The basics of scientific calibration in CMB experiments

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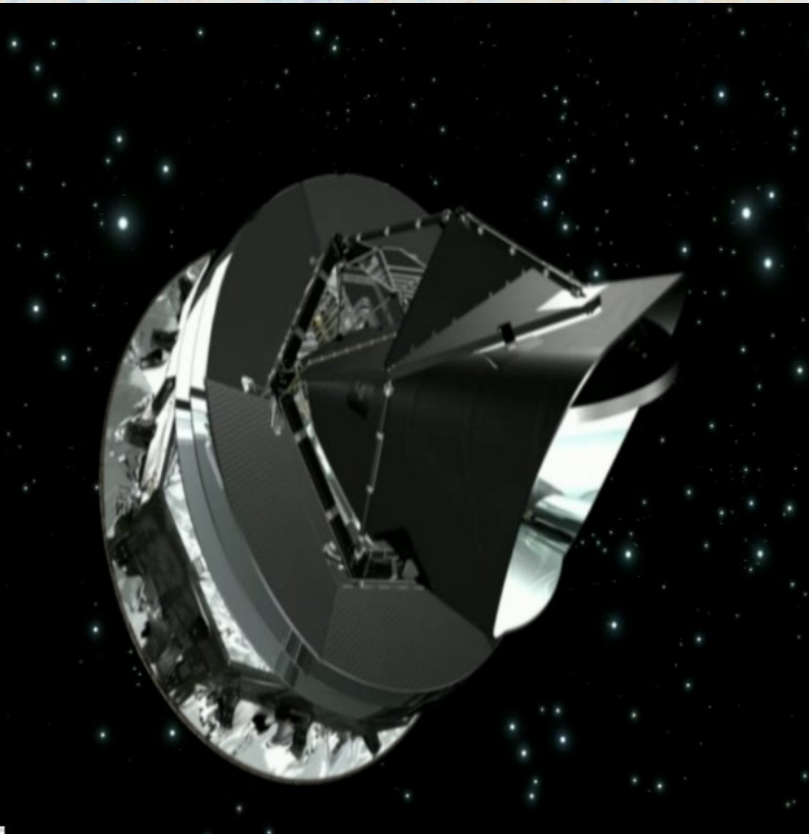
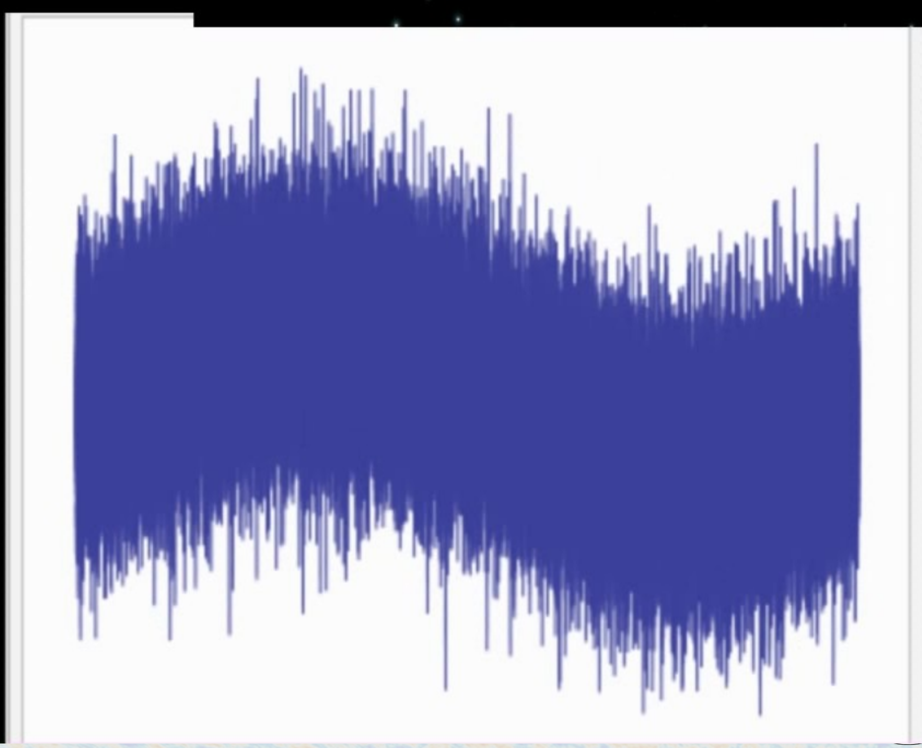
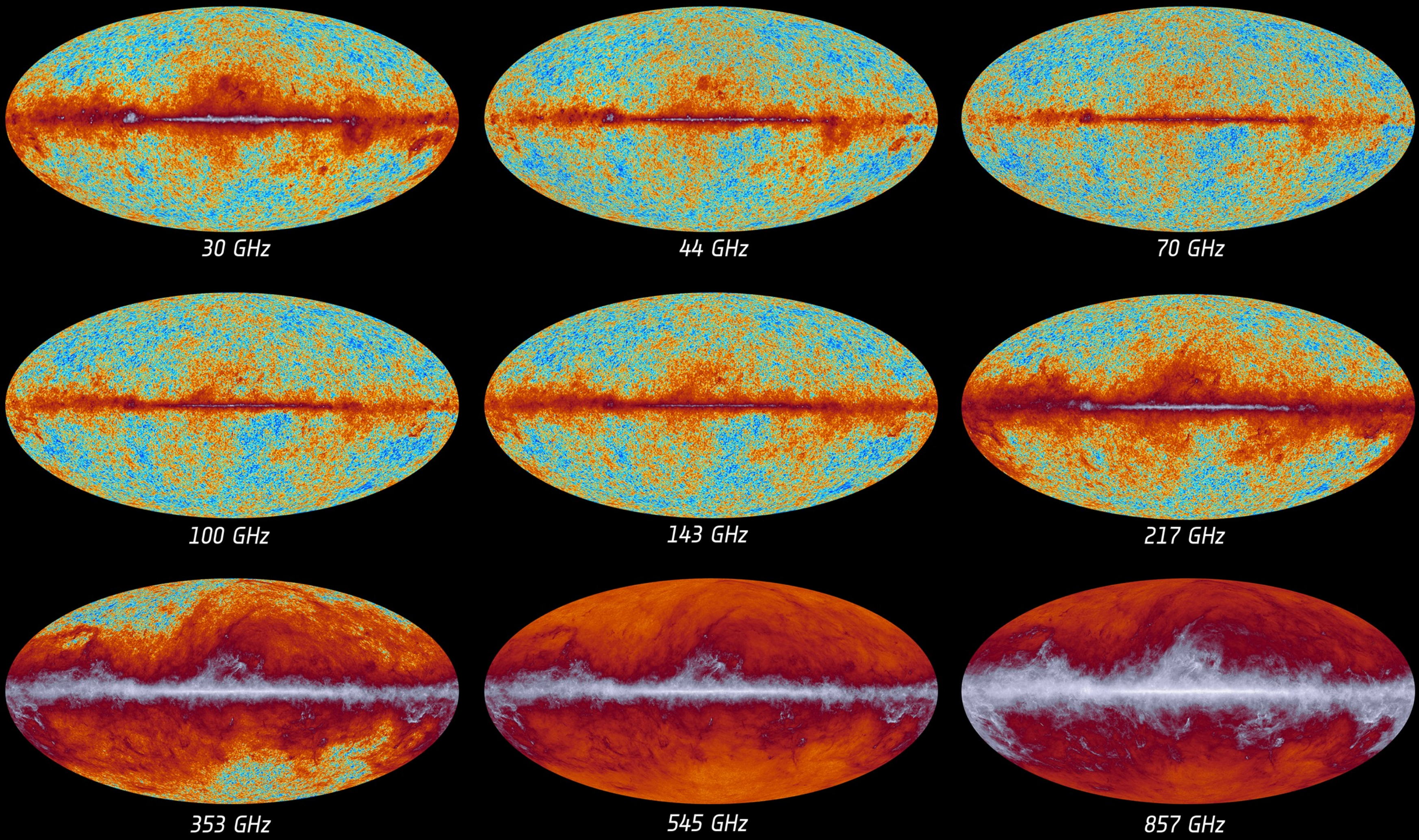
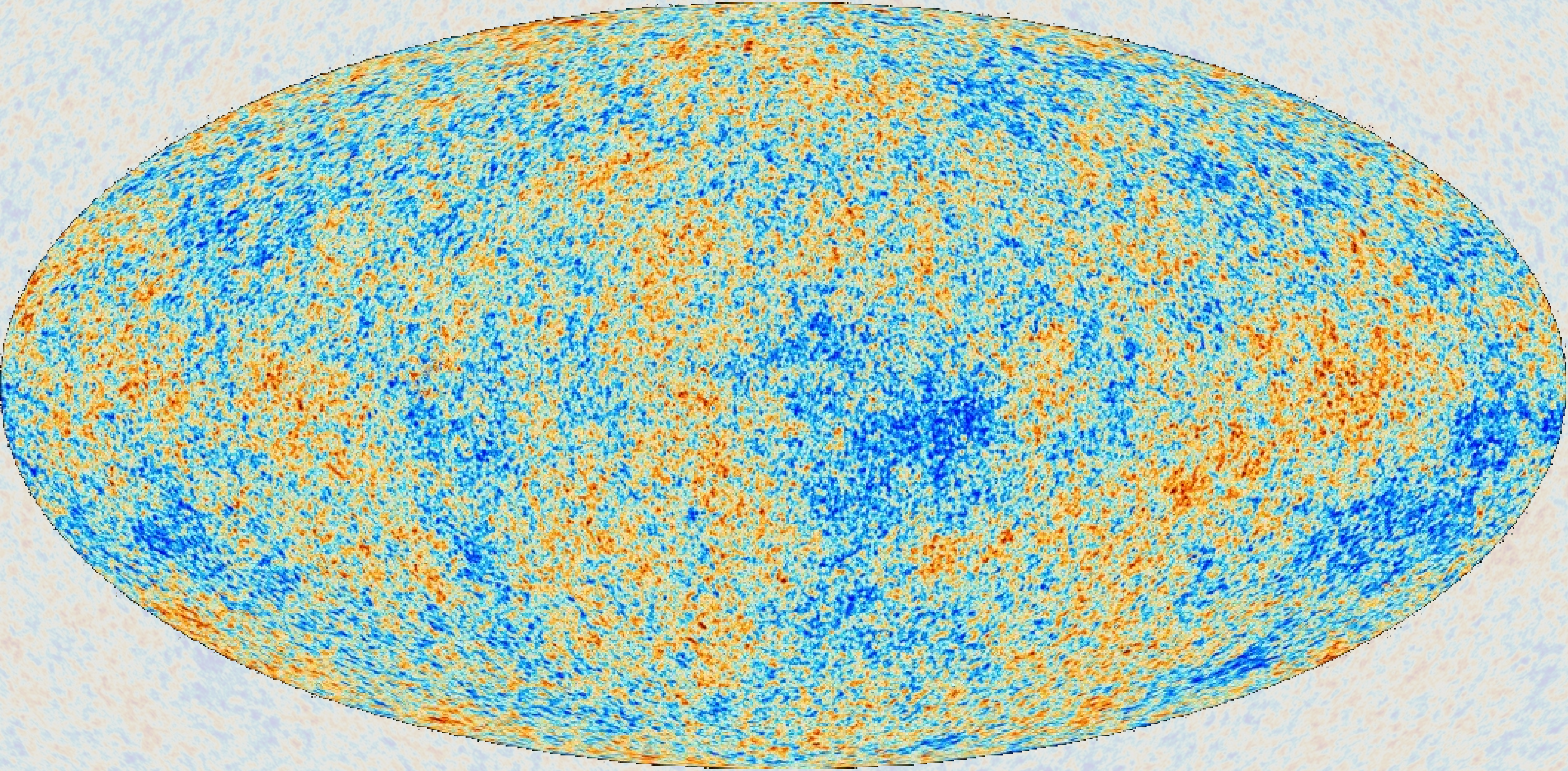




The sky as seen by Planck



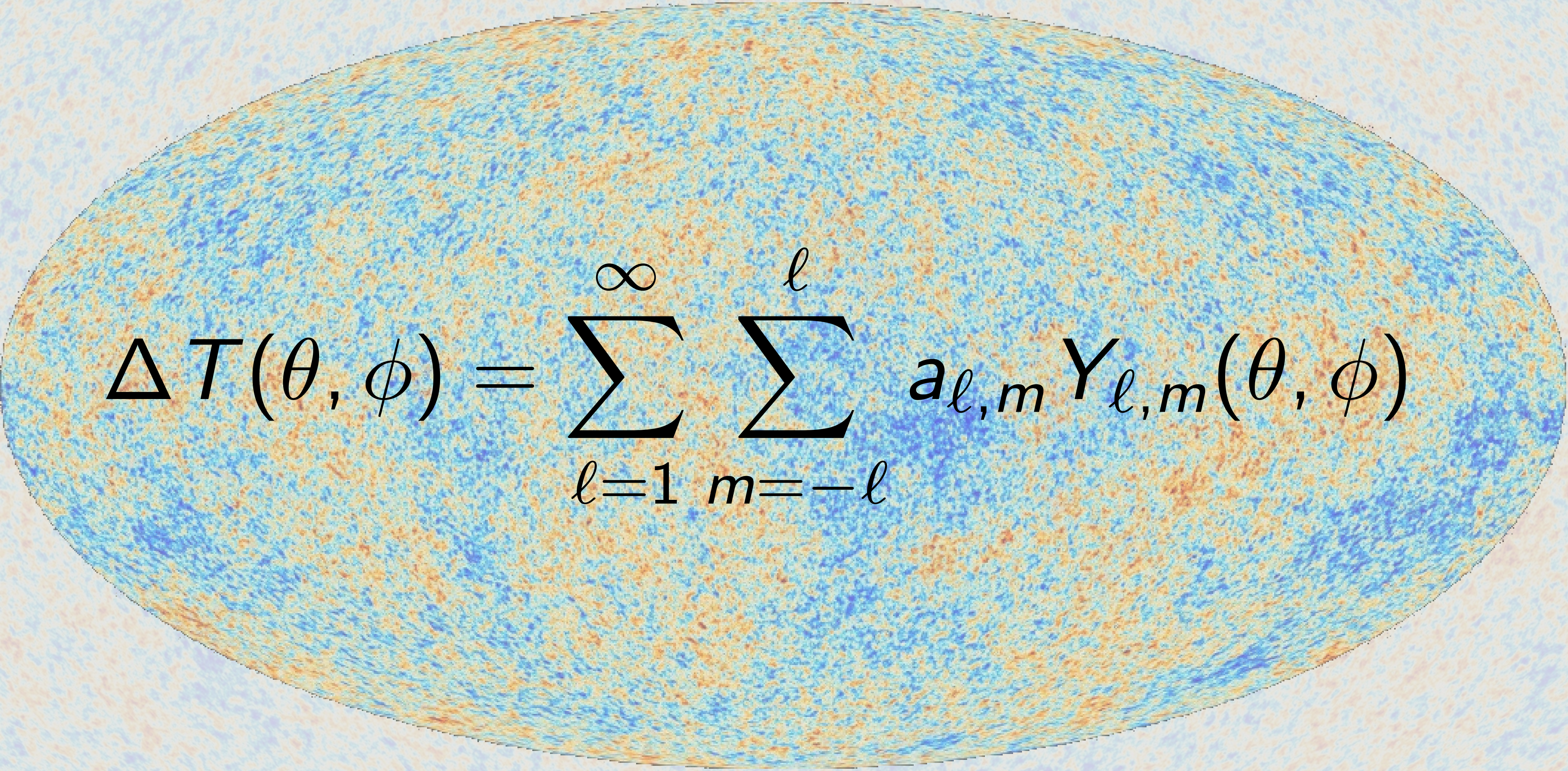
The first and oldest light in the universe



Aniello Mennella

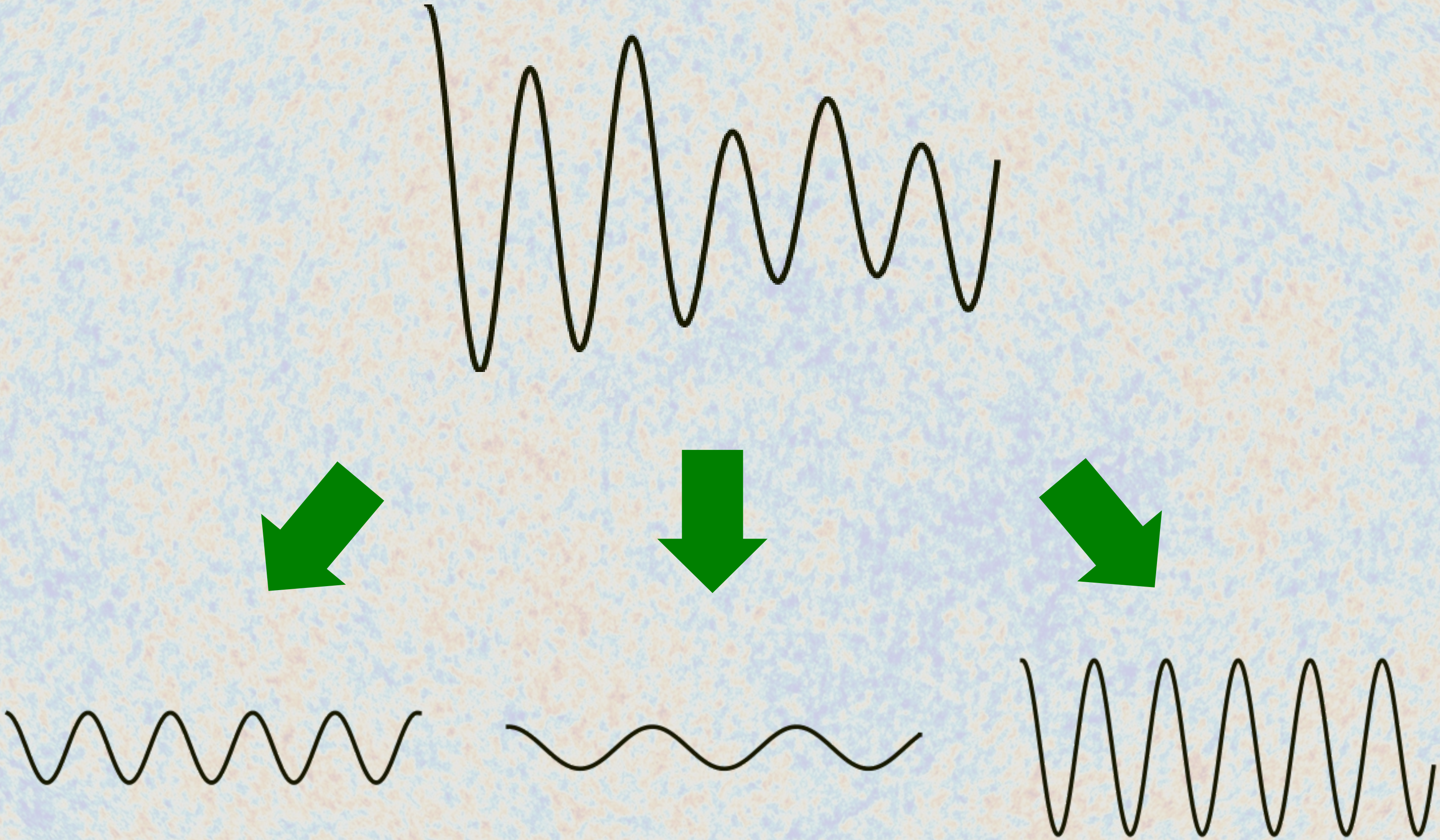
Observing the Universe with the Cosmic Microwave Background
L'Aquila, April 22-26, 2014

Spherical harmonics

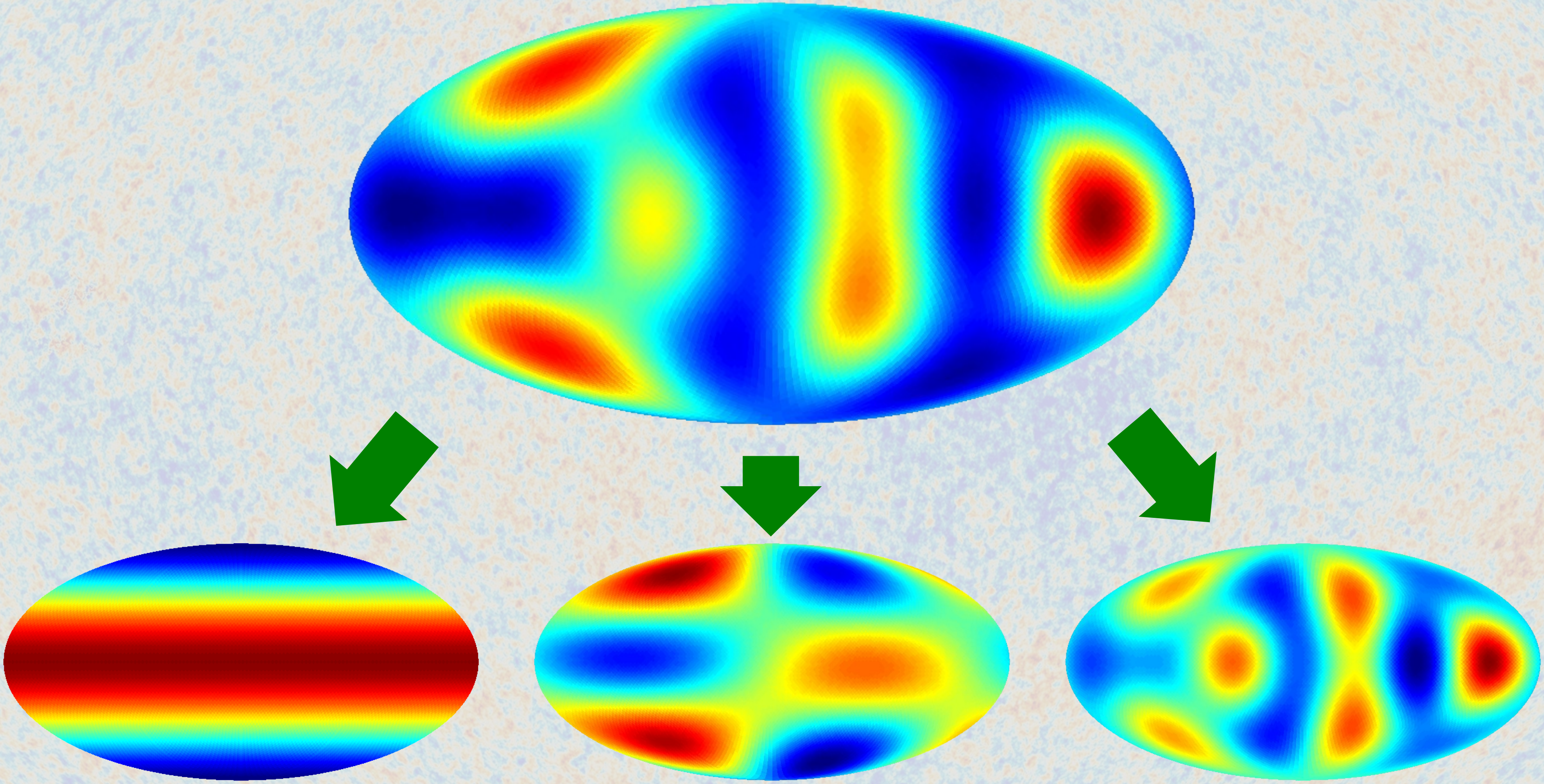


$$\Delta T(\theta,\phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell,m} Y_{\ell,m}(\theta,\phi)$$

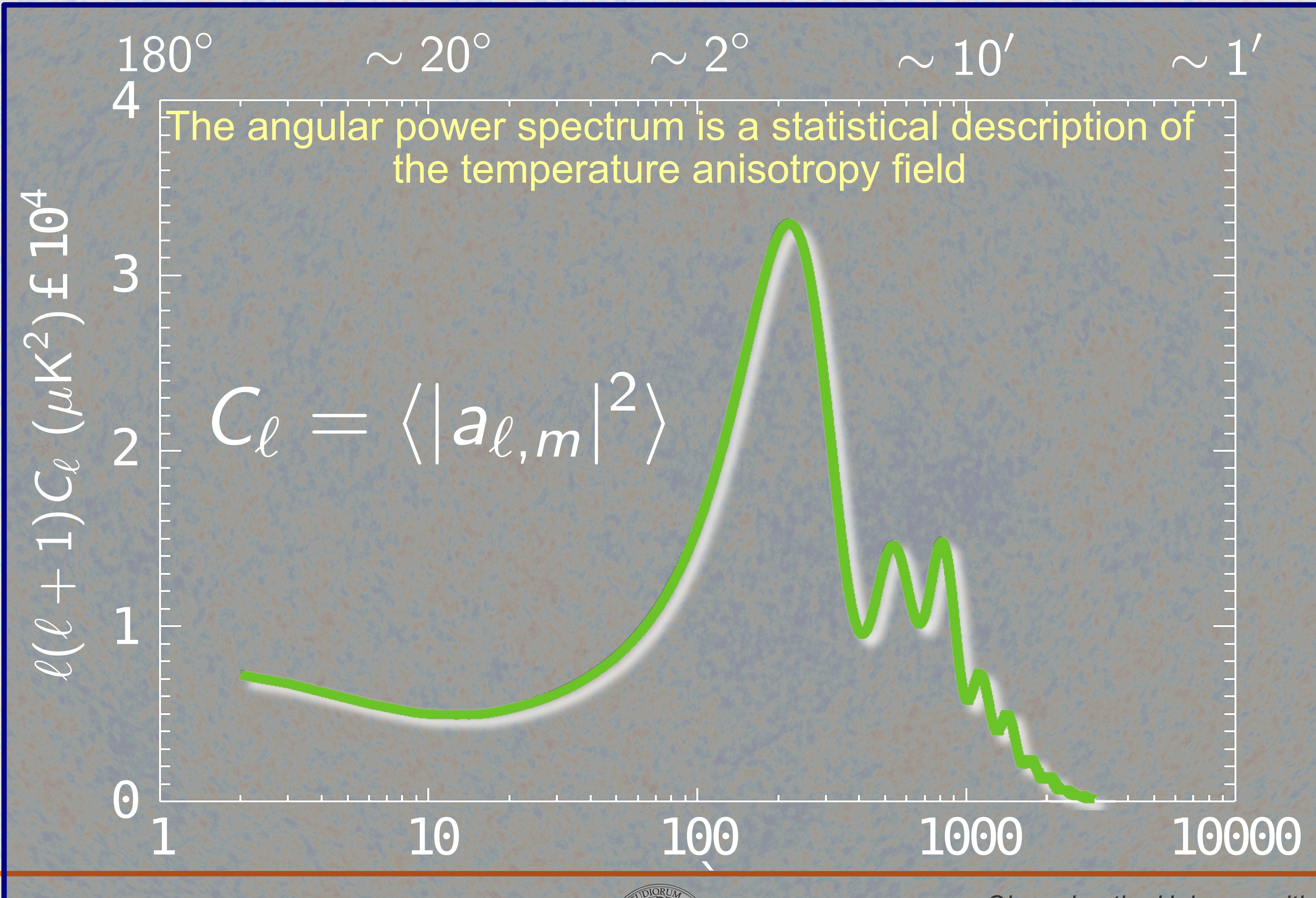
Fourier analysis



Spherical harmonics



The angular power spectrum



- How do we convert time-ordered data in Volt or in other units into a map in Kelvin?

- What instrument properties must be taken into account in the calculation of the “true” power spectrum?

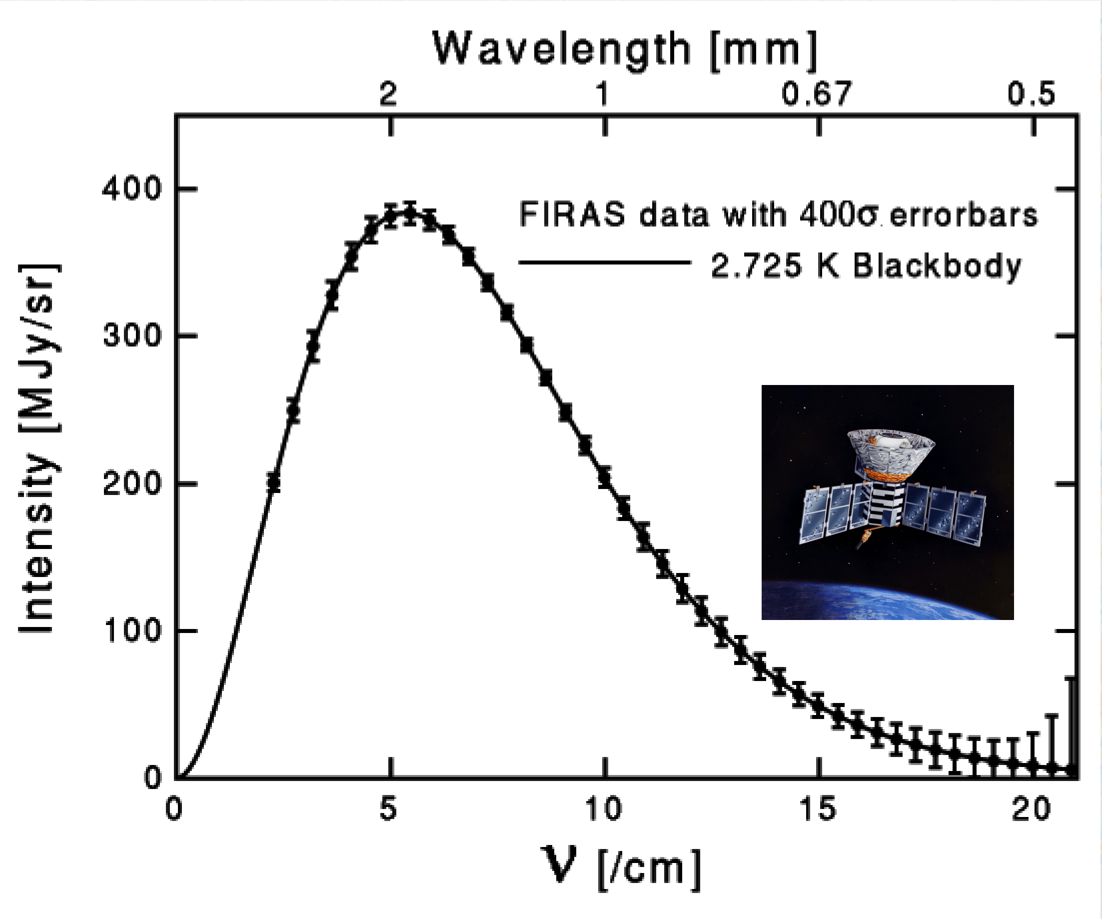
- How do we transform correctly a calibrated map into a power spectrum?

We need step back and define few basic quantities, conventions and instrumental properties which are common in CMB experiments

Why do we measure the field intensity in Kelvin?

The spectrum of the CMB

- The CMB has the spectrum of a black body
- This is one of the strongest evidences of the matter-radiation thermal equilibrium in the primordial universe



The surface brightness

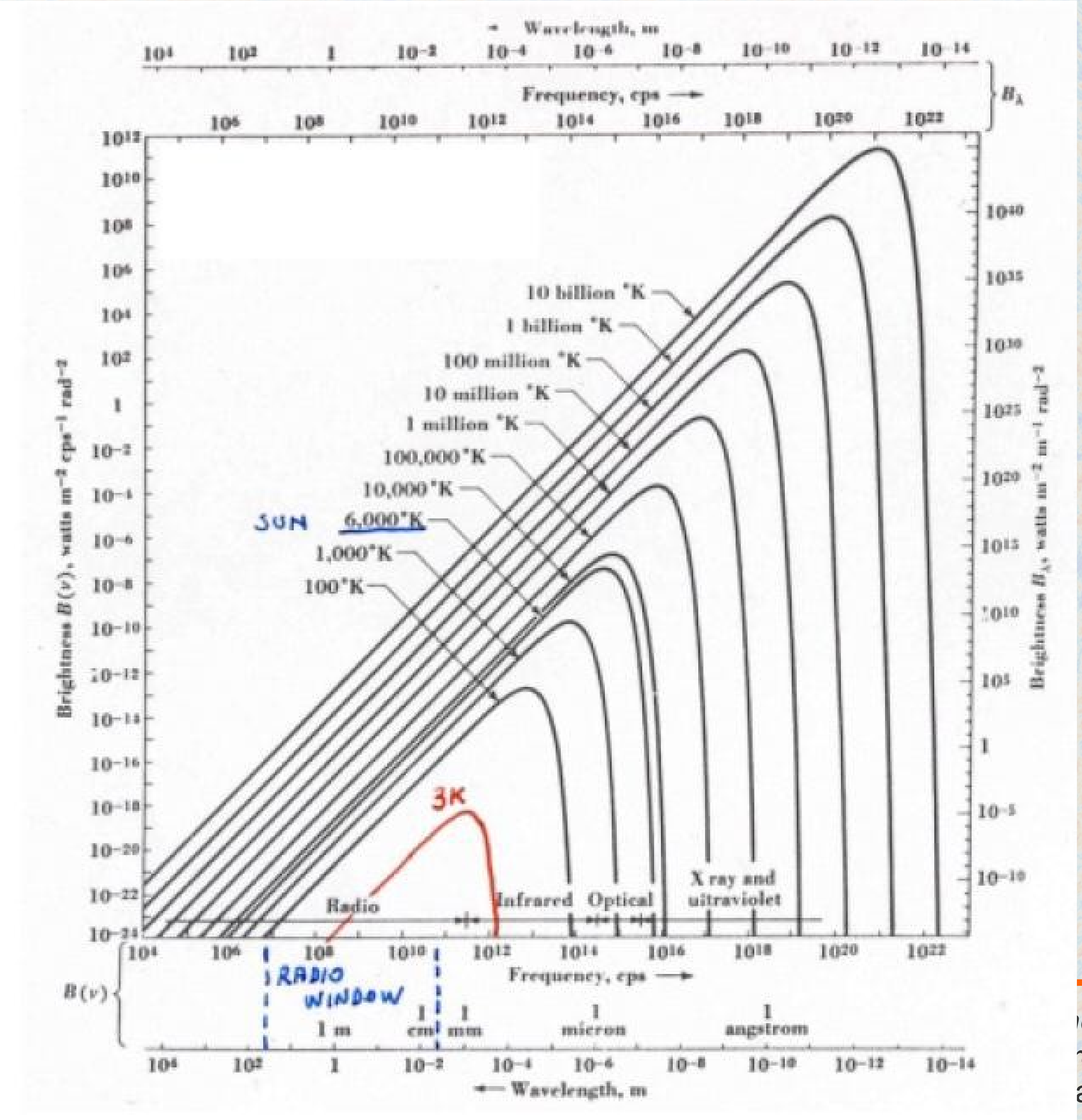
- The power per unit frequency, per unit area and per unit solid angle is defined **brightness.** It is:

$$B(\nu) = \frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]^{-1}$$

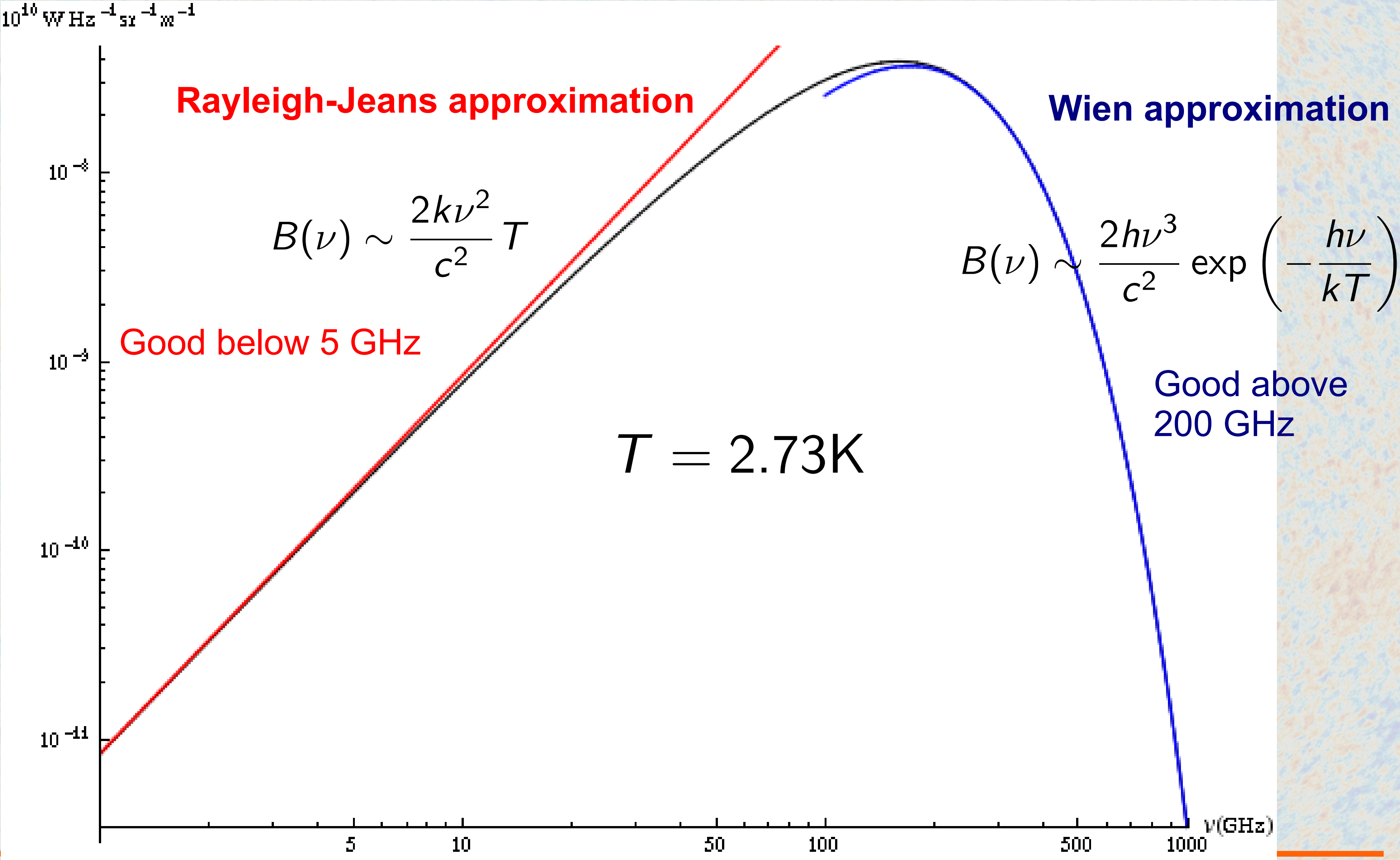
we measure it in W m^{-2} Hz^{-1} sr^{-1}

- Notice that it depends only on the *temperature* of the emitting body

The black body spectrum at different temperatures



Approximations of the black body spectrum



The brightness temperature

- Let us consider a source with brightness $B(\nu)$. We can write the following relationship:

$$T_B = \frac{c^2}{2k\nu^2} B(\nu)$$

T_B is called **brightness temperature**

Exercises

1. Show that T_B has the units of a temperature

2. Show that **if** $h\nu / kT << 1$ **and** $B(\nu)$ is a black-body spectrum **then** T_B corresponds to the *thermodynamic temperature* of the source

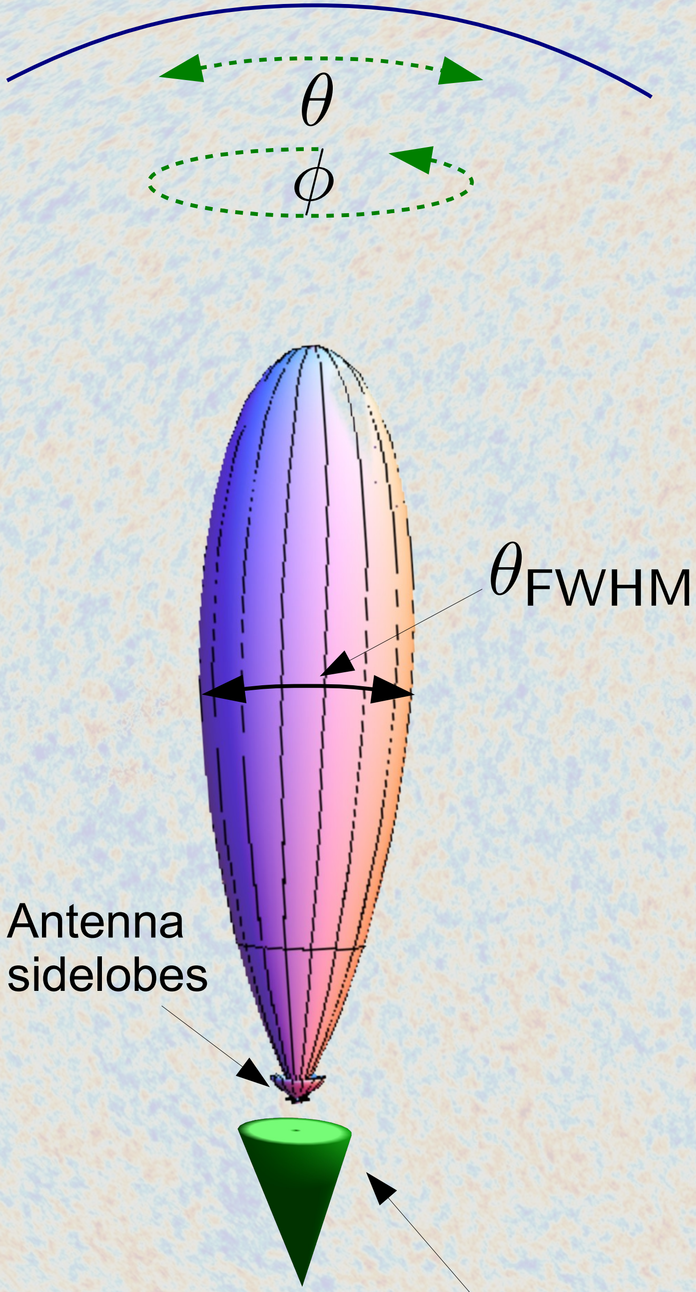
The brightness temperature

- We have seen that there is a relationship between the power emitted by a source and its brightness temperature
- For black-body emissions at enough low frequencies the brightness temperature **is** the thermodynamic temperature
- For black-body emissions at a general frequency ν, the relationship between brightness temperature and thermodynamic temperature is:

$$\boxed{T_B = \frac{x}{e^x - 1} T_{\mathrm{th}} \ \text{ where } \ x \equiv h\nu / kT}$$

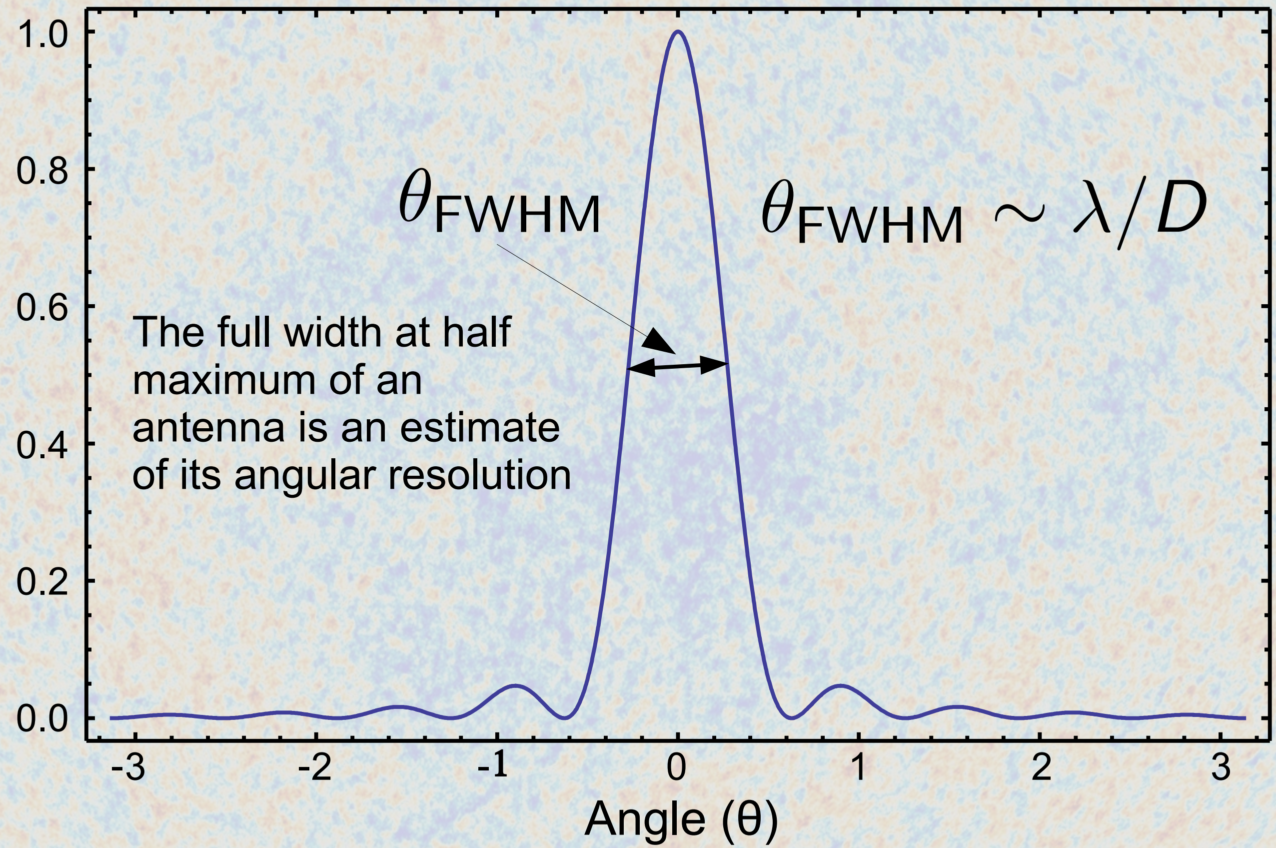
How the instrument optics observes a source in the sky

The antenna beam pattern

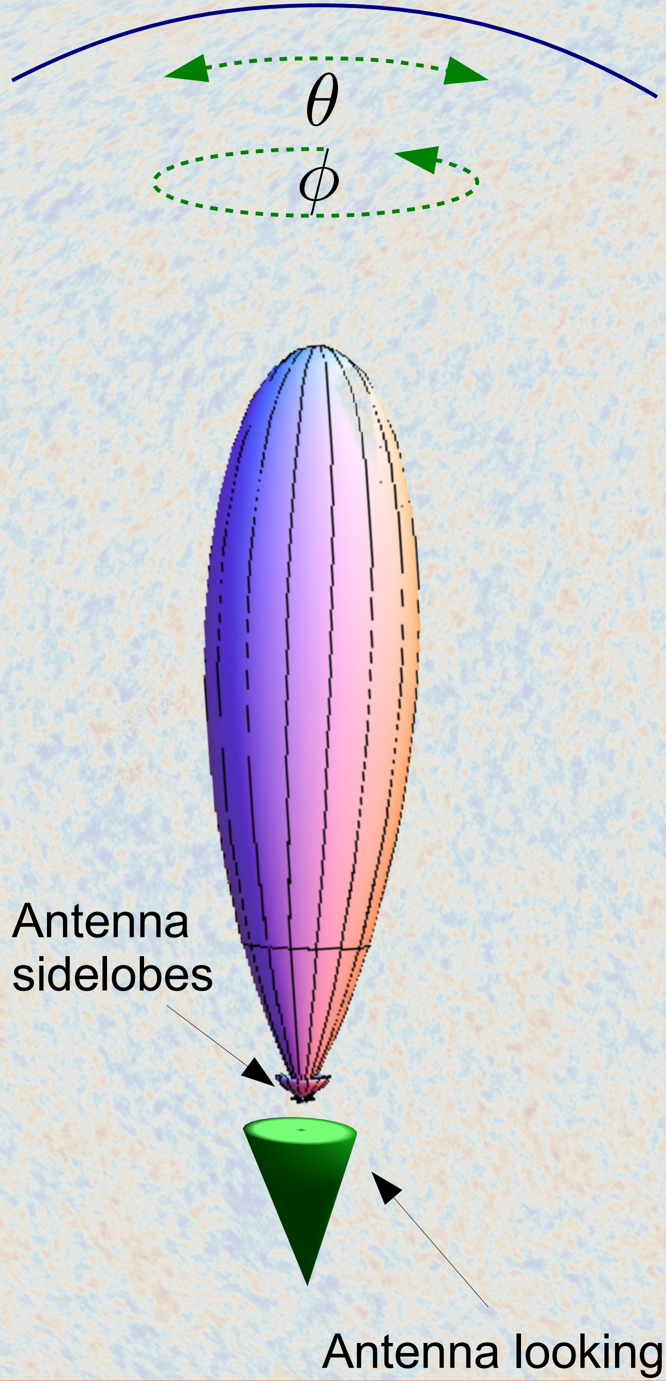


Antenna looking at the sky

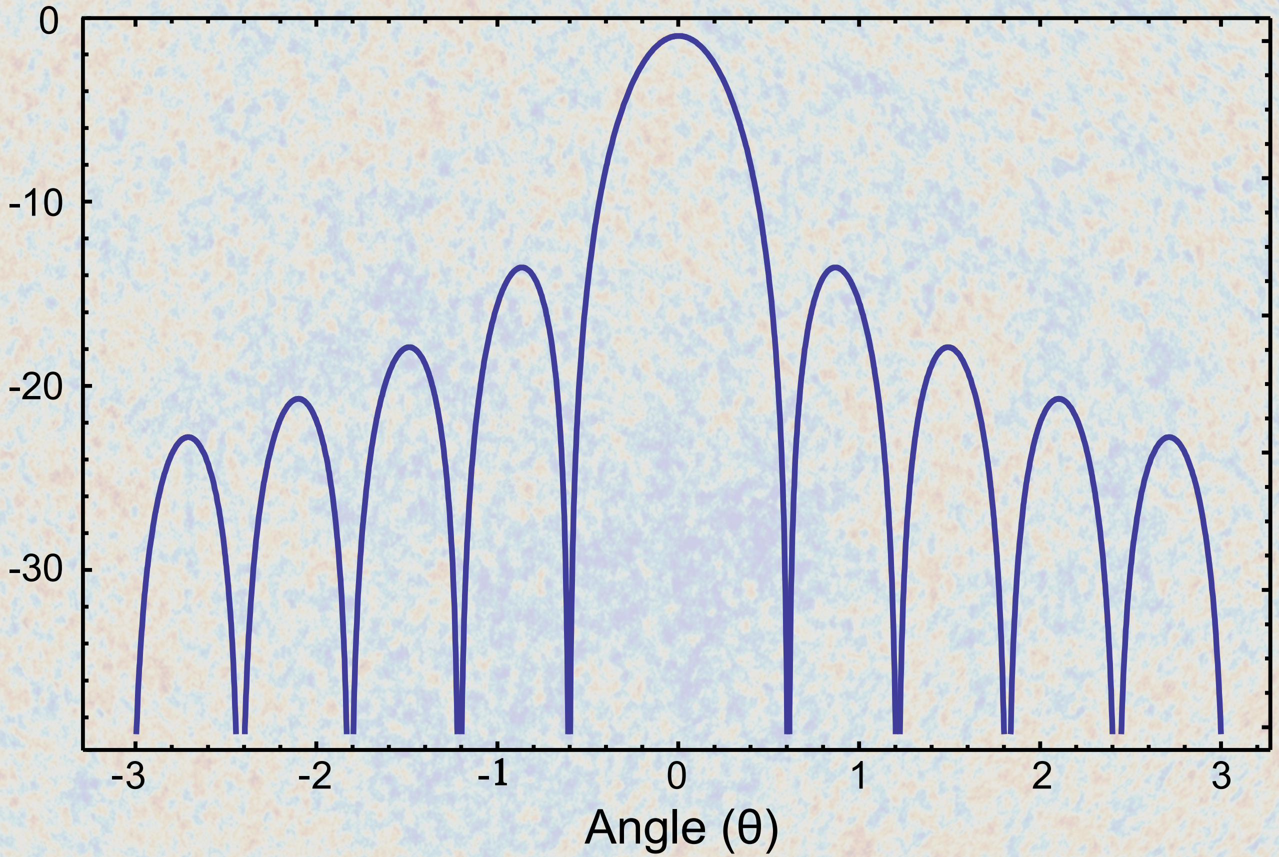
Power normalized to maximum ($P_n \equiv P\,/\,P(0,0)$)



The antenna beam pattern



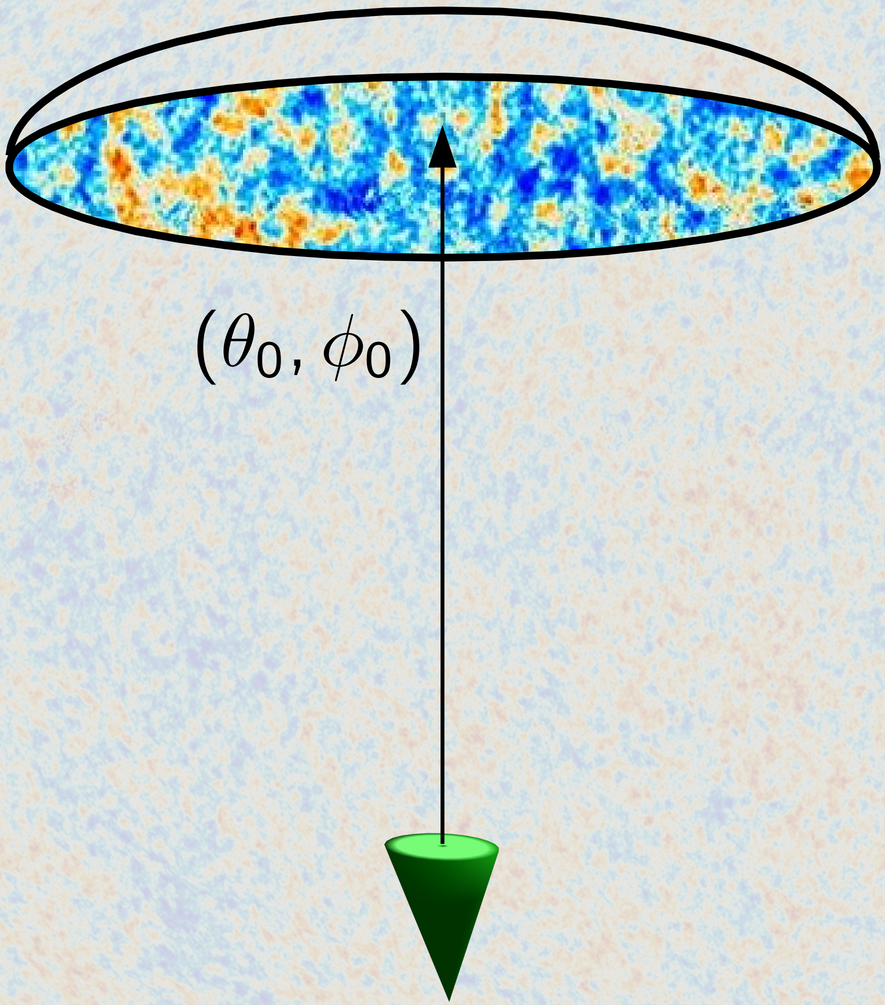
Normalised power in dB: $P(\mathrm{dB}) = 10\log_{10}(P_n)$



$$\boxed{\Omega_a = \int_{4\pi} P_n(\theta,\phi)\,d\Omega}$$

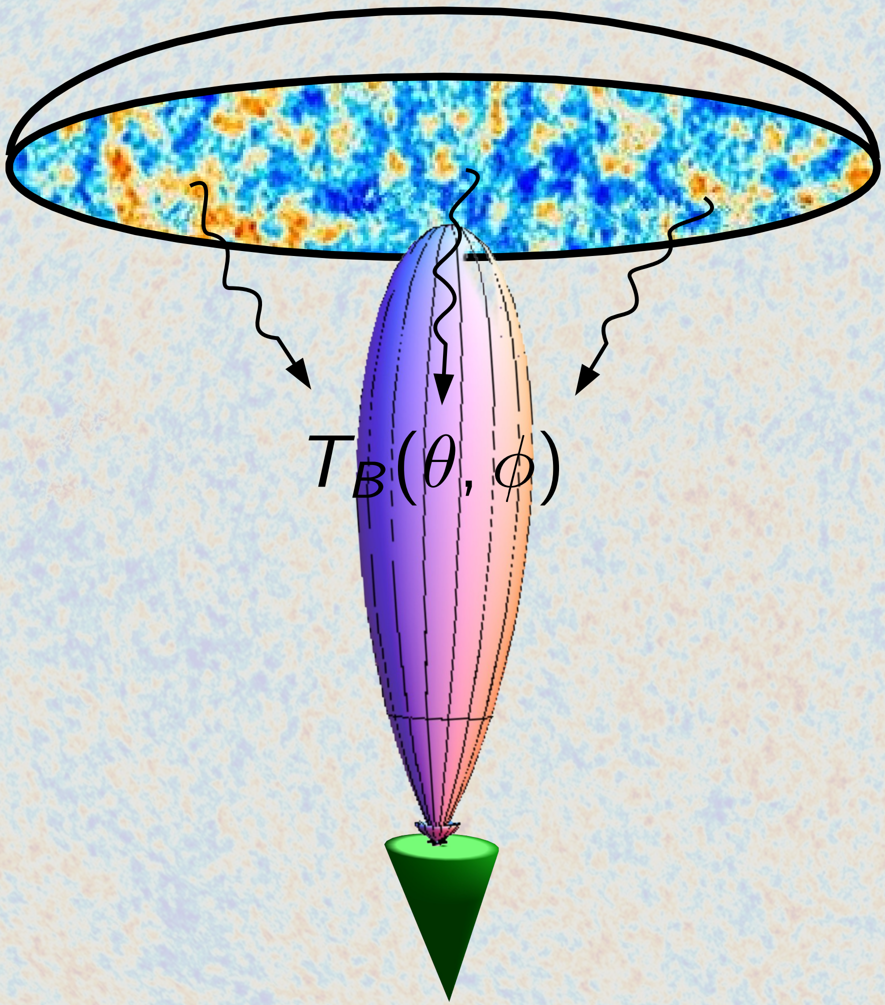
The antenna temperature

Measured power from the sky



- Let us consider an antenna (connected to a receiver) looking at an extended source with brightness temperature $T_B(\theta,\varphi)$
- Now we ask: what is the power measured by the receiver when the antenna points in a direction (θ_0,φ_0)?

Measured power from the sky



- Let us consider an antenna (connected to a receiver) looking at an extended source with brightness temperature $T_B(\theta,\varphi)$
- Now we ask: what is the power measured by the receiver when the antenna points in a direction (θ_0,φ_0)?
- Because of the angular response (beam pattern) of the antenna **the signal coming from all directions** contribute to the measured power
- Of course the signal coming from the direction of the antenna optical axis will contribute more to the measured power than other directions

Measured power from the sky

- At every given frequency, ν, the detected power is:

$$W = (1/2) A \int_{4\pi} B_\nu(\theta,\phi) P_n(\theta,\phi) d\Omega$$

Half of the power is detected because a receiver will generally detect only one polarisation

Antenna area

Surface brightness

Antenna beam pattern

Measured power from the sky

- At every given frequency, ν, the detected power is:

$$W = (1/2)A \int_{4\pi} B_\nu(\theta,\phi) P_n(\theta,\phi) d\Omega$$

The detected power is the convolution of the source brightness with the antenna beam pattern

- If we have a receiver that is sensitive in a frequency interval (bandwidth) Δν, the above equation becomes:

$$W = (1/2)A \int_{\Delta\nu} \int_{4\pi} B_\nu(\theta,\phi) P_n(\theta,\phi) d\Omega \, d\nu$$

The antenna temperature

- Now let us write B_ν in terms of the brightness temperature, T_B

$$B_\nu(\theta,\phi) = \frac{2k\nu^2}{c^2} T_B(\theta,\phi,\nu)$$

- The measured power, W, becomes:

$$W = \int_{\Delta\nu}\int_{4\pi} \frac{k\nu^2 A}{c^2} T_B(\theta,\phi,\nu) P_n(\theta,\phi,\nu) d\Omega\, d\nu$$

The antenna temperature

- Considering that $\boxed{\Omega_a = \lambda^2 / A = c^2 / \nu^2 A}$

  and that $\Omega_a = \int_{4\pi} P_n(\theta,\phi)\,d\Omega$

  we obtain

This is a fundamental relationship in antenna theory, which we will not demonstrate now

$$W = k \int_{\Delta\nu} \boxed{\frac{\int_{4\pi} T_B(\theta,\phi,\nu) P_n(\theta,\phi,\nu)\,d\Omega}{\Omega_a(\nu)}} d\nu$$

We call this term: **antenna temperature**

Detected power

$$\boxed{W = k\int_{\Delta\nu} T_a(\nu)d\nu \approx kT_a\Delta\nu}$$

The detected power is proportional to the antenna temperature.

The above approximation is valid if T_a does not vary too much within the bandwidth

Photometric calibration

What is photometric calibration?

- The raw output of a receiver is not in terms of Watt, but it is given in Volt or in some other **uncalibrated** unit.
- For example, in Planck-LFI, the (almost) raw output is a differential voltage which is proportional to the CMB fluctuation in antenna temperature units

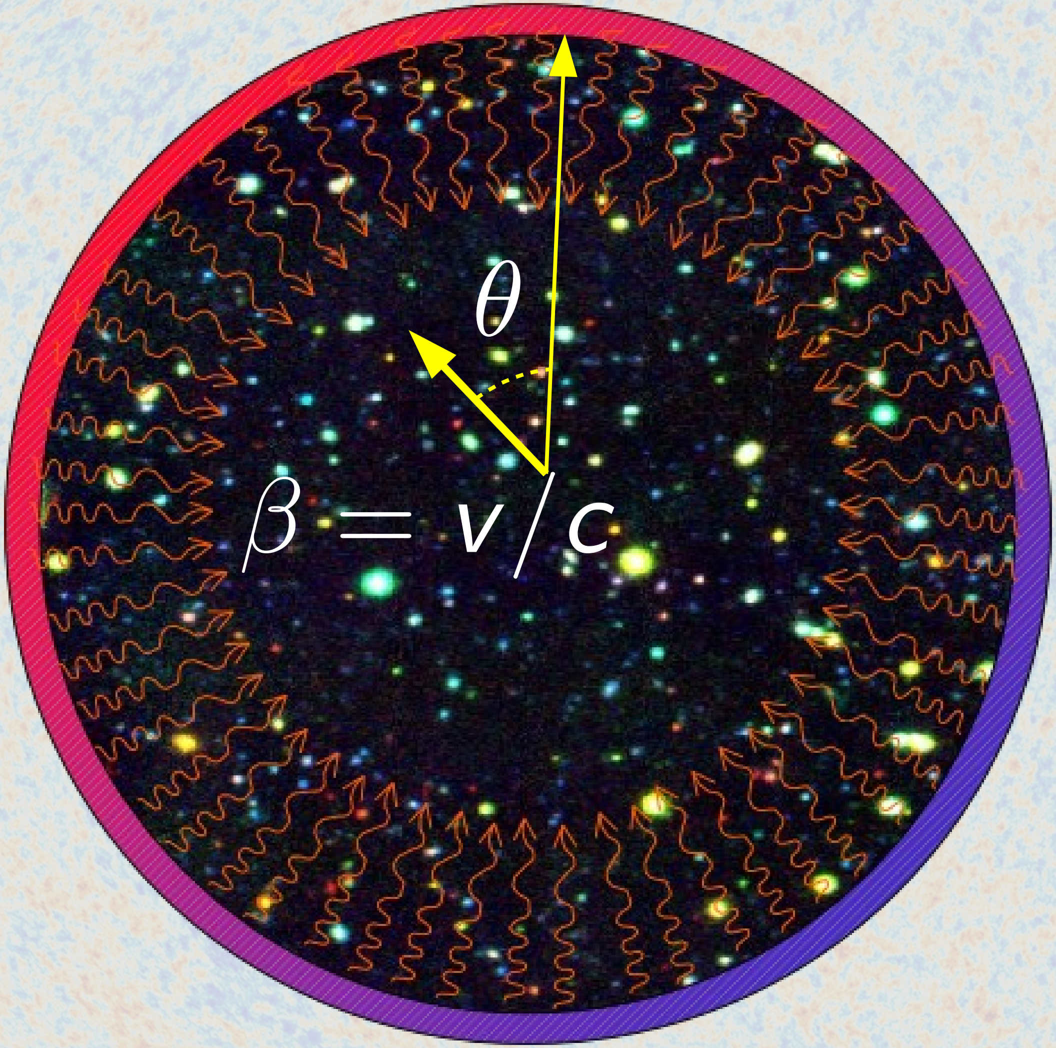
$$\Delta V_{\mathrm{out}} = G(t)\,\Delta T_{\mathrm{a,CMB}}$$

Notice that *G* is not constant, because the receiver amplifiers do not have a constant gain, but it fluctuates and changes with time

What is photometric calibration?

- Photometric calibration is the process by which we convert, at each time, the raw receiver output into antenna temperature units
- To do this we need, at each time, a very accurate **calibrator**, a signal that we know the emission of and that we can relate to our output
- With the CMB the best calibrator is right there all over the sky: it is the **CMB dipole anisotropy**

The CMB dipole

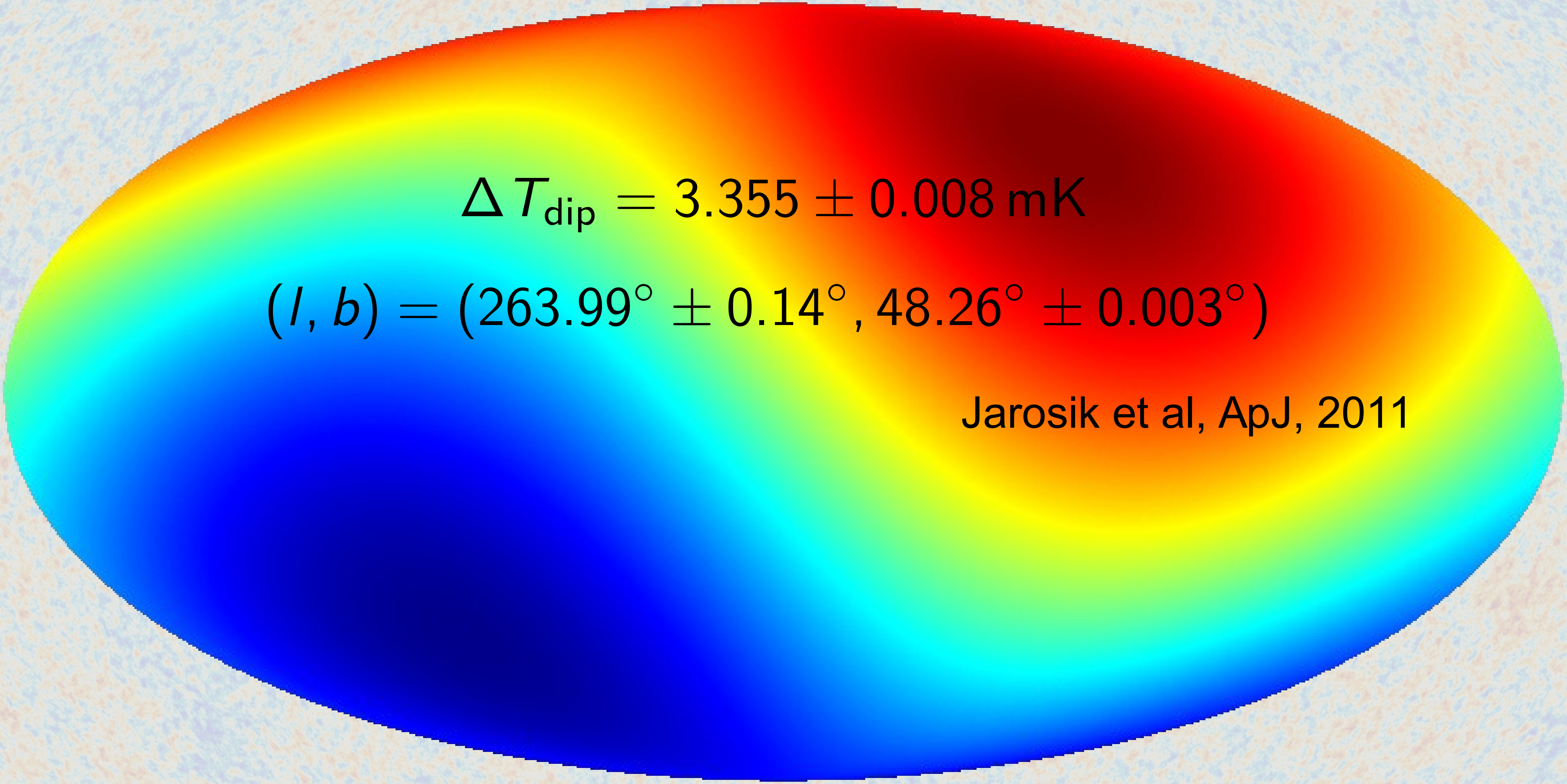


- Because our reference frame moves with respect to the CMB there is a Doppler effect
- Received CMB photons are "hotter" in the direction of our movement and "colder" in the opposite direction
- Observing at an angle θ with respect to our movement we have that the detected temperature is

$$T(\theta) = T_0 \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta}$$

$$= T_0(1-\beta\cos\theta) + O(\beta^2)$$

The CMB dipole



$$\Delta T_{\mathrm{dip}} = 3.355 \pm 0.008\,\mathrm{mK}$$

$$(l, b) = (263.99^\circ \pm 0.14^\circ, 48.26^\circ \pm 0.003^\circ)$$

Jarosik et al, ApJ, 2011

– 3.354 mK 3.354 mK

Continuous calibration

- The dipole is well-known and always visible in the sky
- Therefore at first approximation we can write our raw signal as:

$$\Delta V_{\mathrm{out}}(t) = G(t) \times \Delta T_{\mathrm{a,dipole}}(t)$$

knowing $\Delta T_{\mathrm{a,dipole}}(t)$ we can recover G at each time, t

- Notice that the dipole term is in antenna temperature, so **the knowledge of the beam is required**

Things are generally more complicated....

- The sky contains also other signals → iterative calibration
- The receiver data is noisy → statistical uncertainties
- There are other effects rather than gain variation, offsets, systematic effects, etc → strategies to trace systematic variations
- The knowledge of the beam pattern is required → see next

Optical calibration

What is optical calibration?

- We need know the antenna pattern for two reasons:

  1. *To calibrate (see previous discussion)*
  2. *To recover the "true" power spectrum*

- Optical calibration is the process by which we know the angular response of the instrument optics

The beam window function

- Let us recall that the CMB power spectrum comes from the spherical harmonic expansion of the CMB

$$\Delta T(\theta,\phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell,m} Y_{\ell,m}(\theta,\phi)$$

$$C_\ell = \langle |a_{\ell,m}|^2 \rangle$$

But this is valid for the true sky. The measured sky is the convolution of the true sky with the beam pattern

The beam window function

- The angular power spectrum from the measured sky is related to the *true* angular power spectrum by:

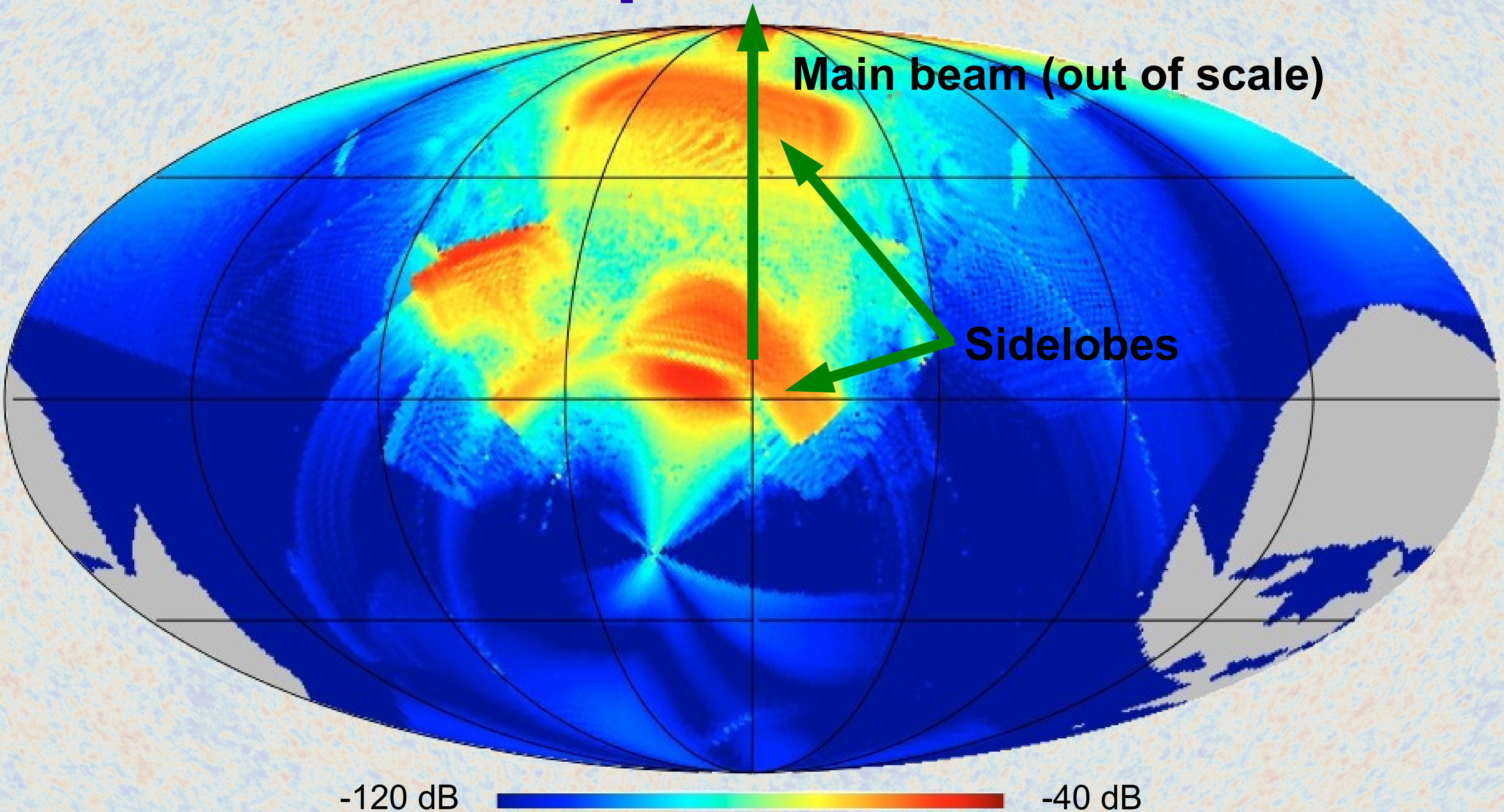
$$C_\ell^{\mathrm{meas}} = W_\ell \, C_\ell^{\mathrm{true}}$$

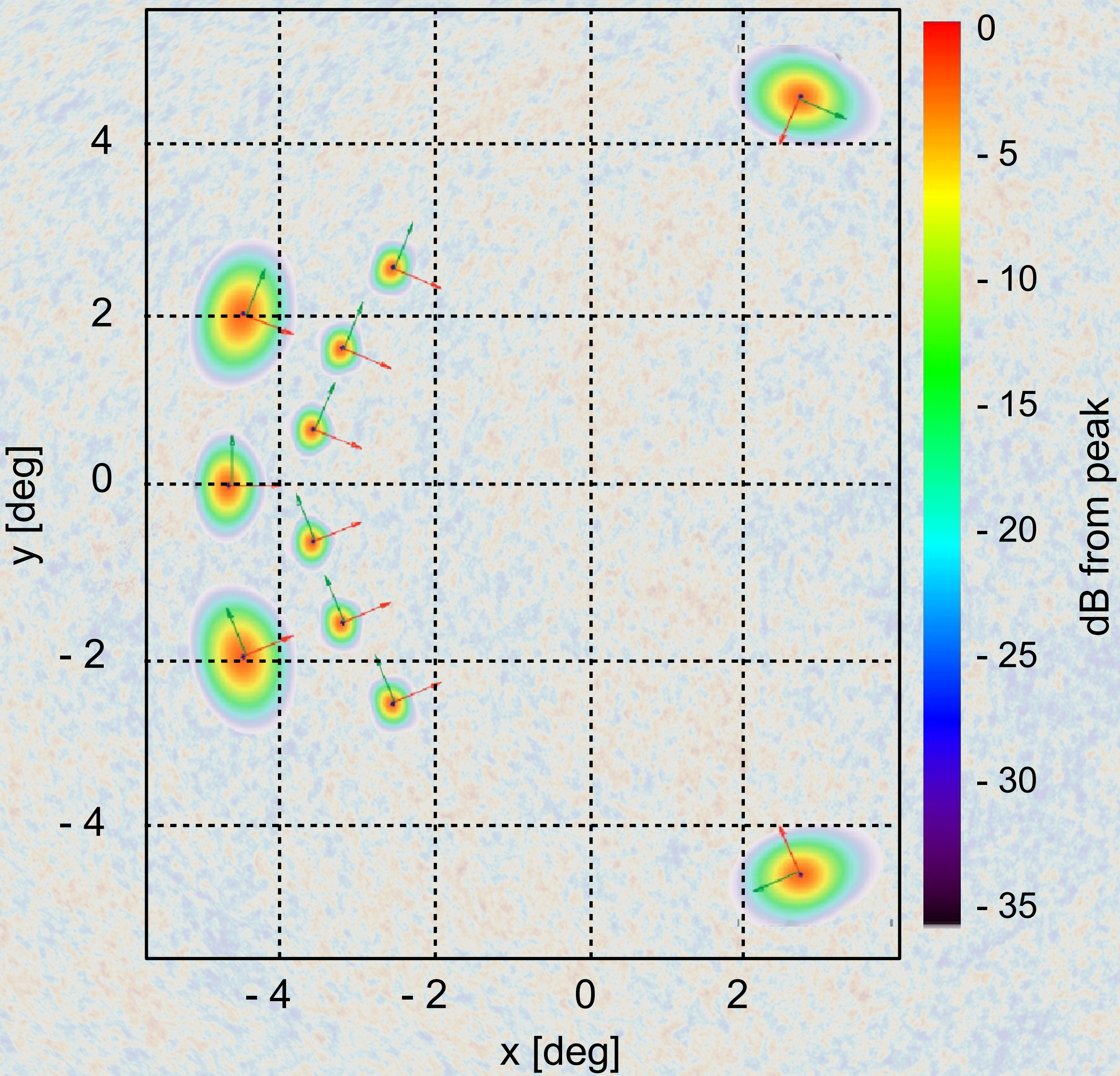
where W_ℓ is the **beam window function** and can be obtained once the antenna beam pattern is known

How do we know beam patterns?

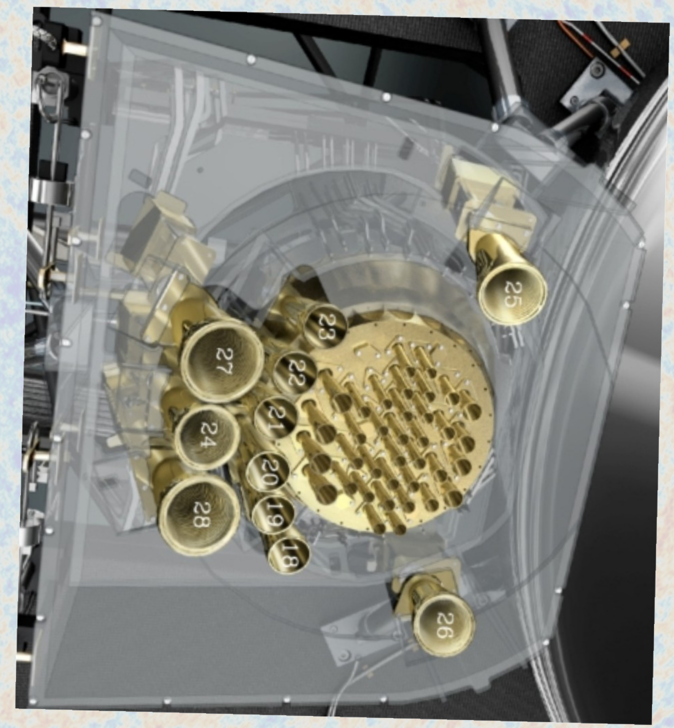
- From ground measurements (very difficult measurements, often limited)
- From simulations (computer-intensive, but allow detailed response at all angles)
- From in-flight data (scanning point sources)

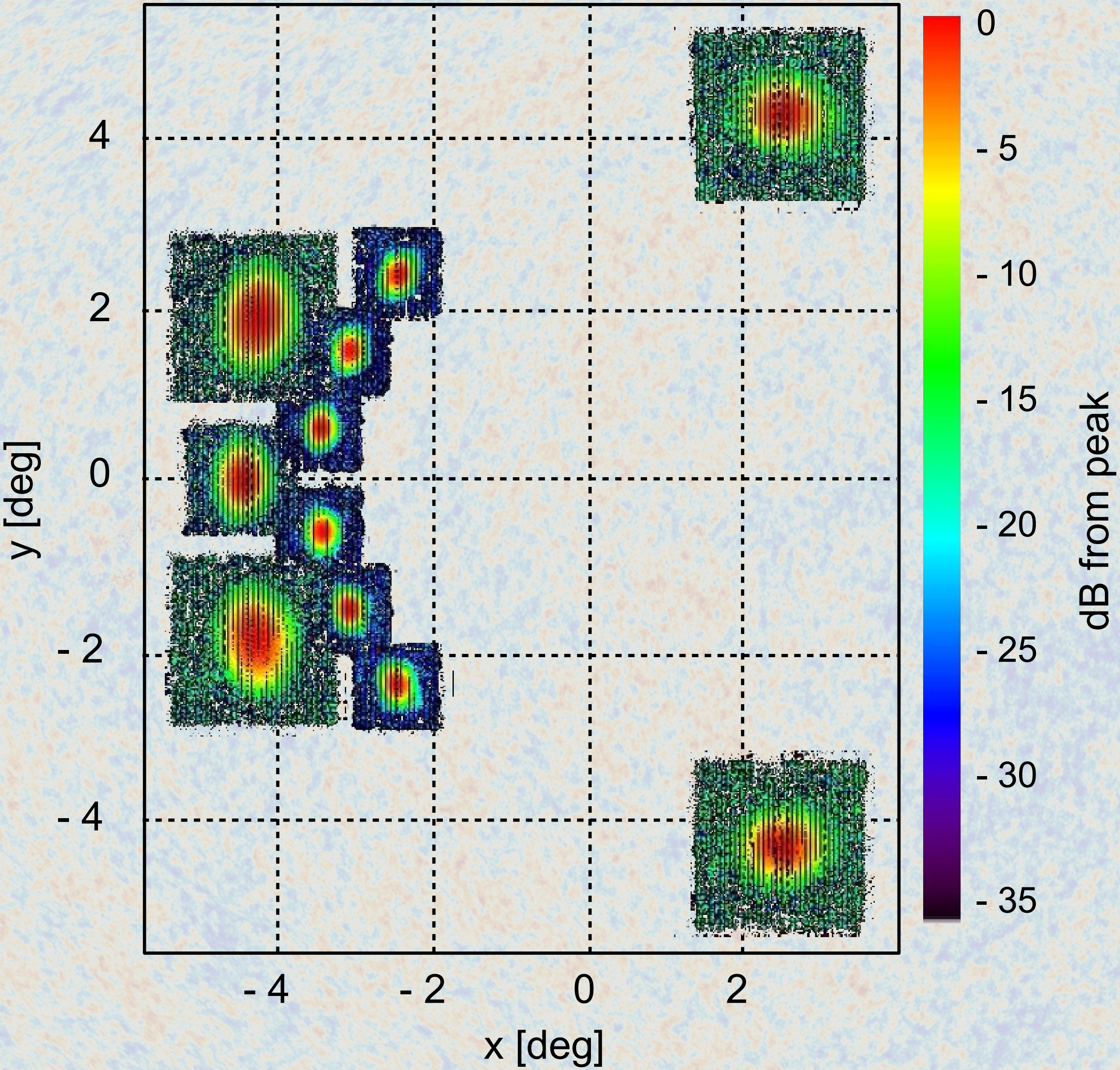
LFI 30 GHz simulated beam pattern



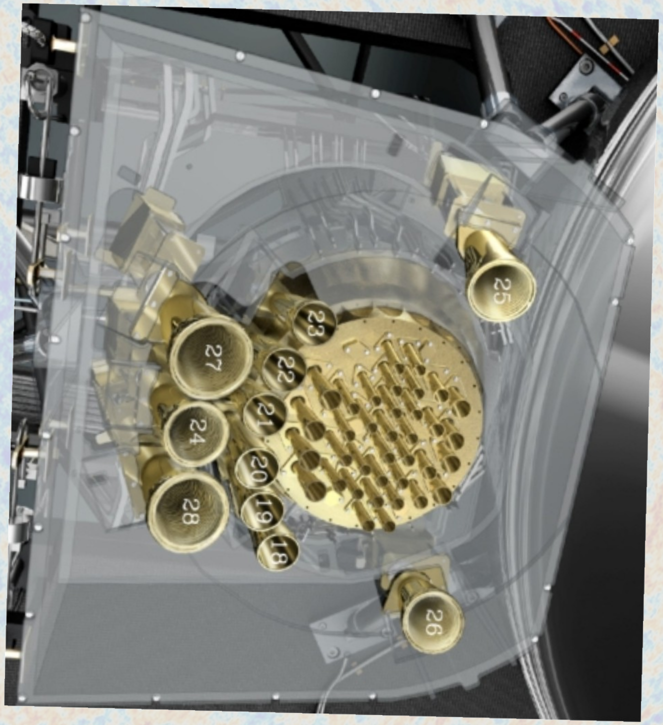


Simulated main beams of the LFI receivers in the focal plane



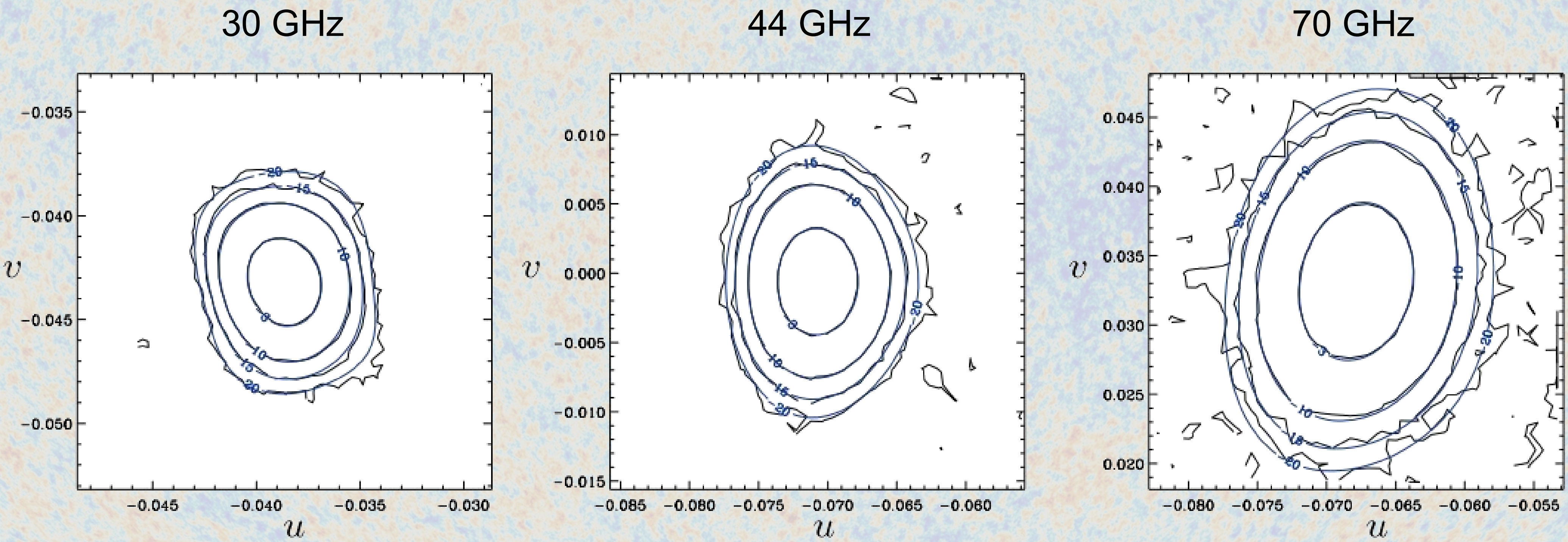


Main beams of the LFI receivers in the focal plane measured with Jupiter during the first year



Beam simulations and measurements

- Comparison of simulated (blue) and measured (black) beam contours for three LFI receivers



Conclusions

- Calibration is a key process in CMB data analysis. Science exploitation requires data calibrated with high accuracy and free from systematic effects
- Knowledge of the instrument beam is crucial. Uncertainties at that level can be a cause of very strong and long-lasting headaches
- CMB photometric calibration in temperature benefits from the existence of the CMB dipole. Nevertheless it is far from being a trivial exercise.
- The bottom of all lines is, as always,

Know your instrument!