





# Solar Wind: the Legacy of Helios and the promises of Solar Orbiter



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Ground based and space instruments for researches in Solar-Terrestrial Physics 6-10 June 2016, L'Aquila, Italy

### Outline

### State of the art on solar wind observations (Helios data)

- Solar wind and IMF macrostructure
- Differences between fast and slow wind and radial evolution

### Open issues

□ The upcoming **Solar Orbiter** mission

### Helios program

Most of our knowledge about solar wind plasma and magnetic field in the inner heliosphere is due to Helios 1-2 s/c developed by the Federal Republic of Germany (FRG) in a cooperative program with NASA



Programme realized in only 5 years!

1969: contract between FRG and NASA approved

10 December 1974: Helios 1 launched

### **Helios instruments**



Nr.	Thema	Experimentatoren	Institut	
1	Sonnenwind	H. ROSENBAUER, R. SCHWENN	MPI für Physik und Astrophysik, Institut für extraterrestrische Physik, Garching	Plasma Experiment 1.
2020		J. H. WOLFE	NASA Ames Research Center	Magnetic Field
2/4	Interplanetares	G. MUSMANN, G. DEHMEL,	TU Braunschweig, Institut für Geophysik und Meteorologie	
3	Magnetfeld	N. F. NESS, L. F. BURLAGA F. MARIANI	NASA Goddard Space Flight Center Universität Rom, Istituto di Fisica	Experiments 2. 3. 4.
5	Elektrische	D. A. GURNETT	Un. of Jowa, Dep. of Physics and	Plasma Wave
	Felder,	P. J. Kellogg	Astronomy, Jowa City Un. of Minnesota, School of Physics and Astronomy, Minneapolis	Experiment 5.
	Radiowellen	R. R. WEBER	NASA Goddard Space Flight Center	
6	Kosmische	H. KUNOW, G. GREEN, R. MÜLLER, G. WIBBERENZ	Un. Kiel, Institut für Reine und Angewandte Kernphysik	Cosmic Radiation
7	Strahlung	J. H. TRAINOR, K. G. MCCRACKEN F. B. MCDONALD E. C. ROELOF, B. J. TEEGARDEN	NASA Goddard Space Flight Center Un. of New Hampshire SCIRO, Melbourne, Australien	Experiment 6. 7.
8	Strahlung mitt-	E. KEPPLER, G. UMLAUFT,	MPI für Aeronomie, Lindau	Low-Energy Electron
	lerer Energie	B. WILKEN WILLIAMS	ESSA, Boulder	and Ion Spectrometer 8
9	Zodiakallicht	C. LEINERT, H. LINK, E. PITZ	MPI für Astronomie, Heidelberg	
10	Mikro- meteoriten	E. GRUN, P. GAMMELIN, J. KISSEL	MPI für Kernphysik, Heidelberg	Zodiacal Light
11	Relativitäts- theorie	W. KUNDT, O BÖHRINGER W. G. MELBOURNE, I. D. ANDERSON	Un. Hamburg, Institut f. Theor. Physik JPL, Pasadena	Photometer 9.
12	Faraday Rotation	H. VOLLAND, M. BIRD G. S. LEVY	Un. Bonn, Radioastron. Institut JPL, Pasadena	Micrometeorid
12 Z	Elektronen- dichte der Korona	P. Edenhofer, E. Lüneburg	DFVLR Oberpfaffenhofen	Andryser 10.

### Helios lifetime during solar cycles 20 - 21



made from hourly averages

helios 2 # events minimum SW speed helios 2 # events ascending SW speed helios 2 # events maximum SW speed

Best data coverage during primary missions to the Sun during 1975 and 1976

Bimodal nature of solar wind particularly clear during solar minimum

### **First Helios observations**

Decreasing distance

@ 1 AU

- Broad high-speed streams with  $v_p \approx 700$  km/s,  $n_p \approx 3$  cm<sup>-3</sup>,  $T_p \approx 2.10^5$  K
- Regions of slow solar wind with  $v_p$  ~ 400 km/s,  $n_p$  ~ 10 cm^{-3} and  $T_p$  ~ 4.10^4 K, highly variable

With decreasing distance, we observe a steepening of the fast streams' leading edges



(Schwenn, 1990)

### First detailed studies on the dynamical interaction between fast and slow wind



 The interactions of slow and fast solar wind at the leading edges of high speed streams causes compression to high plasma densities and deflections of the flow on both sides: westward in the slow plasma and estward in the fast plasma



Fast wind interacts with the slow wind ahead creating a compression region called *stream-interface* 



### The crossing of the HCS at short heliocentric distances



Figure 5. An idealized coronal streamer and its stalk, which forms the plasma sheet in interplanetary space. The radial extension of the boundaries of the streamer are responsible for the observed density halo. The profile of path-integrated density is also shown.



[Bavassano et al., 1997]

### Importance of separating fast from slow wind

□ Fast and slow wind features should never be averaged together.

«Asking for the average solar wind might appear as silly as asking for the taste af an average drink. What is the average between wine and beer? Obviously a mere mixing – and averaging means mixing – does not lead to a meaningful result.

Better taste and judge separately and then compare, if you wish.»

[Rainer Schwenn, Solar Wind 5, 1982]

### Differences in the Alfvénic character of the fluctuations





### Differences in the $\delta \underline{B}$ - $\delta \underline{V}$ alignment



 $\vec{b}$  and  $\vec{v}$  quite aligned within fast wind best alignment ~ 20-30 min

### Differences in the spectral signature



### **Differences in the correlation length**







IMF power spectrum at 1 AU (Low freq. from Bruno el al, 1985; high freq. Tail from Leamon et al, 1999)

 $10^{7}$ 10<sup>6</sup> 10<sup>5</sup>

10<sup>4</sup>

10<sup>3</sup>

10<sup>2</sup>

10 10<sup>c</sup>

10<sup>-1</sup>  $10^{-2}$ 

10<sup>-3</sup>

 $10^{-4}$ 

 $10^{-7}$ 

fraction of AU

power density [ $\gamma^2/Hz$ ]

Laboratory experiment with low temperature helium gas flow











Cascade à la Richardson



(Frisch, 1995)

### **Navier-Stokes equations**



Elsässer variables

 $\vec{z}^{\pm} = \vec{v} \pm \vec{b} = \vec{v} \pm \vec{B} / \sqrt{4\pi\rho}$ 

### Differences in the power associated to e<sup>+</sup> and e<sup>-</sup>



### Differences in the level of normalized crosshelicity



[adapted from Marsch and Tu, 1990]

### Differences in the level of magnetic and kinetic energy content



[adapted from Marsch and Tu, 1990]

### Power law spectra $\rightarrow$ scale invariance $\rightarrow$ self-similarity

A typical IMF power spectrum in interplanetary space at 1 AU [Low frequency from *Bruno et al.*, 1985; high freq. tail from *Leamon et al*, 1999]



### What is intermittency?

#### Data show that solar wind PDFs DO NOT RESCALE

♣ Large scales  $\rightarrow$  Gaussian PDF

 $\clubsuit$  Small scales  $\rightarrow$  peaked PDF fatter tail, extreme events more probable

The evolution of the shape of the PDF can be measured by the *Flatness*  $F_{\tau}$ (fourth order moment) of the distribution itself. An estimate of this parameter can be obtained directly from the structure functions:  $S_{\tau}^{p} = \langle x(t+t)-x(t) \rangle^{p} \rangle$ 



### **Different Flatness**



### **Radial dependences with Helios**

All these features evolve with the radial distance from the Sun in the fast wind





Coronal conditions particularly steady at the Sun allowed to observe the 'same' corotating stream at different heliocentric distances [Villante (1980) and Bavassano *et al.* (1982)]



Unique chance to study the radial evolution of fluctuations while looking at plasma coming from the same solar source



### Radial evolution of solar wind turbulence



## Evolution in the power associated to e<sup>+</sup> and e<sup>-</sup>

#### FAST

For increasing distance:

- e<sup>+</sup> decreases towards e<sup>-</sup>
- spectral slope evolves towards -5/3

### SLOW

- No much radial evolution
- spectral slopes always close to -5/3



### Evolution of $\delta \underline{B} - \delta \underline{V}$ alignment



FAST

- Since  $e^+ \rightarrow e^-$ ,  $\delta \underline{B} \delta \underline{V}$  alignment decreases during expansion
- Best alignment for younger turbulence (0.3AU)

#### **SLOW**

 No alignment for slow wind, as expected from fully developed turbulence (|δZ<sup>+</sup>|=|δZ<sup>-</sup>|)

### **Evolution of intermittency**

flatness factor  $F_{\tau}$  as a function of scale and radial distance from the sun computed for magnetic field vector fluctuations

![](_page_28_Figure_2.jpeg)

- slow wind is more intermittent than fast wind
- fast wind shows a clear radial trend which is missing in the slow wind.

![](_page_29_Picture_0.jpeg)

 $\sigma_{c} = \frac{e^{+} - e^{-}}{e^{+} + e^{-}} = \frac{2 < v \cdot b >}{e^{v} + e^{b}}$  $\sigma_{R} = \frac{e^{v} - e^{b}}{e^{v} + e^{b}}$  $\sigma_{C}^{2} + \sigma_{R}^{2} \le 1$ 

![](_page_30_Figure_0.jpeg)

### Radial evolution of MHD turbulence in terms of $\sigma_R$ and $\sigma_C$ (scale of 1hr)

![](_page_30_Figure_2.jpeg)

 $\sigma_{c} = \frac{e^{+} - e^{-}}{e^{+} + e^{-}} = \frac{2 < v \cdot b >}{e^{v} + e^{b}}$  $\sigma_{R} = \frac{e^{v} - e^{b}}{e^{v} + e^{b}}$  $\sigma_{C}^{2} + \sigma_{R}^{2} \le 1$ 

![](_page_31_Figure_0.jpeg)

### Radial evolution of MHD turbulence in terms of $\sigma_R$ and $\sigma_C$ (scale of 1hr)

![](_page_31_Figure_2.jpeg)

A new population appears, characterized by magnetic energy excess and low Alfvénicity

![](_page_32_Figure_0.jpeg)

### Radial evolution of MHD turbulence in terms of $\sigma_R$ and $\sigma_C$ (scale of 1hr)

![](_page_32_Figure_2.jpeg)

this might be a result of turbulence evolution or the signature of underlying advected structure

![](_page_33_Figure_0.jpeg)

![](_page_33_Figure_1.jpeg)

Helios 2 observations

Different situation in Slow Wind:

- no evolution
- second population already present at 0.3 AU

### Solar wind turbulence is mainly made of two 'ingredients'

(Mariani et al., 1973; Thieme et al., 1988, 1989; Tu et al., 1989, 1997; Tu and Marsch, 1990, 1993; Bieber and Matthaeus, 1996; Crooker et al., 1996; Bruno et al., 2001, 2003, 2004; Chang and Wu, 2002; Chang, 2003; Chang et al., 2004; Tu and Marsch, 1992, Chang et al., 2002, Borovsky, 2006, 2009, Li, 2007, 2008, *Tu and Marsch*, 1991; *Bruno and Bavassano*, 1991, *Bieber et al*, 1996; see more refs. in *Bruno and Carbone*, 2013)

- Alfvénic fluctuations which propagate
- Structures advected by the wind or locally generated

![](_page_34_Figure_4.jpeg)

(Bruno et al, 2001)

### Solar wind turbulence is mainly made of two 'ingredients'

(Mariani et al., 1973; Thieme et al., 1988, 1989; Tu et al., 1989, 1997; Tu and Marsch, 1990, 1993; Bieber and Matthaeus, 1996; Crooker et al., 1996; Bruno et al., 2001, 2003, 2004; Chang and Wu, 2002; Chang, 2003; Chang et al., 2004; Tu and Marsch, 1992, Chang et al., 2002, Borovsky, 2006, 2009, Li, 2007, 2008, *Tu and Marsch*, 1991; *Bruno and Bavassano*, 1991, *Bieber et al*, 1996; see more refs. in *Bruno and Carbone*, 2013)

- Alfvénic fluctuations which propagate
- Structures advected by the wind or locally generated

![](_page_35_Figure_4.jpeg)

### **Typical VDF and heating**

717 km/s

781 km/s

Fast wind

(Marsch et al, 1982)

![](_page_36_Figure_2.jpeg)

Slow wind

Fast wind does not expand adiabatically Preferencial heating perpendicular to local B direction

![](_page_36_Figure_5.jpeg)

Thermal anisotropies hide kinetic processes not fully understood

### **Kinetic aspects**

Wave-particle interactions are the key to understand ion kinetics in the corona and solar wind

![](_page_37_Figure_2.jpeg)

Alfvénic fluctuations might play an important role in determining the speed of minor ions.

- $\Delta V_{\alpha p}$  increases with  $V_{sw}$
- $\Delta V_{\alpha p}^{-}$  increases approaching the sun
- $\Delta V_{\alpha p}^{\mu r}$  is of the order of  $V_A$
- No radial dependence for slow wind

![](_page_37_Figure_8.jpeg)

ions in resonance with transverse ion-cyclotron waves, propagating parallel to the magnetic field, undergo merely pitch-angle diffusion which shapes the VDF

### Radial evolution of solar wind turbulence

![](_page_38_Figure_1.jpeg)

No dissipation range with Helios data since temporal resolution does not allow to investigate this range.

[Bruno & Carbone, 2013]

### Radial evolution of the 'kinetic' break

![](_page_39_Figure_1.jpeg)

Magnetic field spectral densities relative to measurements recorded by Messenger (at 0.42 and 0.56 AU), Helios 2 (at 0.29, 0.65 and 0.89 AU), Wind at the Lagrangian point L1, and Ulysses at 1.4 AU within highspeed streams observed in the ecliptic.

Recently, it has been found that the break position is in remarkable agreement with the ion-cyclotron resonant frequency condition.

Different relevant lengths can be associated with the heating phenomenon, depending on the particular dissipation mechanism we consider.

(Telloni et al, ApJ 2015)

![](_page_40_Picture_0.jpeg)

## Solar Orbiter's novelties respect to previous missions

solar orbiter

Solar Orbiter will be the first spacecraft since Helios to sample the inner heliosphere at distances as close to the Sun as  $60 R_s$ . The main goal is to study the link between solar sources and in situ measurements. To do that:

- ✓ It will be equipped with in-situ instruments significantly more capable than those flown on Helios, as well as with remote-sensing instruments for the observation of the corona and photosphere.
- ✓ Its orbital design allows the spacecraft to achieve approximate co-rotation with the Sun for periods of several days, measuring the solar wind plasma and magnetic field in-situ while simultaneously observing their source regions on the Sun.
- ✓ Increasing inclination up to more than 30° with respect to the solar equator allows out-of-ecliptic measurements.

![](_page_41_Figure_0.jpeg)

solar orbiter

![](_page_42_Picture_0.jpeg)

### Solar Orbiter detailed science objectives

Objective 1: What drives the solar wind and where does the heliospheric magnetic field originate?

- 1.1 What are the source regions of the solar wind and heliospheric magnetic field?
- 1.2 What mechanisms heat and accelerate the solar wind?
- 1.3 What are the sources of solar wind turbulence and how does it evolve?

Objective 2: How do solar transients drive heliospheric variability?

- 2.1 How do CMEs evolve through the corona and inner heliosphere?
- 2.2 How do CMEs contribute to solar magnetic flux and helicity balance?
- 2.3 How and where do shocks form in the corona?

Objective 3: How do solar eruptions produce energetic particle radiation that fills the heliosphere?

- 3.1 How and where are energetic particles accelerated at the Sun?
- 3.2 How are energetic particles released from their sources and distributed in space and time?
- 3.3 What are the seed populations for energetic particles?

Objective 4: How does the solar dynamo work and drive connections between the Sun and the heliosphere?

- 4.1 How is magnetic flux transported to and re-processed at high solar latitudes?
- 4.2 What are the properties of the magnetic field at high solar latitudes?
- 4.3 Are there separate dynamo processes acting in the Sun?

Payload: In-Situ Instruments							
100	metis						
EPD	Energetic Particle Detector	J. Rodríguez- Pacheco(E)	Composition, timing and distribution functions of energetic particles				
MAG	Magnetometer	T. Horbury (UK)	High-precision measurements of the heliospheric magnetic field				
RPVV	Radio & Plasma Waves	M. Maksimovic (F)	Electromagnetic and electrostatic waves, magnetic and electric fields at high time resolution				
SWA	Solar Wind Analyser	C. Owen (UK)	Sampling protons, electrons and heavy ions in the solar wind				

#### Payload: Remote-Sensing Instruments

EUI	Extreme Ultraviolet Imager	P. Rochus (B)	High-resolution and full-disk EUV imaging of the on-disk solar corona
METIS	Coronagraph	E.Antonucci (l)	Visible and (E)UV imaging of the off-disk corona
РНІ	Polarimetric & Helioseismic Imager	S. Solanki (D)	High-resolution vector magnetic field, line-of-sight velocity in photosphere, visible imaging
SoloHI	Heliospheric Imager	R. Howard (USA)	Wide-field visible imaging of the solar corona and wind
SPICE	Spectral Imaging of the Coronal Environment	European- led	EUV spectroscopy of the solar disk and near-Sun solar corona
STIX	Spectrometer/Telecope for Imaging X-rays	S. Krucker (CH)	Imaging spectroscopy of solar X-ray emission

### Solar Wind Analyser Plasma Suite

![](_page_44_Picture_1.jpeg)

The Solar Wind Plasma Analyzer (SWA) consists of a suite of 3 sensors:

- the Electron Analyser System (EAS),
- the Proton-Alpha Sensor (PAS) and
- the Heavy Ion Sensor (HIS),

together with a common DPU.

First suite of coordinated in situ measurements made inside 1 AU, which include <u>mass</u> <u>composition</u> as well as <u>high resolution 3-D</u> <u>velocity distributions</u> (ions and electrons).

solar orbiter

### solar orbiter

### Proton and Alpha Sensor (PAS)

![](_page_45_Picture_2.jpeg)

#### • Energy range from 0.2 – 20 keV/q, with $\Delta$ E/E ~7.5%

• FoV: elevation  $\pm 22.5^{\circ} \Delta \Theta = 5^{\circ}$ , azimuth  $-24^{\circ} \div 42^{\circ} \Delta \Phi = 6^{\circ}$ 

#### High temporal resolution

- Full 3D VDF sampled at 1 sec (NM)
- **Moments** (number density, bulk speed, pressure tensor) of the proton distribution at 4s (NM)
- Reduced 3-D distributions up to 14 Hz (BM)

Scientific objectives

- kinetic and fluid properties of the bulk solar wind plasma and dominant physical processes (e.g.: wave- particle interactions, origin and dissipation of turbulence, etc);
- dynamics and evolution of stream interactions, shocks and CMEs

![](_page_45_Picture_12.jpeg)

### **HELIOS-Solar Orbiter comparison**

SWA:

□ sampling capabilities three orders of magnitude faster than Helios.

 $\hfill\square$  First time exploration of the dissipation range with 3D VDF

![](_page_46_Figure_4.jpeg)

![](_page_46_Picture_5.jpeg)

### Structures advected by the wind

![](_page_47_Figure_1.jpeg)

Going close to the sun and sampling for long enough time intervals will allow to go through the advected structure of the wind

### Conclusion

![](_page_48_Picture_1.jpeg)

- ✓ Helios still represents a unique dataset to study the radial evolution of solar wind fluctuations in the inner heliosphere
- Our scientific community has been working with Helios data for the past 40 years mainly using data from the primary missions of Helios 1 and 2 (about 4 months each), publishing hundreds of papers.

- SolO is a discovery mission: remote & in situ packages
  @ 0.28 AU, corotation, high latitude
- SolO will answer fundamental questions relevant to both solar and stellar physics.
- The Solar Wind Analyser will investigate kinetic and fluid properties of the bulk solar wind plasma and dominant physical processes allowing to investigate for the first time composition and the dissipation range close to the Sun.

![](_page_48_Picture_7.jpeg)

#### The spectral cascade ends up in what looks like a "dissipation range "

![](_page_49_Figure_1.jpeg)

typical IMF power spectrum in at 1 AU [Low frequency from Helios (Bruno et al., 1985), high freq. tail from WIND (Leamon et al, 1998)]

- Correlative Scale/Integral Scale:
  - the largest separation distance over which eddies are still correlated. i.e. the largest turb. eddy size.
- Taylor scale:
  - The scale size at which viscous dissipation begins to affect the eddies.
  - Several times larger than Kolmogorov scale
  - it marks the transition from the inertial range to the dissipation range.
- Kolmogorov scale:
  - The scale size that characterizes the smallest dissipation-scale eddies

![](_page_49_Figure_11.jpeg)

(Batchelor, 1970)

different relevant lengths can be associated with the heating phenomenon, depending on the particular dissipation mechanism we consider

I Characteristic scales which could be related to the observed spectral break are:

Proton inertial length  $\lambda_i = 2\pi c / \omega_p$ Proton Larmor radius  $\lambda_L = 2\pi v_{th} / \Omega_p$ where  $\begin{aligned} \omega_p &= (4\pi n q^2 / m_p)^{1/2} & \text{proton plasma frequency [rad/s]} \\ \Omega_p &= q B / (m_p c) & \text{proton cyclotron frequency [rad/s]} \end{aligned}$ since  $c / \omega_p = v_A / \Omega_p$ proton inertial length can be expressed as  $\lambda_i = 2\pi v_A / \Omega_p \implies \lambda_i \cong \lambda_L$ 

### Heavy ions will help to identify the source regions of CME's

![](_page_51_Figure_1.jpeg)

![](_page_51_Figure_2.jpeg)

solar orbiter

SWA measurements of

- electron pitch angle distribution,
- alpha/proton ratio,
- freeze-in temperature (e.g. Fe),
- O and Fe charge state ratios will establish firm links between coronal sources of CME's and their in-situ counterparts.

![](_page_52_Picture_0.jpeg)

### Origin of the solar wind

![](_page_52_Figure_2.jpeg)

 The transition between fast and slow wind is sharply detected by
 elemental and charge composition which remains unchanged during wind expansion.

Small scale properties of coronal hole boundaries can be detected

TABLE I. A list of the most abundant elements present in the solar photosphere arranged according to their FIP.

Element	Ζ	FIP (eV)	Photospheric abundance (relative to H)
He	2	24.6	$9.0 \times 10^{-2}$
Ne	10	21.6	$1.2 \times 10^{-4}$
Ar	18	15.8	$3.6 \times 10^{-6}$
N	7	14.5	$1.1 \times 10^{-4}$
Н	1	13.6	1
0	8	13.6	$8.5 \times 10^{-4}$
Cl	17	13.0	$1.9 \times 10^{-7}$
С	6	11.3	$3.6 \times 10^{-4}$
Р	15	10.5	3.7×10 <sup>-7</sup>
S	16	10.3	$1.9 \times 10^{-5}$
Si	14	8.1	3.6×10 <sup>-5</sup>
Fe	26	7.9	$3.2 \times 10^{-5}$
Mg	12	7.6	3.8×10 <sup>-5</sup>
Ni	28	7.6	$1.8 \times 10^{-6}$
Mn	25	7.4	$3.4 \times 10^{-7}$
Cr	24	6.8	$4.8 \times 10^{-7}$
Ca	20	6.1	$2.2 \times 10^{-6}$
Al	13	6.0	$3.0 \times 10^{-6}$
Na	11	5.2	$2.1 \times 10^{-6}$
K	19	4.3	$1.4 \times 10^{-7}$

Slow wind has higher oxygen freeze-in temperature Slow wind has higher FIP effect (enrichment of Mg/O, Mg has a lower FIP with respect to O)

### Power law spectra $\rightarrow$ self-similarity

A typical IMF power spectrum in interplanetary space at 1 AU [Low frequency from *Bruno et al.*, 1985; high freq. tail from *Leamon et al*, 1999]

![](_page_53_Figure_2.jpeg)

#### Scale invariance implies selfsimilar PDFs

#### Power law brings scale invariance

A given observable  $v(\ell)$  is invariant for a scale transformation  $\ell \rightarrow r\ell$  if there exists a parameter  $\mu(r)$  such that  $v(\ell)=\mu(r)v(r\ell)$ . The solution of this relation is a power law:  $v(\ell)=C\ell^h$  where  $h=\log \mu(r)$ .

![](_page_53_Figure_6.jpeg)

From small to large scales

If we introduce a scale transformation  $\ell \rightarrow r\ell$ , we obtain:  $\delta v_{r\ell} \sim r^h \delta v_\ell$  that implies that PDF( $\delta v_{r\ell}$ ) ~ PDF( $r^h \delta v_\ell$ ). Van Atta and Park (1975) showed that, using standardized variables like  $y_\ell = \delta v_\ell / \langle (\delta v_\ell)^2 \rangle^{1/2}$ , we obtain: PDF( $y_\ell$ )=PDF( $y_{r\ell}$ ).