Particle transport in the heliosphere – part 2

Gaetano Zimbardo

With contributions by S. Perri, P. Pommois, P. Veltri

University of Calabria, Rende, Italy

International Space Science School

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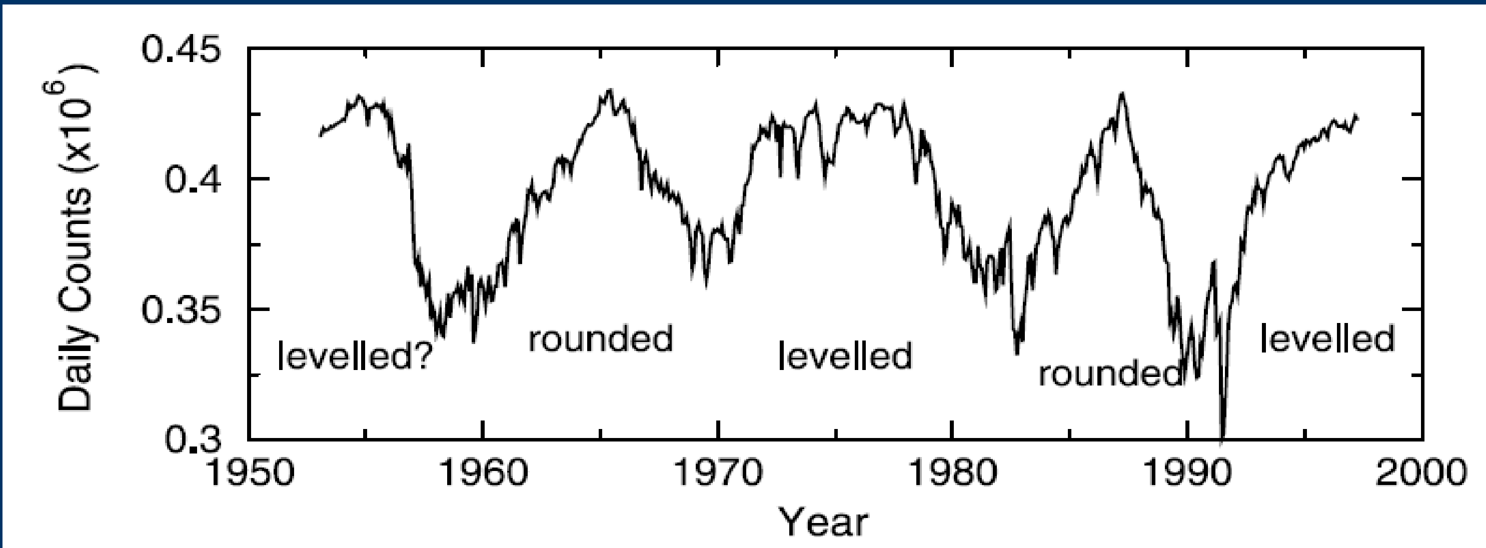
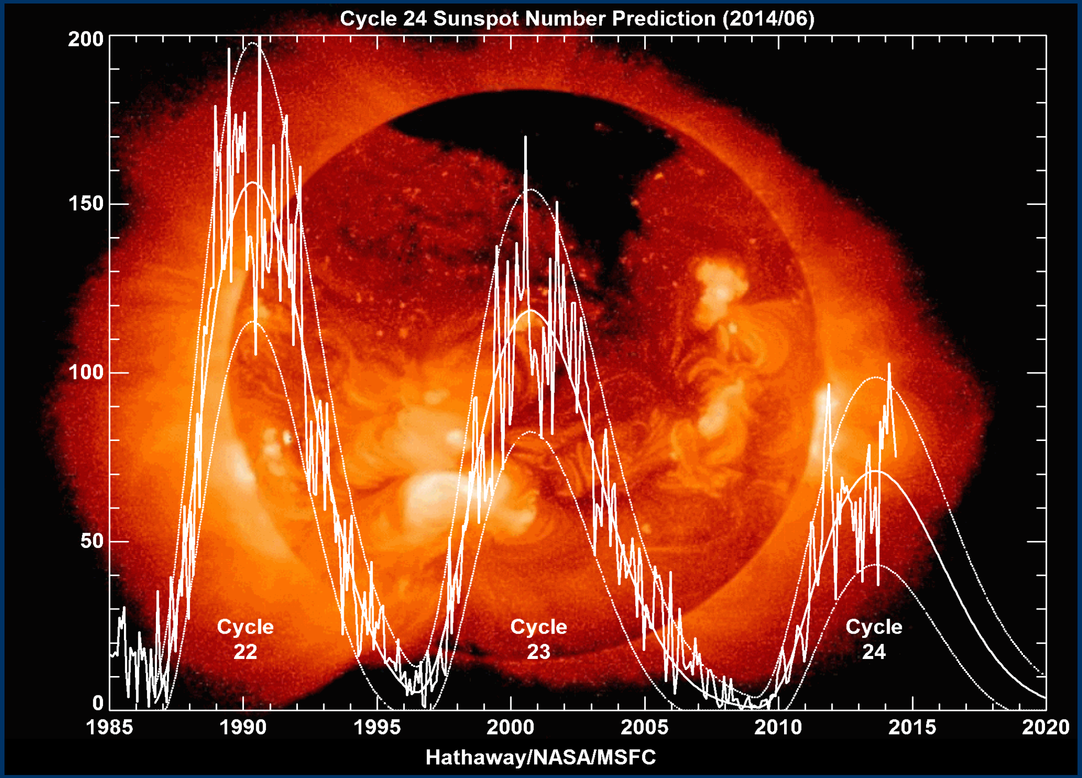


Daily Counts (x10^6)
0.45
0.4
0.35
0.3
1950
1960
1970
1980
1990
2000
Year
levelled?
rounded
levelled
rounded
levelled



Cycle 24 Sunspot Number Prediction (2014/06)
200
150
100
50
0
1985
1990
1995
2000
2005
2010
2015
2020
Cycle 22
Cycle 23
Cycle 24
Hathaway/NASA/MSFC

- In part 1 we considered the effects of magnetic turbulence on perpendicular diffusion:
- i) SEP dropouts
- ii) energetic particle propagation at high latitudes

- Now we concentrate on parallel transport, which depends sensitively on the particle pitch angle scattering due to resonant fluctuations

  - We shall see that anomalous, nondiffusive regimes are possible for both parallel and perpendicular transport of energetic particles ...

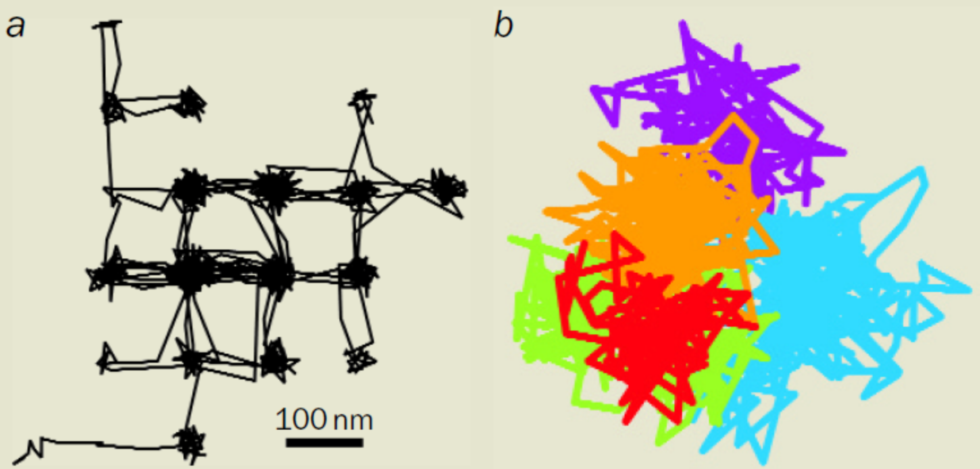
Normal diffusion: $\langle \Delta x^2 \rangle = 2D\,t$

Anomalous diffusion: $\langle \Delta x^2 \rangle \propto D_\alpha t^\alpha$

Anomalous diffusion spreads its wings

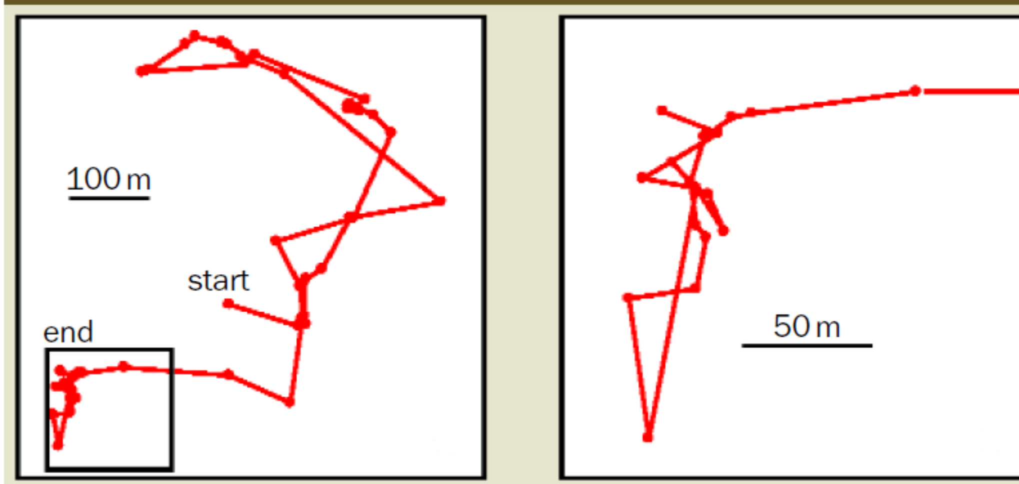
Joseph Klafter and Igor M Sokolov

3 Subdiffusion in cells



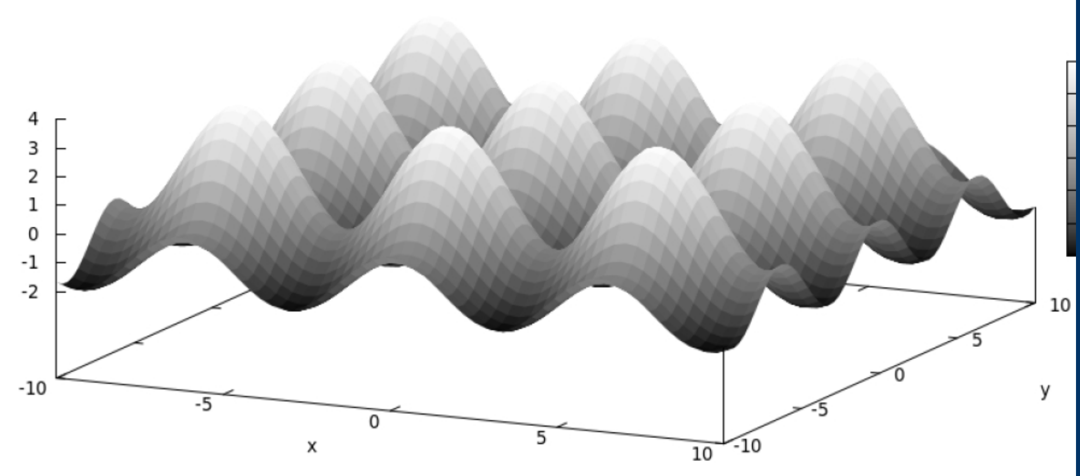
Researchers have found that the way proteins diffuse across cell membranes can be described by anomalous diffusion that is slower than the normal case. (*a*) This is a simulation of such a random walk, which shows a 2 ms timeframe over which a protein “hops” between 120 nm^2 compartments thought to be formed by the cell’s cytoskeleton. (*b*) The experimental trajectories of proteins in the plasma membrane of a live cell (shown in a 0.025 ms timeframe) provide evidence for this trapping nature,

4 Superdiffusion in monkey behaviour

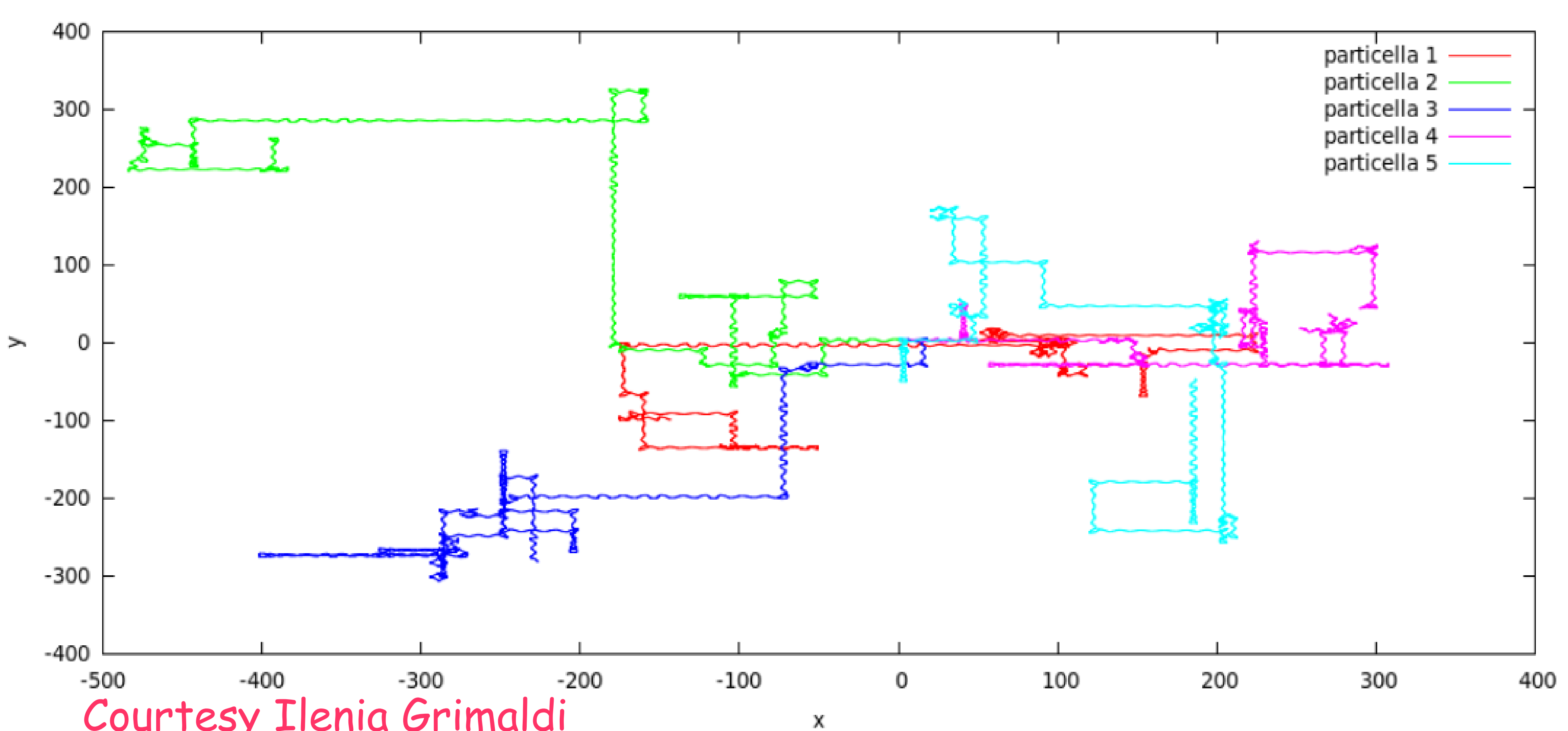


The typical trajectories of spider monkeys in the forest of the Mexican Yucatan peninsula display steps with variable lengths, which correspond to a diffusive process that is faster than that of normal diffusion. An example of such a trajectory is shown on the left. A magnified part of it is shown on the right; this image looks qualitatively similar to the larger-scale trajectory, which is an important property of Lévy walks. Similar behaviour is found in the

Motion in egg-crate potential: $U = A + B\left[\cos(\hat{x}) + \cos(\hat{y})\right] + C\, \cos(\hat{x})\cos(\hat{y})$

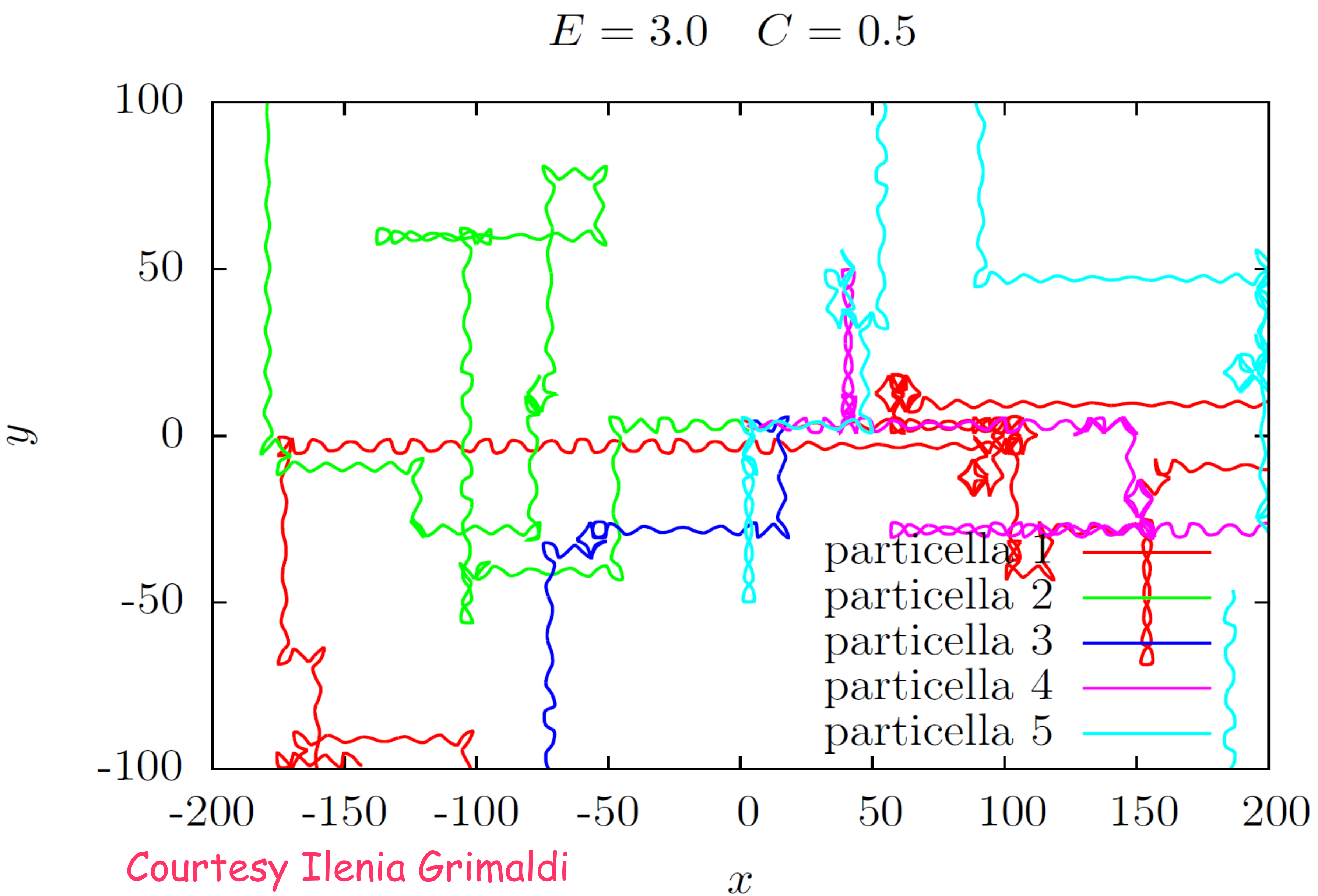


Geisel, Zacherl, Radons, PRL, 1987.

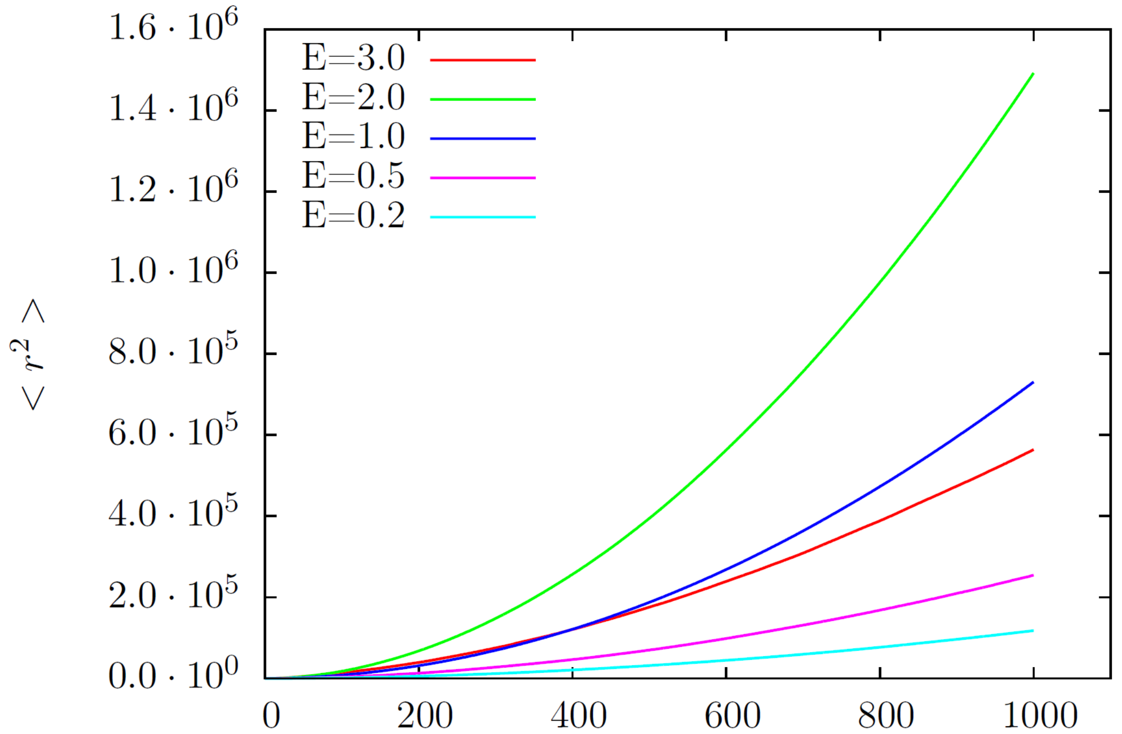


Courtesy Ilenia Grimaldi

Trajectories blow-up:



Courtesy Ilenia Grimaldi



Computing the mean square displacement for 1000 particles ...

$$< r^2 > \sim t^{\alpha}$$

$$\log < r^2 > = \alpha \log t + \log D_{\alpha}$$

E_{tot}	$\alpha \pm \Delta\alpha$	$D_{\alpha} \pm \Delta D_{\alpha}$
3	$1,6781 \pm 0,0006$	$5,2200 \pm 0,0196$
2	$1,9161 \pm 0,0003$	$2,6695 \pm 0,0050$
1	$1,9602 \pm 0,0001$	$0,9632 \pm 0,0010$
0, 5	$1,8529 \pm 0,0003$	$0,7030 \pm 0,0014$
0, 2	$1,8926 \pm 0,0004$	$0,2473 \pm 0,0007$

When does normal diffusion fail?

Normal diffusion:

$$D = \int_0^\infty \langle v(0)v(t)\rangle dt \approx v^2\tau_c$$

$$D \sim \lambda^2/\tau$$

Superdiffusion when the mean free path diverges:

$$\lambda^2 \equiv \langle \ell^2 \rangle = \int \ell^2 \psi(\ell) d\ell$$

i.e., when the velocity correlation function has long tails ...

$$C_L(\tau) \equiv \langle v(t)v(t+\tau)\rangle$$

Subdiffusion when the mean waiting time diverges:

$$\tau = \int t w(t) dt$$

Brownian versus Levy random walks: propagators are different

Gaussian walk

$$\langle \Delta x^2 \rangle = 2D\,t$$

$$n(x,t) = \frac{1}{\sqrt{4\pi D t}} \exp\left(-\frac{x^2}{4Dt}\right)$$

Lévy walks

$$\langle \Delta x_i^2 \rangle = 2\boldsymbol{\kappa}_i t^{\gamma_i}$$

$$P(k,t) \simeq \exp\left(\frac{C_2 |k|^{\mu-1}}{\tau} t\right)$$

$$\xi = \hat{x}/\hat{t}^{1/(\mu-1)} \ll 1$$

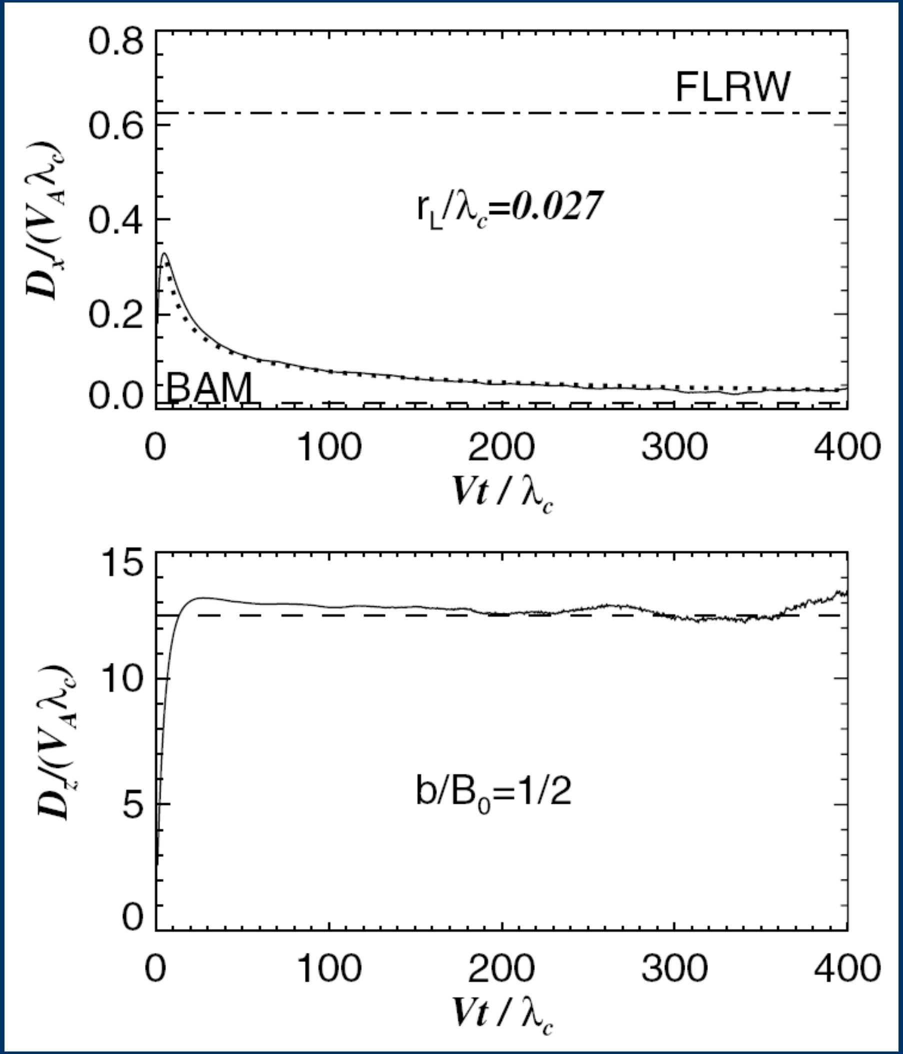
$$P(x,t) \simeq \frac{a_0}{t^{1/(\mu-1)}} \exp\left[-a_1 \left(\frac{x}{t^{1/(\mu-1)}}\right)^2\right]$$

$$\xi \gg 1 \text{ but } |x| < vt$$

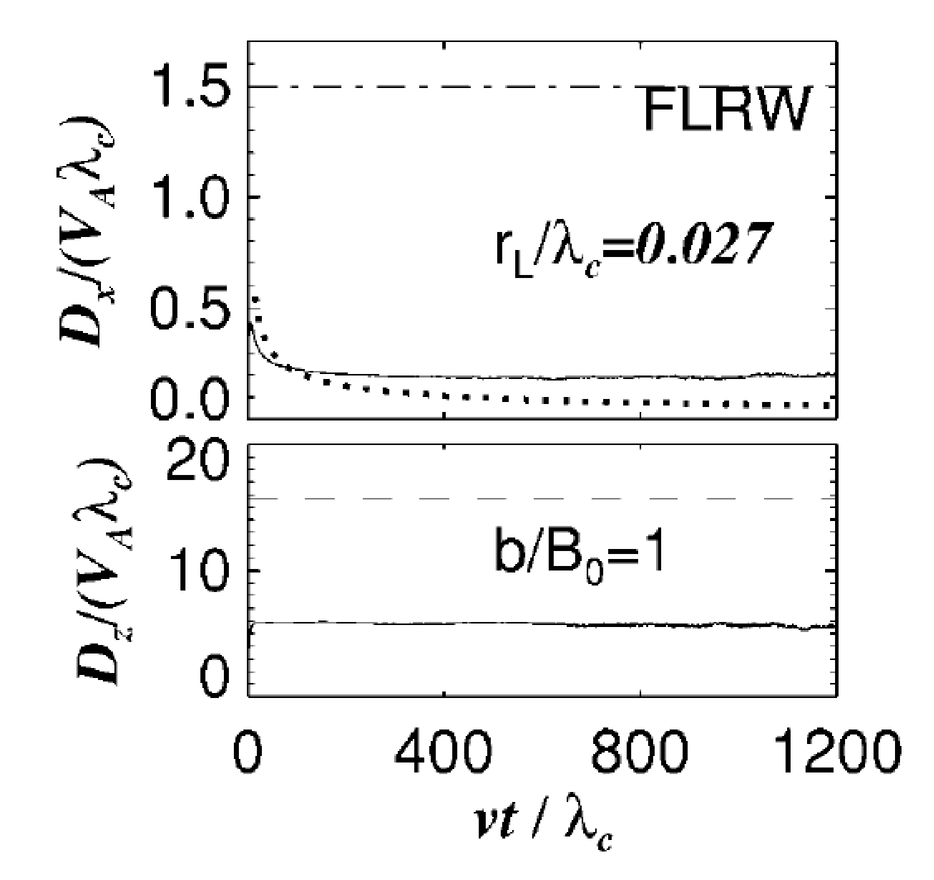
$$P(x,t) \simeq \frac{\Gamma(\mu)}{\pi} \sin\left[\frac{\pi}{2}(\mu-1)\right] \frac{|C_2|}{\tau} \frac{t}{|x|^{\mu}}$$

While $P(x,t) = 0$ for $|x| > vt$

Perpendicular subdiffusion considered by Qin, Matthaeus, and Bieber by computing the running diffusion coefficients:



Slab turbulence, perpendicular subdiffusion (Qin et al., GRL, 2002)



Composite turbulence (mostly 2D), recovery of diffusion (Qin et al., ApJL, 2002)

Numerical Simulations

The magnetic field is represented as a superposition of a constant field and a fluctuating field

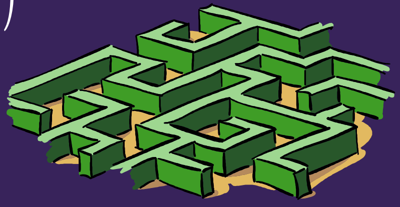
$$\mathbf{B}(\mathbf{r}) = \mathbf{B_0} + \delta\mathbf{B}(\mathbf{r})$$

where

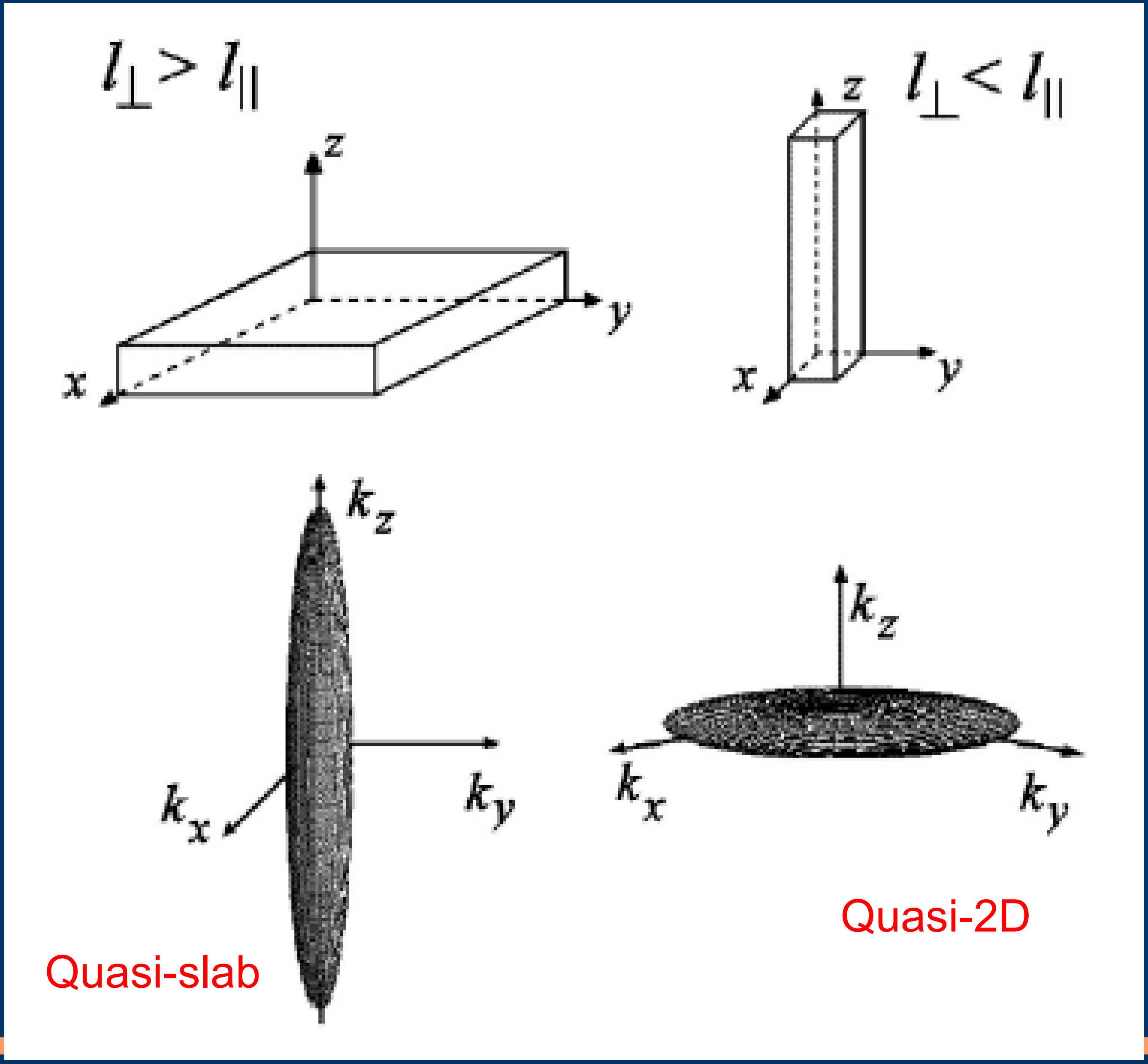
$$\mathbf{B}_0 = B_0 \hat{e}_z \qquad \delta\mathbf{B}(\mathbf{r}) = \sum_{\mathbf{k},\sigma} \delta B(\mathbf{k})\, e^{(\sigma)}(\mathbf{k}) \exp i\left[\mathbf{k}\cdot\mathbf{r} + \phi_{\mathbf{k}}^{(\sigma)}\right]$$

with

$$e^1(\mathbf{k}) = i\frac{\mathbf{k}\times\mathbf{B}_0}{\left|\mathbf{k}\times\mathbf{B}_0\right|}, \qquad e^2(\mathbf{k}) = i\frac{\mathbf{k}}{\left|\mathbf{k}\right|}\times e^1(\mathbf{k})$$



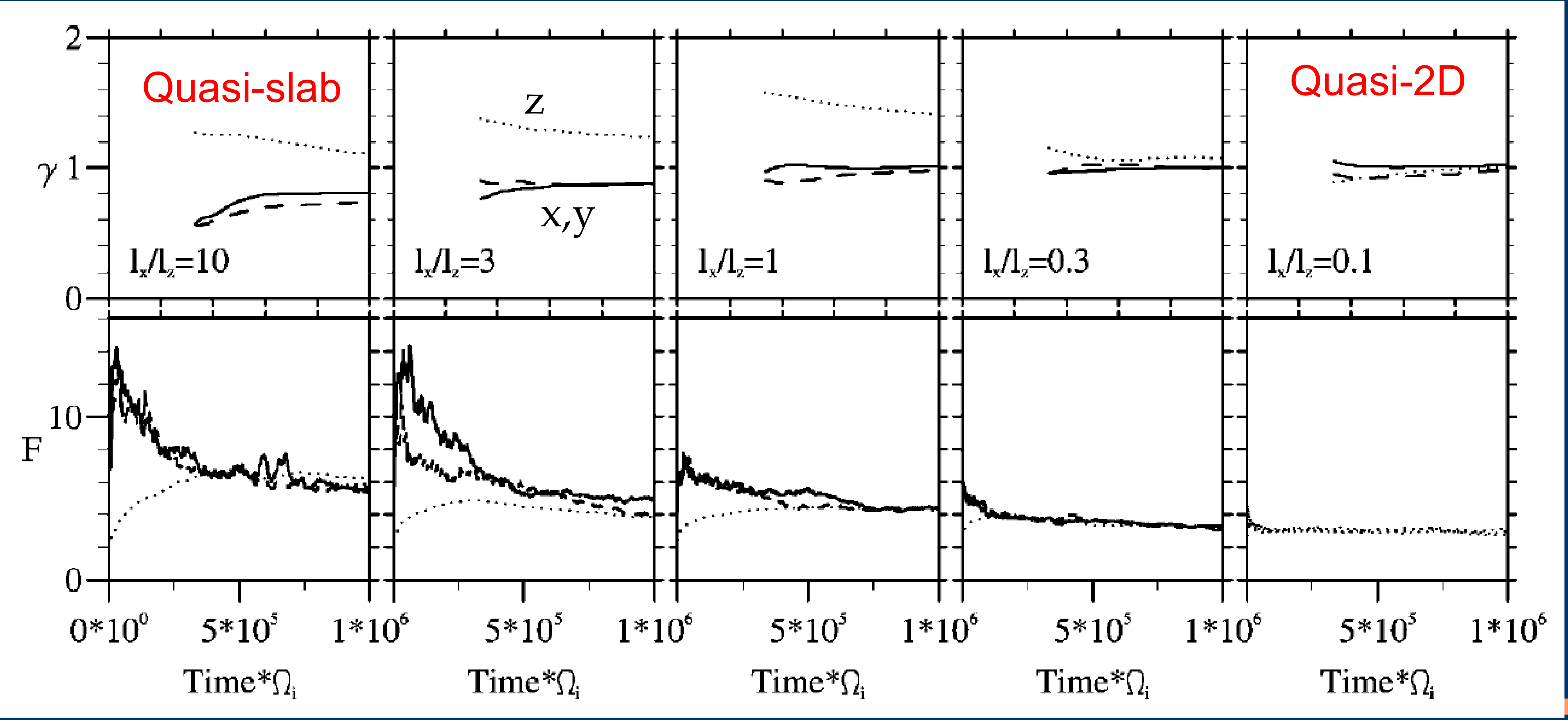
Magnetic Turbulence Anisotropy in physical and phase space



Anomalous transport depends on the turbulence anisotropy: (from Zimbardo et al., ApJL, 2006)

$$\langle \Delta x_i^2 \rangle = 2\kappa_i t^{\gamma_i}$$

$$F_i = \langle \Delta x_i^4 \rangle / \langle \Delta x_i^2 \rangle^2$$



Parallel superdiffusion and perpendicular subdiffusion also found by Shalchi and Kourakis, Astron. Astrophys, 2007

Test particle simulation with 20% slab and 80% 2D composite turbulence model

$$\langle (\Delta z(t))^2 \rangle_P \sim t^{b_\parallel + 1}$$

$$\kappa_{zz} \sim t^{b_\parallel}$$

$$\kappa_{xx} \sim t^{b_\perp}$$

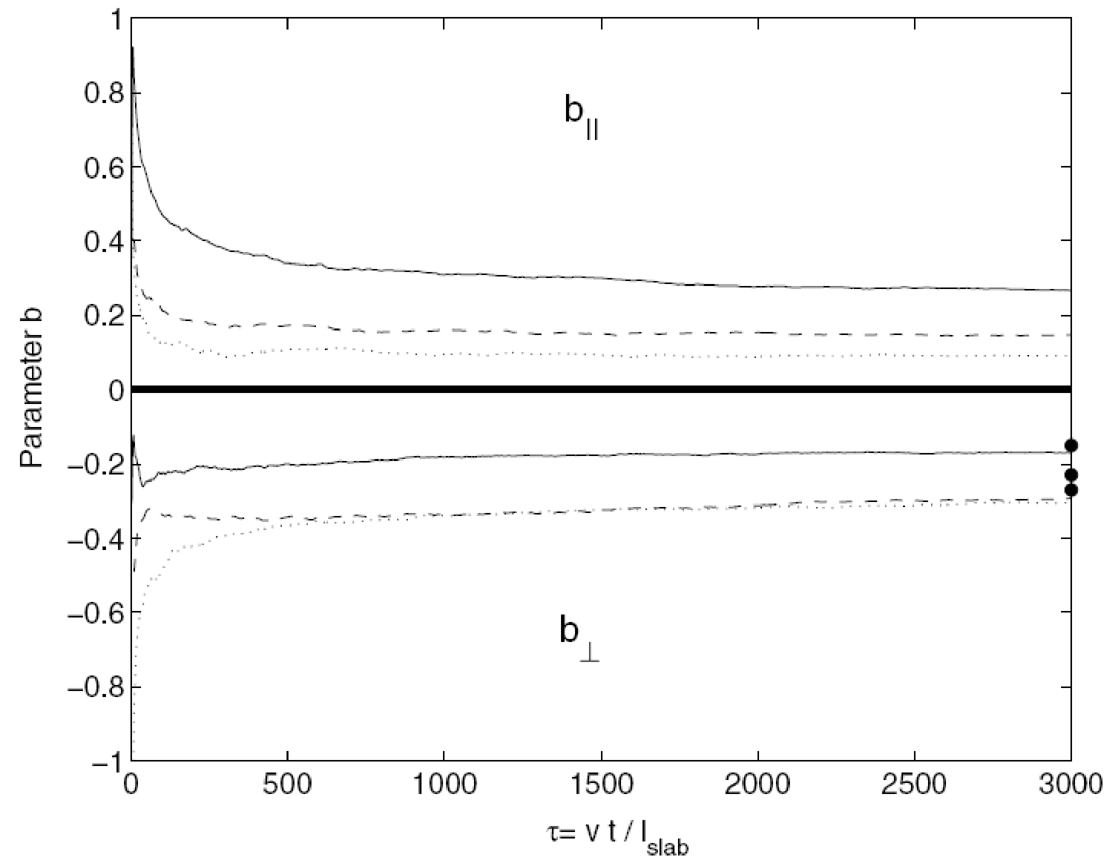
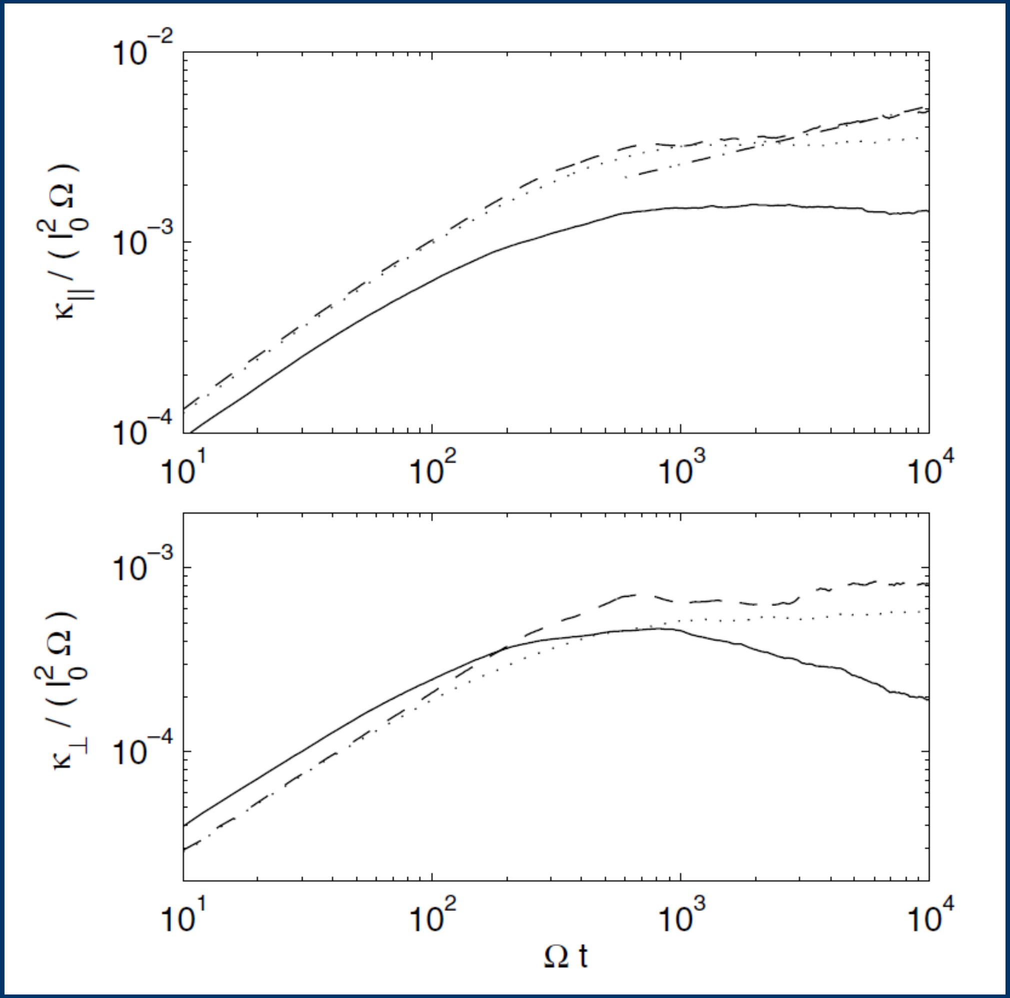


Fig. 2. The parameters $b_\parallel$ and $b_\perp$ as a function of time for different values of the dimensionless rigidity: $R = 10^{-3}$ (dotted line), $R = 10^{-2}$ (dashed line), and $R = 10^{-1}$ (solid line). The dots denote the values predicted by the GCD-model. Clearly we find a weakly superdiffusive behavior of parallel transport ($b_\parallel > 0$) and a weakly subdiffusive behavior of perpendicular transport ($b_\perp < 0$).

Parallel superdiffusion and perpendicular subdiffusion also found by Tautz, PPCF, 2010

- Perpendicular subdiffusion for magnetostatic slab turbulence (solid lines)
- The inclusion of time dependent electric and magneti fields leads to superdiffusion (dashed lines) with $\langle (dz)^2 \rangle = K\, t^{1.31}$

- The reported results are obtained with independent models and implementations of magnetic turbulence

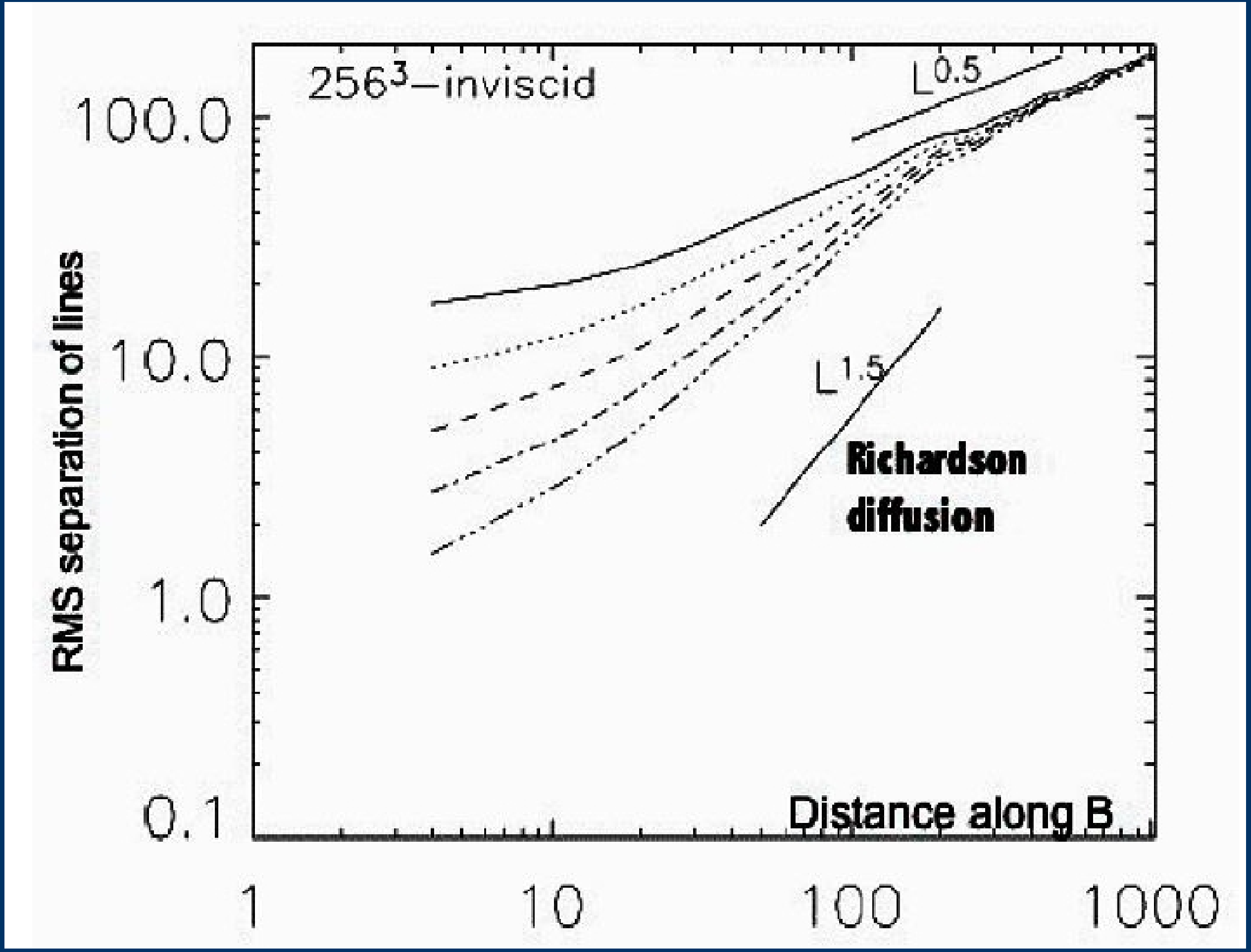


Superdiffusion of cosmic rays "a la Richardson" recently considered by Lazarian and Yan, ApJ (2014):

$$v(\ell) \sim v_0(\ell/L)^{1/3}$$

$$\frac{d}{dt}\ell(t) \sim (\epsilon\ell)^{1/3}$$

$$\ell^2 \sim \epsilon t^3$$



Bend-over of RMS at the interstellar turbulence injection scale, 100 pc

A useful parameter is the Kubo number R = (dB/B)(l_z /l_x)

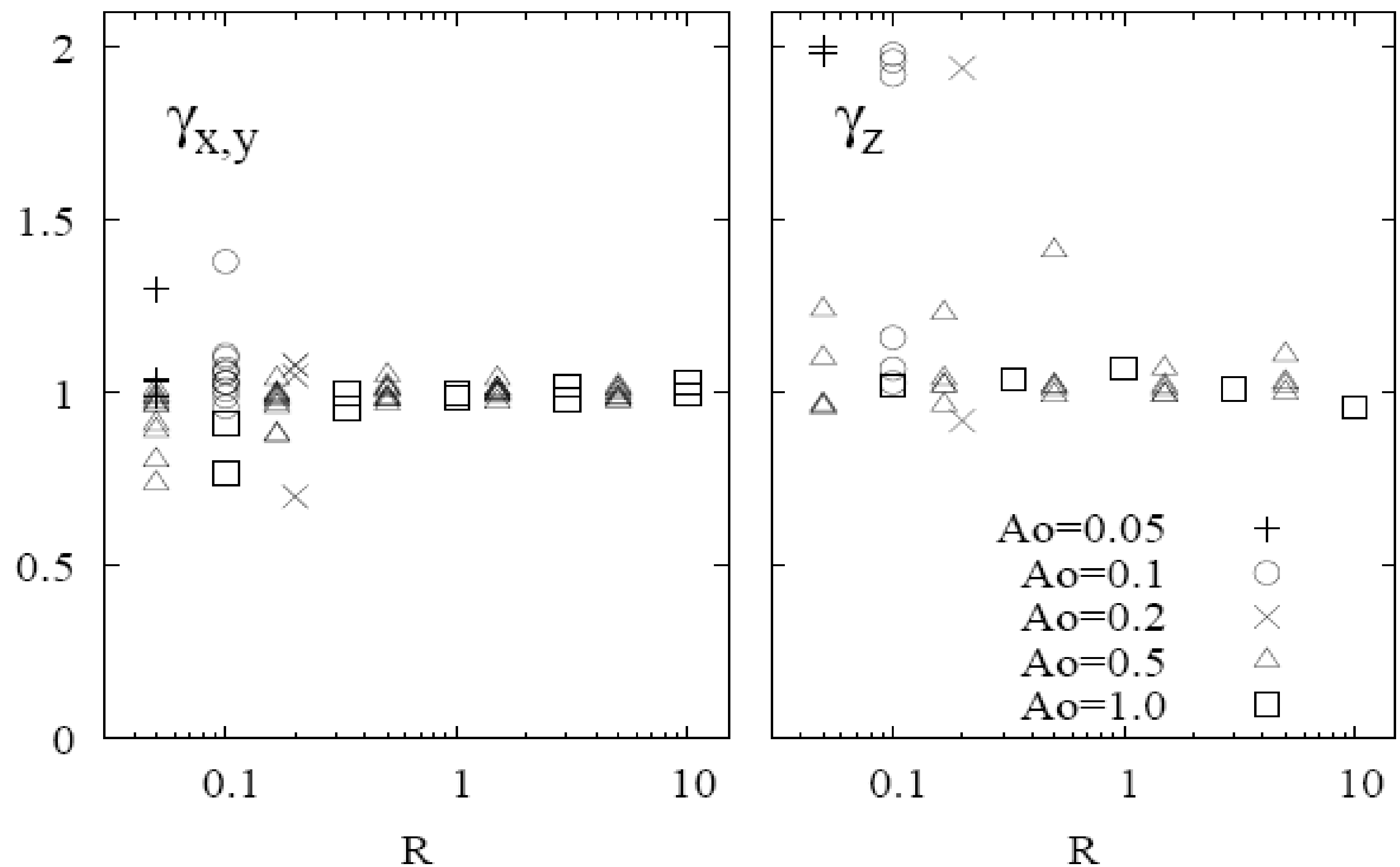
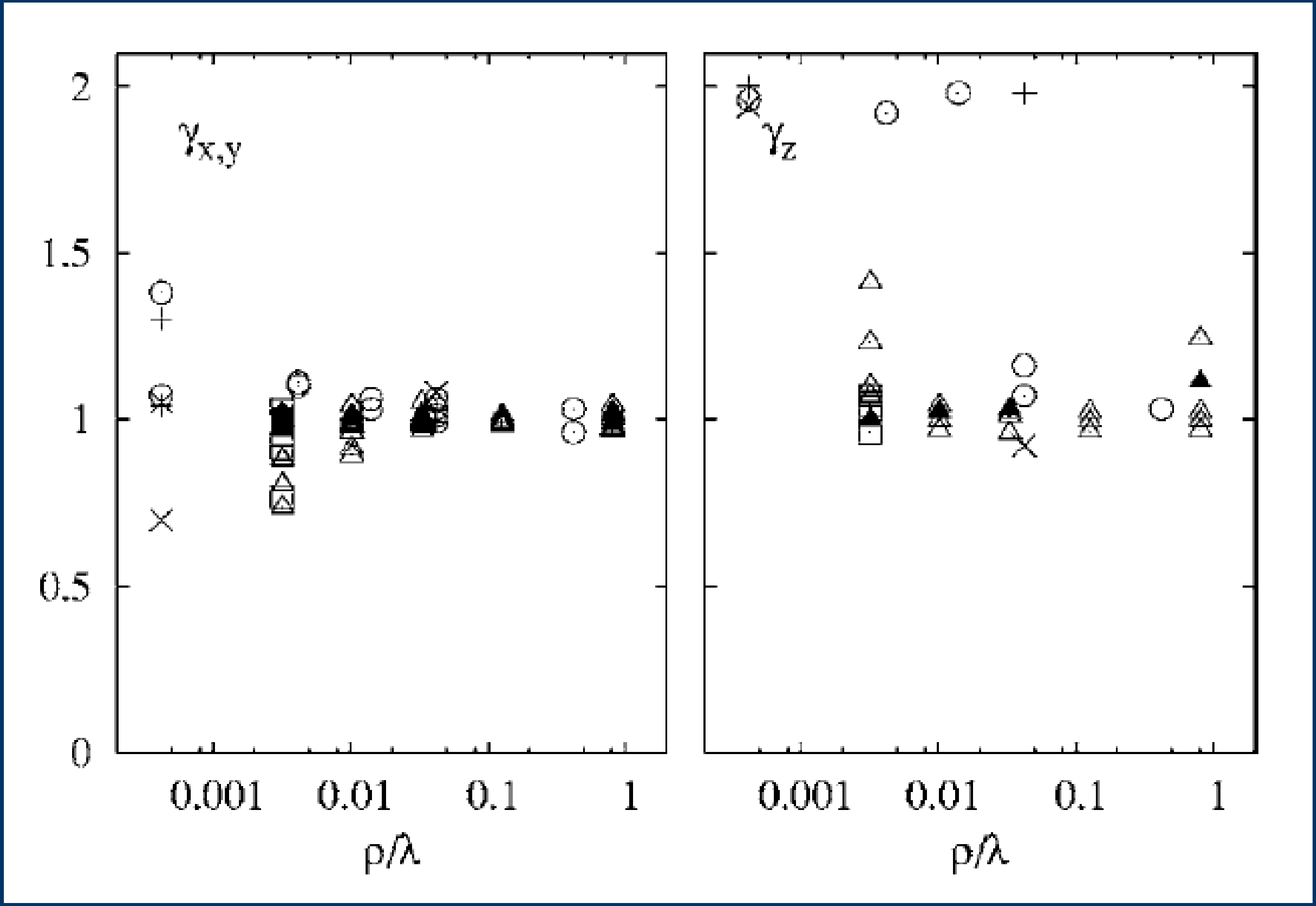


Figure 9: Anomalous transport exponents γ_x, γ_y, γ_z versus the Kubo number R. The different turbulence levels $\delta B/B_0$ are indicated by $+$: $\delta B/B_0 = 0.05$, $\odot$: $\delta B/B_0 = 0.1$, $\times$: $\delta B/B_0 = 0.2$, $\triangle$: $\delta B/B_0 = 0.5$, : $\delta B/B_0 = 1.0$.

From Pommois et al., Phys. Plasmas, 2007

The transport regime also depends on the ratio rho/lambda (Pommois et al., Ph. Pl., 2007)

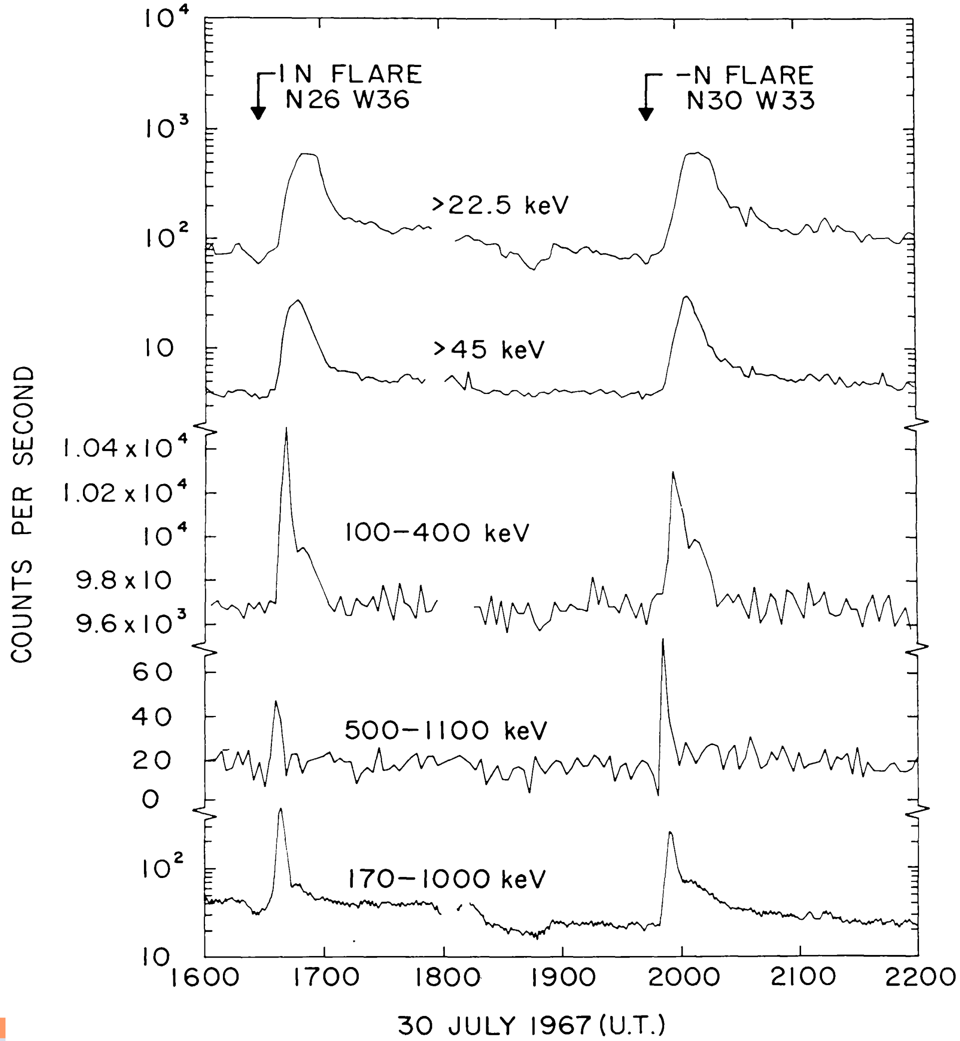


Particle transport is not necessarily diffusive:

NON-RELATIVISTIC SOLAR ELECTRONS

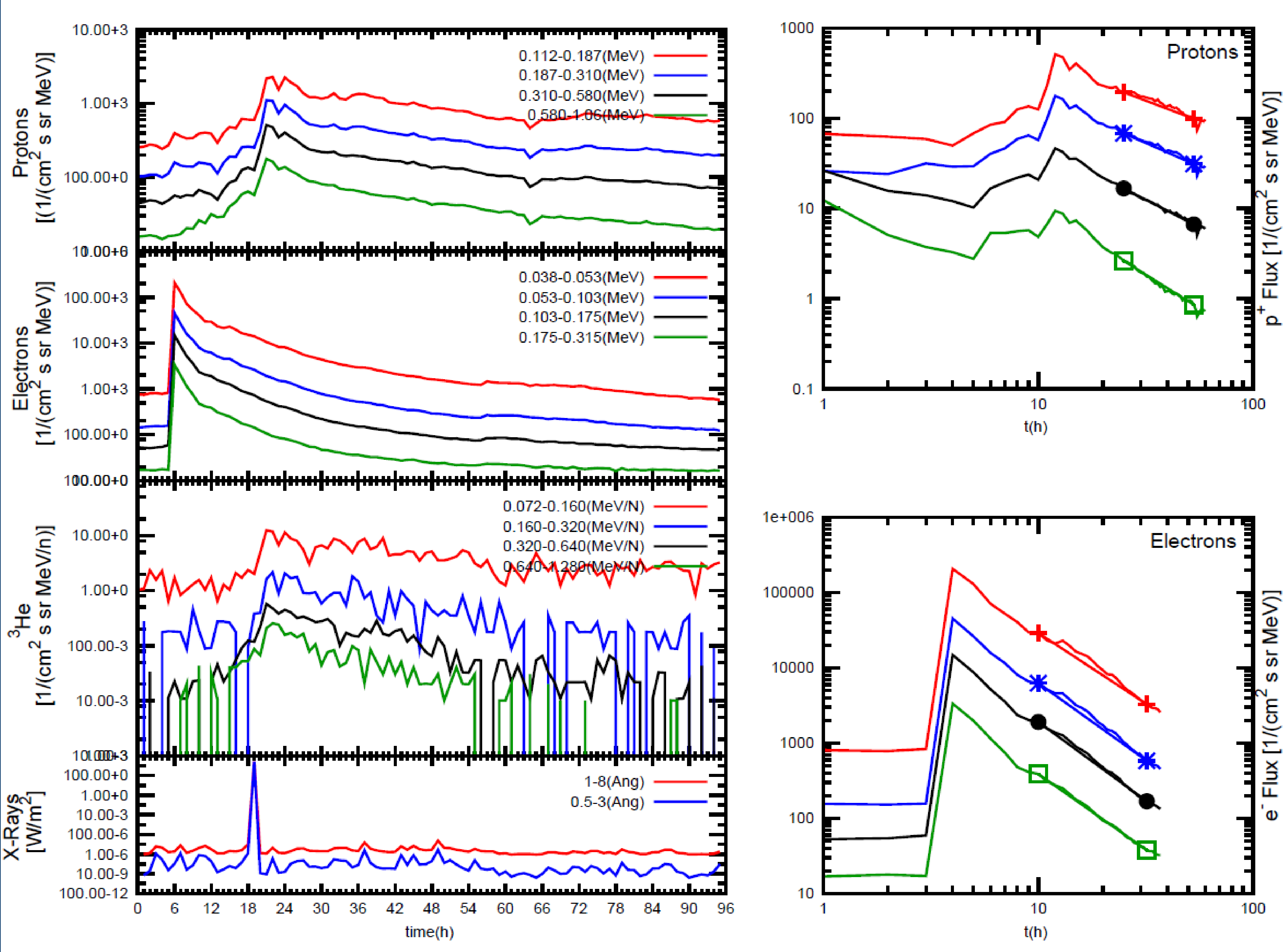
R. P. LIN

Space Sciences Laboratory, University of California, Berkeley, Calif. 94720, U.S.A.



"Non-relativistic electrons exhibit a wide variety of propagation modes in the interplanetary medium, ranging from diffusive to essentially scatter-free."
(Lin, SSRv, 1974)

Study of impulsive event by Trotta and Zimbardo (2011)



Superdiffusion from analysis of energetic particle profiles measured by spacecraft

- The flux of energetic particle can be expressed by means of the probability of propagation from (x′, t′) to (x, t):

$$f(x, E, t) = \int P(x - x', t - t') f_{\rm sh}(x', E, t') dx' dt'$$

- We consider particles emitted at an infinite planar shock moving with velocity V_sh: $f_{\rm sh}(x', E, t') = f_0(E)\, \delta\,(x' - V_{\rm sh}\, t')$

Normal diffusion,
Gaussian propagator:

$$f(x, E, t) \propto \exp(-V_{\rm sh} x/D)$$

(e.g., Fisk and Lee, ApJ, 1980)

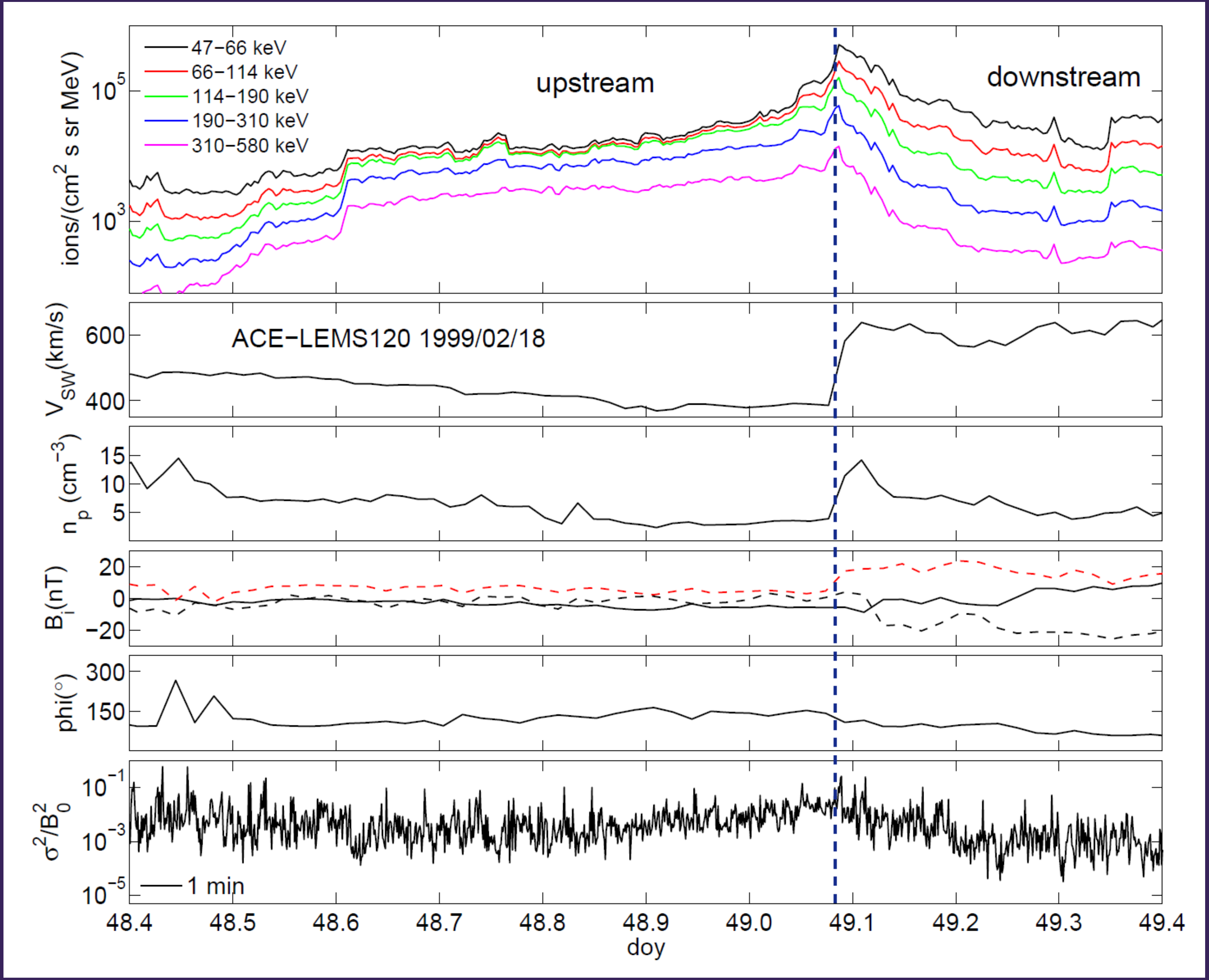
Superdiffusion,
power law propagator:

$$f(0, E, t) \sim V_{\rm sh}^{-\mu} (-t)^{2-\mu} \propto \frac{1}{(-t)^{\gamma}}$$

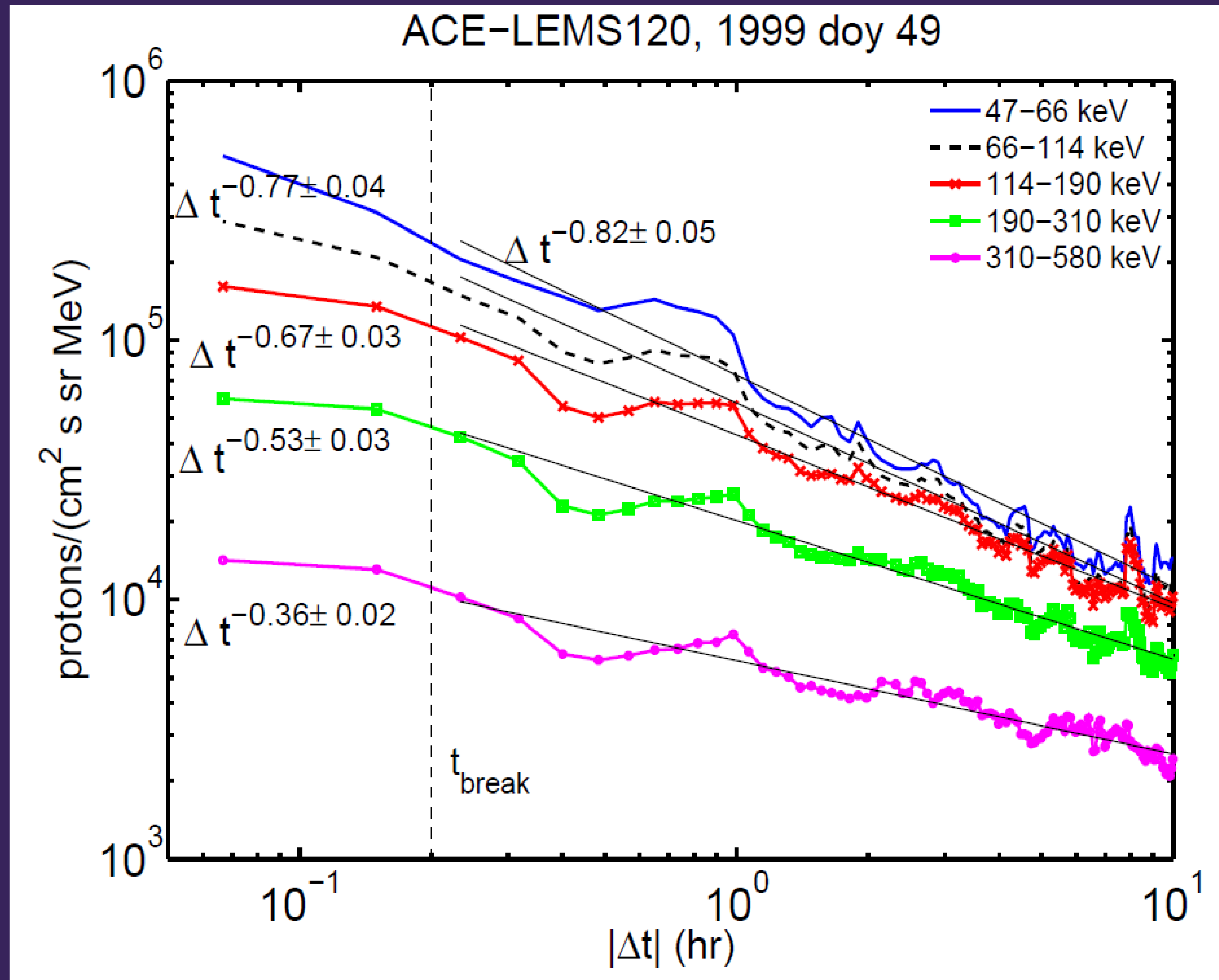
Perri and Zimbardo, 2007, 2008

Power law profile with ◦Ť ≭ ɱ valid for x >> l_0; the break in the power law allows to determine $D_{\sim}$ and l_0.

Interplanetary shock observed at 1 AU by ACE spacecraft in February 1999



A power law intensity profile fits the upstream data very well …

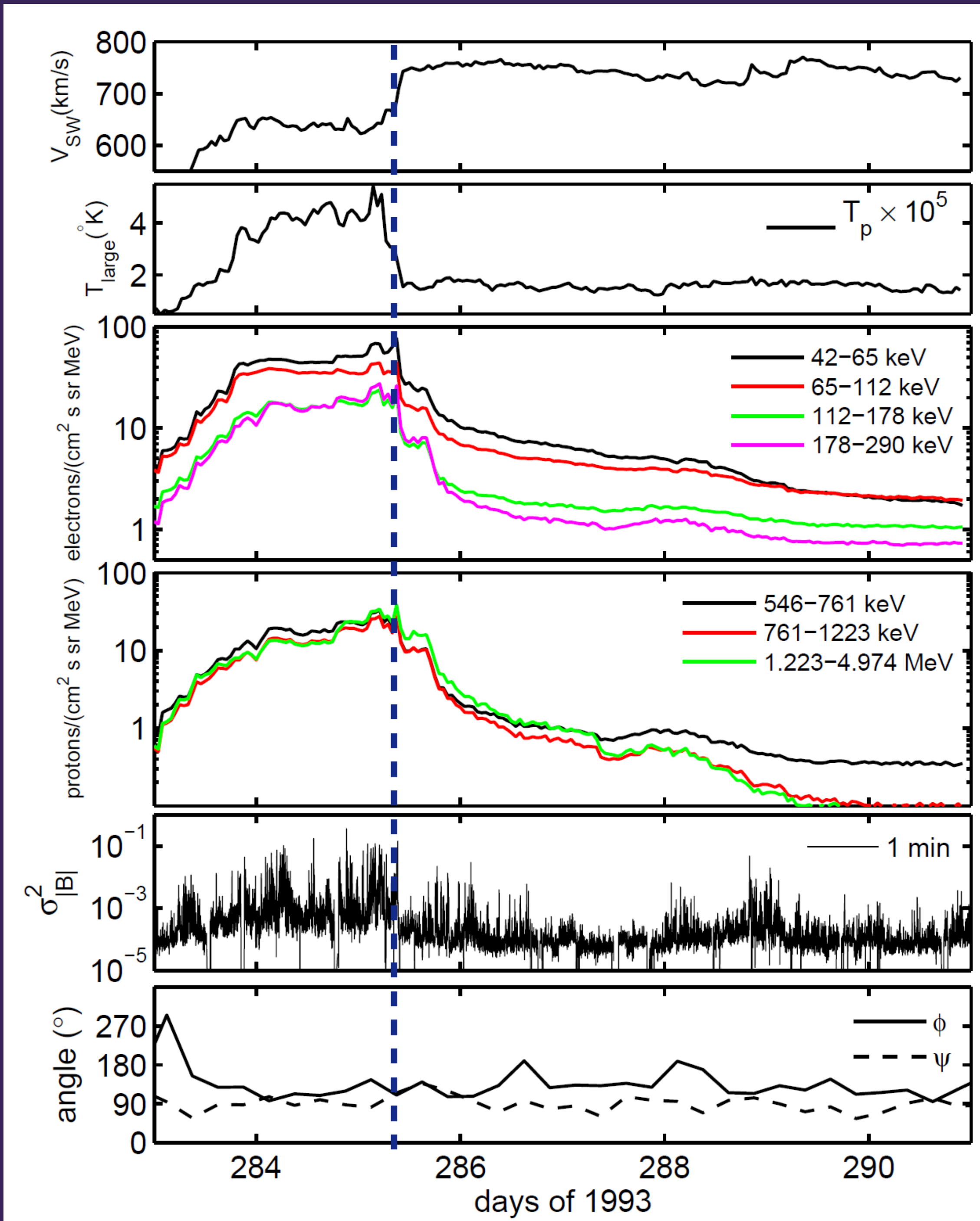


Superdiffusion exponent

alpha = 2 - gamma

Perri and Zimbardo, ApJ, submitted.

Ulysses s/c shock crossing of October 11, 1992

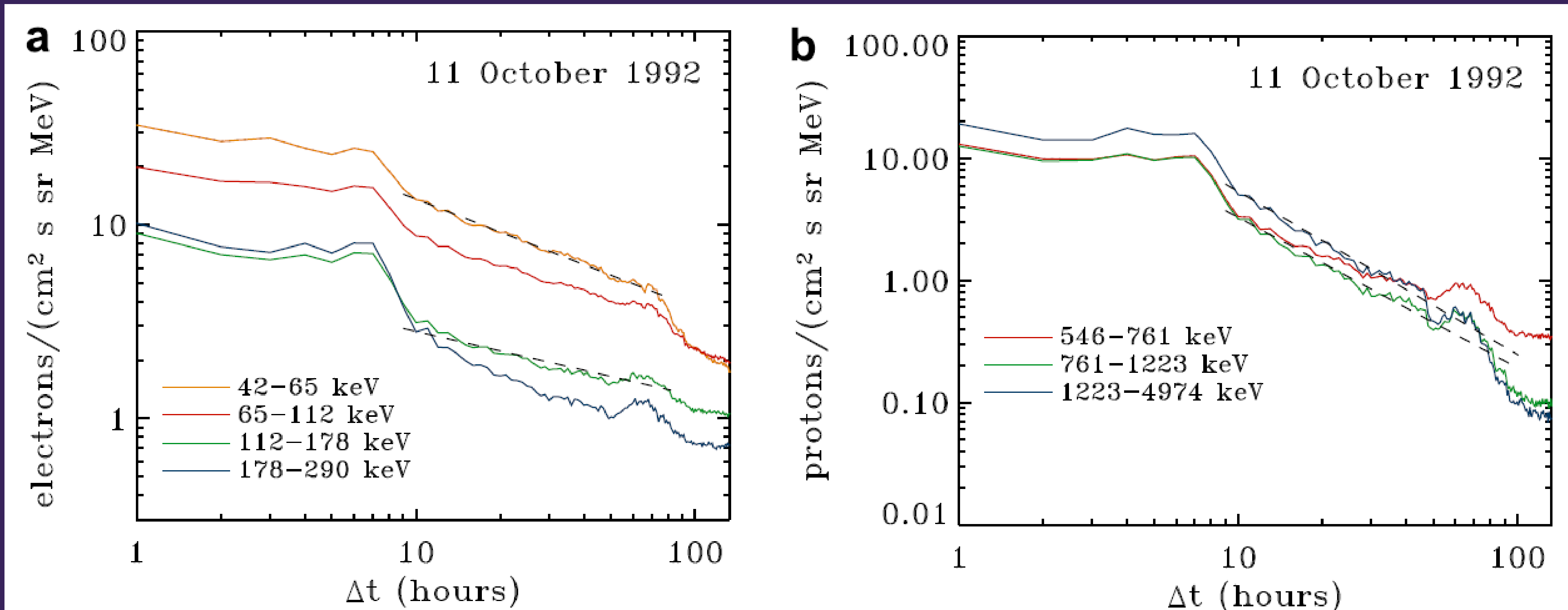


- Both protons and electrons are accelerated at CIR shocks:
- (data from CDAweb, thanks to Lancaster and Tranquille, PI D. McComas, L. Lanzerotti)

The pitch angle scattering rate depends on the wave magnetic field power:

$$D_{\alpha\alpha}^{\pm} = \left(\frac{B_\mathrm{w}}{B_0}\right)^2 \Omega_\mathrm{g}^{\pm}$$

Event of October 11, 1992



Statistical parameters for the fits of the electron time profiles.

S/C	DD/MM/YYYY	Energy (keV)	γ	α
Ulysses	11/10/1991	42–65	0.56 ± 0.08	1.44
		65–112	0.44 ± 0.09	1.56
		112–178	0.3 ± 0.1	1.70
		178–290	0.4 ± 0.2	1.60

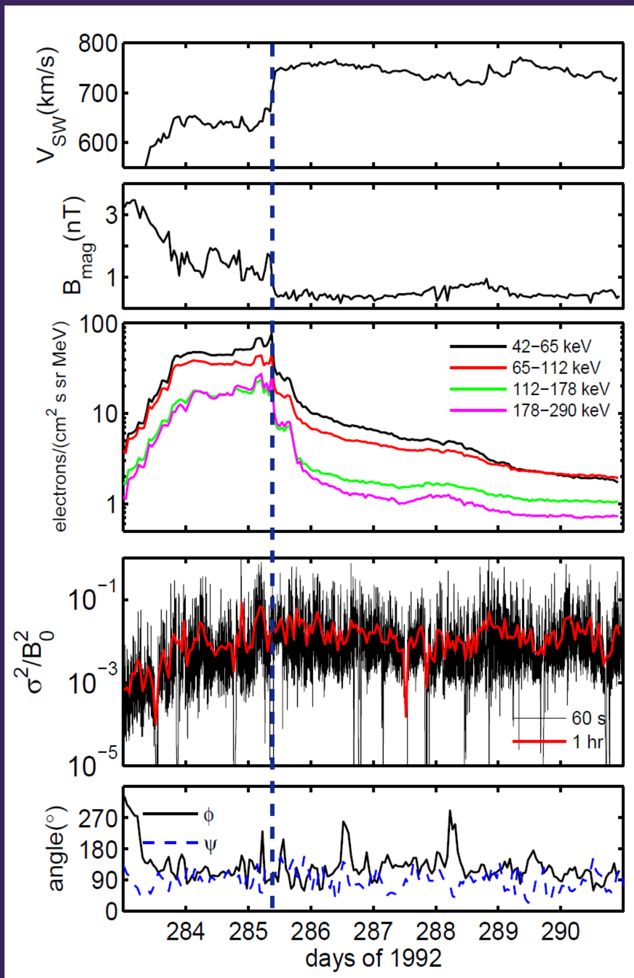
Electron transport is superdiffusive! Proton transport is normal diffusive (from Perri and Zimbardo, Adv. Spa. Res. 2009)

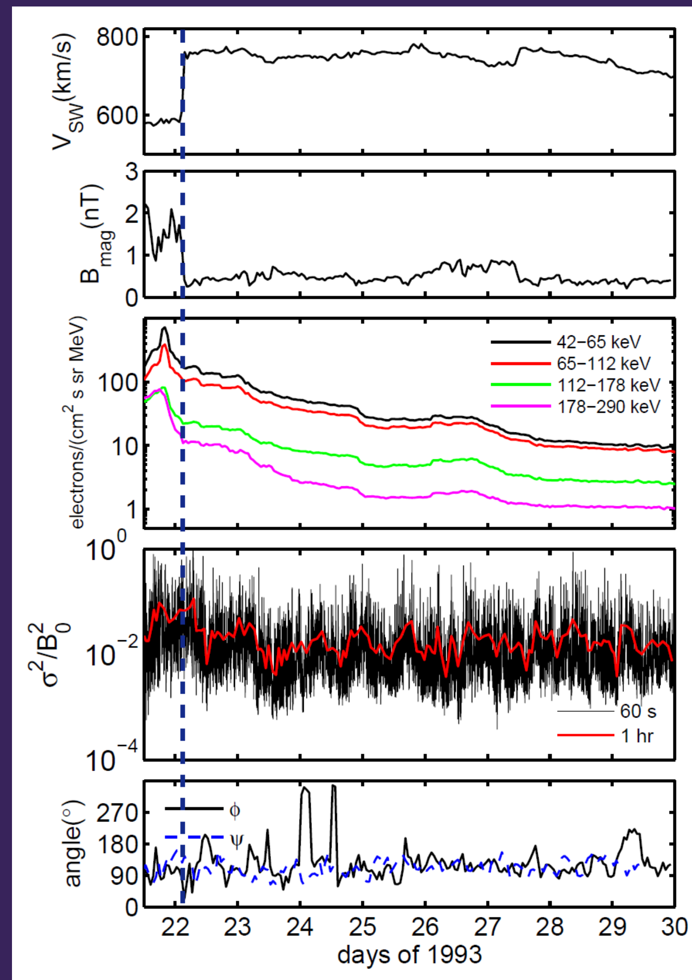
àt=|t-t_{sh}|

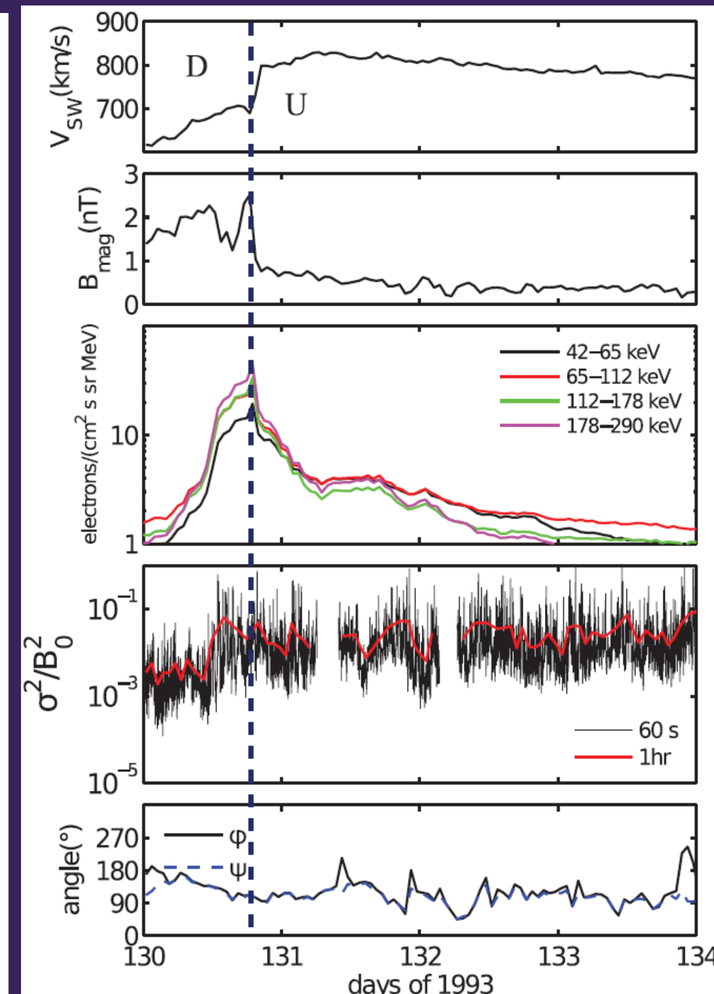
Power law J=A(àt)$^{-°}$

Exponential J=K exp(-ì àt)

Could the "power law" profile be due to a spatially varying diffusion coefficient? … No!





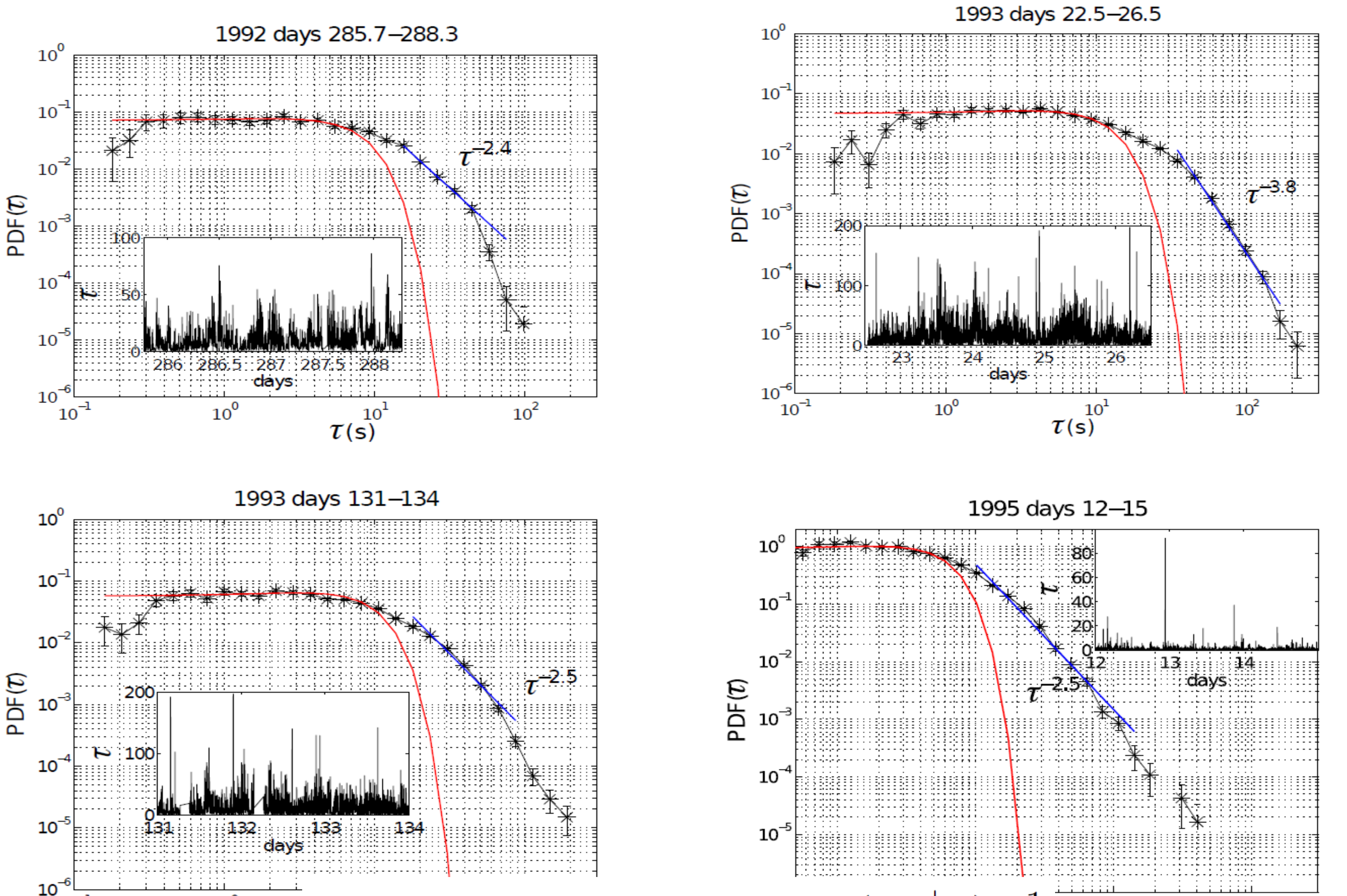


Pitch angle scattering rate:

$$D_{zz} = V^2 \int_{-1}^{1} \mu_1 \left[\int^{\mu_1} \frac{(1-\mu^2)}{\langle (\Delta\mu)^2 \rangle / \Delta t} \, d\mu \right] d\mu_1$$

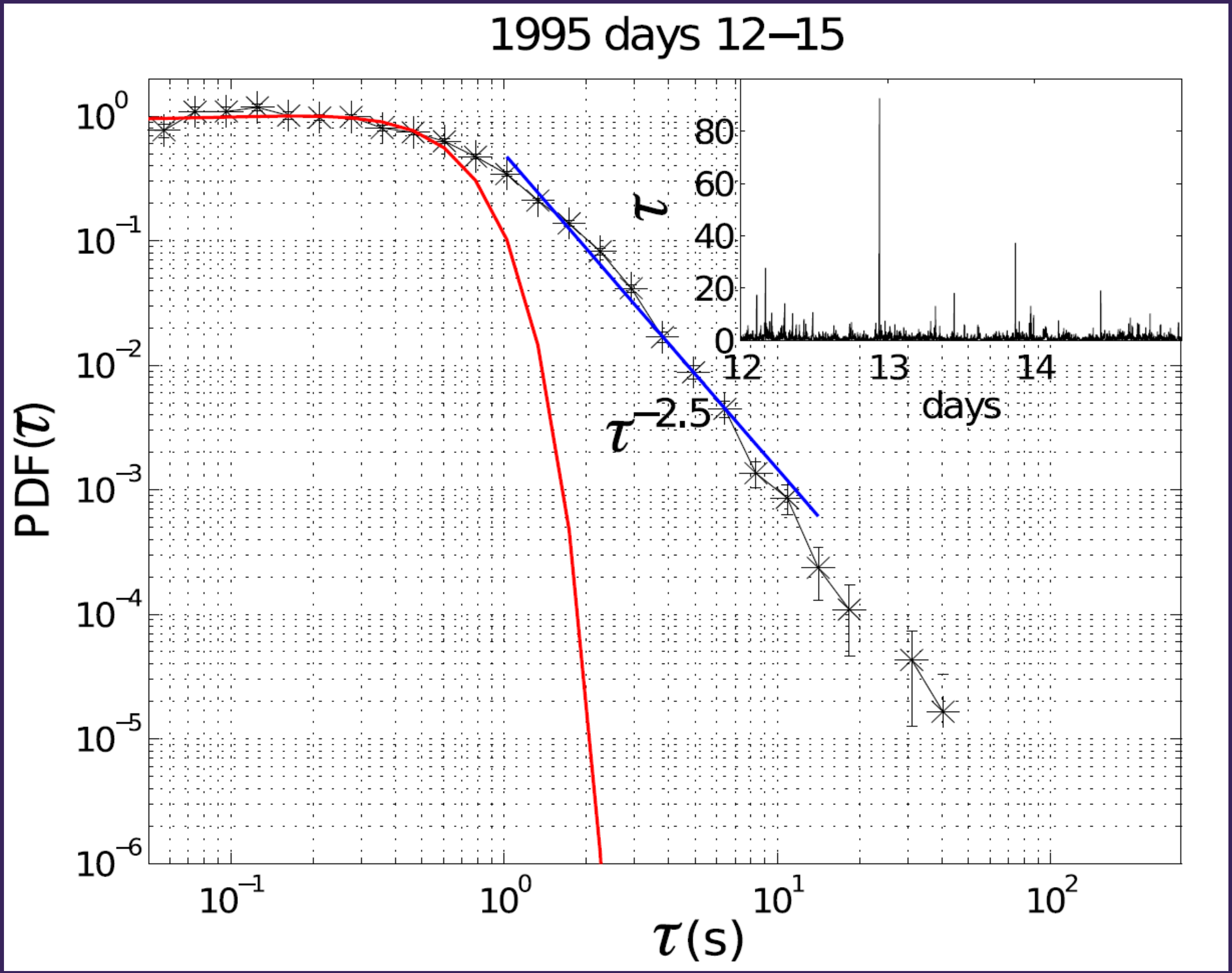
$$D^{\pm}_{\alpha\alpha} = \left(\frac{B_w}{B_0} \right)^2 \Omega_{\pm}$$

Power law distribution of pitch angle scattering times



$$\text{scattering times } \tau = (D^{\pm}_{\alpha\alpha})^{-1}$$

Distribution of pitch-angle scattering times in the fast solar wind (from Perri and Zimbardo, Astrophys. J., 754, 2012).



$$\psi(\ell,\tau) = A|\ell|^{-\mu}\delta(\ell - v_{\|}\tau)$$

$$\psi(\tau) = A|v_{\|}|^{-\mu}\tau^{-\mu}$$

How does superdiffusion modify particle acceleration?

- Superdiffusive Shock Acceleration (SSA),
  Perri and Zimbardo, ApJ, 750 (2012);
  Zimbardo and Perri, ApJ, 778 (2013)
- Work based on two excellent papers:
- Duffy, Kirk, Gallant, Dendy, Astron. Astrophys., 302, L21 (1995)
- Kirk, Duffy, Gallant, Astron. Astrophys., 314, 1010 (1996)
- It is shown that the change in the energy spectral index depends on the scaling properties of the propagator.

Sketch of derivation of SSA energy spectral index

From standard DSA ...

$$\tilde{\gamma} = \frac{P_{\rm esc}}{\Delta E/E} = \frac{3}{V_1/V_2 - 1} \equiv \frac{3}{r-1},$$

$$P_{\rm esc} = \frac{n_2 V_2}{n_0 v/4}$$

n_2V_2 = far downstream particle flux;
$n_0v/4$ = flux of particles crossing from Up to Down

Density as a function of propagator ...

$$n(x,t) = \int P(x-x', t-t') S_{\rm sh}(x',t') dx' dt',$$

$$S_{\rm sh}(x',t') = \Phi_0(E)\delta(x' - V_{\rm sh}t')$$

$$P(x,t) = \tilde{C}\frac{f(\xi)}{t^{1/(\mu-1)}}$$

scaling variable $\xi = \hat{x}/\hat{t}^{1/(\mu-1)}$

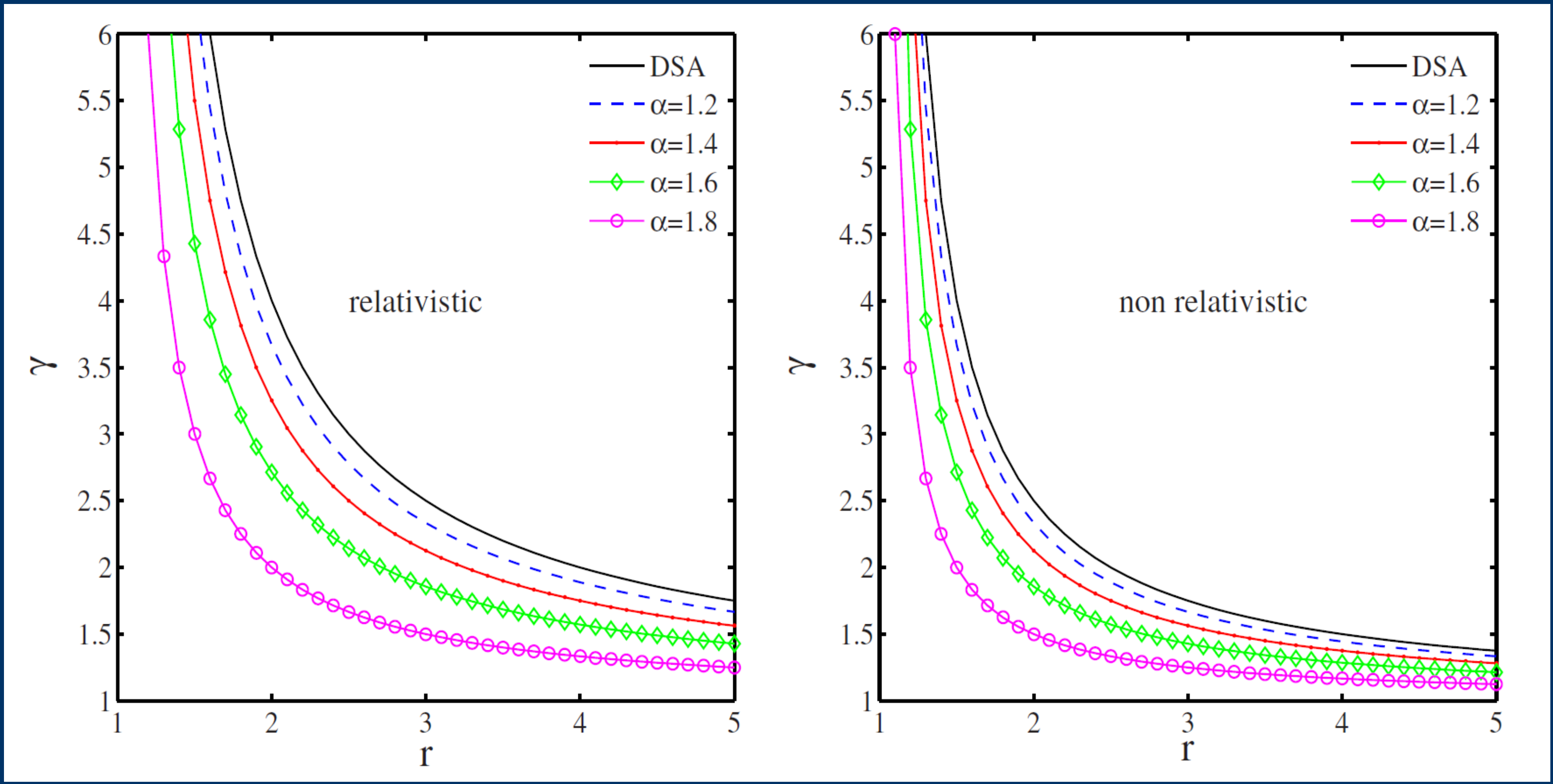
$$n_0 = n_d \frac{V_d}{V_{\rm sh}}\left(\frac{\mu-1}{\mu-2}\right)\int_0^\infty \ell_0 f(\xi)\,d\xi \rightarrow \frac{n_2}{n_0} = 2\frac{\mu-2}{\mu-1}$$

Spectral index of differential energy distribution dN/dE

	DSA	SSA
Relativistic	$\gamma = \dfrac{r+2}{r-1}$	$\gamma = \dfrac{6}{r-1}\dfrac{2-\alpha}{3-\alpha} + 1$
Non relativistic	$\gamma = \dfrac{2r+1}{2(r-1)}$	$\gamma = \dfrac{3}{r-1}\dfrac{2-\alpha}{3-\alpha} + 1$

In general, superdiffusive shock acceleration allows to obtain harder spectra than DSA.

Spectral index as a function of compression ratio for several superdiffusion exponents

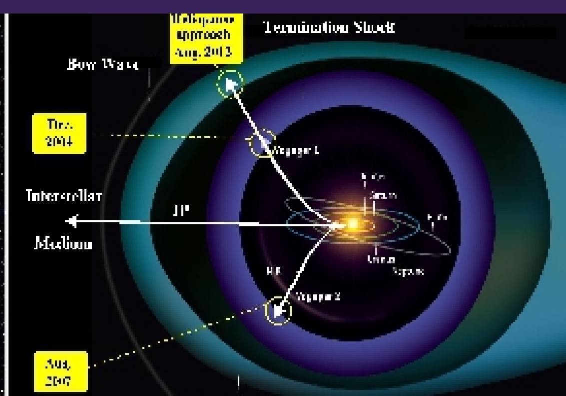


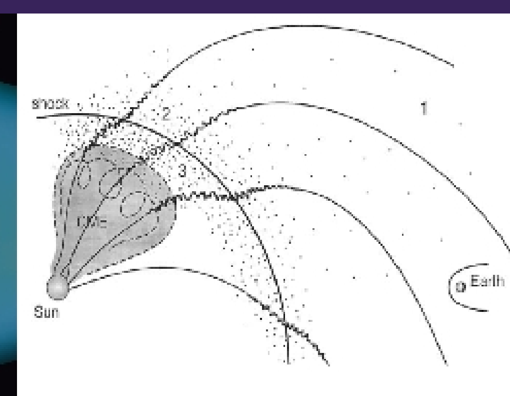
From Zimbardo and Perri, ApJ, 2013

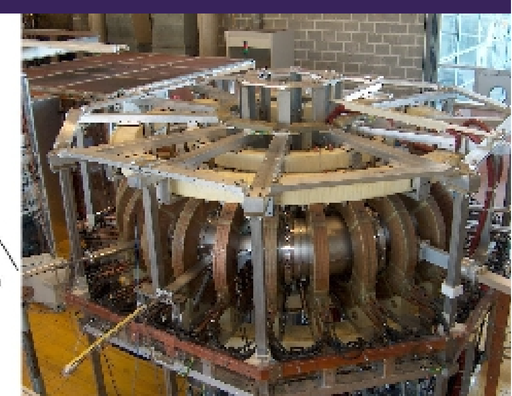
41st COSPAR scientific assembly 2016
(Istanbul, August 2016)

“Event D1.4: Anomalous transport of energetic particles in the heliosphere and the galaxy”

Organized by G. Zimbardo, F. Effenberger, S. Perri and H. Fichtner





Summary and suggestions for future work

The theoretical modeling of anomalous transport has been quickly revised.

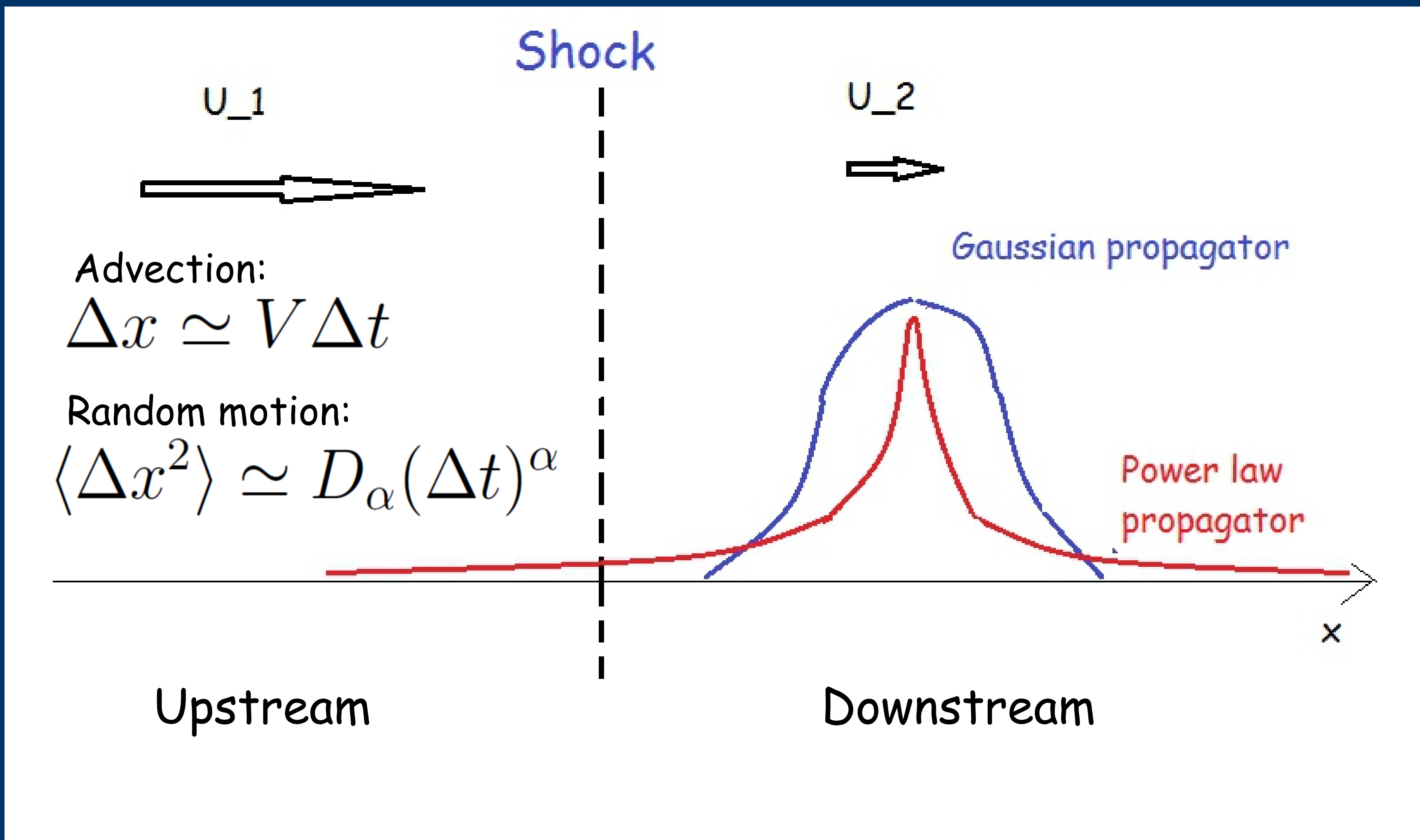
Numerical simulations leading to subdiffusion and superdiffusion have been presented.

A method to extract the anomalous diffusion parameters from spacecraft data has been presented.

The theory of diffusive shock acceleration has been extended to the superdiffusive case: from DSA to SSA.

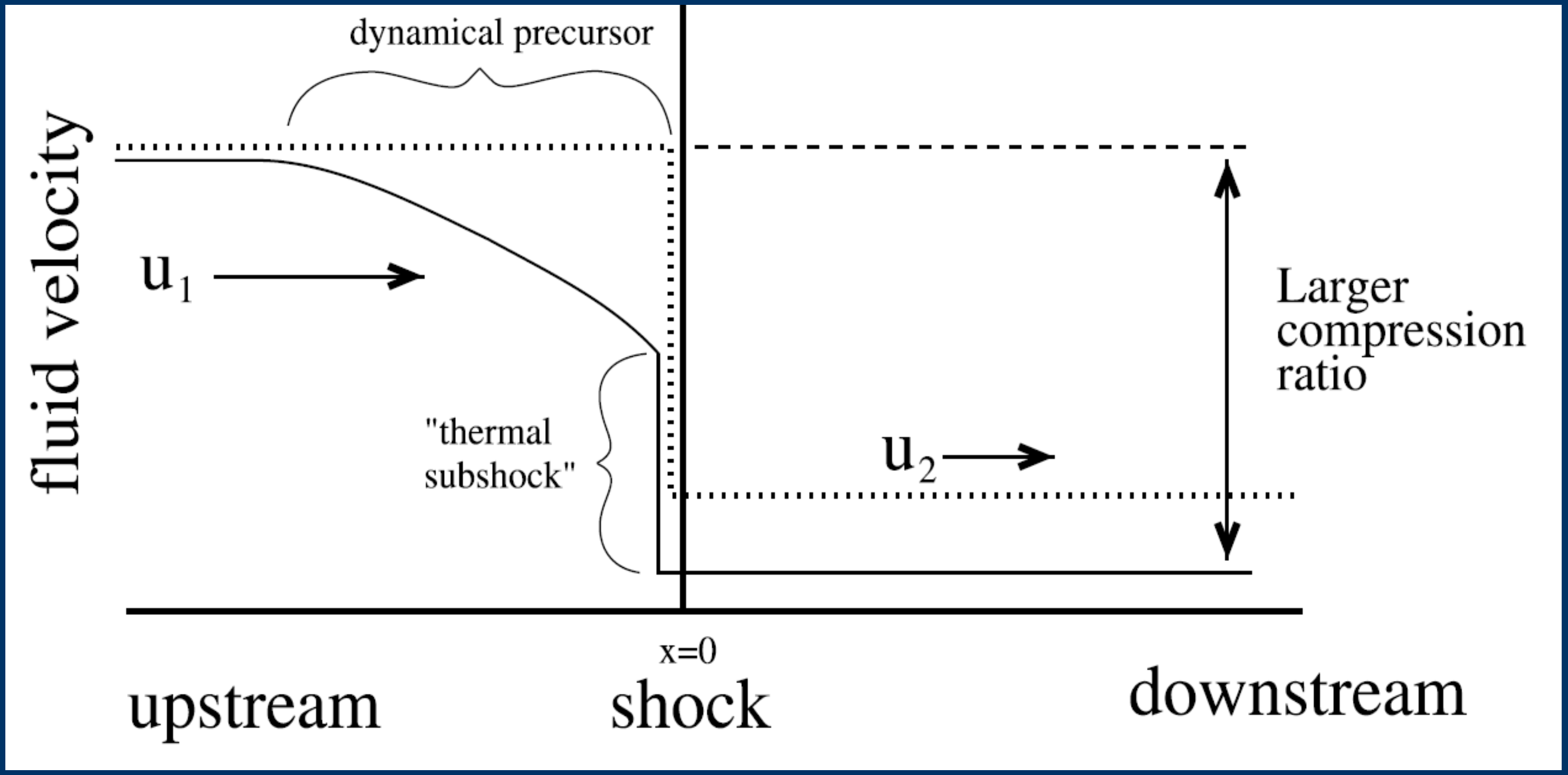
More work is needed to understand in which cases wave particle interactions lead to anomalous diffusion. In particular, nonlinear regimes and particle trapping in the waves have to be considered.

What is the origin of the different spectral index? It is the larger return probability due to the power law tails of the propagator

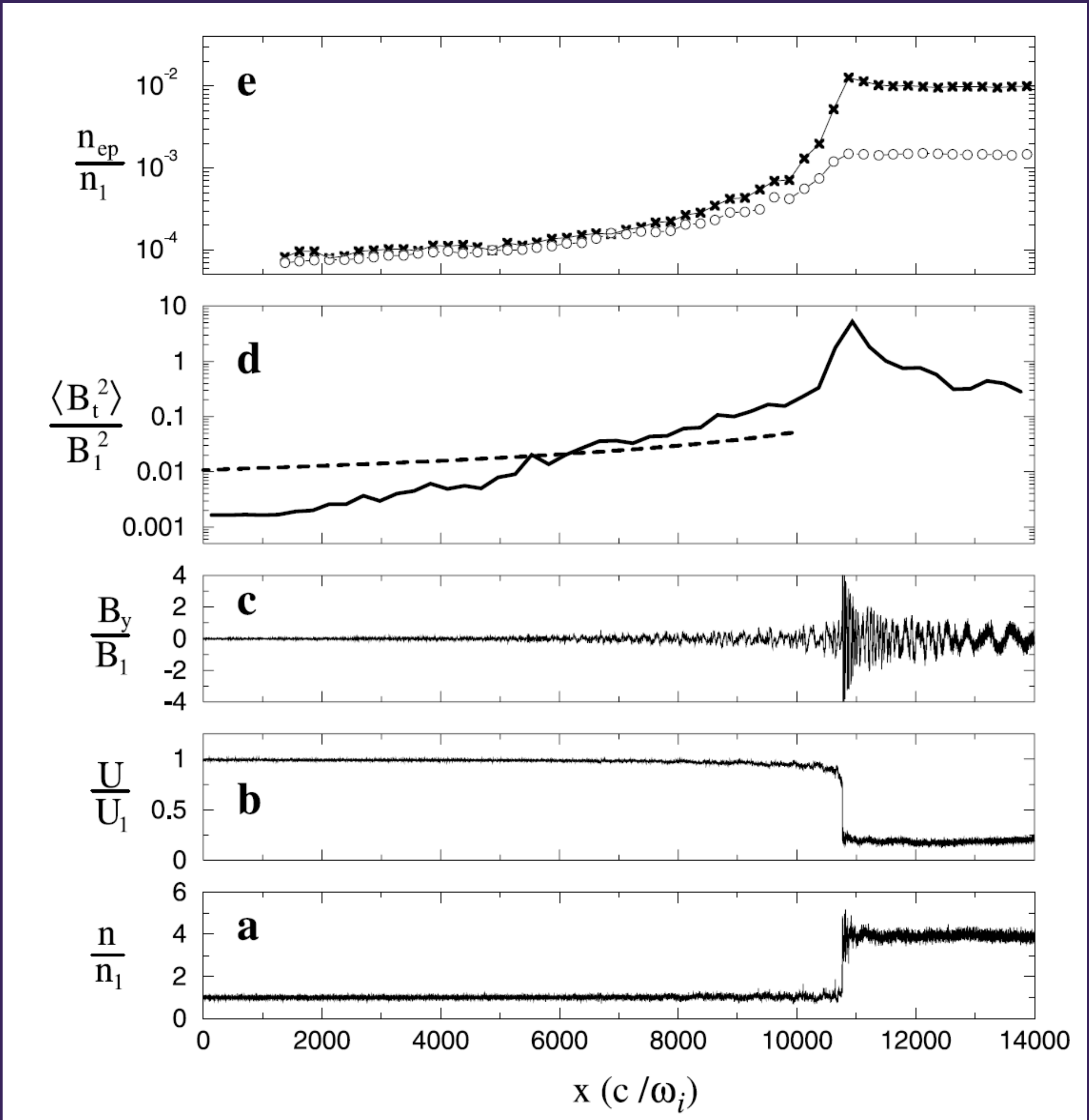


The effective compression ratio can be increased by considering and extended cosmic ray precursor. This lead to a spectral flattening for high energies, but this is not the case of GeV electrons which are accelerated close to the shock.

SSA leads to harder spectra even with r = 4



Can the "power law" profile be due to a spatially varying diffusion coefficient?



Numerical simulations by Giacalone, ApJ (2004), show that the magnetic fluctuation level decreases with the shock distance, implying a weaker pitch-angle scattering.