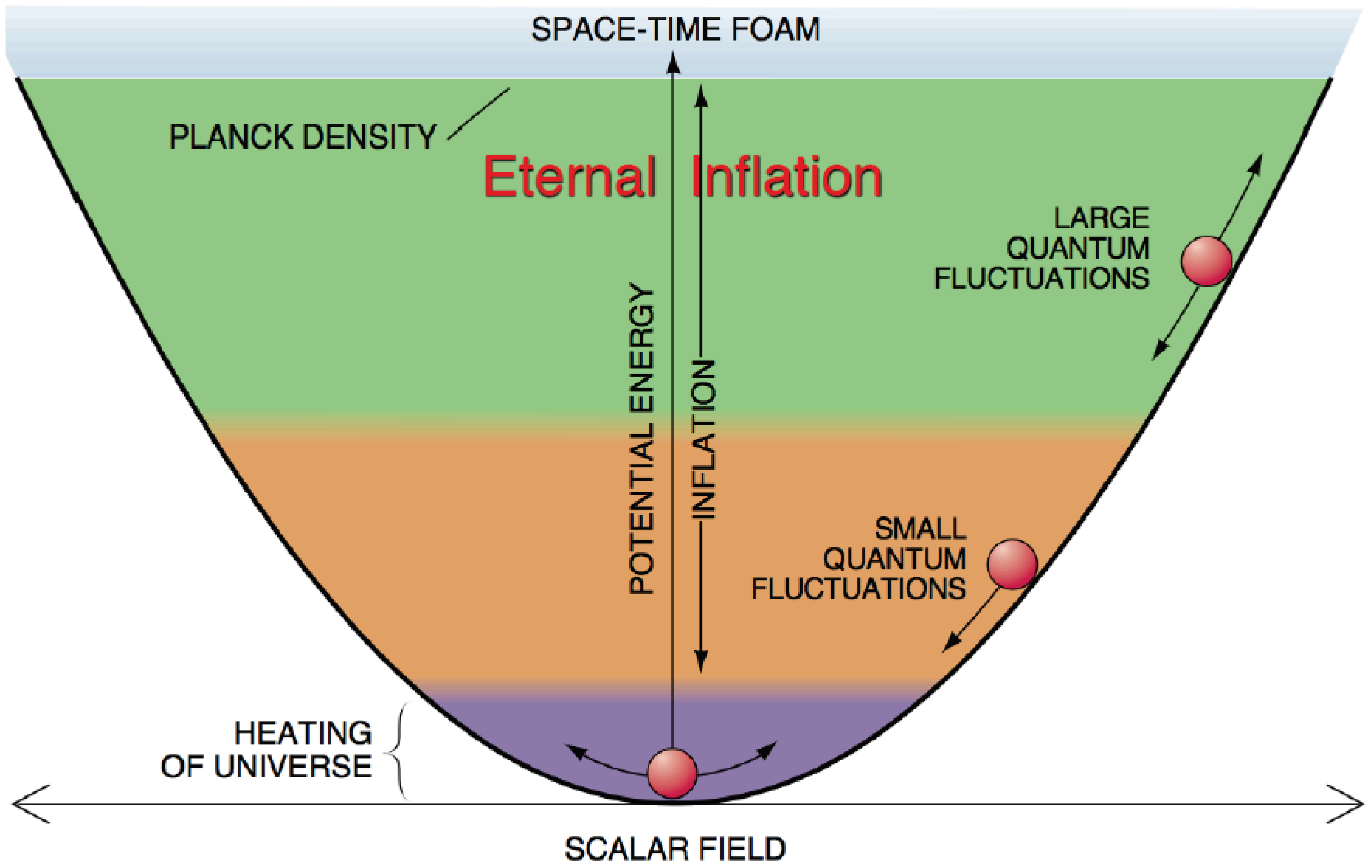
- **Tensor-to-scalar perturbation ratio**

$$r = \frac{\mathcal{P}_{\rm T}}{\mathcal{P}_\zeta} = 16\epsilon$$

- **Consistency relation** (valid for *all* single field slow-roll inflation, easily generalizable to
- non-canonical kinetic term) $r = -8n_T$

# Chaotic inflation

No need for false vacua, thermal equilibrium, and phase transitions. Just use any sufficiently flat potential.




Credits: Andrei Linde 2013

# Why Quantum?

J. Wheeler

Without inflation, the universe is small. Without quantum, the universe is empty.

According to inflationary theory, all matter in the universe was produced due to quantum creation of particles in the post-inflationary reheating process, and the large scale structure of the universe was produced due to inflationary quantum fluctuations.

Credits: Andrei Linde 2013

"… for the practical purposes of describing the observable part of our Universe one can still speak about the big bang, just as one can still use Newtonian gravity theory to describe the Solar system with very high precision."

Andrei Linde 1995

# Testable predictions of inflation

✓ <u>Cosmological aspects</u>

- *Critical density Universe*
- *Almost scale-invariant and nearly Gaussian, adiabatic density fluctuations*
- *Almost scale-invariant stochastic background of relic gravitational waves*

✓ <u>Particle physics aspects</u>

- *Nature of the inflaton*
- *Inflation energy scale*

# Curvature power-spectrum from inflation

- The Planck baseline model assumes purely adiabatic scalar perturbations at very early times, with a (dimensionless) curvature power-spectrum parameterized by

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1 + (1/2)(dn_s/d\ln k)\ln(k/k_0)}$$

with $n_s$ and dns/dlnk = const. The baseline model assumes no "running", i.e. $dn_s/d \ln k = 0$. The pivot scale $k_0 = 0.05\ \mathrm{Mpc}^{-1}$. With this choice, $n_s$ is not strongly degenerate with the amplitude parameter $A_s$.

# Tensor-mode power-spectrum

- The *Planck* analysis also considers extended models with a significant amplitude of primordial gravitational waves (tensor modes) with (dimensionless) tensor mode spectrum parameterized as a power-law with

$$\mathcal{P}_{\mathrm{t}}(k) = A_{\mathrm{t}} \left( \frac{k}{k_0} \right)^{n_{\mathrm{t}}}$$

- We define $r_{0.05} = A_t/A_s$, the primordial tensor-to-scalar ratio at $k = k_0$. Our constraints are only weakly sensitive to the tensor spectral index, $n_t$ (assumed to be close to zero), and we adopt the theoretically motivated single-field inflation consistency relation $n_t = - r_{0.05}/8$, rather than varying $n_t$ independently.

# Initial (inflationary) conditions

- The Planck nominal mission temperature anisotropy measurements, combined with the WMAP large-angle polarization, constrain the scalar spectral index to

$$n_s = 0.9603 \pm 0.0073,$$

ruling out exact scale invariance at over 5σ. Planck establishes an upper bound on the tensor-to-scalar ratio at

$$r < 0.11 \text{ (95\% CL)}.$$

The Planck data shrink the space of allowed standard inflationary models, preferring potentials with $V'' < 0$. Exponential potential models, the simplest hybrid inflationary models, and monomial potential models of degree $n \geq 2$ do not provide a good fit to the data. Planck does not find statistically significant running of the scalar spectral index, obtaining

$$dn_s / d \ln k = -0.0134 \pm 0.0090.$$

# Energy scale of inflation (pre-Bicep2)

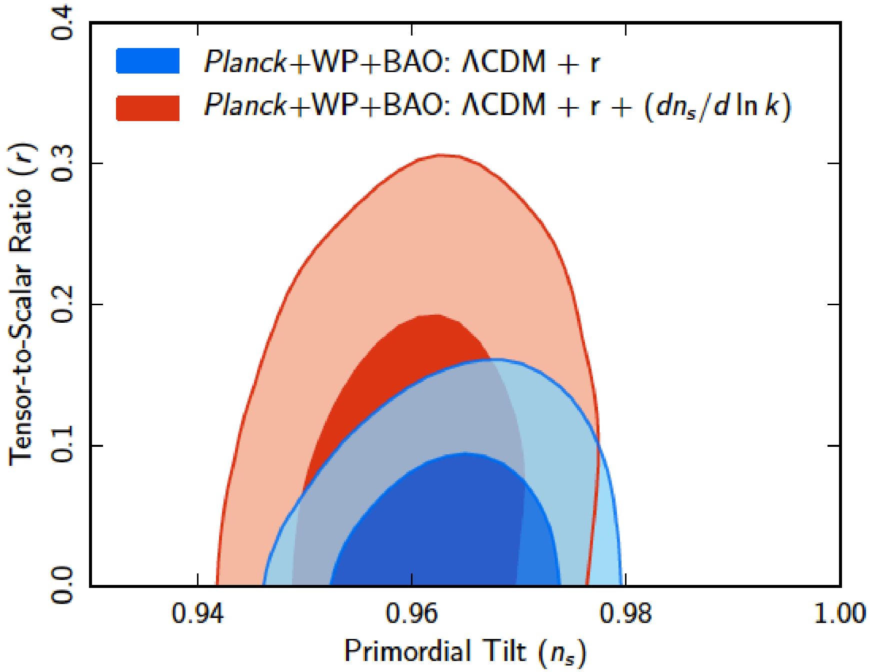
- The Planck constraint on the tensor to scalar ratio *r* corresponds to an upper bound on the energy scale of inflation

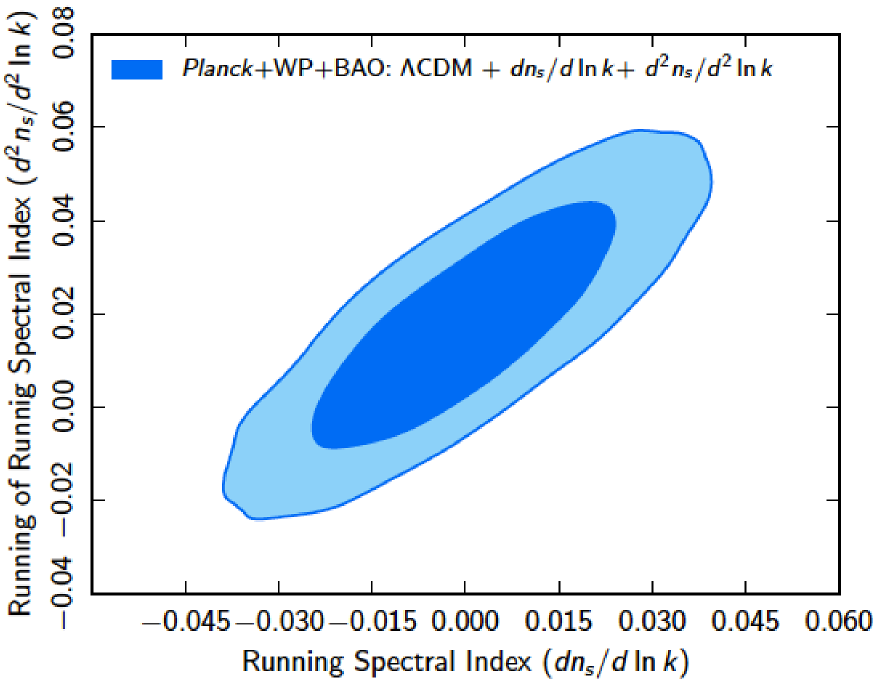
$$V_* = \frac{3\pi^2 A_s}{2}\, r\, M_{pl}^4 = (1.94 \times 10^{16}\ \mathrm{GeV})^4 \frac{r_*}{0.12}$$

at 95 % CL. This in turn is equivalent to an upper bound on the Hubble parameter during inflation

$$H_*/M_{pl} < 3.7 \times 10^{-5}.$$

# Initial (inflationary) conditions




Marginalized joint 68% and 95% CL regions
for (r, $n_s$), using Planck+WP+BAO with and without a running spectral index

Marginalized joint 68% and 95% CL regions for ($d^2n_s$=d ln $k^2$; d $n_s$=d ln k) using Planck+WP+BAO.

# *Planck* constraints on inflation

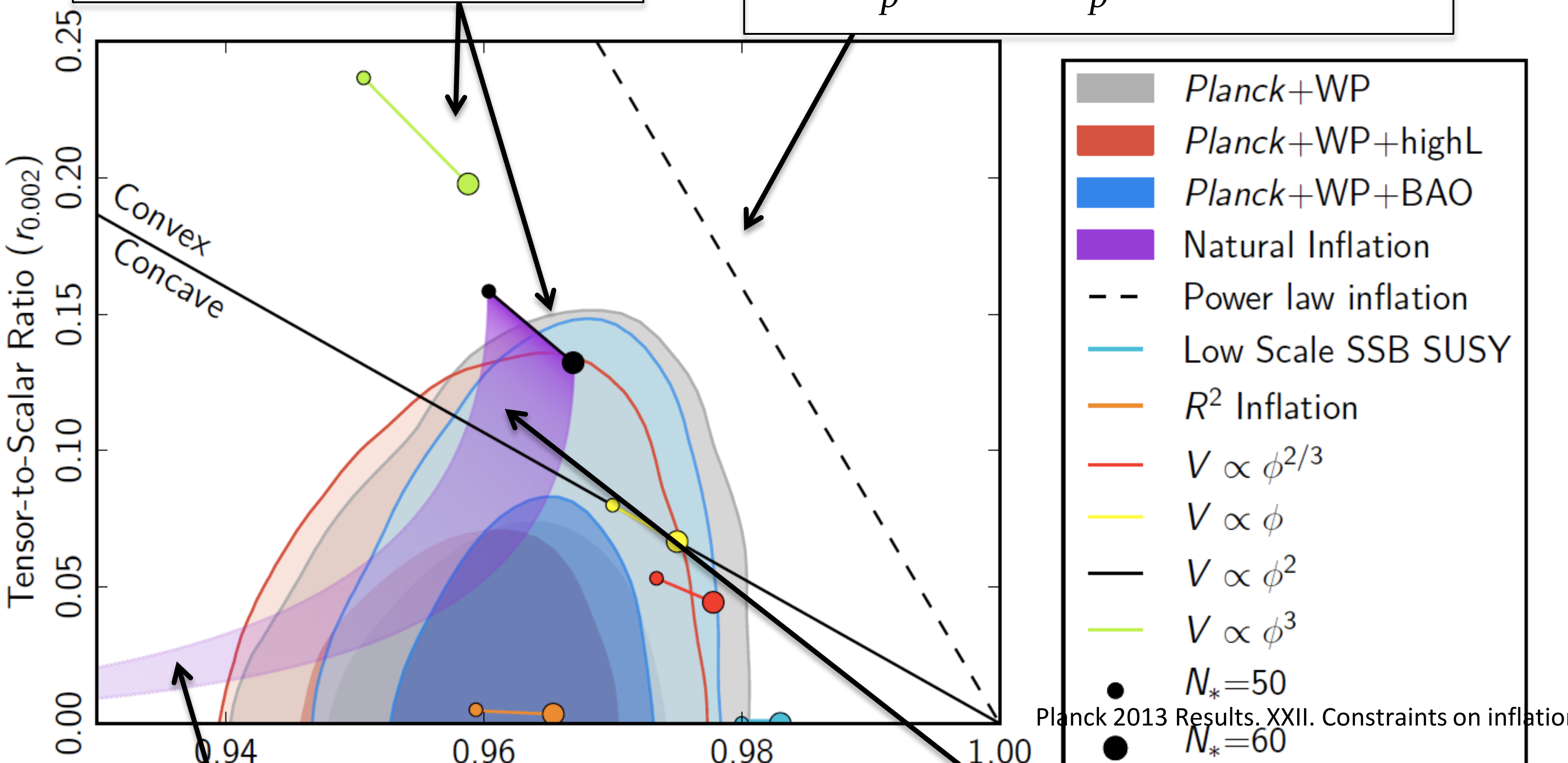
Large field models $V(\phi) \propto \phi^{\alpha}$

$$r = \frac{4\alpha}{N} \qquad 1 - n = \frac{\alpha + 2}{2N}$$

Exponential models

$$V(\phi) \propto \exp[-\sqrt{2/p}\ \phi / M_{Pl}] \to a(t) \propto t^{p}$$

$$r = \frac{16}{p} \qquad 1 - n = \frac{2}{p}$$




Small field models $V(\phi) \propto 1 - (\phi/\mu)^p, \ p \geq 3$

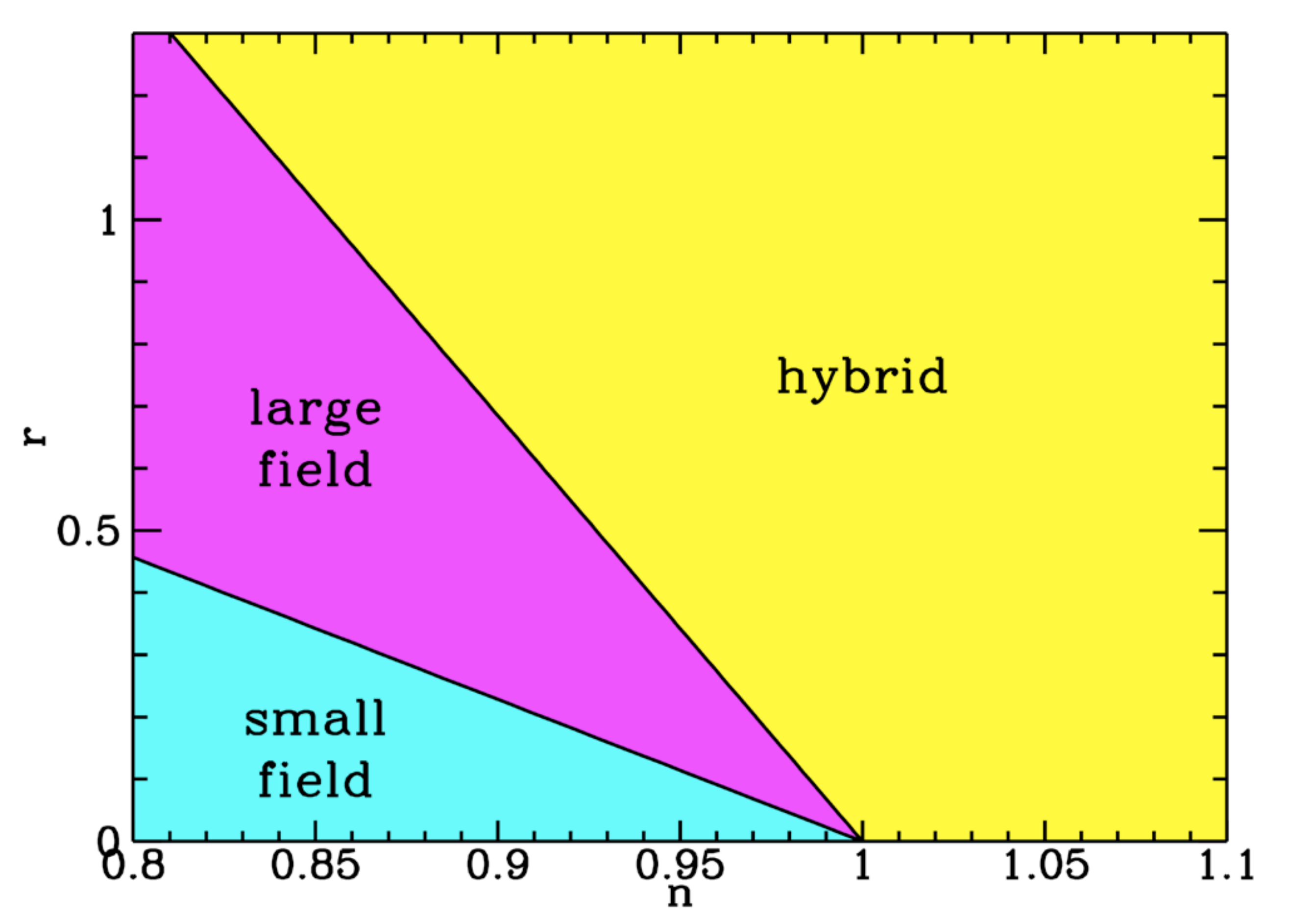
$$r \sim 0 \quad 1 - n = \frac{2}{N}\frac{(p-1)}{(p-2)}$$

for example p = 3 out of 95% *CL*

Natural inflation $V(\phi) \propto 1 + \cos(\phi / f)$

consistent for $f \geq 5 M_{Pl}$

# Classifying inflationary models




e.g., Kinney et al.
astro-ph/0007375

# Two simple but very important examples

**``Large field'' models**

$$V(\phi) \propto \phi^{\alpha}$$

typical of ``caothic inflation scenario''
(Linde `83)

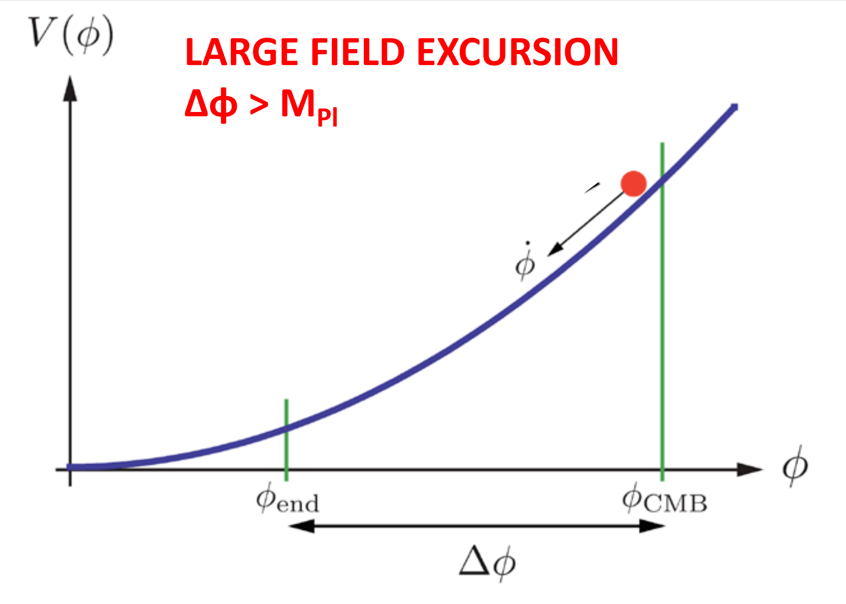
$$V(\phi) \propto \exp[\phi/\mu]$$

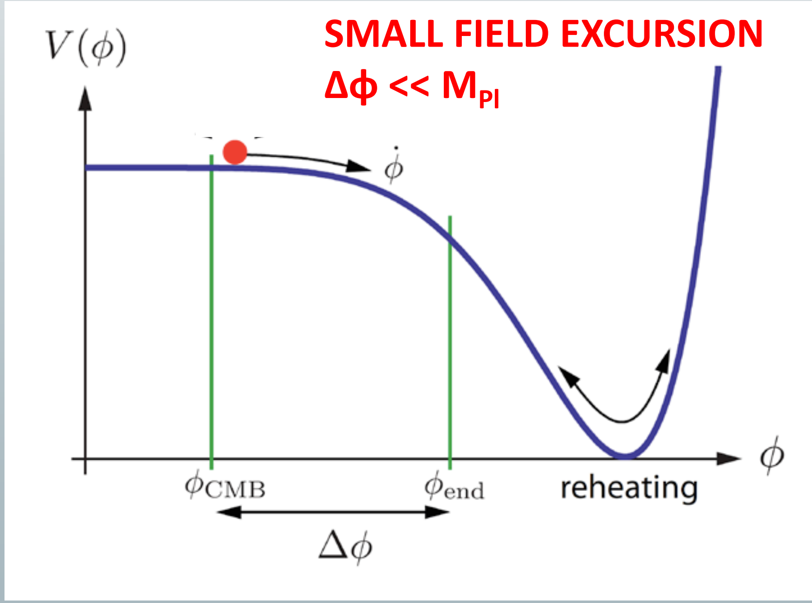
``power law inflation'' (Lucchin, Matarrese'85)

**``Small field'' models**

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right] \qquad \phi < \mu < M_{Pl}$$

from spontaneous symmetry breaking or Goldstone, axion models (Linde; Albrecht, Steinhardt `82; Freese et al '90)



# Consequences for Inflation models (*Planck*)

- exponential, simplest hybrid inflation and monomial ($V(\phi)=\phi^n$, n=2,4) do not fit well observations, e.g. n=4 outside the 99.7% CL, n=2 outside 95% CL
- Axion monodromy inspired models, as above, with n=1,2/3, lie within 95% CL, and on the boundary of 95% CL respectively.
- Hilltop ($1-\phi^p/\mu^p$) con p=2 in agreement, within 95% CL
- natural inflation ($1+\cos[\phi/f]$) consistent, for $f \geq 5\ M_{Pl}$
- $R^2$ Model (Starobinsky '80) consistent with data

# Gravity-wave background from inflation

- As originally noticed by Starobinski (1979) an early period of quasi-de Sitter evolution leaves its imprint in terms of a low-amplitude *stochastic background of gravitational waves* (see also Grishchuck 1975, Rubakov et al. 1982, Fabbri & Pollock 1982, Abbott & Wise 1984) which originated from quantum vacuum fluctuations of (linearized) spin-2 gravitational perturbations ("gravitons"), left the horizon during inflation (hence remaining frozen and unobservable) and rentered the horizon recently, hence becoming potentially observable as classical tensor perturbations of space-time.

- The detection of these primordial gravitational waves represents the "smoking gun" proof of the validity of the inflationary theory, otherwise very hard to "falsify"; other crucial specific imprints being: the existence of perturbations with a super-horizon seed (***detected!***), specific non-Gaussian signatures of primordial perturbations (***strongly constrained by Planck, which strongly supports the simplest inflation models***).

# Primordial gravitational waves

GWs are tensor perturbations of the metric. Restricting ourselves to a flat FRW background (and disregarding scalar and vector modes)

$$\mathbf{ds^2=a^2(\tau)[- d\tau^2 + (\delta_{ij} + h_{ij}(\underline{x},\tau))\, dx^i\, dx^j]}$$

where $h_{ij}$ are tensor modes which have the following properties

$h_{ij} = h_{ji}$ (symmetric)

$h^i_{\ i} = 0$ (traceless)

$h^i_{\ j|i} = 0$ (transverse)

and satisfy the equation of motion

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \nabla^2 h_{ij} = 0$$

**‘ = d/d$\tau$**

# Primordial gravitational waves

GWs have only (9→6-1-3=) 2 independent degrees of freedom, corresponding to the 2 polarization states of the graviton

$$h_{ij}(\vec{x},\tau) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \varphi(\vec{x},\tau)\varepsilon_{ij}(\vec{k})$$

$$\varphi'' + 2\frac{a'}{a}\varphi' + k^2\varphi = 0$$

polarization tensor

free massless, *minimally coupled* scalar field

dynamical behaviour:

**k « aH** (outside the horizon) **φ ≈ const + decaying mode**

**k » aH** (inside the horizon**)** **φ ≈ $e^{\pm ik\tau}/a$** gravitational waves: freely stream, experiencing redshift and dilution, just like free photons)

# Inflaton dynamics and the level of gravity waves

"Large field'' models can produce a high level of gravity waves ($r>0.01$)
"Small field'' models produce a low level of gravity waves ($r<0.01$)

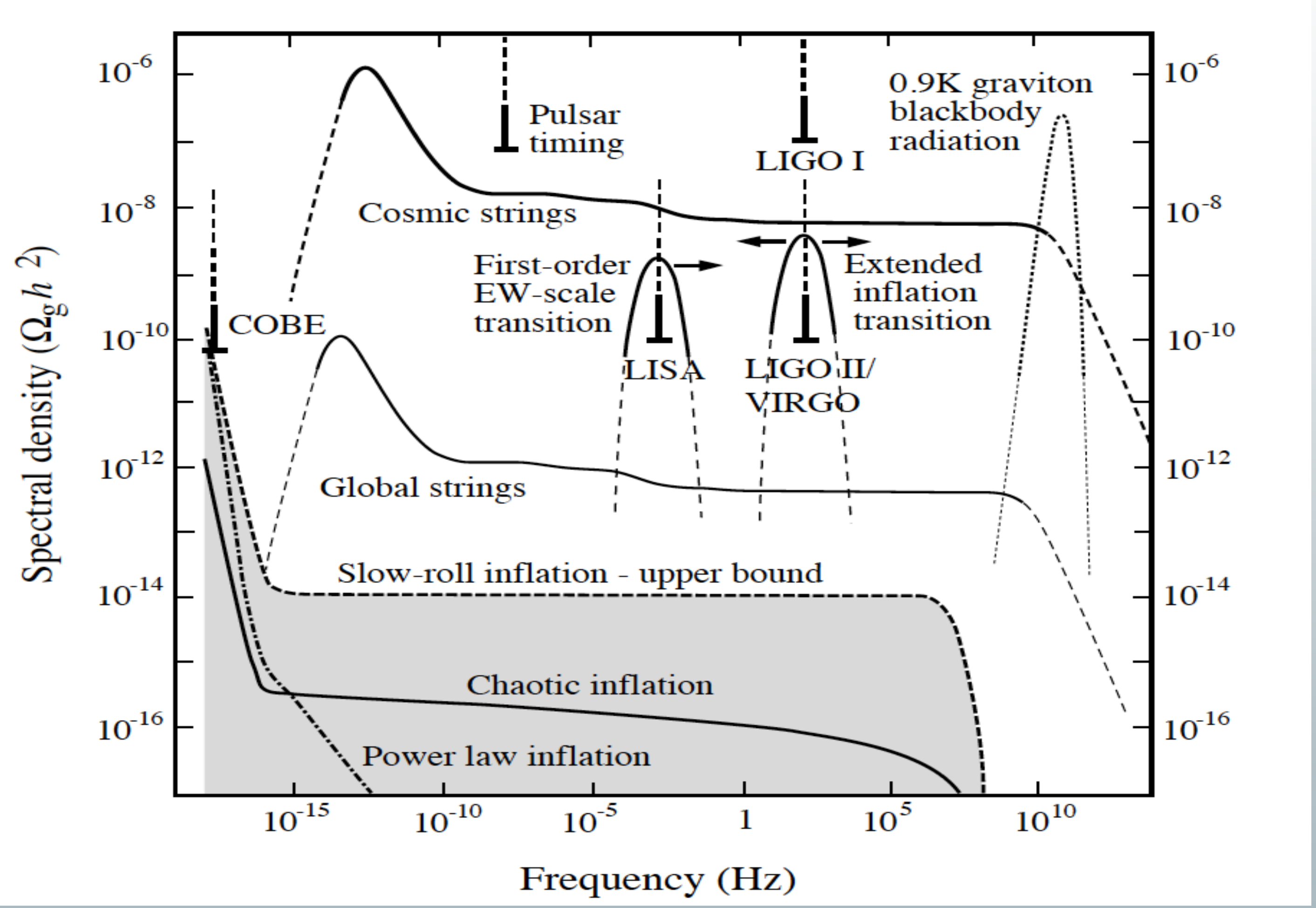
$$\boxed{\frac{\Delta\phi}{m_{\rm Pl}} \simeq \left(\frac{N}{30}\right) \times \left(\frac{r}{0.01}\right)^{1/2}}$$

$30 \leq N \leq 60.$

So the bigger the field excursion during inflation the bigger the amplitude of gravity waves

# The search for primordial GW

Note: this is an "historical" plot, used for illustration purposes only!

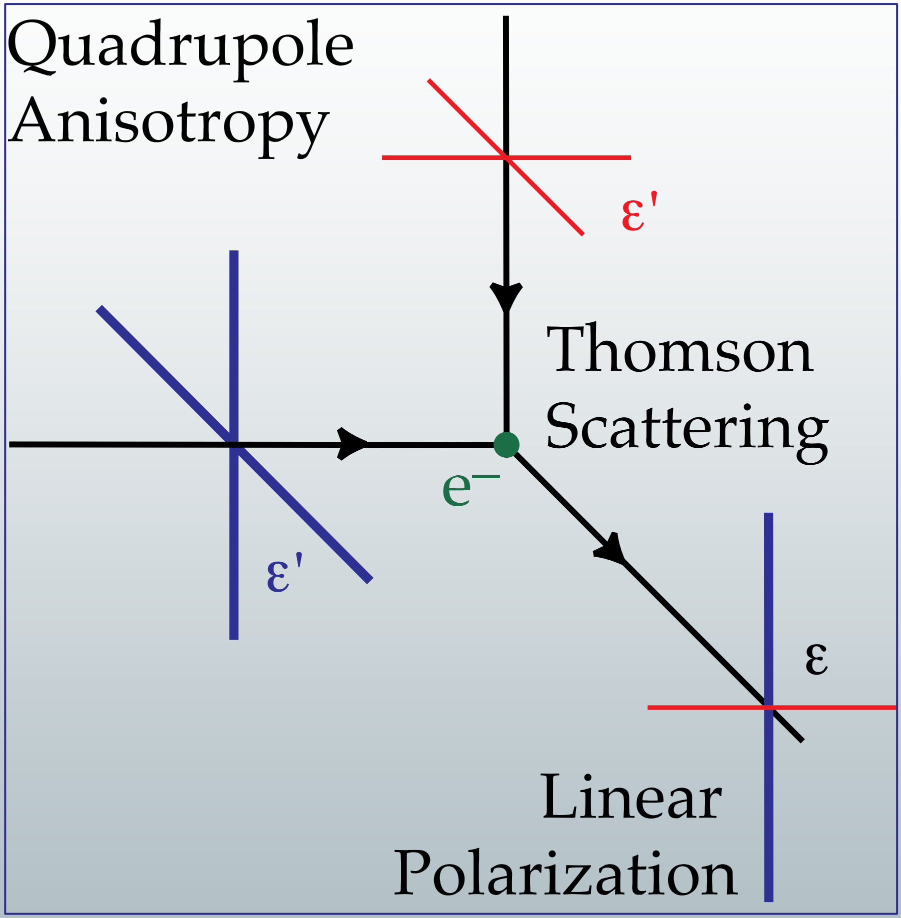



*Battye & Shellard 1996*

# The search for primordial GW

- The primordial GW amplitude is maximal at horizon reentering → search for primordial GW background effects on CMB temperature anisotropy and polarization
- CMB temperature anisotropy mixes up scalar and tensor modes (hence indirect upper bound by e.g. *Planck*)
- Tensor modes (and vector modes too, if present) induce a specific polarization type ("B-mode") which can't be induced by scalar perturbations (which produce the "E-mode" only)
- However, a cosmological foreground B-mode is non-linearly induced by the conversion of E-modes into B-modes owing to gravitational lensing from LSS (***recently detected by SPT and POLARBEAR!***) → accurate "delensing" required & GWs detectable only if their amplitude is above a certain level ...

# CMB polarization




$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi} \left| \hat{\varepsilon}' \cdot \hat{\varepsilon} \right|$$

- Thomson scattering generates linear polarized radiation if the intensity of the incident radiation present a quadrupole moment

- An incident quadrupole can arise from

  1. Anisotropies in the density of photons surrounding the electron (scalar perturbations)
  2. **A quadrupolar stretching of space due to a passing gravitational wave**

Assume we observed polarization in the CMB. Can we tell whether the source is a scalar or a tensor?

# E, B polarization modes

- A vector can always be decomposed into a curl-free (electric) and a divergenceless (magnetic) component.

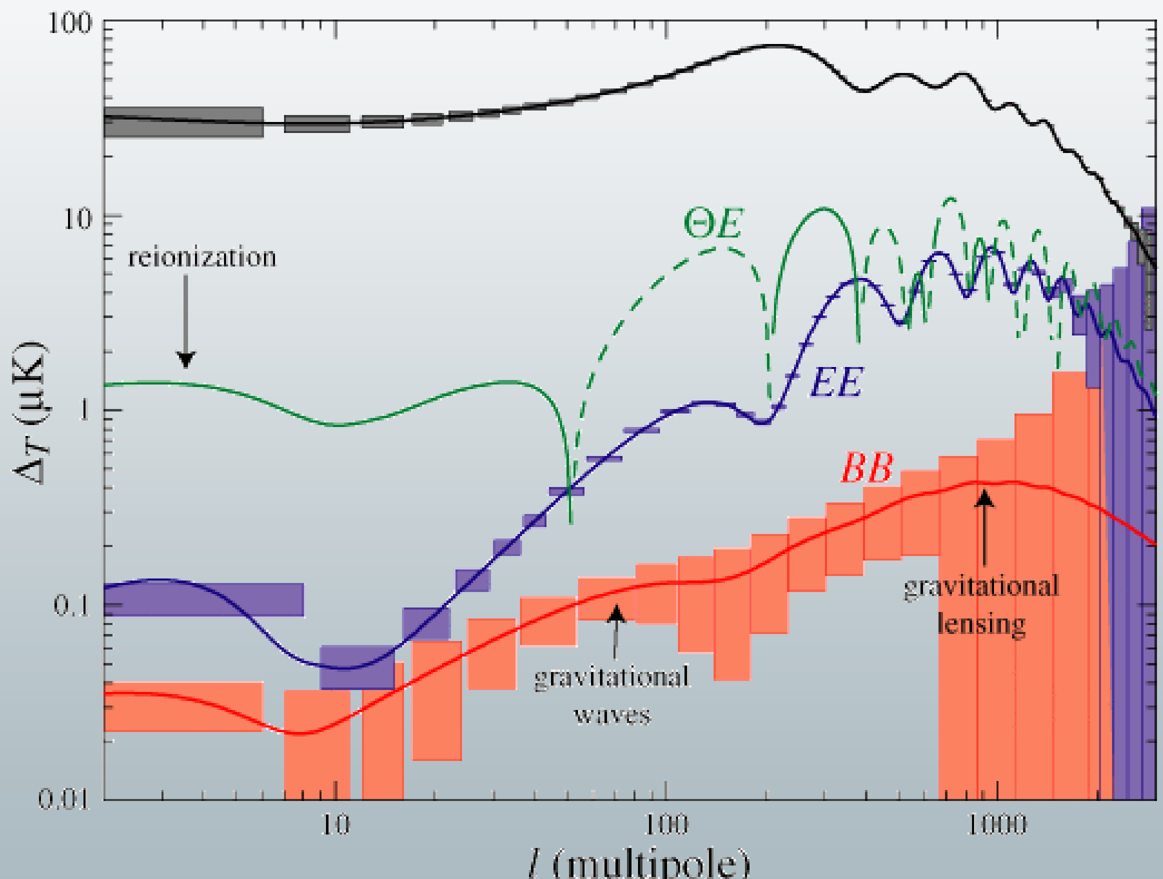
$$\vec{v} = \vec{\nabla}\phi + \vec{\nabla} \times \vec{A}$$

- P=(Q,U) does not transform as vector but as a trace-free symmetric 2x2 tensor. A decomposition similar to the vector case still exists but it involves *second* (covariant) derivatives of two scalar fields called the E and B mode, in analogy with the vector case

- The usefulness of the E-B decomposition of CMB polarization will be clear shortly. as an anticipation: **scalar (density) perturbations can generate only an E-mode, while tensor (GW) perturbations source both E and B modes.**

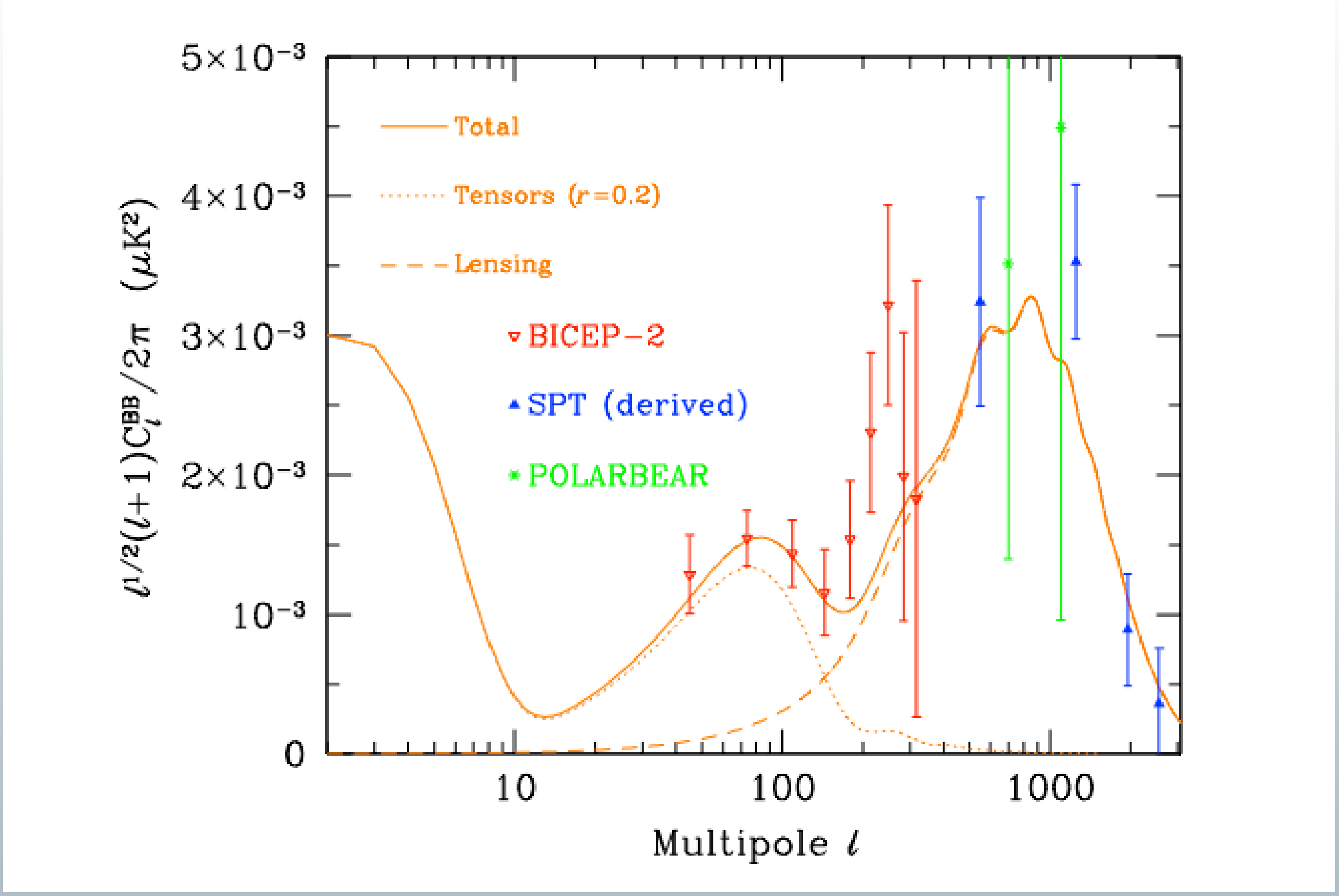
# Power spectra

(If) **T, E, B** are Gaussian scalar fields on the sphere → they are entirely defined by their **angular power spectrum**.




$\ell$ defines the wave angular frequency, corresponding to an angular scale $\vartheta \sim \frac{\pi}{\ell}$

# Bicep 2 vs. other observations




Bicep2: $r = 0.2^{+0.07}_{-0.05}$ - r=0 excluded at the 5.9 $\sigma$ level

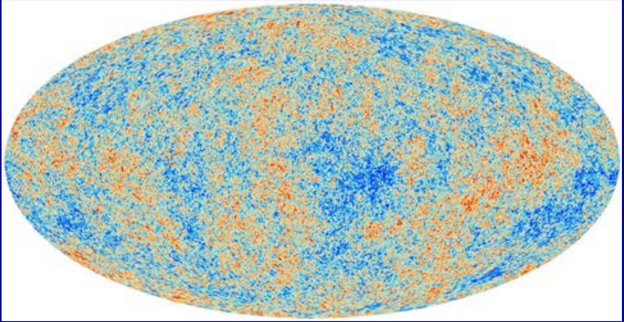
# Comparison with *Planck*

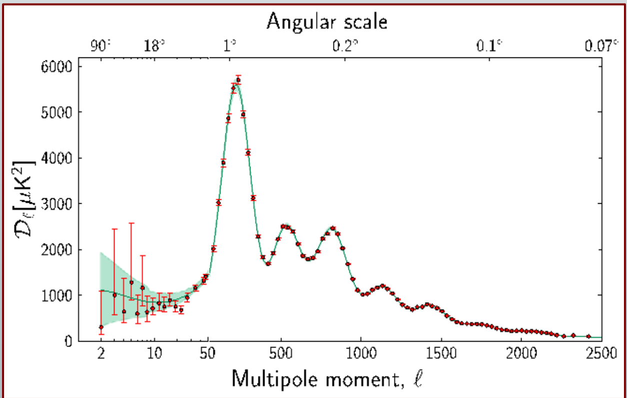
Tensor perturbations provide a contribution to the TT power spectrum. *Planck* could use this to set a constraint on r (in good agreement, and improving on previous WMAP constraint).

Foreground subtraction using DDM2 model (353 Ghz *Planck*-based) yields

$$r = 0.2^{+0.07}_{-0.05}$$ Bicep2 (BB) $\boxed{r = 0.16^{+0.06}_{-0.05}}$

$$r < 0.11 \ (95\% \ c.l.)$$ *Planck* (TT)






- There is some tension between the two measurements. It can be alleviated allowing a running of the scalar spectral index. However the required level of running is not easy to realize in an inflationary context.

- Note however that dust subraction already brings r from BICEP2 down to r=0.16. The preliminary plot from Keck array also shows that high-l outliers disappear. That might bring r down further

# Future prospects

- The primordial B-mode detection by Bicep2 looks robust

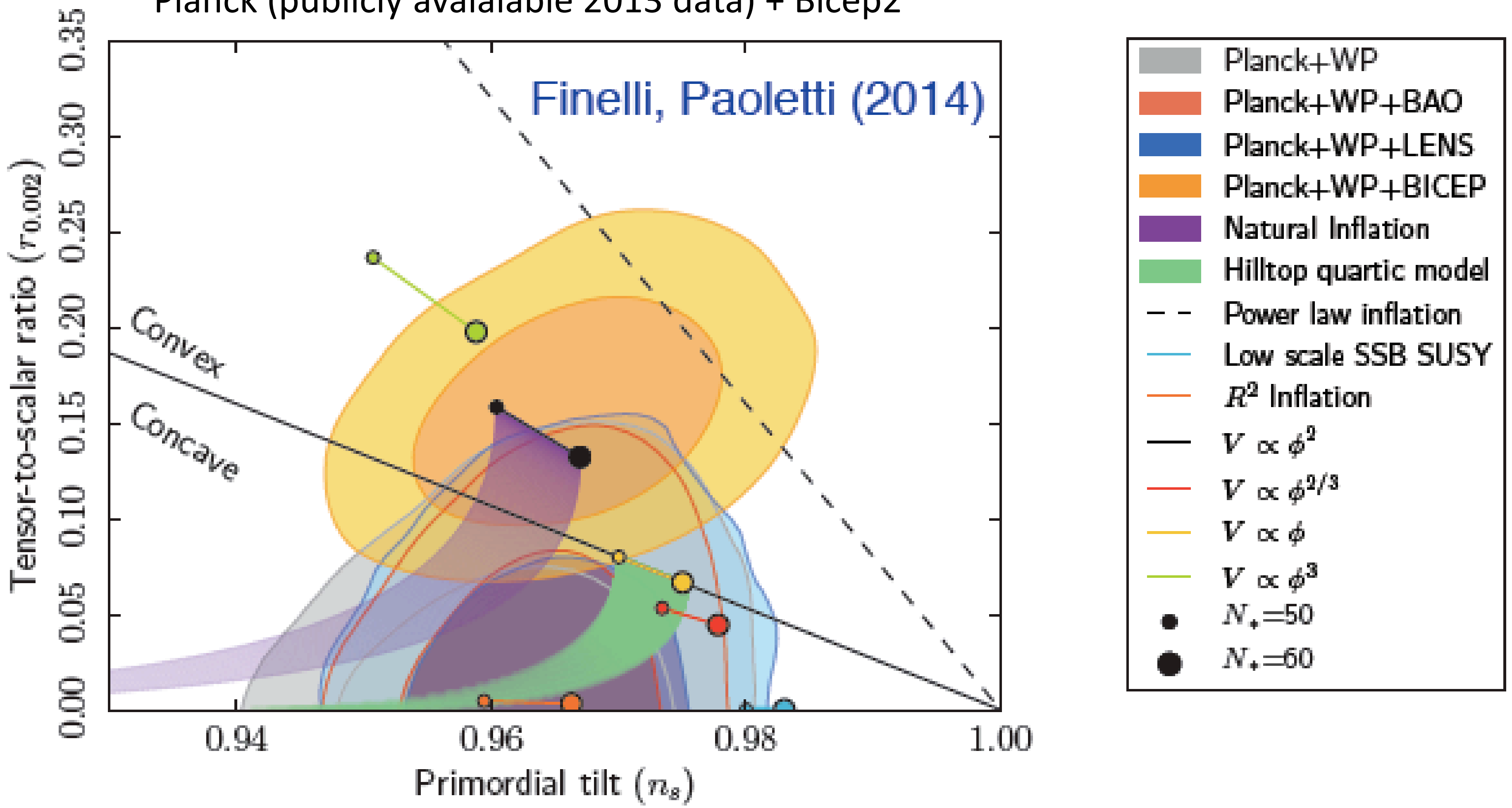
- Additional and more accurate measurements from the ground will come from Keck array and Bicep3.

- *Planck* can play a crucial role in confirming the discovery:

  - *Full sky* measurements → low-l reionization BB spectrum bump is accessible
  - *Multi-frequency* measurements. Accurate characterization of B-mode dust emission

# After BICEP2 ... consequences for inflation models

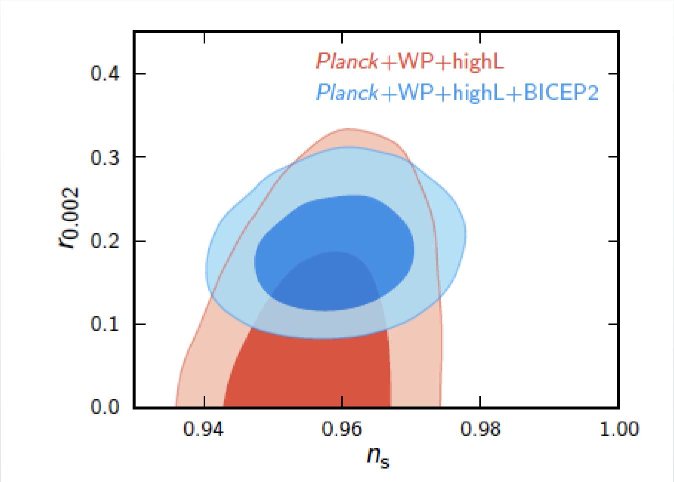
Planck (publicly avaialable 2013 data) + Bicep2



# Is there a tension between *Planck* and Bicep2?

If the tension is real → various ways to reconcile the two observations

Negative *running of the spectral index*




$$\frac{dn_s}{d\ln k} \sim -2\%$$

BUT THIS WOULD RULE OUT ALL SIMPLEST MODELS OF INFLATION
(they typically predict running $O(10^{-3})$).

- Negative cross-correlation of tensors and scalars (allowed in anisotropic inflation models), e.g. *Contaldi et al. arXiV:1403.4596*
- Blue tilt of GW (Giusarma et al. 2014; Smith et al. 2014; Hu et al. 2014)
- (step) Feature in the inflaton potential, e.g. Miranda et al. *arXiV:1403.5231*

# Consequences for high energy physics

- Inflation is probing the GUT scale!
  Tremendously high-energy scales never achievable in laboratories

$$V^{1/4} = 1.94 \times 10^{16} \left(\frac{r}{0.12}\right)^{1/4} \mathrm{GeV}$$

- Inflation is providing a clear evidence of physics beyond the Standard Model of particle physics

- *Who* is the inflaton??
  Now this question has become more and more pressing (most probably it is not the Higgs field).

# Consequences for inflationary models

BICEP2 *strongly* reduces the number of inflationary models that agree with data

- Low-energy scale inflation models are strongly RULED OUT

- Higgs-inflation (Bezrukov & Shaponiskov 2008)
  tries to identify the Higgs of the SM with the inflaton
  (needs non-minimal coupling with gravity)

  Prediction: $r \approx 0.0034$ → RULED OUT *(some fairly contrived variants still alive)*

- $R^2$ inflation (Starobinsky '80) *(connected to Higgs inflation by a Weyl rescaling)*

  Prediction: $r \approx 0.0034$ → RULED OUT

... many more inflation models ruled out

# Consequences for inflationary models

- A simple quadratic potential $V(\phi)=\frac{m^2}{2}\phi^2$ (Linde ’82) perfectly sits within 1σ-regions.

- ``Natural’’ inflation (Freese et al. 90):
  flat potential arises naturally as result of a shift symmetry

$$V(\phi)=V_0\left[1-\cos\left(\frac{\phi}{\mu}\right)\right]$$

  consistent with data.

# Sensitivity of Inflation to fundamental physics?

*The question is: "is the excursion of the field $\Delta\phi > M_{PL}$ a problem for inflation models?"*

*Not necessarily. Rather, it could be an opportunity to probe high-energy physics and the physics behind inflation.*

*Rely on some symmetry criterion: e.g. approximate shift-symmetry $\rightarrow$ flatness of inflaton potential*

# The future: a new era of gravity-wave based cosmology!

- Measure the tensor spectral index

$$\mathcal{P}_{\mathrm{T}}(k) = \frac{128}{3}\frac{V}{M_{\mathrm{Pl}}^4}\left(\frac{k}{k_0}\right)^{n_{\mathrm{T}}}$$

Tensor spectral index: $n_{\mathrm{T}} = -2\epsilon$

- Test the consistency relation (``the holy grail of inflation''):

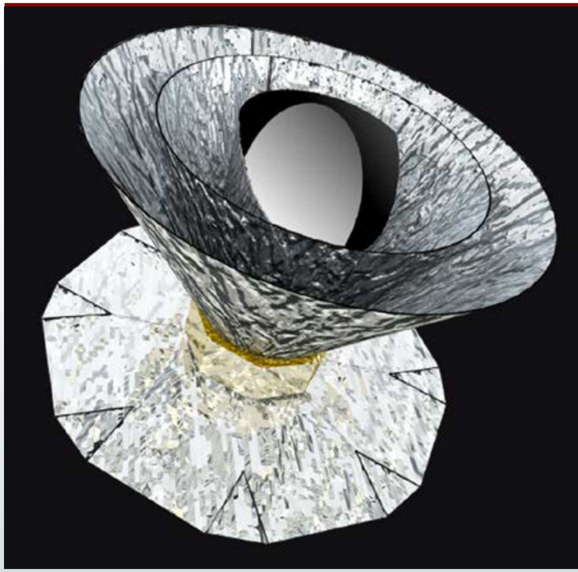
$$r = -8n_T$$

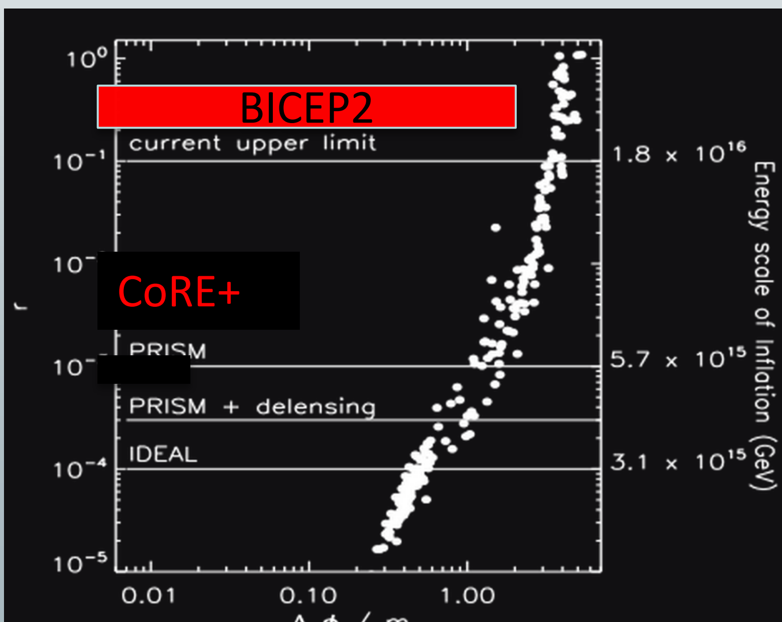
generalized to inflaton w. non-canonical kinetic term
$r = -8c_s n_T$ with $c_S > 0.02$ (Planck 2013)

- Try to measure higher-order correlators of the tensor perturbations, like the 3-point function of tensors <hhh> → graviton interactions (upper bounds obtainable by *Planck* 2014)

- Try to constrain deviation from GR at very high-energies
→ Implications for GW detectors?

# What Next? CoRE+ → an opportunity






- The *ultimate* CMB polarization mission
- A "Cosmic Origins" explorer will be submitted to ESA in response to a M4 call for proposal (next Fall?)
- Timeframe: late 2020s
- At least 30 times more sensitive than Planck
- Wide frequency coverage (e.g. 50 - 800 GHz) to leverage out foreground contamination
- Near total control of systematics – no suborbital probe can achieve this
- If Bicep2 result will hold to scrutiny, you will need a CoRE+ mission to squeeze inflation science out of tensor modes
- If Bicep2 results will not hold, the lesson learnt will call even more strongly for a satellite mission, the only chance to squeeze r to $O(10^{-3})$ and measure tensor spectral index $n_T$ to high accuracy, hence being able falsify inflationary tensor consistency relation

# Conclusions on gravity waves

- Bicep2 observed the imprints on CMB polarization of gravitational waves originated by quantum vacuum oscillations (→ gravitons) in the very early Universe
- Strong support for inflation theory
- Energy scale of inflation (GUT scale) determined
- Probe of physics at $10^{13}$ times the energy scale of LHC → evidence for physics beyond the SM
- Far reaching consequences for detailed models of inflation (hints for UV completion?)
- This discovery calls for confirmation by independent observations, such as *Planck* CMB polarization data analysis can soon (*Fall 2014*) provide.

# Primordial non-Gaussianity:

# a new route to falsify Inflation ...

- Strongly non-Gaussian initial conditions studied in the eighties.
- New era with $f_{NL}$ models from inflation (Salopek & Bond 1991; Gangui, Lucchin, Matarrese & Mollerach 1994: $f_{NL} \sim 10^{-2}$; Verde, Wang, Heavens Kamionkowski 2000; Komatsu & Spergel 2001; Acquaviva, Bartolo, Matarrese & Riotto 2002; Maldacena 2002; + many models with (much) higher $f_{NL}$).
- Primordial NG emerged as a new "smoking gun" of (non-standard) inflation models, which complements the search for primordial GW.

# ... and to test the physics of the Early Universe

- The NG amplitude and shape measures deviations from standard inflation, perturbation generating processes after inflation, initial state before inflation, ...

- Inflation models which would yield the same predictions for scalar spectral index and tensor-to-scalar ratio might be distuinguishable in terms of NG features.

- We should aim at "reconstructing" the inflationary action, starting from measurements of a few observables (like $n_S$, $r$, $n_T$, $f_{NL}$, $g_{NL}$, etc. ...), just like in the nineties we were aiming at a reconstruction of the inflationary potential.

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# Non-Gaussianity from inflation: theory and observations

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# Simple-minded NG model ...

# has become reality

Many primordial (inflationary) models of non-Gaussianity can be represented in configuration space by the simple formula (Salopek & Bond 1990; Gangui et al. 1994; Verde et al. 1999; Komatsu & Spergel 2001)

$$\Phi = \phi_L + f_{NL} * (\phi_L^2 - \langle\phi_L^2\rangle) + g_{NL} * (\phi_L^3 - \langle\phi_L^2\rangle \phi_L) + \ldots$$

where $\Phi$ is the large-scale gravitational potential (more precisely $\Phi = 3/5\ \zeta$ on superhorizon scales, where $\zeta$ is the gauge-invariant comovign curvature perturbation), $\phi_L$ its linear Gaussian contribution and $f_{NL}$ the dimensionless *non-linearity parameter* (or more generally *non-linearity function*). The percent of non-Gaussianity in CMB data implied by this model is

$$\text{NG \%} \sim 10^{-5}\ |f_{NL}|$$

$$\sim 10^{-10}\ |g_{NL}|$$

$< 10^{-4}$ from CMB & LSS

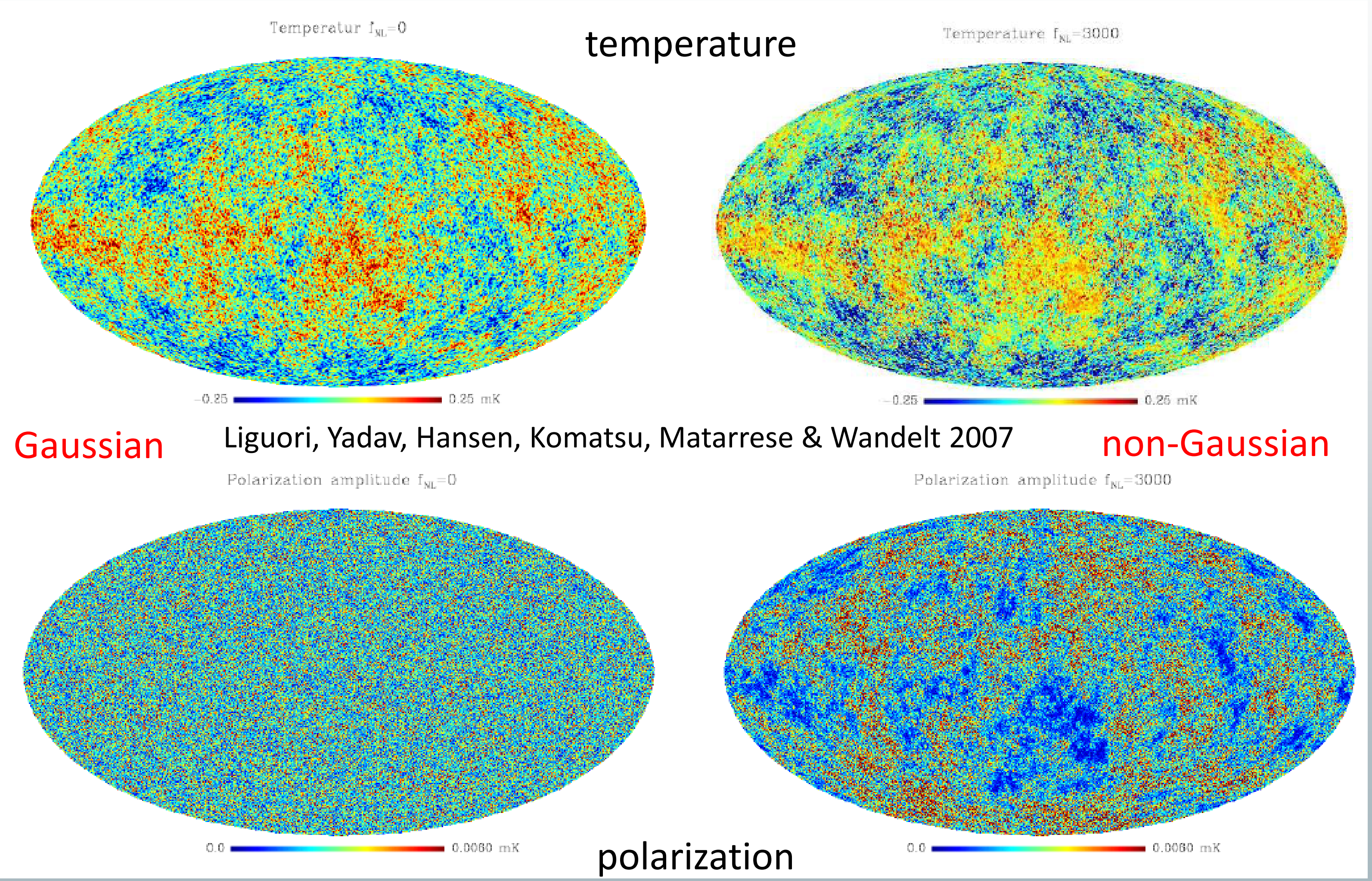
$10^{-5}$ from *Planck*

$< 10^{-4}$ from CMB & LSS

"non-Gaussianity = non-dog"
(Ya.B. Zel'dovich)

# NG CMB simulated maps

temperature




Gaussian

Liguori, Yadav, Hansen, Komatsu, Matarrese & Wandelt 2007

non-Gaussian

polarization

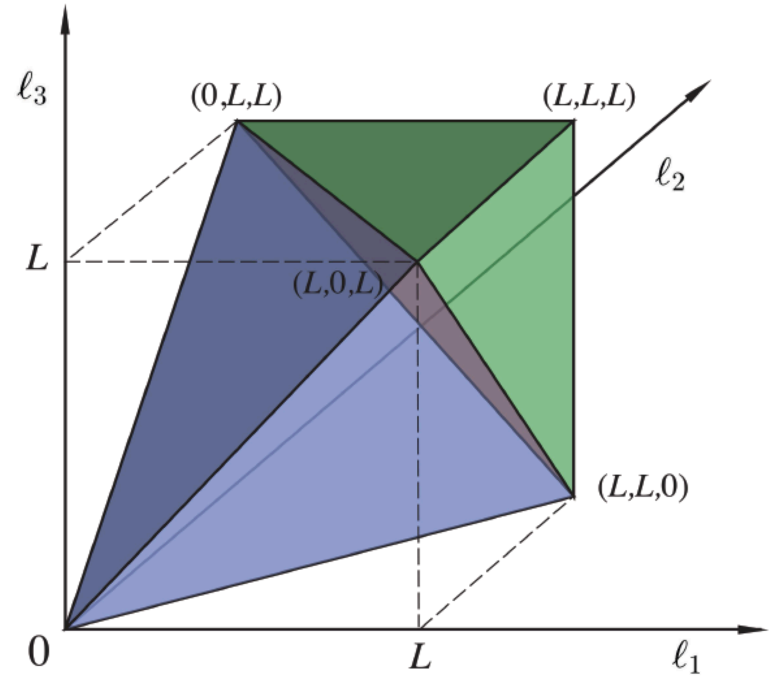
# CMB bispectrum

$$B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \equiv \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle$$

$$= \mathcal{G}^{\ell_1 \ell_2 \ell_3}_{m_1 m_2 m_3} \, b_{\ell_1 \ell_2 \ell_3}$$

Gaunt integrals

$$\mathcal{G}^{\ell_1 \ell_2 \ell_3}_{m_1 m_2 m_3} \equiv \int Y_{\ell_1 m_1}(\hat{\boldsymbol{n}})\, Y_{\ell_2 m_2}(\hat{\boldsymbol{n}})\, Y_{\ell_3 m_3}(\hat{\boldsymbol{n}})\, d^2\hat{\boldsymbol{n}}$$
$$= h_{\ell_1 \ell_2 \ell_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix},$$



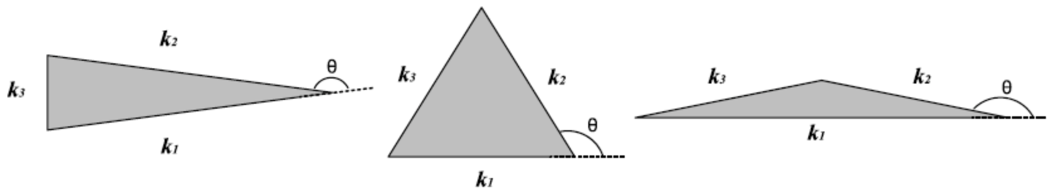

Triangle condition: $\ell_1 \le \ell_2 + \ell_3$ for $\ell_1 \ge \ell_2, \ell_3$, +perms.

Parity condition: $\ell_1 + \ell_2 + \ell_3 = 2n\,, \quad n \in \mathbb{N}\,,$

Resolution: $\ell_1, \ell_2, \ell_3 \le \ell_{\max}\,, \quad \ell_1, \ell_2, \ell_3 \in \mathbb{N}\,.$

# the NG shape information

*... there are more shapes of non-Gaussianity (from inflation) than ... stars in the sky*



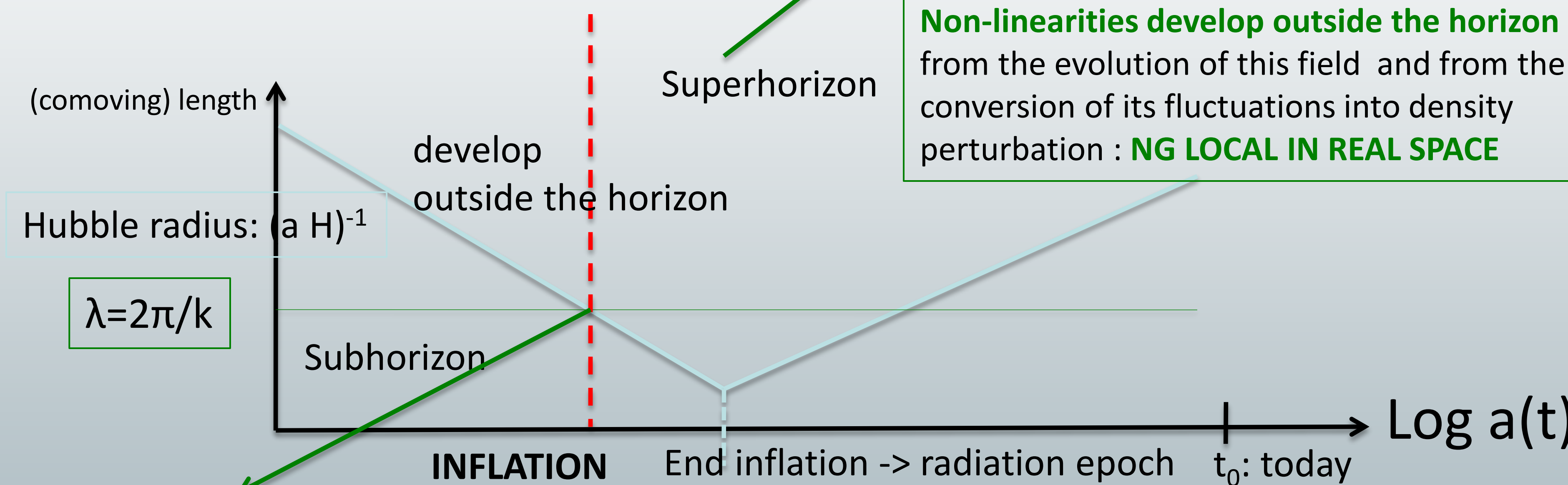
(1) Squeezed (2) Equilateral (3) Folded

+ many more

# Classes of inflation models: NG imprints

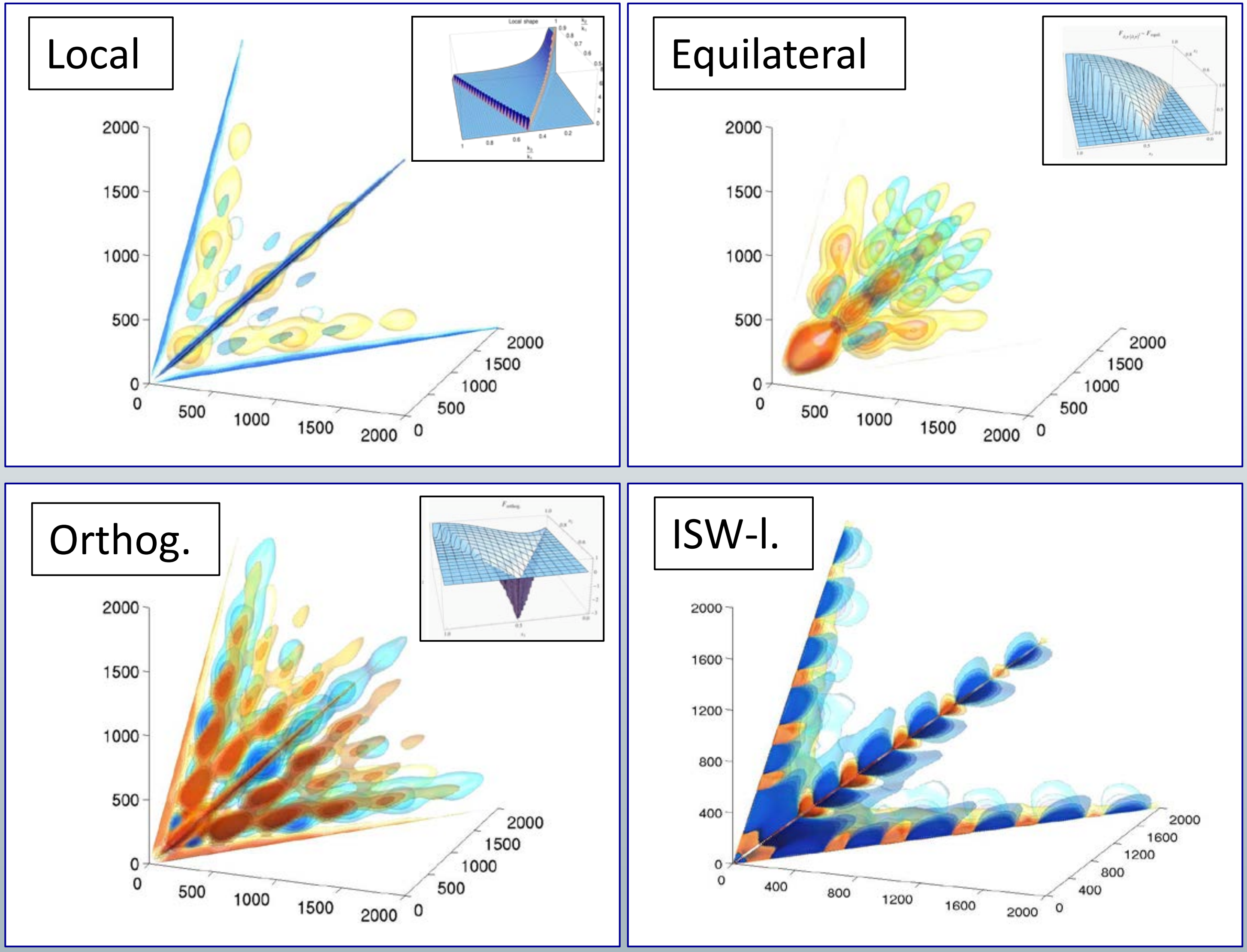
The (comoving) **Hubble radius (a H)$^{-1}$** ("effective horizon") decreases exponentially during inflation

**MULTI-FIELD INFLATION:**
Additional light fields, different from the inflaton contribute to the curvature perturbation:
**Non-linearities develop outside the horizon** from the evolution of this field and from the conversion of its fluctuations into density perturbation : **NG LOCAL IN REAL SPACE**




**INFLATION MODELS WITH HIGHER DERIVATIVE INTERACTIONS**
NG generated at horizon crossing: **correlation is among modes with comparable wavelength** which leave the horizon nearly at the same time. These interactions become exponentially irrelevant when the modes leave the horizon because (time and spatial derivatives become small)
**EQUILATERAL NG**

# Shapes


Local
2000
1500
1000
500
0
0
500
1000
1500
2000
Equilateral
Orthog.
ISW-l.
400
800
1200
1600

# Bispectrum shapes

- ***local*** shape: Multi-field models, Curvaton, Ekpyrotic/cyclic, etc. ...

- ***equilateral*** shape: Non-canonical kinetic term, DBI, K-inflation, Higher-derivative terms, Ghost, EFT approach

- ***orthogonal*** shape: Distinguishes between variants of non-canonical kinetic term, higher-derivative interactions, Galilean inflation

- ***flat*** shape: non-Bunch-Davies initial state and higher-derivative interactions, models where a Galilean symmetry is imposed. The flat shape can be written in terms of equilateral and orthogonal.

# Optimal $f_{NL}$ bispectrum estimator

$$\hat{f}_{NL} = \frac{1}{N} \sum B^{m_1 m_2 m_3}_{l_1 l_2 l_3} \left[ \left(C^{-1}a\right)^{m_1}_{l_1} \left(C^{-1}a\right)^{m_2}_{l_2} \left(C^{-1}a\right)^{m_3}_{l_3} - 3 C^{-1}_{l_1 m_1 l_2 m_2} \left(C^{-1}a\right)^{m_3}_{l_3} \right]$$

The theoretical template needs to be written in separable form. This can be done in different ways and *alternative implementations differ basically in terms of the separation technique adopted and of the projection domain*.

- KSW (Komatsu, Spergel & Wandelt 2003) separable template fitting + Skew-$C_l$ extension (Munshi & Heavens 2010)
- Binned bispectrum (Bucher, Van Tent & Carvalho 2009)
- Modal expansion (Fergusson, Liguori & Shellard 2009)

*Sub-optimal estimators also applied:*
*Wavelet decomposition (Martinez-Gonzalez et al. 2002; Curto et al. 2009) & Minkowski Functionals (Ducout et al. 2013)*

# Optimal $f_{NL}$ bispectrum estimator

$$\hat{f}_{NL} = \frac{1}{N} \sum B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \left(C^{-1}a\right)^{m_1}_{\ell_1} \left(C^{-1}a\right)^{m_2}_{\ell_2} \left(C^{-1}a\right)^{m_3}_{\ell_3} - 3C^{-1}_{\ell_1 m_1 \ell_2 m_2} \left(C^{-1}a\right)^{m_3}_{\ell_3}$$

Leaving aside complications coming from breaking of statistical isotropy (sky-cut, noise…), one can see that we are extracting the three point Function from the data and fitting theoretical bispectrum templates to it

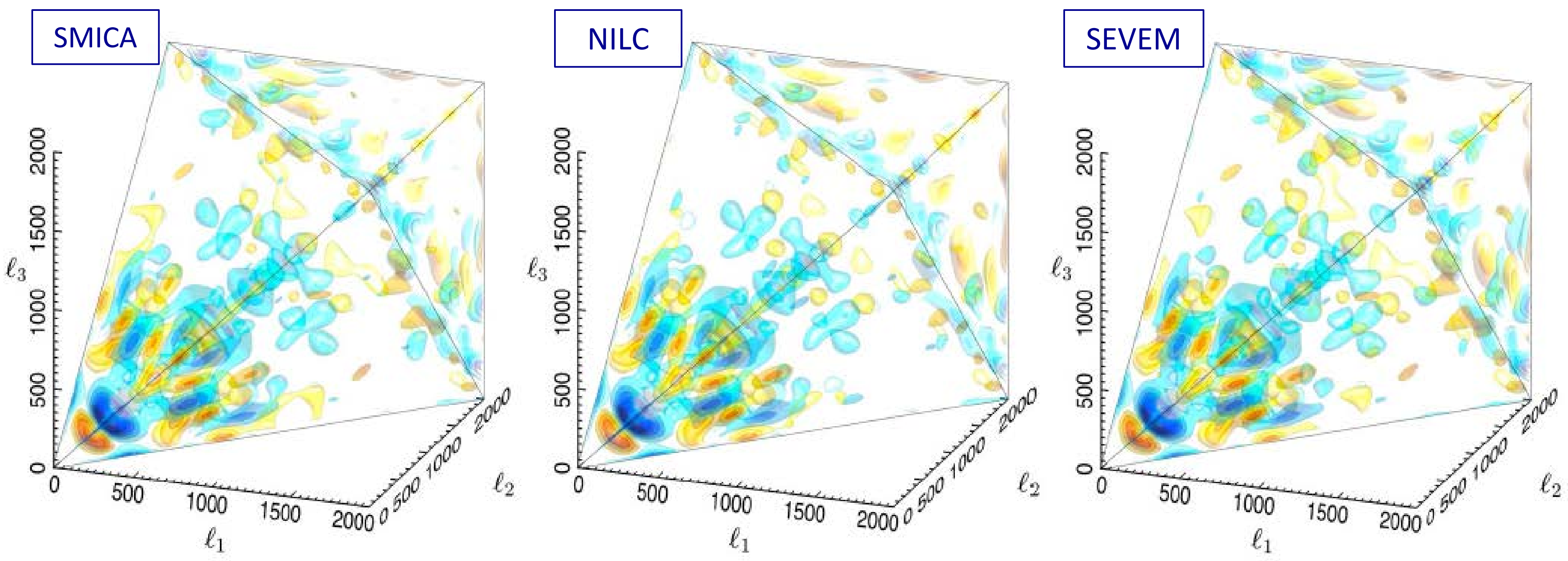
$$\hat{f}_{NL} = \frac{1}{N} \sum_{\ell_i m_i} B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \frac{a^{m_1}_{\ell_1}}{C_{\ell_1}} \frac{a^{m_2}_{\ell_2}}{C_{\ell_2}} \frac{a^{m_3}_{\ell_3}}{C_{\ell_3}}$$

A brute force implementation scales like $\ell^5_{\max}$. Unfeasible at Planck (or WMAP) resolution.

Can achieve massive speed improvement ( $\ell^3_{\max}$ scaling) if the reduced bispectrum is *separable* (Komatsu, Spergel, Wandelt 2003)

$$b_{\ell_1 \ell_2 \ell_3} = \sum_{ijk} X^i_{\ell_1} Y^j_{\ell_2} Z^k_{\ell_3} \Rightarrow B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} = b_{\ell_1 \ell_2 \ell_3} \int Y^{m_1}_{\ell_1}(\Omega) Y^{m_2}_{\ell_2}(\Omega) Y^{m_3}_{\ell_3}(\Omega)$$

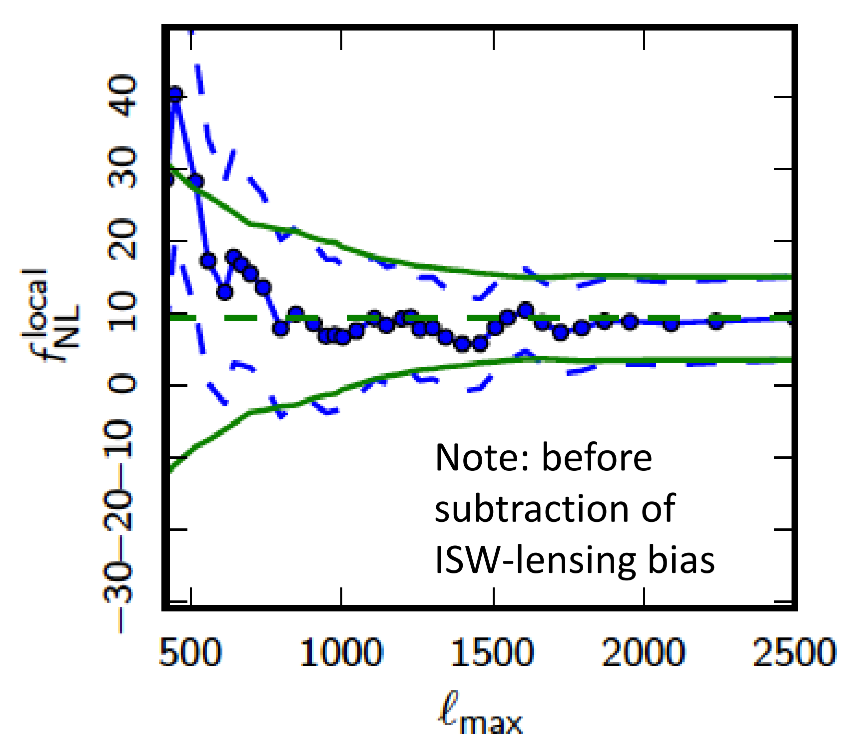
# The *Planck* modal bispectrum




Full 3D CMB bispectrum recovered from the *Planck* foreground-cleaned maps, including SMICA, NILC and SEVEM, using hybrid Fourier mode coefficients, These are plotted in three-dimensions with multipole coordinates $(l_1,l_2,l_3)$ on the tetrahedral domain out to $l_{max}$ = 2000. Several density contours are plotted with red positive and blue negative. The bispectra extracted from the different foreground-separated maps are almost indistinguishable

# Consistency with *WMAP*

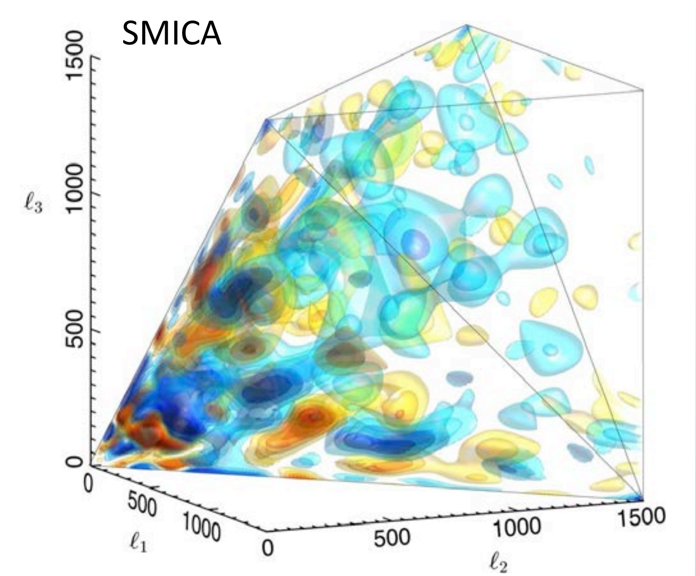
*WMAP* → *Planck*




- By limiting the analysis to large scales (low l), we make contact with *WMAP9* ($f_{NL}^{local} = 37.2 \pm 20$)
- *Planck* now rules out the *WMAP* central value by ≈ 6 sigmas.

# ISW-lensing bispectrum from *Planck*

The coupling between weak lensing and Integrated Sachs-Wolfe (ISW) effects is the leading contamination to local NG. We have detected the ISW lensing bispectrum with a significance of 2.6 σ



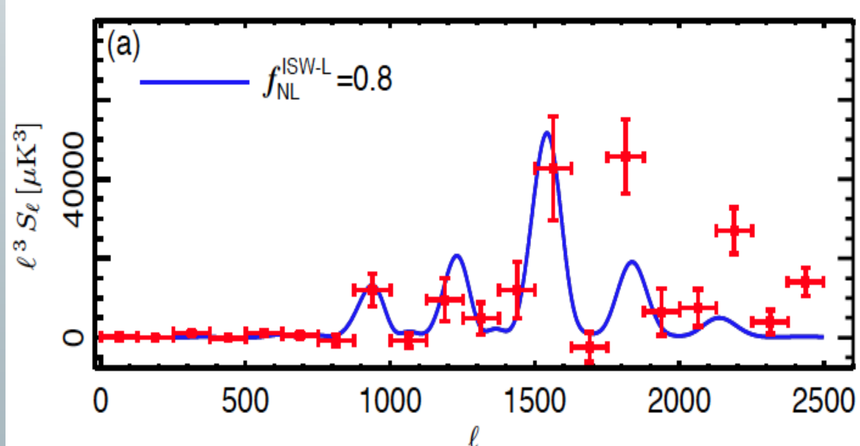

| | SMICA | NILC | SEVEM | C-R |
|---|---|---|---|---|
| KSW . . . . . | 0.81 ± 0.31 | 0.85 ± 0.32 | 0.68 ± 0.32 | 0.75 ± 0.32 |
| Binned . . . . | 0.91 ± 0.37 | 1.03 ± 0.37 | 0.83 ± 0.39 | 0.80 ± 0.40 |
| Modal . . . . | 0.77 ± 0.37 | 0.93 ± 0.37 | 0.60 ± 0.37 | 0.68 ± 0.39 |

Results for the amplitude of the ISW-lensing bispectrum from the SMICA, NILC, SEVEM, and C-R foreground-cleaned maps, for the KSW, binned, and modal (polynomial) estimators; error bars are 68% CL.

| | SMICA | NILC | SEVEM | C-R |
|---|---|---|---|---|
| Local . . . . . . . . . . . . . . . . | 7.1 | 7.0 | 7.1 | 6.0 |
| Equilateral . . . . . . . . . . . . | 0.4 | 0.5 | 0.4 | 1.4 |
| Orthogonal . . . . . . . . . . . . | −22 | −21 | −21 | −19 |

The bias in the three primordial fNL parameters due to the ISW-lensing signal for the 4 component-separation methods.

Skew-$C_l$ detection of ISW-lensing signal




SMICA

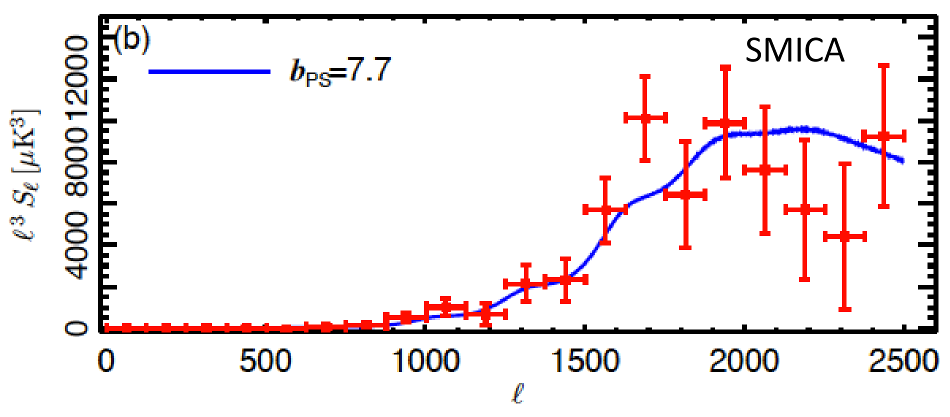
# Point-sources (Poissonian) bispectrum

- Results for the amplitude of the point-source (Poisson) bispectrum (in dimensionless units of $10^{-29}$) from the SMICA, NILC, SEVEM, and C-R foreground-cleaned maps, for the KSW, binned, and modal (polynomial) estimators; error bars are 68% CL. Note that the KSW and binned estimators use $l_{max}$ = 2500, while the modal estimator has $l_{max}$ = 2000.

| | SMICA | NILC | SEVEM | C-R |
|---|---|---|---|---|
| KSW . . . . . . . | $7.7 \pm 1.5$ | $9.2 \pm 1.7$ | $7.6 \pm 1.7$ | $1.1 \pm 5.1$ |
| Binned . . . . | $7.7 \pm 1.6$ | $8.2 \pm 1.6$ | $7.5 \pm 1.7$ | $0.9 \pm 4.8$ |
| Modal . . . . . . | $10 \pm 3$ | $11 \pm 3$ | $10 \pm 3$ | $0.5 \pm 6$ |

Skew-$C_l$ detection of Poissonian point-source bispectrum

Skew-$C_l$s are optimised statistics which retain information on the nature of any NG (Munshi & Heavens 2010)



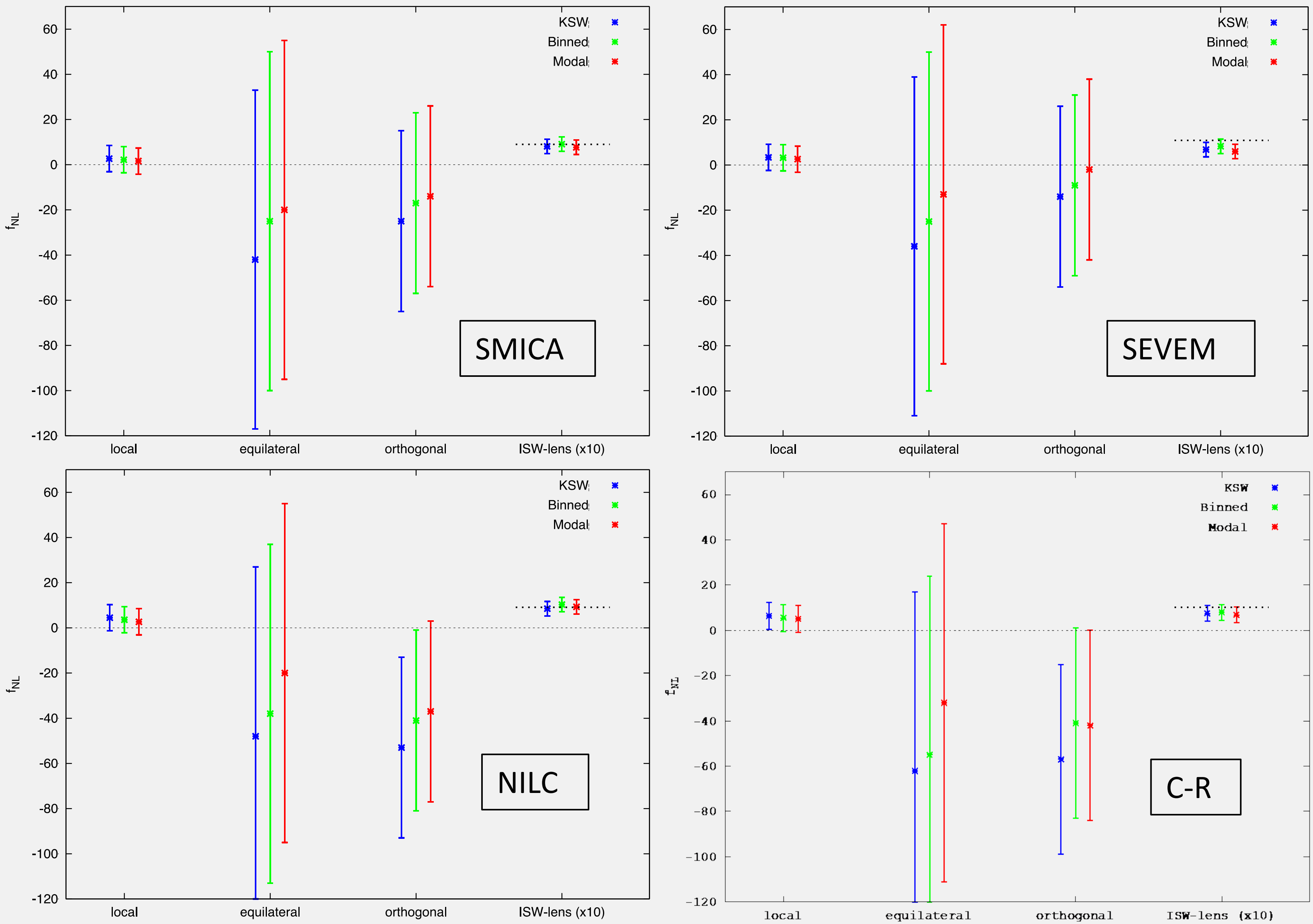
# Results for 3 fundamental shapes (KSW)

- Results for the $f_{NL}$ parameters of the primordial local, equilateral, and orthogonal shapes, determined by the KSW estimator from the SMICA foreground-cleaned map. Both independent single-shape results and results marginalized over the point-source bispectrum and with the ISW-lensing bias subtracted are reported; error bars are 68% CL.

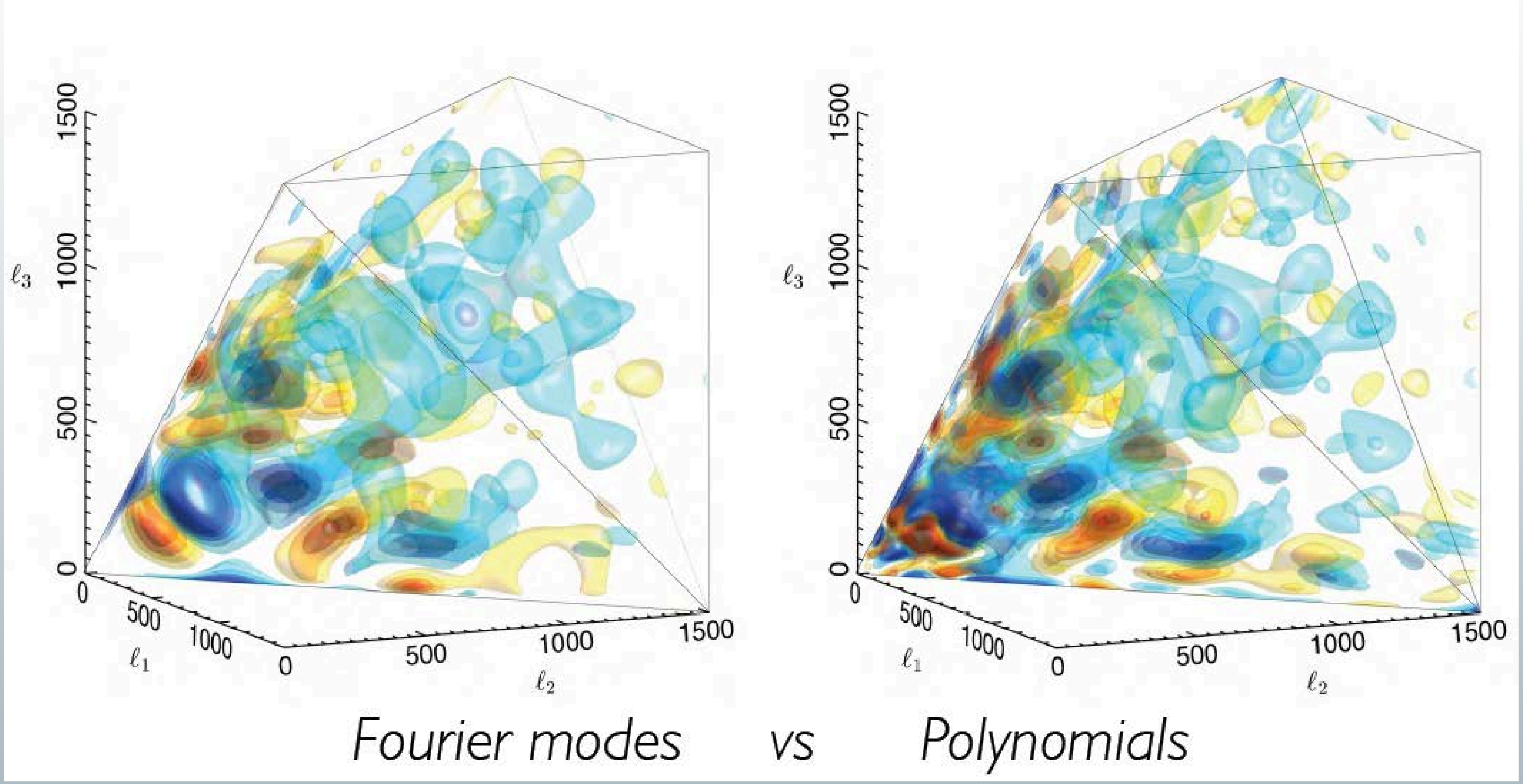
| | Independent KSW | ISW-lensing subtracted KSW |
|---|---|---|
| SMICA | | |
| Local . . . . . . . . . . | $9.8 \pm 5.8$ | **$2.7 \pm 5.8$** |
| Equilateral . . . . . | $-37 \pm 75$ | **$-42 \pm 75$** |
| Orthogonal . . . . . | $-46 \pm 39$ | **$-25 \pm 39$** |

- Union Mask U73 (73% sky coverage) used throughout. Diffusive inpainting pre-filtering procedure applied.

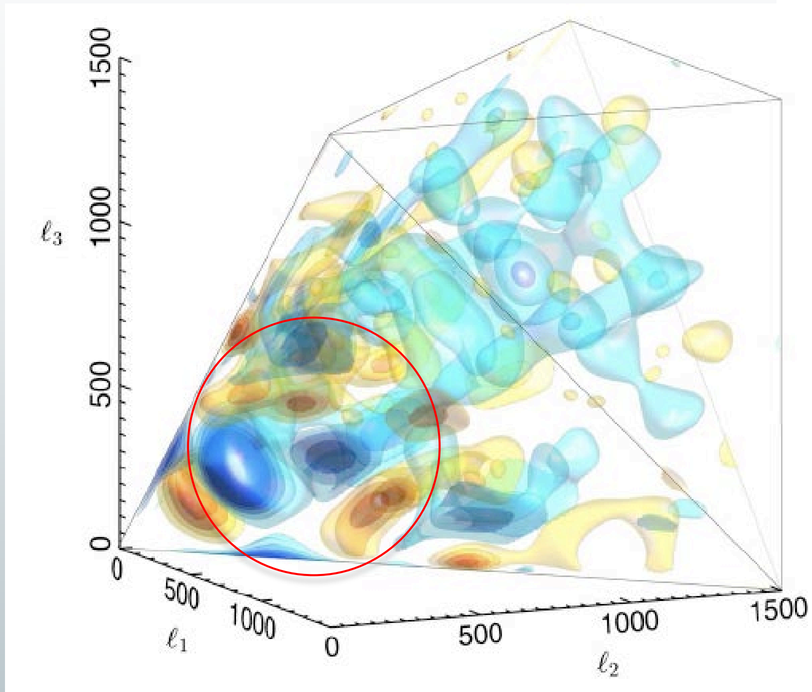
# $f_{NL}$ from *Planck* data

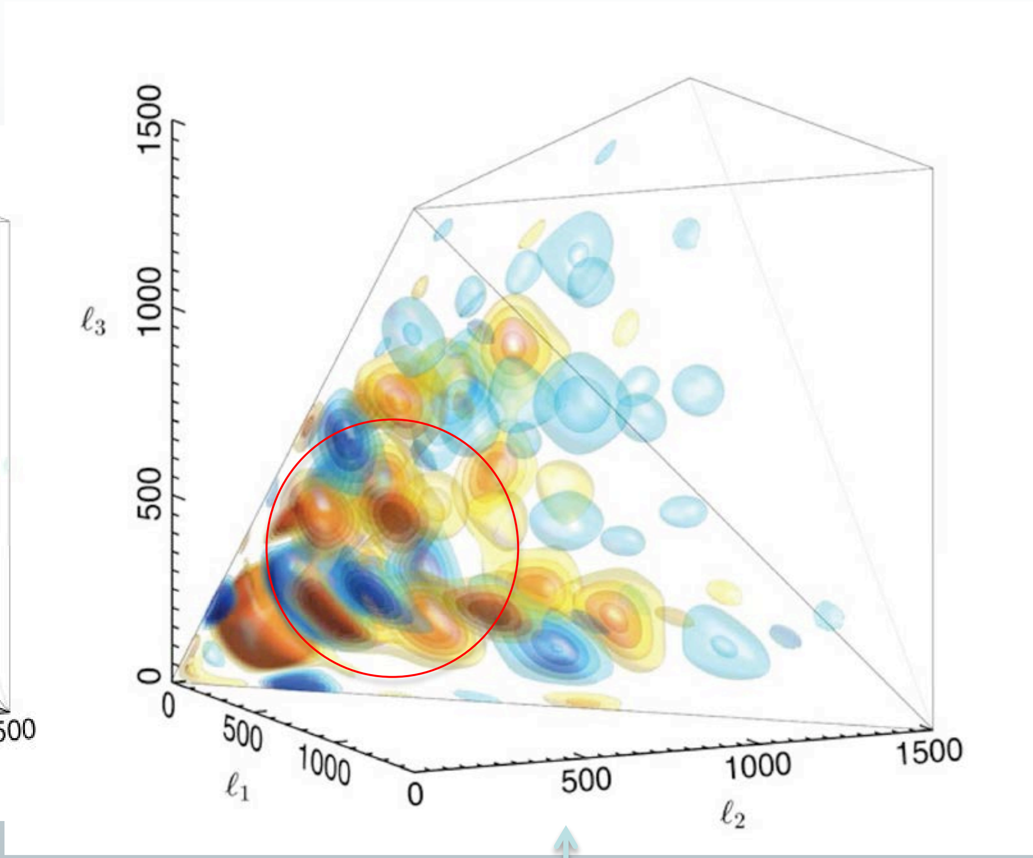


# The *Planck* bispectrum



# Non-standard NG shapes: *Planck* vs. feature models




3D CMB bispectrum as seen by Planck (reconstructed bispectrum)

Bispectrum for the best-fit feature model

# Standard inflation vs. NG

Standard inflation:

- *single scalar field*
- *canonical kinetic term*
- *slow-roll dynamics*
- *Bunch-Davies initiual vacuum state*
- *standard Einstein gravity*

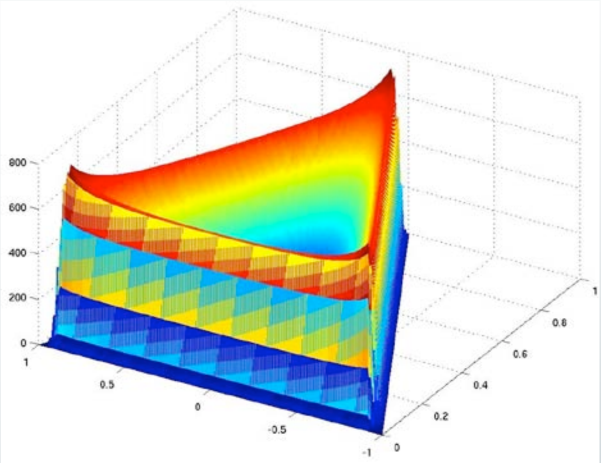
→ no (presently) detectable primordial NG

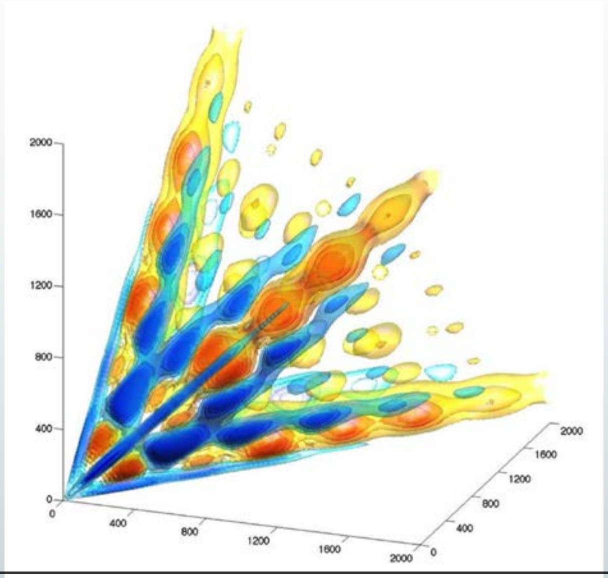
# Some non-standard shapes: excited initial states

Non-Bunch-Davies vacua from trans-Planckian effects or features

Five exemplar flattened models constrained

NBD case




| Flattened model (Eq. number) | Raw $f_{\rm NL}$ | Clean $f_{\rm NL}$ | $\Delta f_{\rm NL}$ | $\sigma$ | Clean $\sigma$ |
|---|---|---|---|---|---|
| Flat model (13) . . . . . . . . . . . . . . . . . . . . . . | 70 | 37 | 77 | 0.9 | 0.5 |
| Non-Bunch-Davies (NBD) . . . . . . . . . . . . | 178 | 155 | 78 | 2.2 | 2.0 |
| Single-field NBD1 flattened (14) . . . . . . . | 31 | 19 | 13 | 2.4 | 1.4 |
| Single-field NBD2 squeezed (14) . . . . . . . | 0.8 | 0.2 | 0.4 | 1.8 | 0.5 |
| Non-canonical NBD3 (15) . . . . . . . . . . . . | 13 | 9.6 | 9.7 | 1.3 | 1.0 |
| Vector model $L = 1$ (19) . . . . . . . . . . . . . | −18 | −4.6 | 47 | −0.4 | −0.1 |
| Vector model $L = 2$ (19) . . . . . . . . . . . . . | 2.8 | −0.4 | 2.9 | 1.0 | −0.1 |

➢ Note we also constrained inflation with gauge fields (vector models)

# Multi-field models of inflation: the curvaton case

$$\Phi(\boldsymbol{x}) = \Phi_L(\boldsymbol{x}) + f_{\rm NL}^{\rm local}(\Phi_L^2(\boldsymbol{x}) - \langle\Phi_L^2(\boldsymbol{x})\rangle)$$

A second scalar field, different from the inflaton and subdominant during inflation, decays after inflation with its fluctuations converted on super-horizon scale to the final gravitational perturbations

$$f_{\rm NL}^{\rm local} = \frac{5}{4r_{\rm D}} - \frac{5r_{\rm D}}{6} - \frac{5}{3}$$

$$r_D = \frac{3\rho_{\rm curvaton}}{3\rho_{\rm curvaton} + 4\rho_{\rm radiation}}$$

$f_{\rm NL}^{\rm loc} = 2.7 \pm 5.8 \quad (68\%{\rm CL})$ → $r_{\rm D} \geq 0.15$ at 95% CL

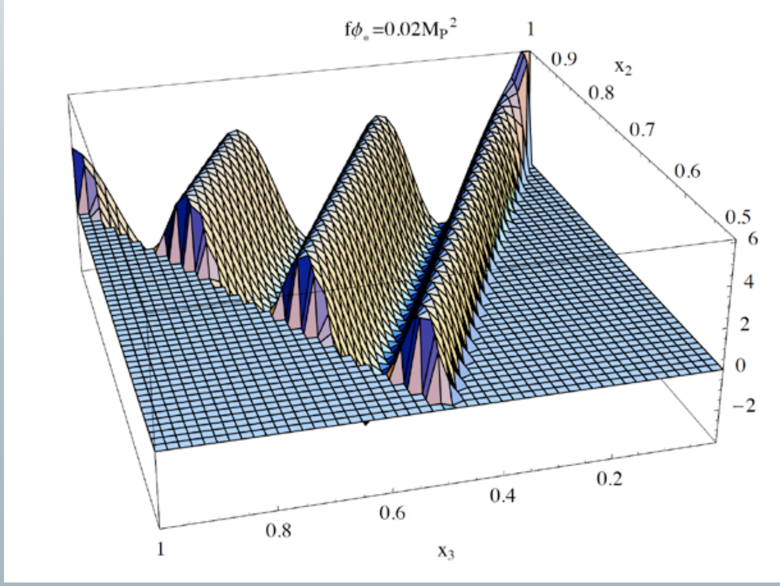
# Models with non-standard NG shapes

***Feature and resonant models***: oscillating bispectra due to

✓ a sharp feature (e.g. step-like) in the inflaton potential
(Wang & Kamionkowski 2000; Chen et al. 07)

✓ periodic features: e.g. ***axion inflation*** $V(\phi)=V_0(\phi)[1+\lambda\cos(\phi/f)]$
(recent interest in axion monodromy inflation motivated by string theory
e.g. McAllister et al. 2010; Siverstein & Westphal 08; Flauger et al 09;
Flauger and Pajer 2011).

Inflaton quantum perturbations can resonate with oscillatory features of the background evolution generating large interactions (NG)



# *Planck* $\tau_{NL}$ constraint

Estimator result $\hat{\tau}_{NL} = 442.$

Gaussian simulations:

$$-452 < \hat{\tau}_{NL} < 835 \text{ at } 95\% \text{ CL } (\sigma_{\tau_{NL}} \approx 335)$$

Consistent with Gaussian null hypothesis (octopole has small weight)

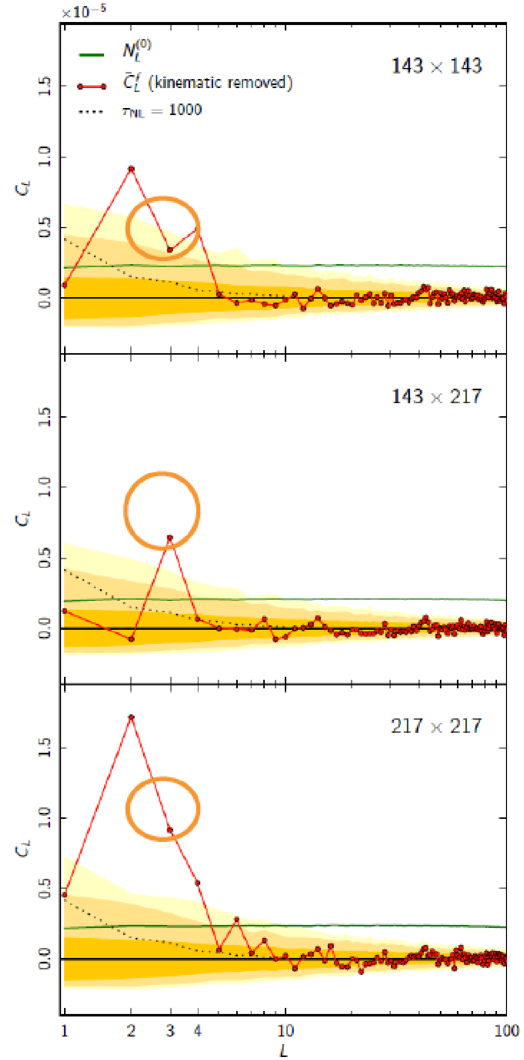
*Note*: signal most L<5 - small number of modes

- Skewed distribution
- Upper limits weaker than you might expect

Conservative upper limit, allowing octopole to be physical using Bayesian posterior

$$\tau_{NL} < 2800 \text{ at } 95\% \text{ CL}$$

Octopole signal varies between frequencies:
(large auto-quadrupole expected from noise bias)



# Conclusions I

- *Planck* has measured with exquisite precision cosmological parameters, including those characterizing primordial (inflationary) perturbations (scalar spectral index, tensor-to-scalar ratio)
- Primordial NG has become a high precision cosmological observable, crucial to unveil the nature of the inflaton field(s), in that it probes the interactions of the inflaton field, i.e. the *physics of the Early Universe*
- *Planck* measurement of primordial non-Gaussianity represents the most stringent test of inflation obtained so far: deviations from primordial Gaussianity are now constrained to be less than 0.01%
- Standard models of slow-roll single-field inflaton have survived; other models ruled out and for many models NG dramatically reduces the parameter space
- *Planck* data bound general single-field and multi-field model parameters, such as the speed of sound, $c_s \geq 0.02$ (95% CL), in an effective field theory parametrization ($c_s \geq 0.07$ *for DBI inflation*), and the curvaton decay fraction $r_D \geq 0.15$ (95% CL). The amplitude of the four-point function in the local model is $\tau_{NL} < 2800$ (95% CL).

# Summary

- The **simplest** inflation models (single-field slow-roll, standard kinetic term, BD initial vacuum state) are favoured by *Planck* data

- Multi-field models are not ruled out but also not detected

- <u>*Taken together, these constraints represent the highest precision tests to date of physical mechanisms for the origin of cosmic structure*</u>

# Future prospects

- <u>short term goals</u>
  - *Improve upper bound on stochastic gravitational-wave background*
  - *Improve $f_{NL}$ limits with polarization & full data*
  - *Look for more non-Gaussian shapes, scale-dependence, etc. ...*
  - *constrain $g_{NL}$*
- <u>long terms goals</u>
  - *detect stochastic gravitational-wave background ...*
  - *reconstruct inflationary action*
  - *if (quadratic) NG turns out to be small for all shapes go on and search for $f_{NL} \sim 1$ non-linear GR effects and second-order radiation transfer function contributions*
  - *what about intrinsic ($f_{NL} \sim 10^{-2}$) NG of standard inflation? CMB polarization + LSS + 21cm background + CMB spectral distortions*