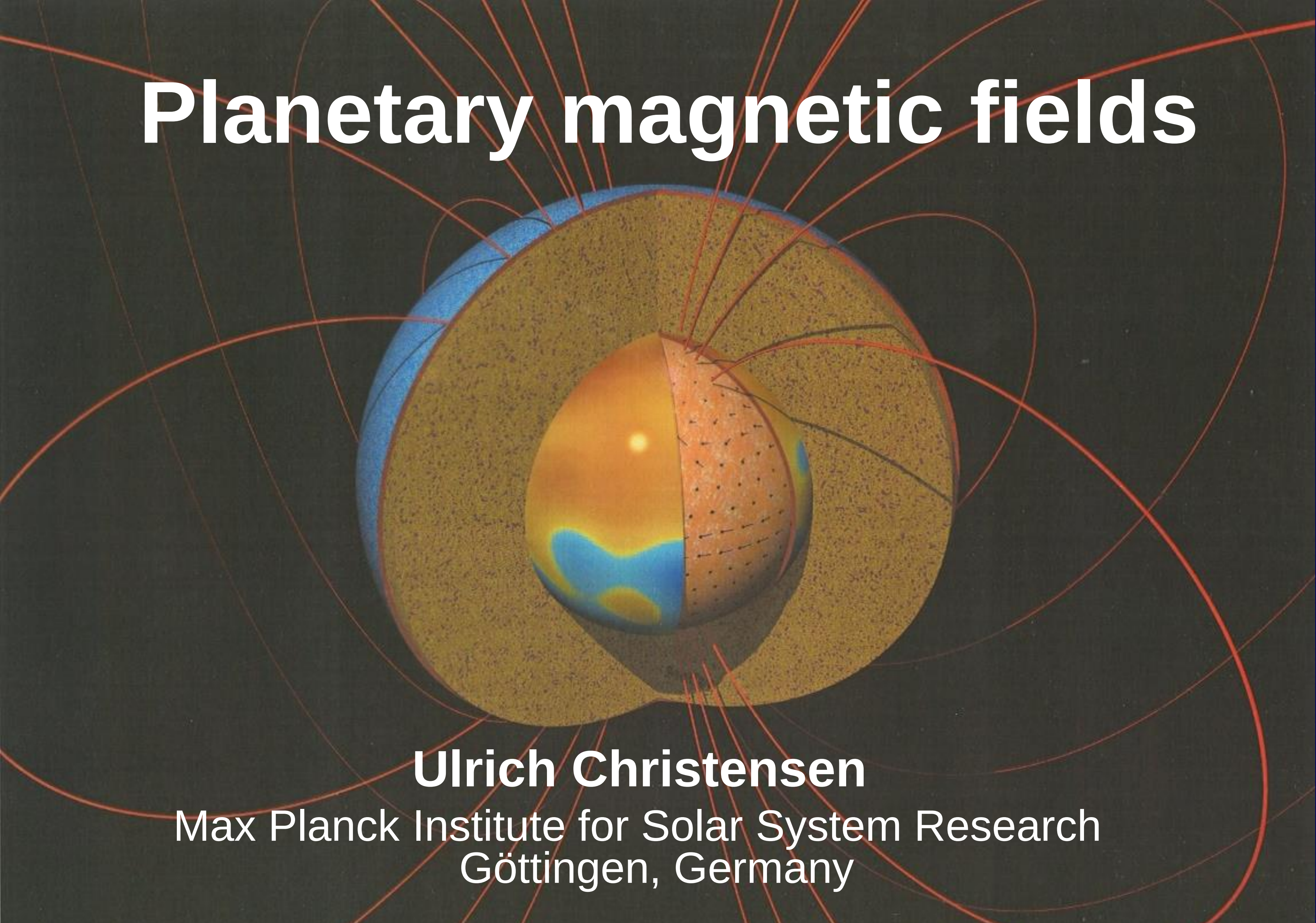

Planetary magnetic fields

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Why interesting ?

- Homogeneous dynamo process is a fundamental physical problem
- Magnetic field controls how planet interacts with its space environment (e.g. solar wind)
- Magnetic field provides window into deep interior of planet, allowing inferences on constitution, dynamics and thermal evolution

Overview

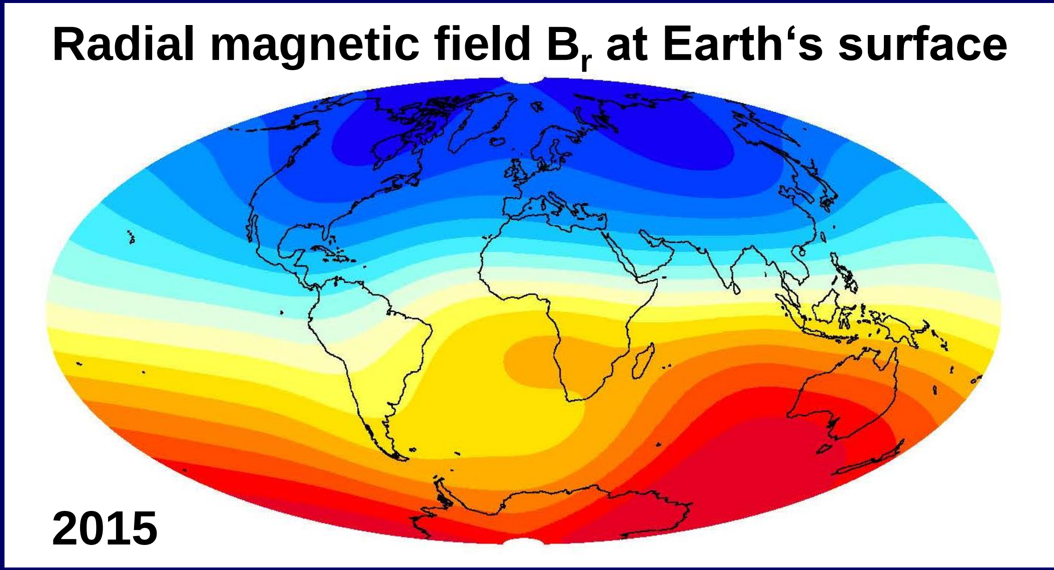
- Properties of Earth‘s magnetic field
- Magnetic fields of solar system planets
- Fundamental requirements for a dynamo
- (A glimpse at the) Theory of dynamos
- Dynamo concepts and models for particular planets (plus a little note on induced magnetic fields)

- Not covered: how magnetic fieldd are measured and separated into its different components

Geomagnetic field

Mapping with high spatial resolution from orbit:

- Magsat (1980)
- Ørsted (1999 -)
- Champ (2000 - 2010)
- SWARM (2013 -)



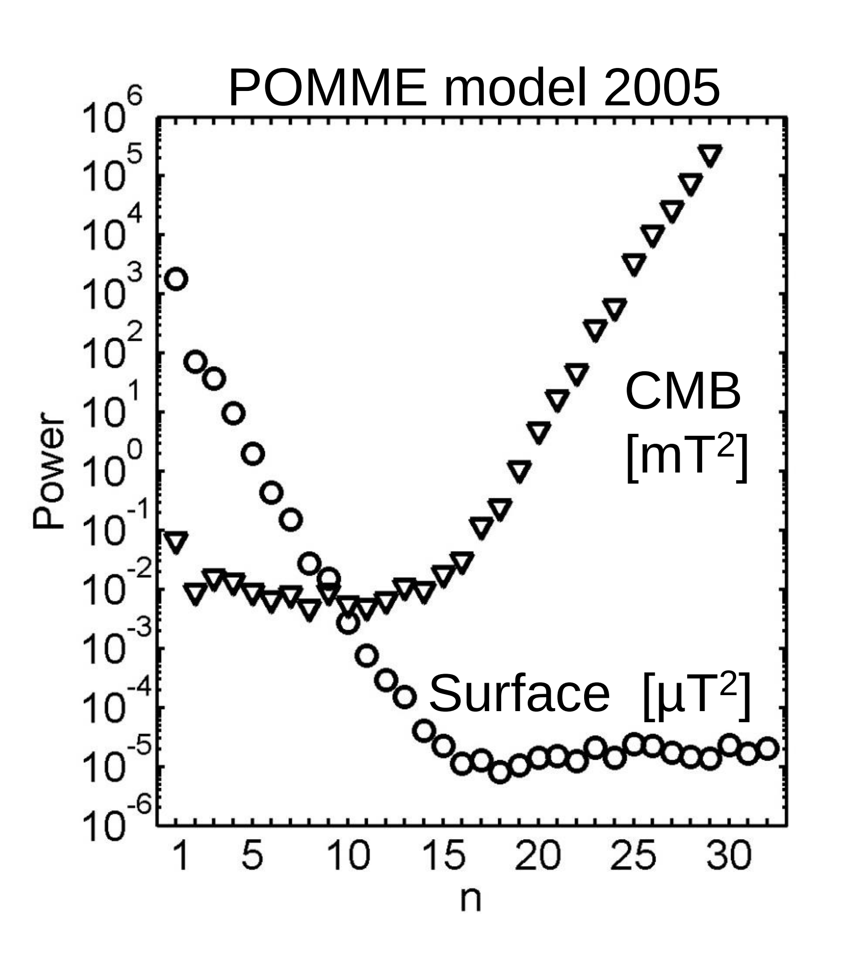
$$\mathbf{B} = -grad\ V \qquad V = R_p \sum_{n=1}^{\infty} \left(\frac{R_p}{r} \right)^{n+1} \sum_{m=0}^{n} P_n^m(\cos\theta)\left(g_n^m \cos m\lambda + h_n^m \sin m\lambda\right)$$

Gauss (1838): Representation of field B by scalar potential V expanded in spherical harmonic functions. Gauss coefficients g_{nm}, h_{nm}

Power spectrum

Degree power

$$P_n = (n+1)\left(\frac{R_p}{r}\right)^{2n+4} \sum_{m=0}^{n} [(g_n^m)^2 + (h_n^m)^2]$$

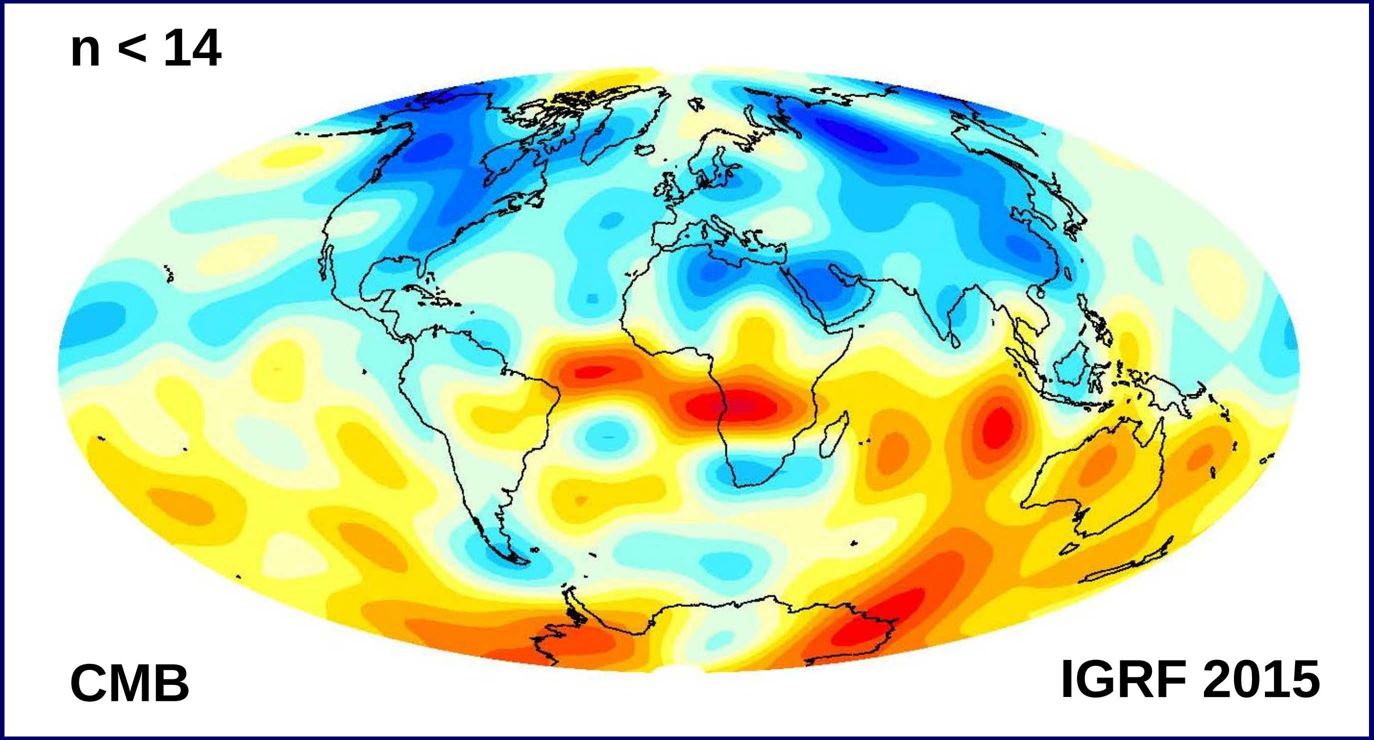


Earth's surface: strong drop up to degree ~ 13, white spectrum beyond

At core-mantle boundary (CMB) in 2900 km depth: Nearly white spectrum (dipole x5 larger) until n ~ 14, blue spectrum beyond.

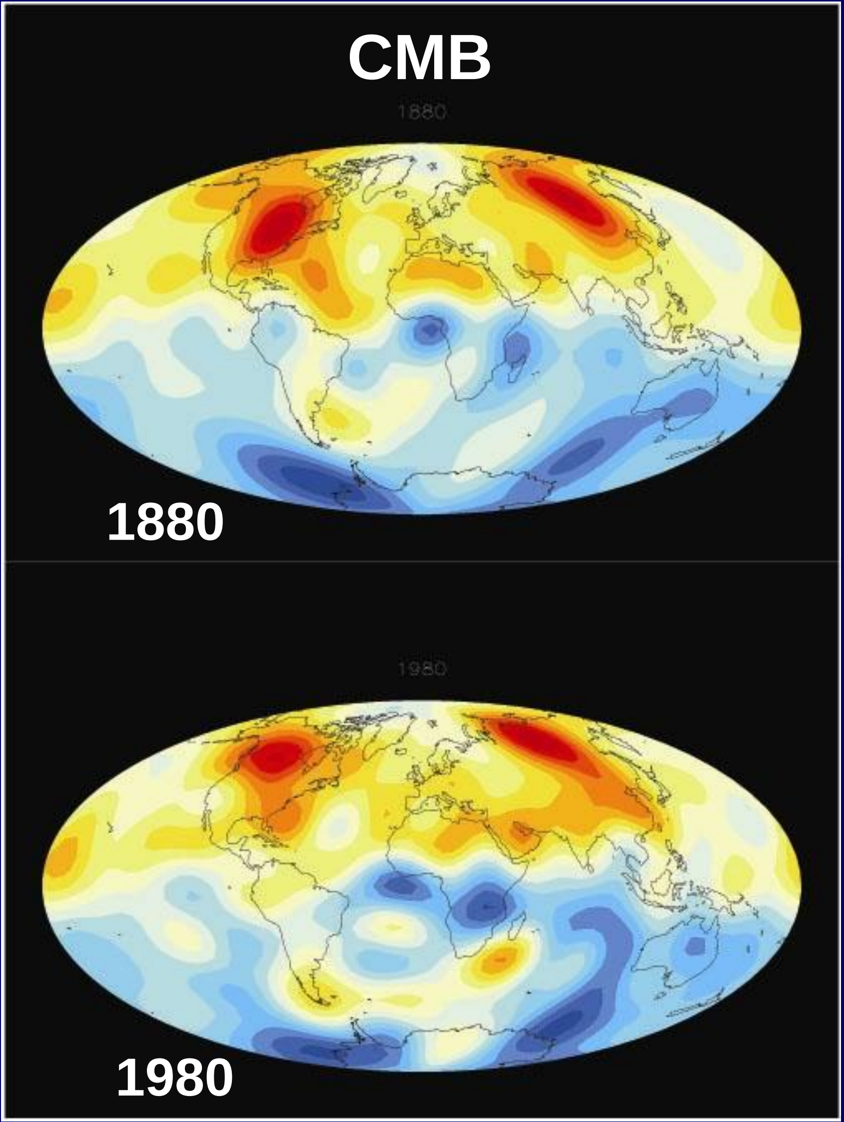
Interpretation: Surface field up to n=14 dominated by core, for n>14 dominated by field of inhomogeneous magnetization of Earth's crust

Field at top of Earth's core



Dipol dominant, multipoles important. Scales < 1500 km unknown.
4 high latitude flux lobes (±65º) @ same longitudes North and South.
Weak flux at rotation poles. Low latitude patches of both polarities.
***rms* – field strength at top of core in degrees 1-13 is 0.39 mT.**
Field strength inside core ~ 2 - 4 mT ?

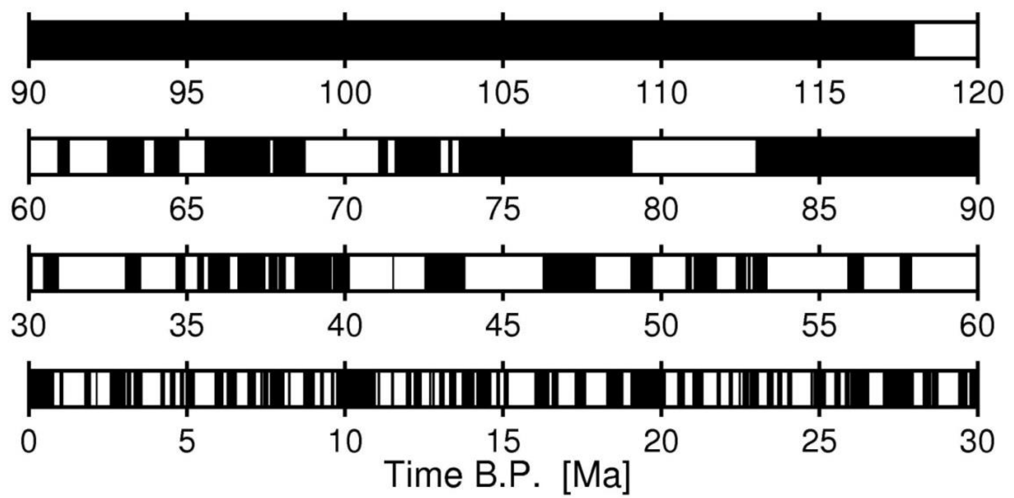
Secular variation



- Dipole dropped by 9% since 1840
- Reconstructions of core field morphology 1590 - present
- Fluctuations of non-dipole parts on time scales 50 – 400 yrs
- Stability of high-latitude flux lobes
- Westward drift below Atlantic and Africa
- Assuming magnetic field is „frozen“ into core fluid ⇒ velocity is of order 0.5 mm/s

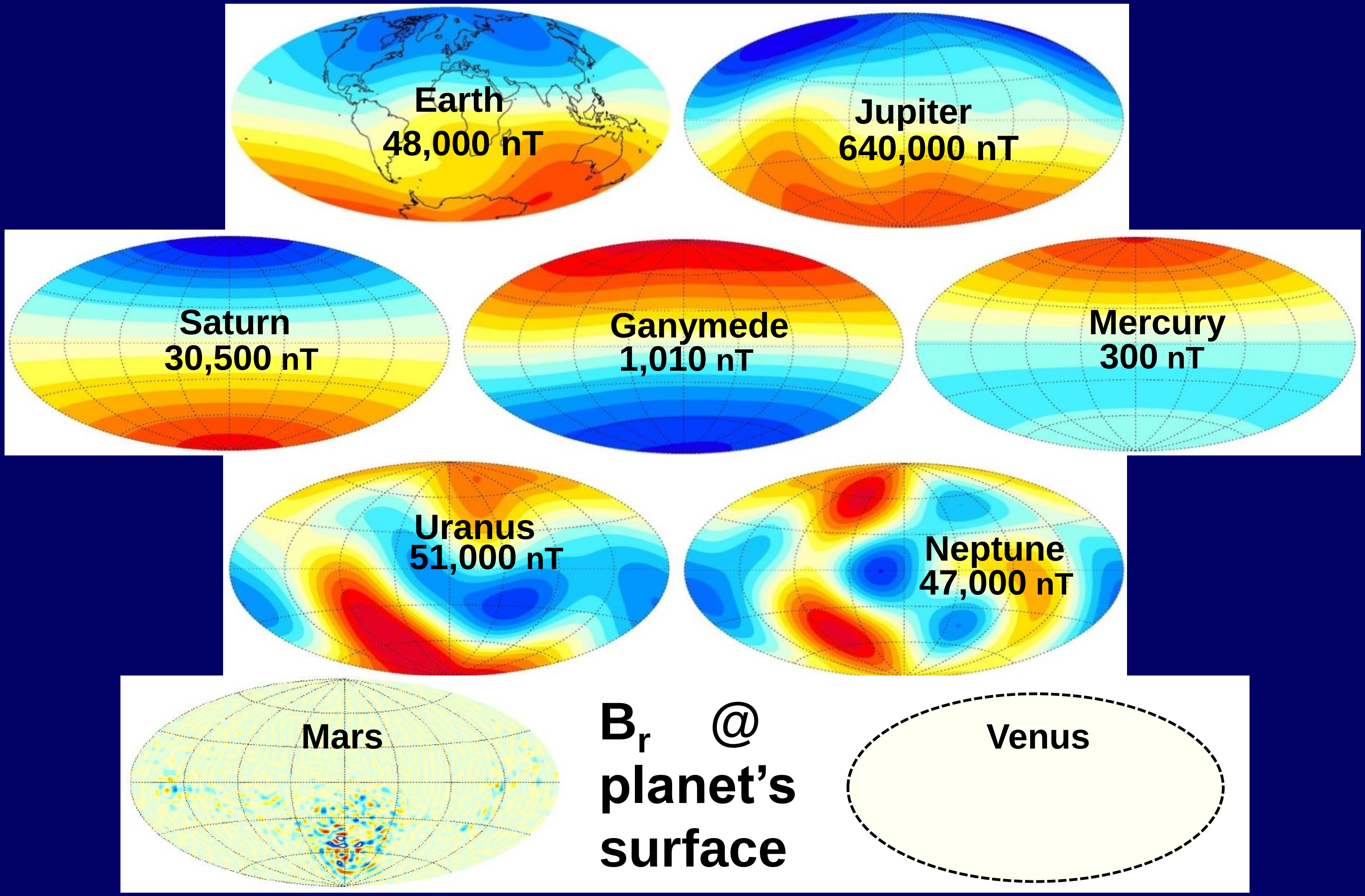
Paleomagnetism

- Ferromagnetic minerals in rocks record direction and intensity of Earth‘s field at time of formation
- Earth‘s field existed since at least 3.5 billion years.
- Intensity fluctuated (factor 2-5), no long-term trend.
- During past ~ 5 Myr, field dominated by axial dipole, moderate contributions from multipoles. For earlier times more difficult to prove (continental drift). Available evidence in favor.
- Dipole polarity reversed stochastically, on average a few times per million years.



Polarity record vs. geological time

Diversity of planetary magnetic fields



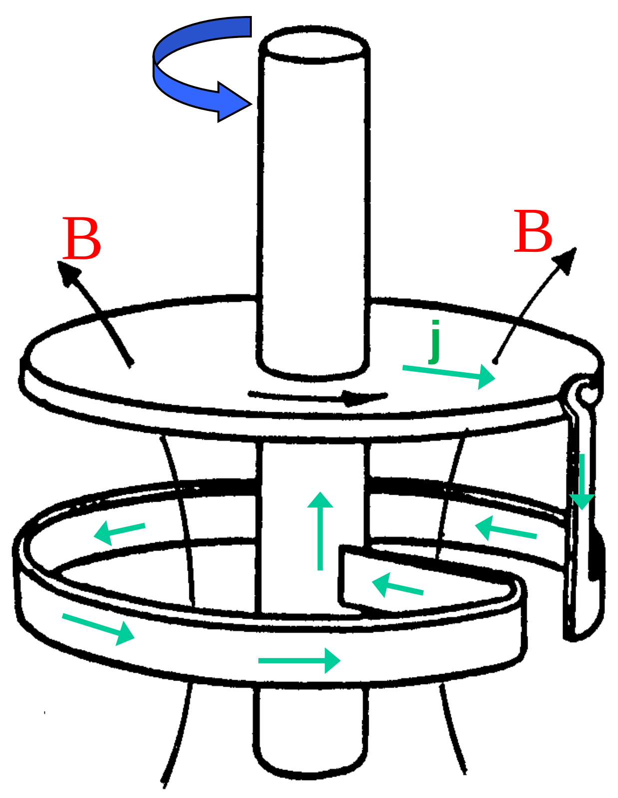
Magnetic fields of solar system planets

Planet	Dynamo	R_c/R_p	B_s [μT]	Dip. tilt	Q/D
Mercury	Yes	0.75	0.35	<1º	0.6
Venus	No	0.55			
Earth	Yes	0.55	44	10.4º	0.14
Moon	In the past	0.2			
Mars	In the past	0.5			
Jupiter	Yes	0.85	640	9.4º	0.10
Saturn	Yes	0.65	31	0º	0.02
Uranus	Yes	0.75	48	59º	1.3
Neptune	Yes	0.75	47	45º	2.7
Ganymede	Yes	0.3 ?	1.0	4º	<0.04

R_c/ R_p: core / planetary radius, B_s: Mean field at planet‘s surface, Q/D: Quadr. / dipole power at R_c

Origin of global planetary fields
$\Rightarrow$ self-sustained dynamo

Disc dynamo



Motion of a conducting fluid in an existing magnetic field induces electrical currents (Faraday's law)

The currents are associated with a magnetic field of their own (Ampère's law).

If the induced field has the right strength and geometry to step in for the field that is needed for the induction process, one speaks of a self-sustained dynamo

Magnetic induction equation

$$\partial \mathbf{B}/\partial t + (\mathbf{u}\cdot\nabla)\mathbf{B} \;=\; (\mathbf{B}\cdot\nabla)\mathbf{u} + \lambda\nabla^2\mathbf{B}$$

advection source/sink diffusion

- Describes evolution of a magnetic field in an electrically conducting fluid moving with a (local) velocity u (u << c).
- Derived from Maxwell‘s equations (without displacement current) and generalized Ohm‘s law for moving incompressible medium.
- $\lambda = 1/(\mu_o\sigma)$: magnetic diffusivity with σ electrical conductivity
- Dimensionless equation by scaling with characteristic length L (e.g. core radius) and characteristic flow velocity U:

$$\partial \underline{\mathbf{B}}/\partial \underline{t} + (\underline{\mathbf{u}}\cdot\underline{\nabla})\underline{\mathbf{B}} \;=\; (\underline{\mathbf{B}}\cdot\underline{\nabla})\underline{\mathbf{u}} + \frac{1}{\mathrm{Rm}}\underline{\nabla}^2\underline{\mathbf{B}}$$

- Magnetic Reynolds number **Rm = UL/λ**

Fundamental requirements for self-sustained dynamo

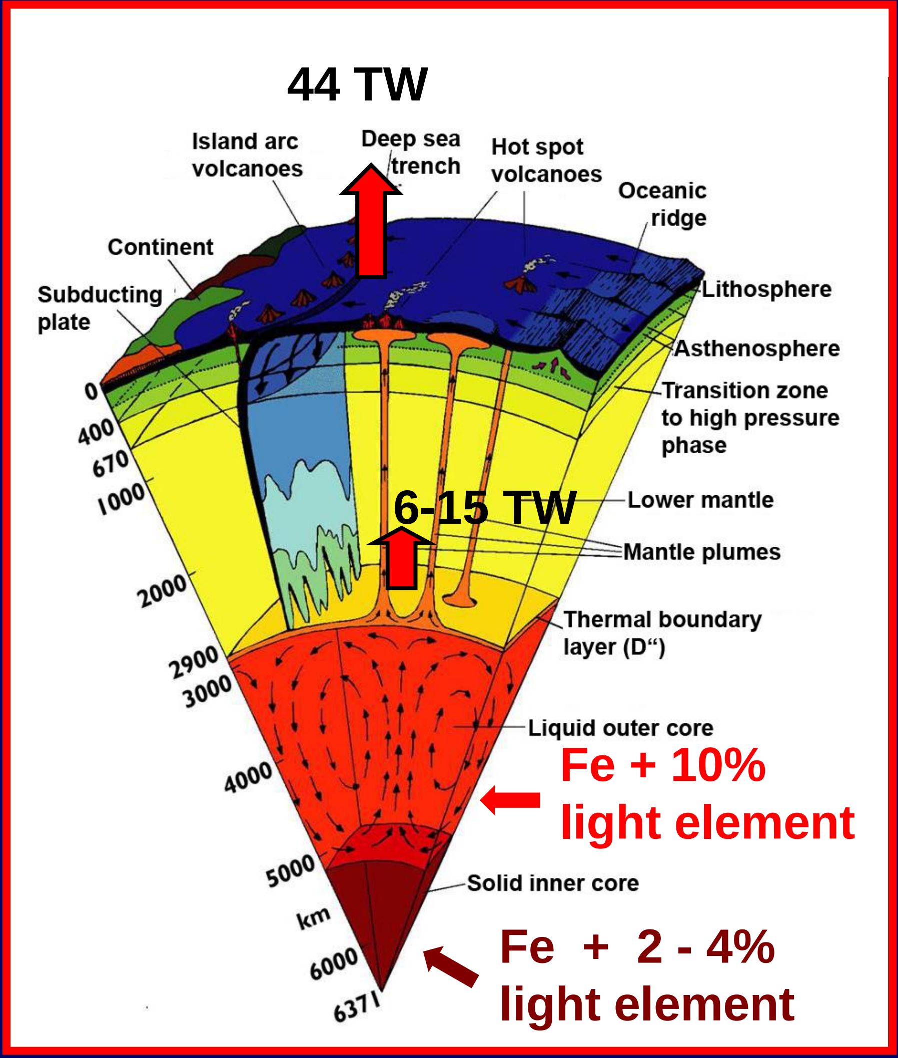
- Electrically conducting fluid layer

- Motion in this layer with a sufficient velocity.
  Magnetic Reynolds number $Rm=UL/\lambda > 50$
  Convection likely source of motion.

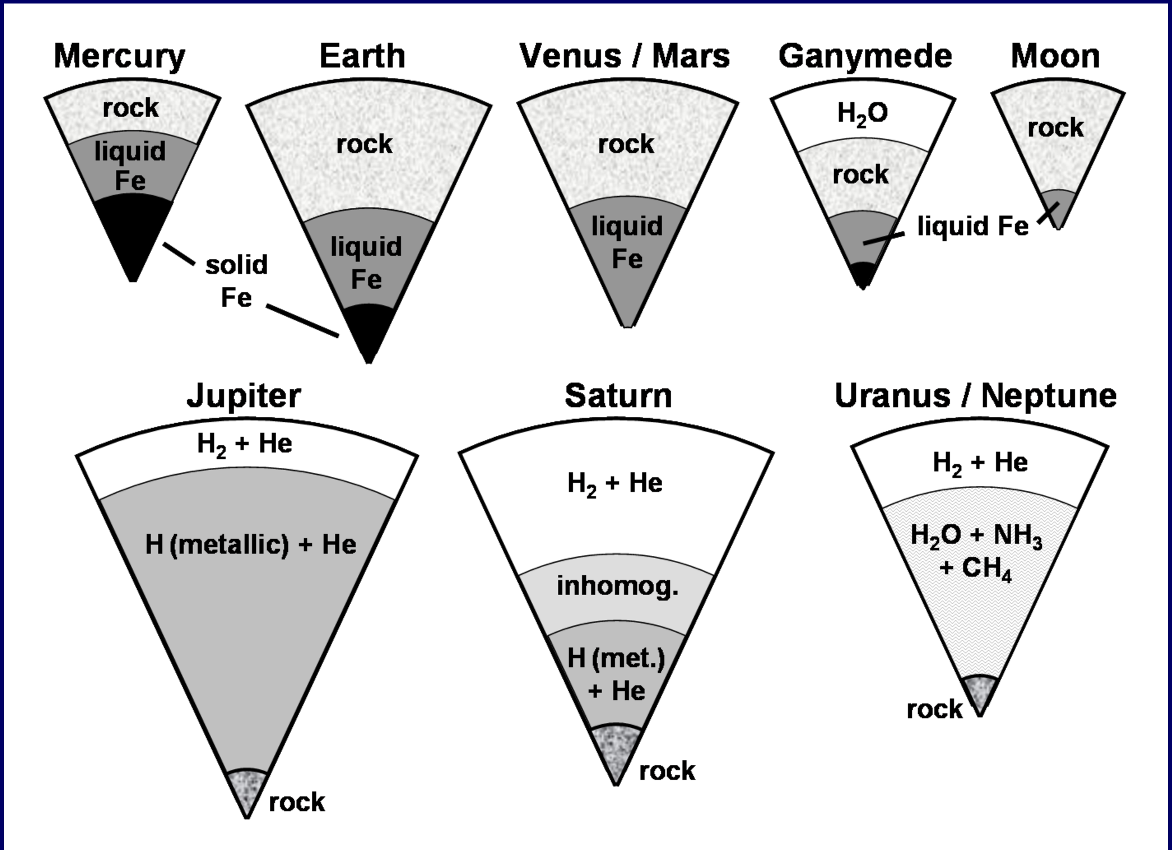
- Motion must have suitable geometry (e.g. helical). Rotation (Coriolis force) important.

Earth: Internal structure & energetics

- Liquid iron core
- Solid inner core (IC) with $0.35R_c$
- ~10% light element (Si, S, O, ...) in outer core, less in inner core
- Earth heat flow 44 TW. Core fraction estimated 6-15 TW
- Core heat flow mostly due to secular cooling. Carried partly by thermal convection.
- Light element enrichment above IC $\Rightarrow$ compositional convection
- For velocity inferred from secular variation, Rm $\approx$ 1000



Planetary interiors: a comparison



Dynamo region:

Liquid iron in Earth-like planets and Ganymede. Solid inner core uncertain.

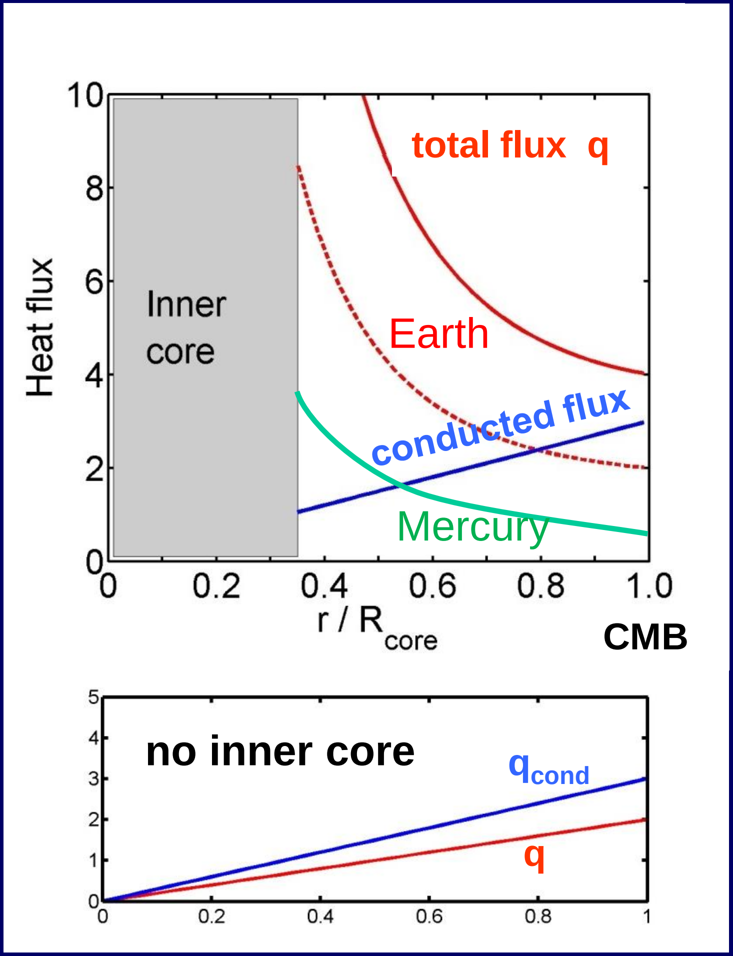
Metallic hydrogen in Jupiter & Saturn

"Ices" with ionic conductivity in Uranus & Neptune

Heat flux: uncertain for rocky planets other than Earth. For gas planets deduced from excess infrared radiation.

Thermal & compositional convection in the cores of terrestrial planets

- **For convection, temperature gradient must exceed adiabatic gradient $(dT/dr)_{ad} = \alpha g(r)T/c_p = T/H_T$**
- **Core heat flux of a terrestrial planet is controlled by the mantle**
- **Significant heat can be conducted along adiabatic temperature gradient Earth's CMB: q_{cond}= 80 - 120 mWm^{-2} Total CMB flux: q = 40 – 100 mWm^{-2}**
- **If growing IC exists, latent heat of freezing contributes to drive thermal convecting. Release of light element drives compositional convection**



Notation: α thermal expansivity, g gravity, T abs. temp., c_p heat capacity, $_{HT}$ temp. scale height

Why Mars & Venus lack a dynamo ?

Core entirely frozen ? **Unlikely**

Thermal evolution modeling; Tidal Love number k_2

Rotation too slow (Venus) ? **Unlikely**

Coriolis force still plays significant role in force balance.

Core not convecting ? **Likely**

**Mantle convection controls heat flow from core.
Lack of plate tectonics implies less efficient cooling of the interior and lower heat flux from the core. Heat flux at top of core probably less than adiabatic conductive flux.
No convection, if planets also lack an inner core.**

But: exsolution of dissolved mantle components (MgO, SiO_2) at top of cooling core driving comp convection ?

Nondimensional equation of motion

$$\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\vec{\nabla}\vec{u}\right) + 2\vec{e}_z \times \vec{u} + \vec{\nabla}P = E\ \nabla^2\vec{u} + Ra^*\ \frac{\vec{r}}{r_o}T\ + (\vec{\nabla}\times\vec{B})\times\vec{B}$$

Inertia **Coriolis** **Pressure** **Viscosity** **Buoyancy** **Lorentz**

Hierarchy of forces in a planetary core:

Coriolis ~ Pressure > Lorentz ~ Buoyancy >> Inertia > Viscosity

(Planetary ratios between forces not necessarily satisfied in numerical dynamo simulations !)

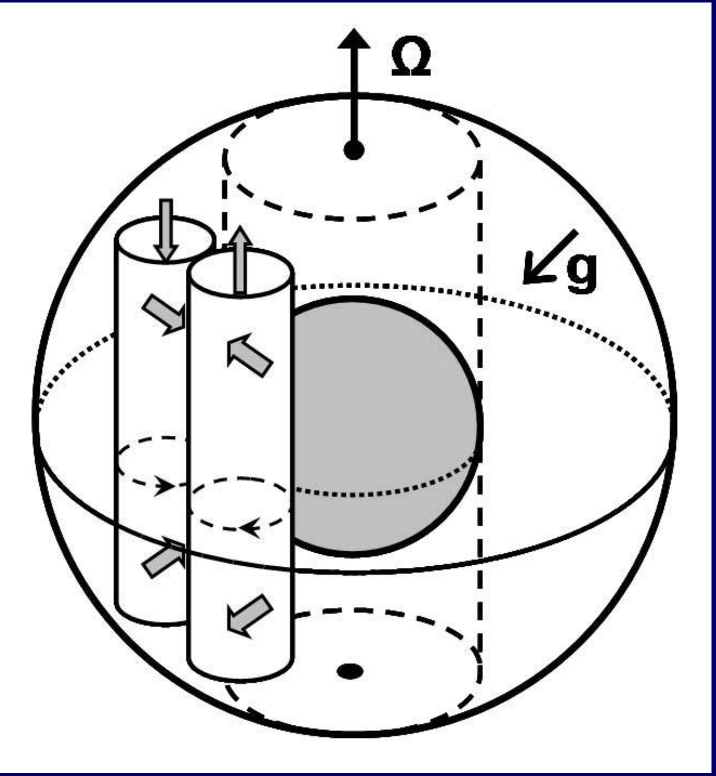
Geostrophic flow in spherical shell

Balance Coriolis force ~ pressure gradient force

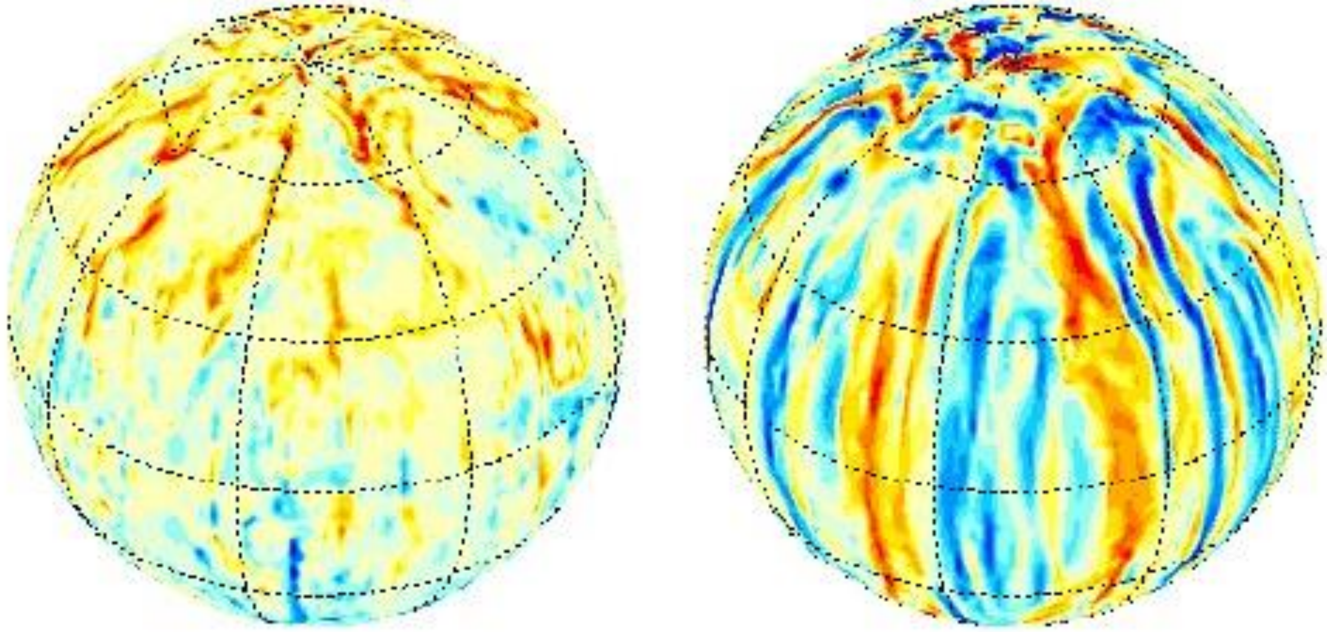
$2\,\rho\,\Omega\times u = \nabla p$ **Take curl** $\Rightarrow$ $(\Omega\bullet\nabla)\,u = 0$

Proudman Taylor theorem

- **When Coriolis and pressure forces dominate, convection in spherical shells in columns aligned with rotation axis outside the inner core tangent cylinder**
- **Must violate P-T-theorem, but does so as little as necessary**
- **Columnar flow is helical: secondary circulation along center of columns**



A numerical geodynamo model

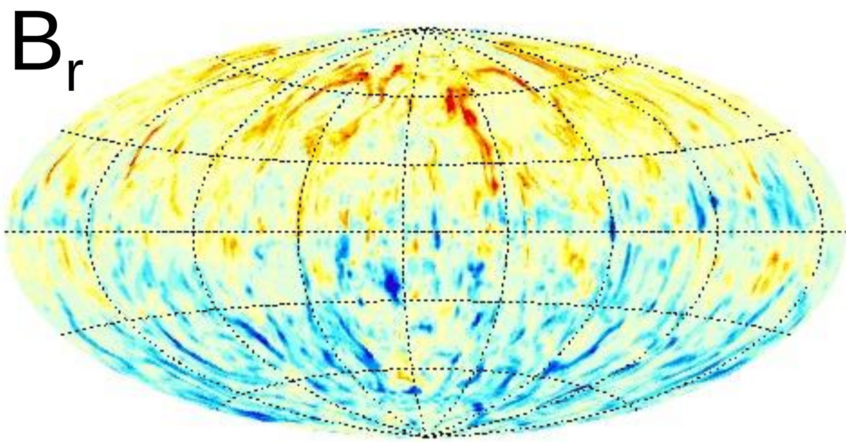


Radial magnetic field @ outer boundary

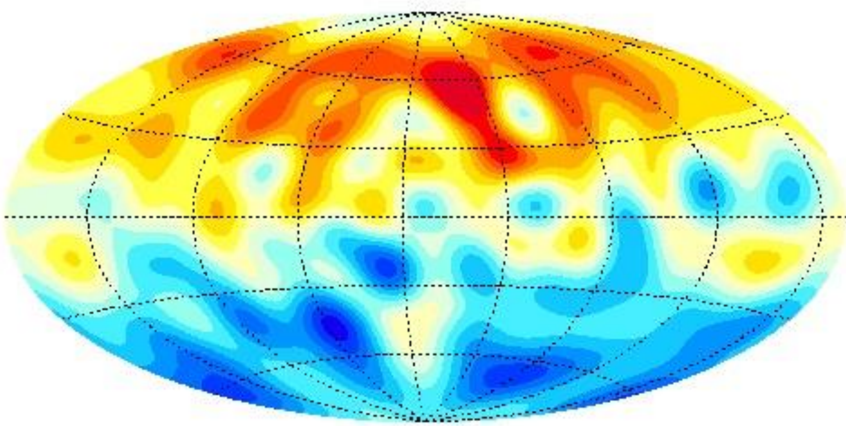
Radial velocity field below outer boundary

- Flow quasi-columnar
- Magnetic field dipole-dominated with small scale structure superimposed

Comparison with Earth: Field morphology

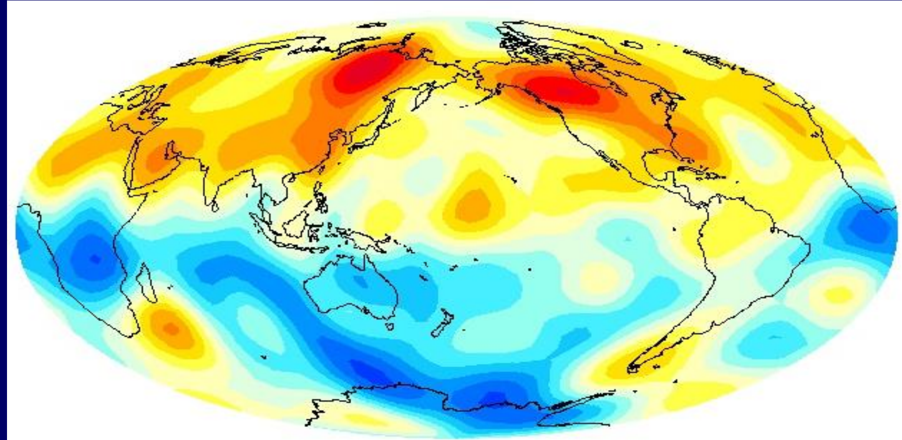


Dynamo model, full resolution



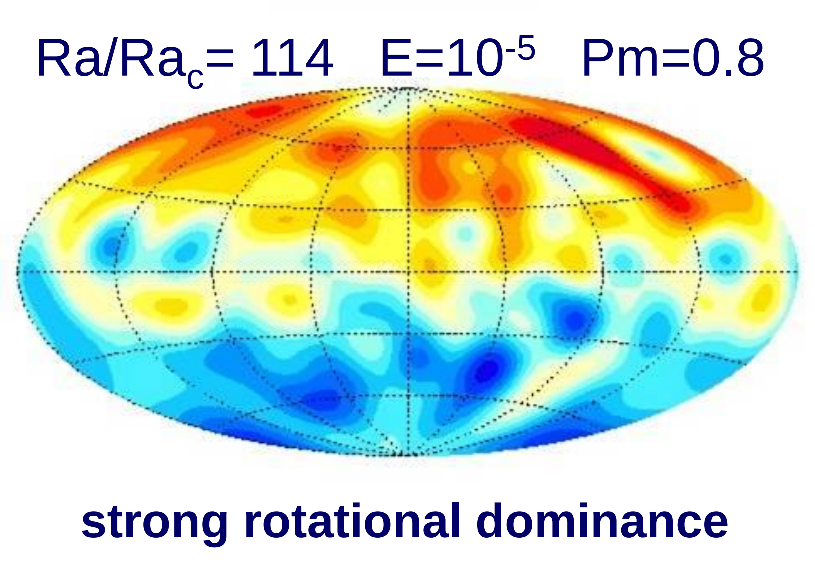
Dynamo model, filtered to n < 14

- **Flux lobes at 60-70º latitude**
- **Weak flux at poles**
- **Flux spots of both polarities at low latitude.**

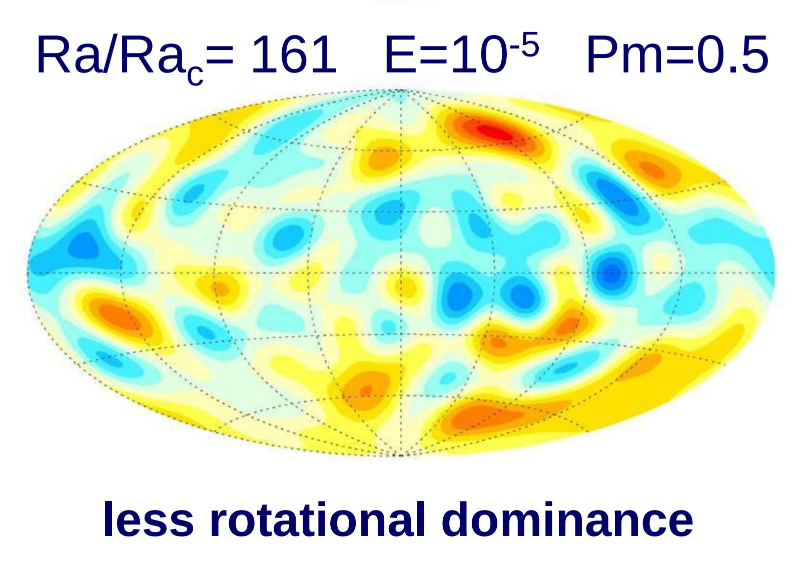


Earth‘s field at core mantle boundary

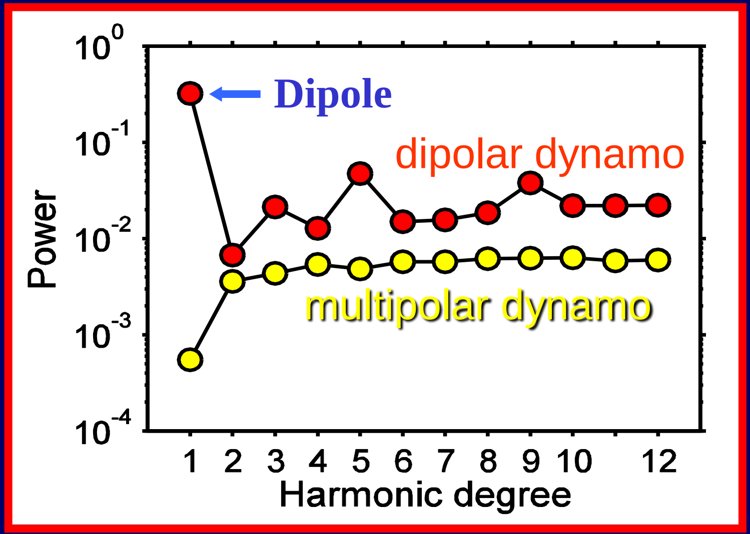
Field morphology: two regimes



strong rotational dominance



less rotational dominance



Power spectrum at dynamo surface nearly white from degrees n=3 to n>12.

Dipolar regime: dipole is clearly stronger than multipoles.

Multipolar regime: dipole is weaker than multipoles.

What controls magnetic field strength?

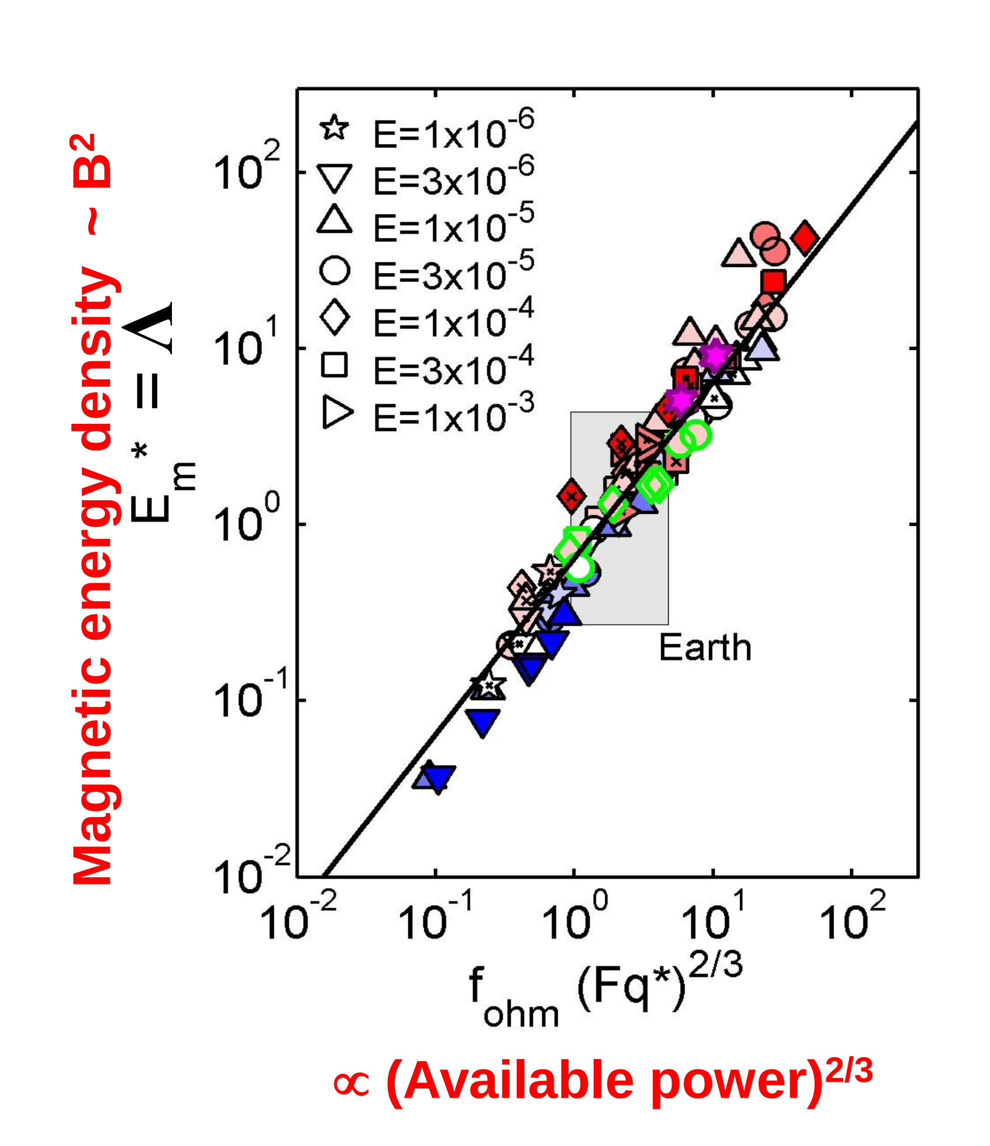
Old hypothesis: Magnetic field strength in a dynamo grows up to the point where Lorentz force is of equal strength as Coriolis force.

$\Rightarrow$ Not supported by numerical simulations

New hypothesis: Magnetic field strength B (inside the dynamo) is controlled by the power P (energy flux) that drives convection:

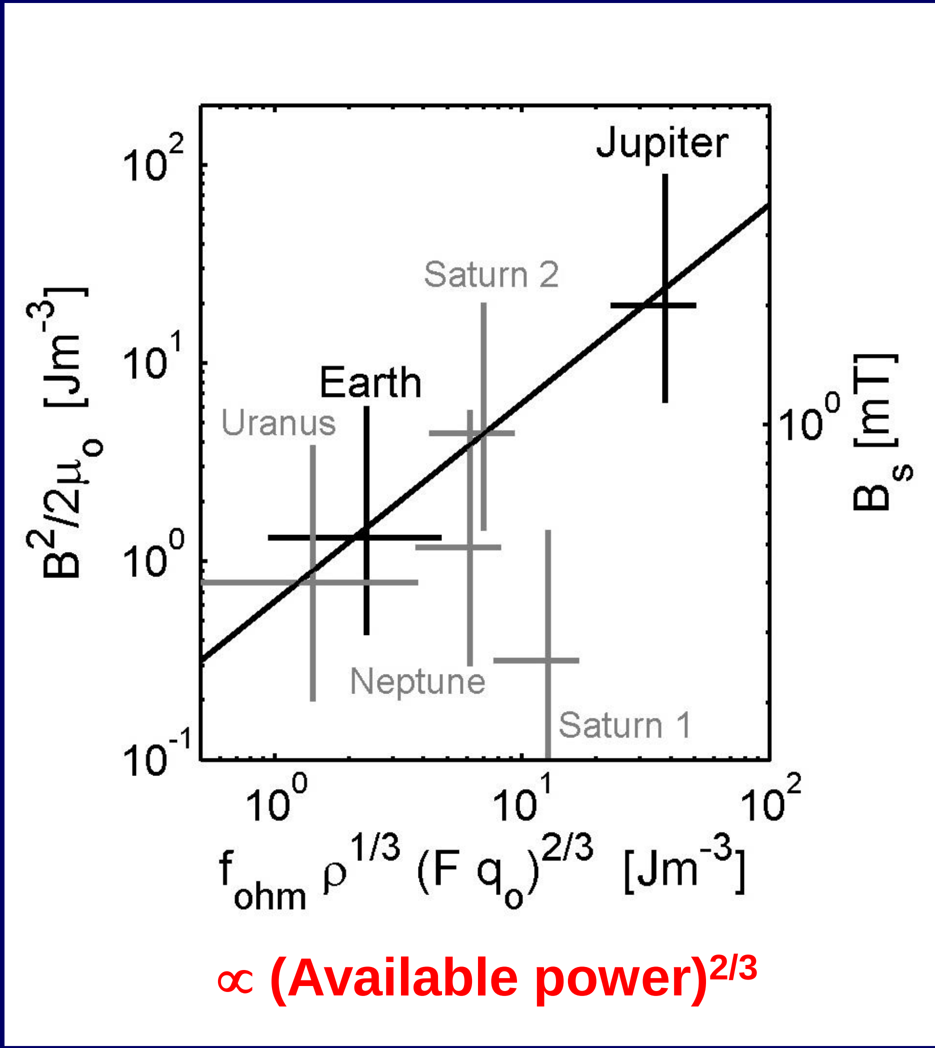
$$B \sim P^{1/3}$$

Test of scaling law vs. simulations



Results of numerical dynamo simulations are in decent agreement with field strength proportional to the cubic root of energy flux

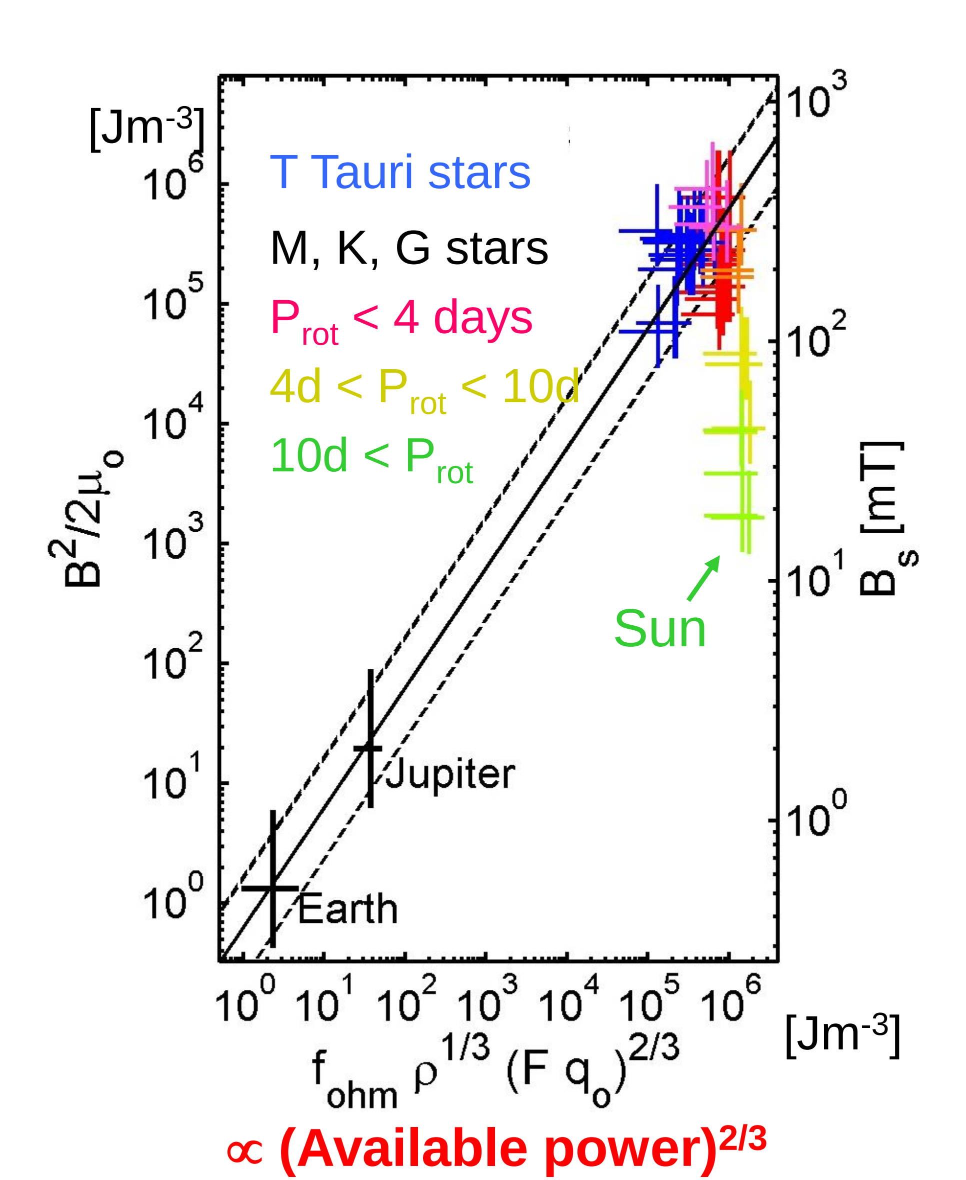
Comparison with planetary fields



Field strength vs. heat flux

Assume ratio between total internal field and dipole field at CMB in range 4 - 15 (from dynamo models)

Comparison with planets and stars

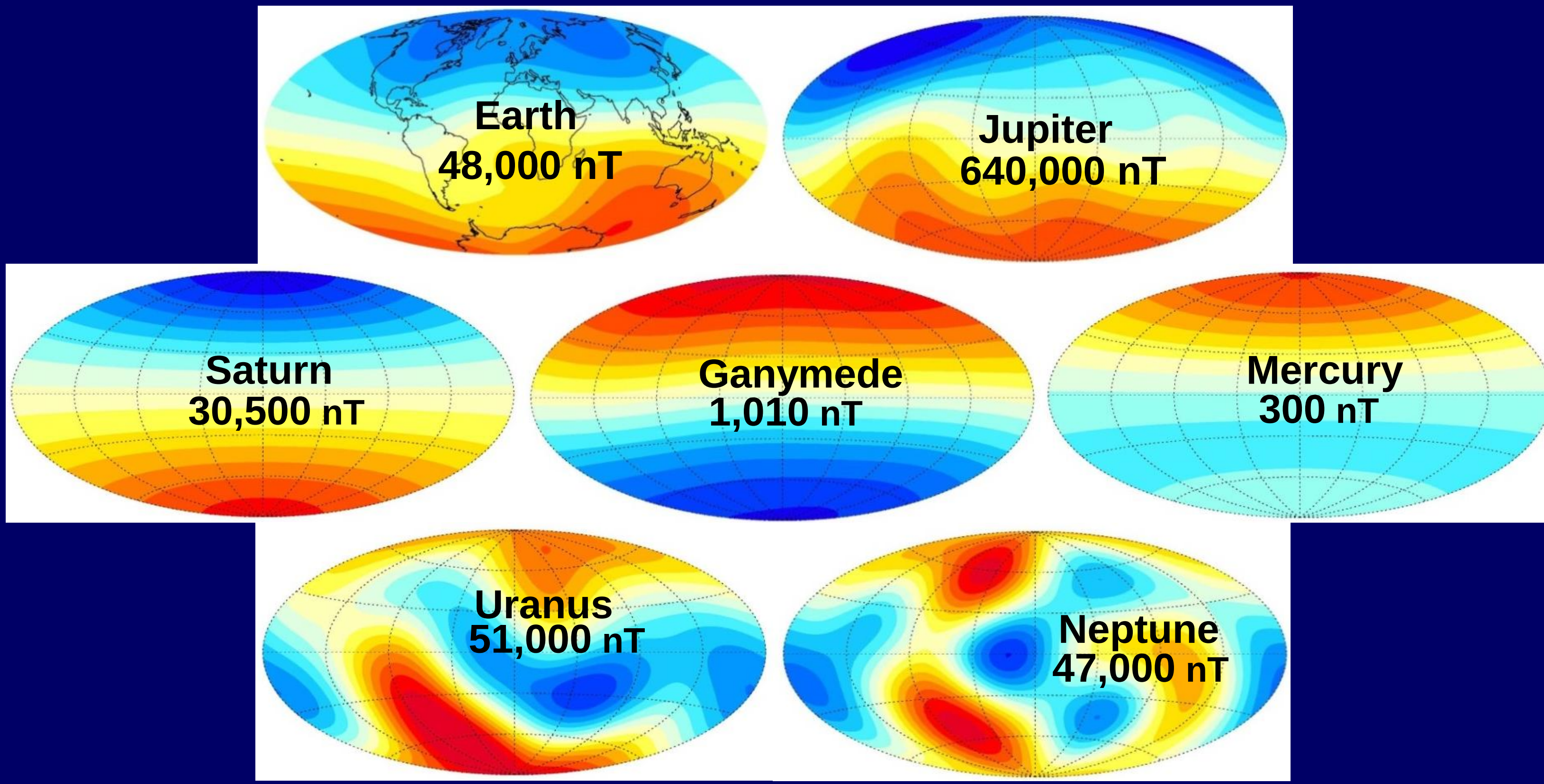


The observed fields of rapidly rotating low-mass stars agree with the prediction as well as that of Jupiter and Earth

⇒ confirmation for scaling law

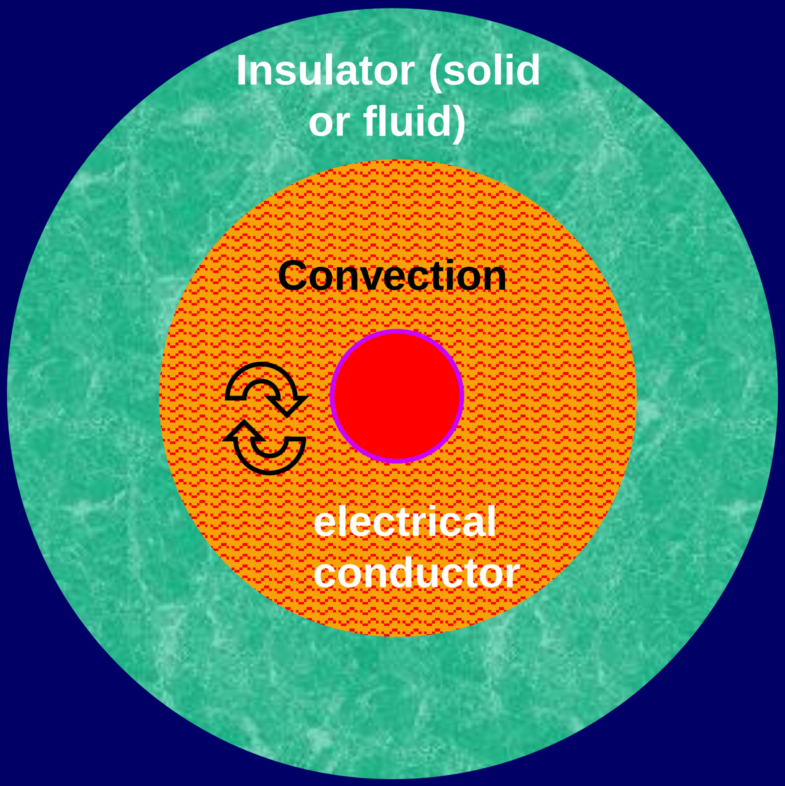
⇒ dynamos in planets and (some) stars may be similar

What can explain the diversity of planetary magnetic fields ?

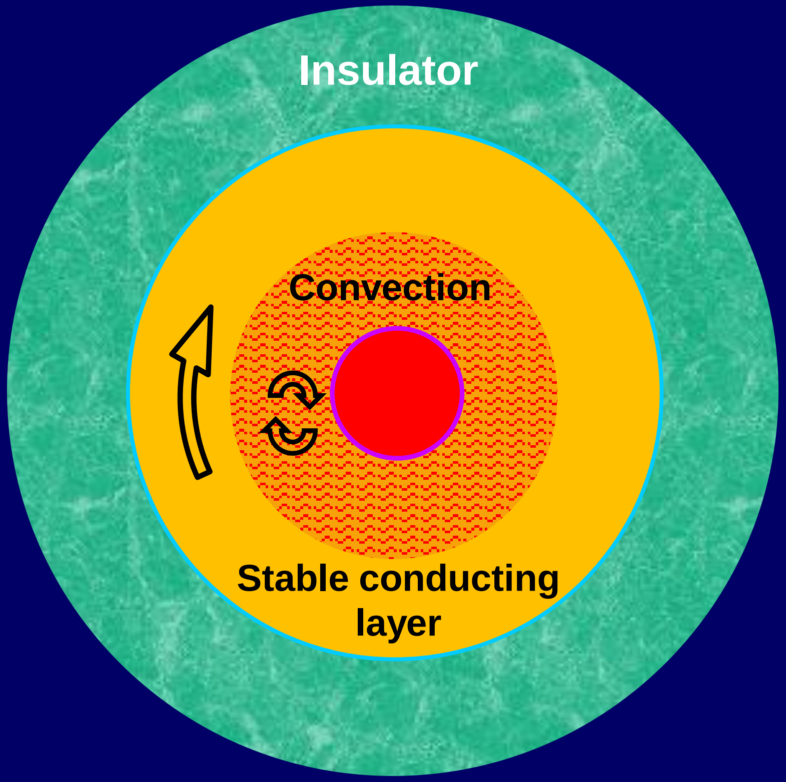


- **Compared to Earth and Jupiter, the fields of Saturn, Mercury and (perhaps) Ganymede are weakish and more axisymmetric**
- **The fields of Uranus and Neptune are multipole-dominated**

Standard model



Stable layer

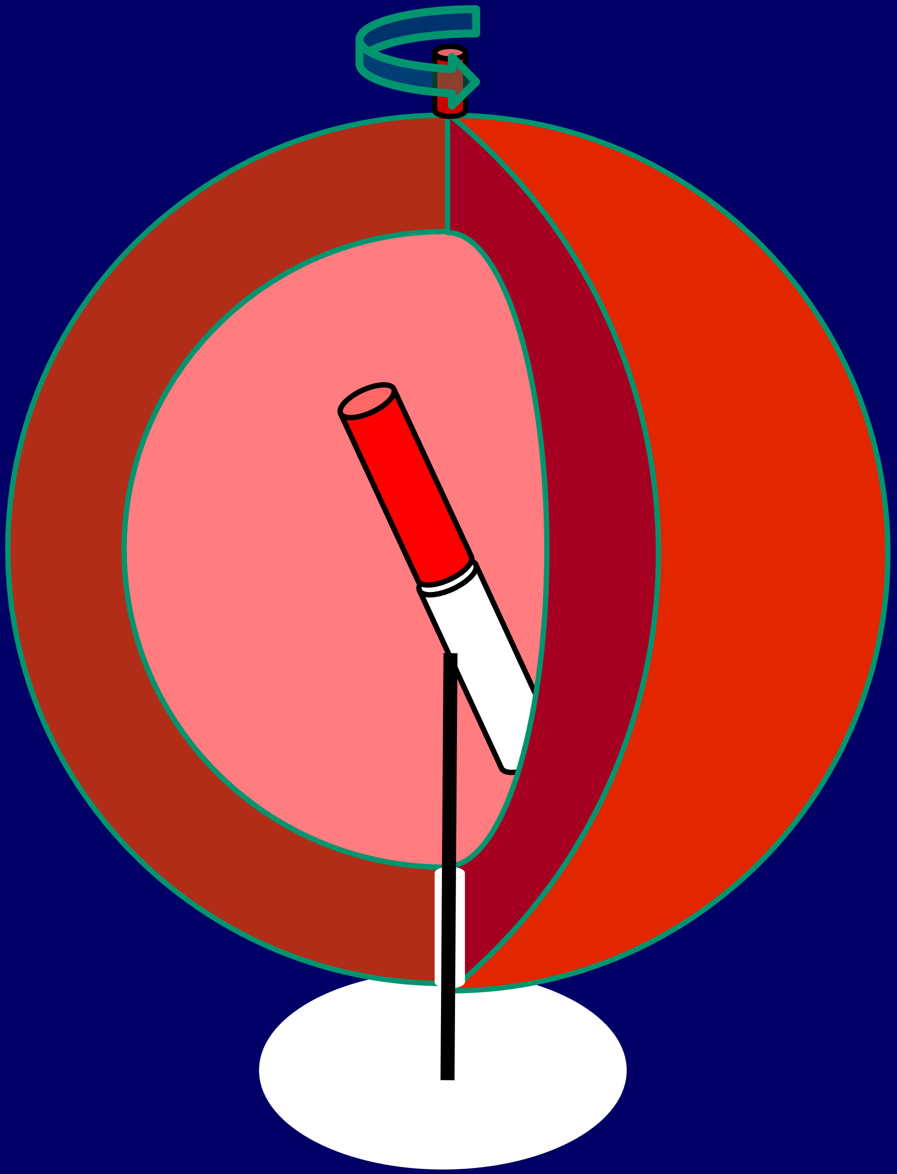


Causes for stable layer

Sub-adiabatic T-gradient in outer parts of core

Compositional stratification (e.g. associated with phase separation)

A toy model



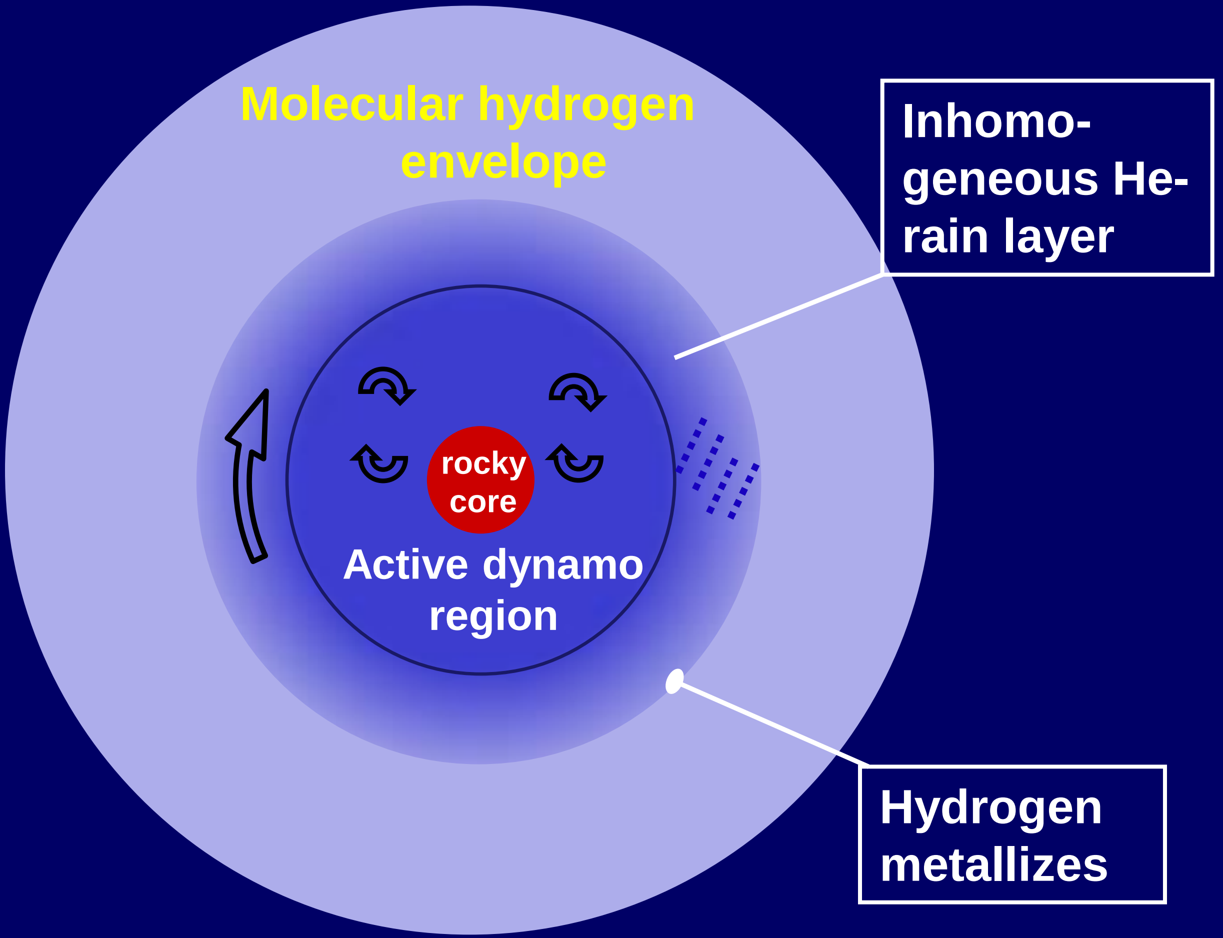
Skin effect damps time-dependent components of the magnetic field which may become invisible on the outside

Saturn: Dynamo below He-rain layer?

Saturn’s magnetic field is extremely axisymmetric:

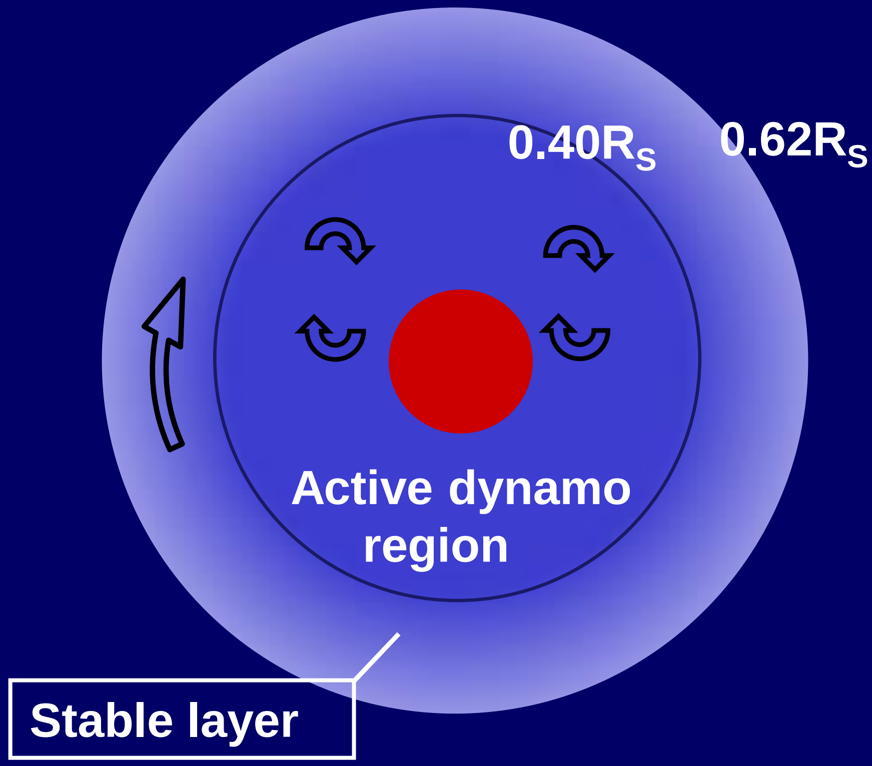
Upper bound on dipole tilt 0.06º

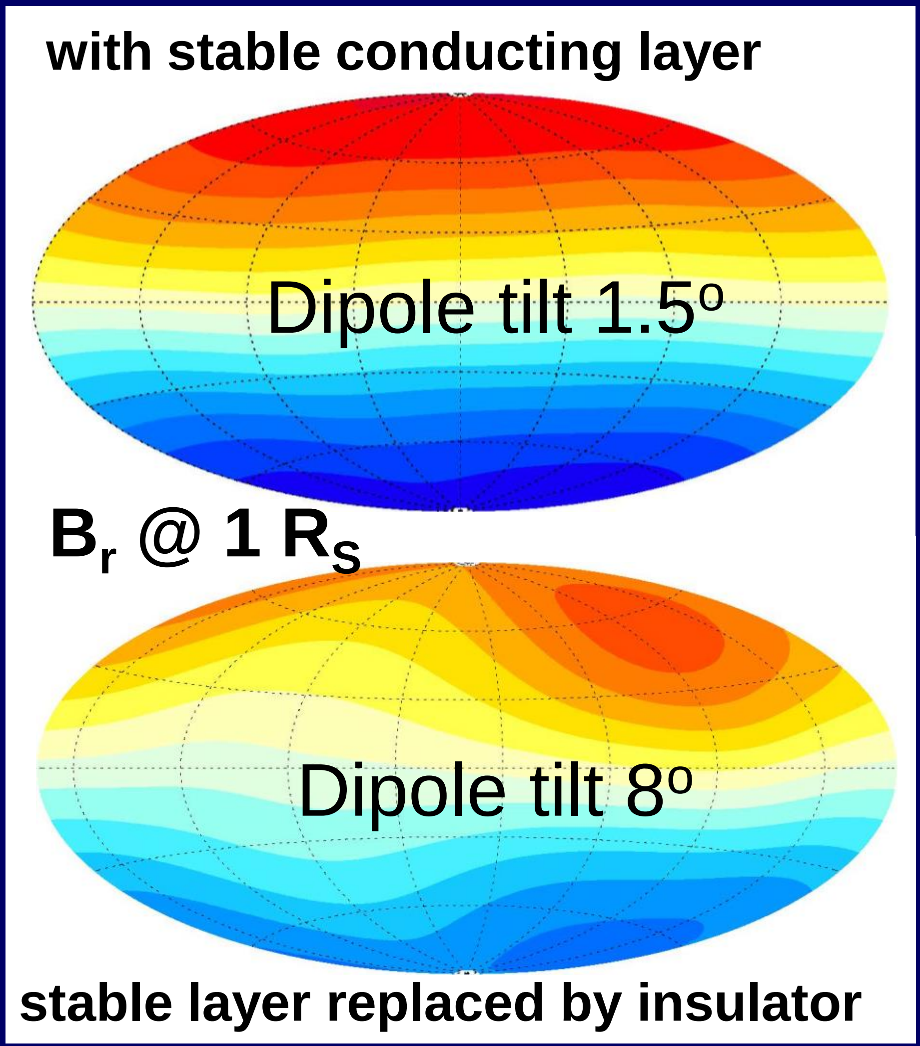
In addition to dipole, only axisymmetric multipoles are needed to fit the measurements



Compared to Jupiter, field strength seems low

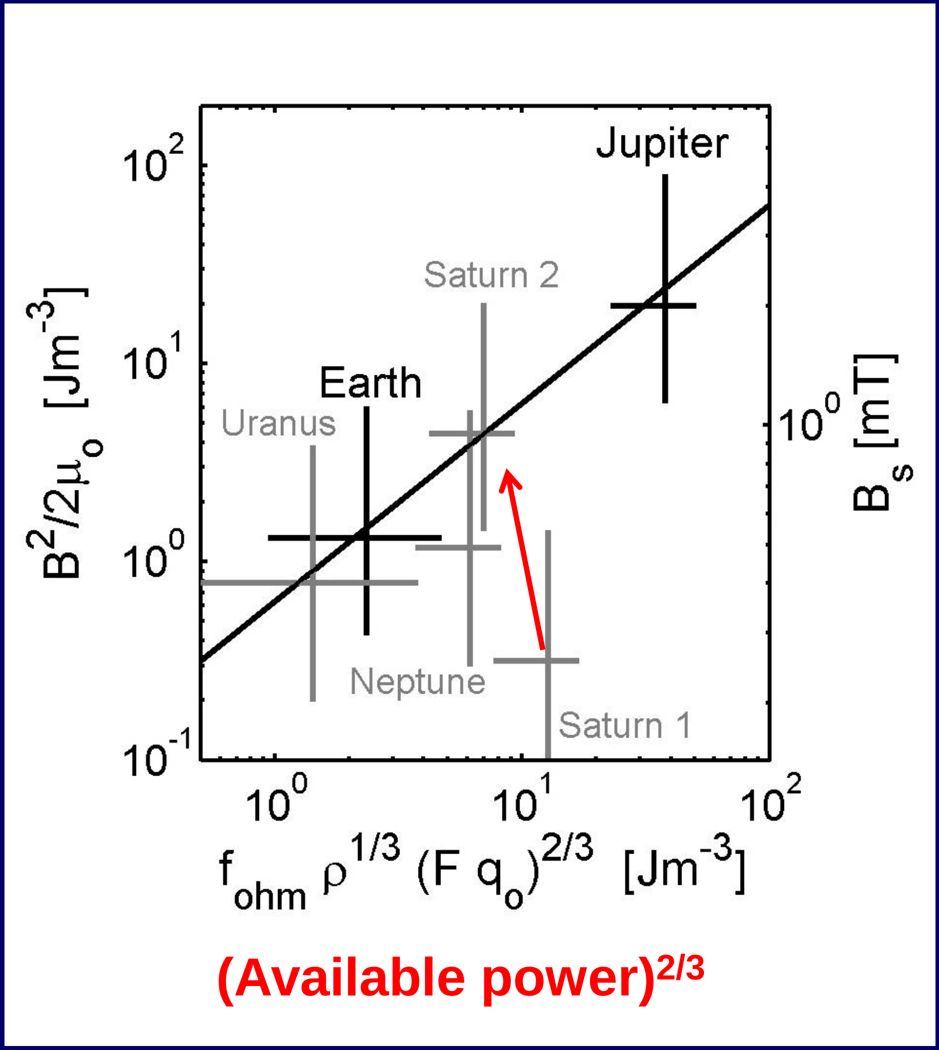
MHD dynamo model with stable layer





Strong non-zonal field component largely filtered out by stable layer

Scaling Saturn‘s field strength



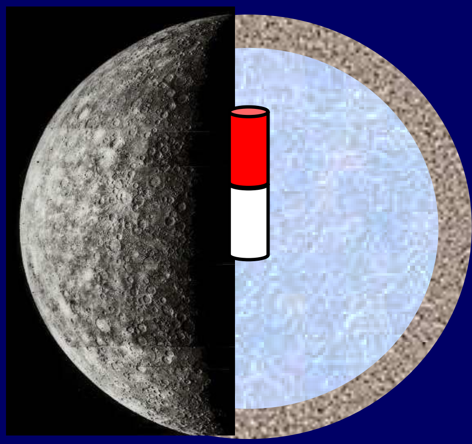
Field strength vs. heat flux

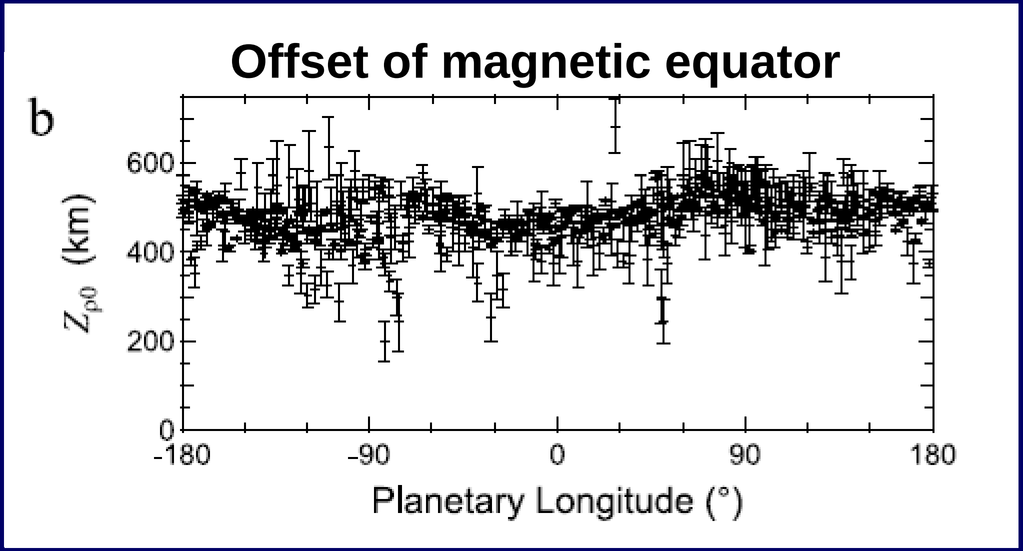
Assume ratio between total internal field and dipole field at CMB in range 4 - 15 (from dynamo models)

Saturn 1: R_c/R_p = 0.62

Saturn 2: R_c/R_p = 0.40

Mercury's magnetic field

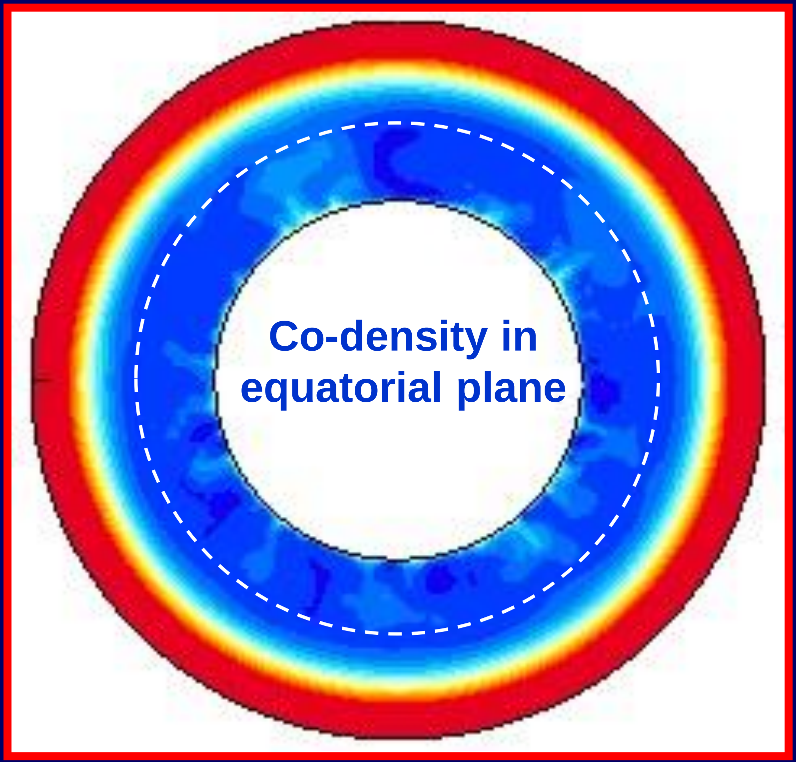


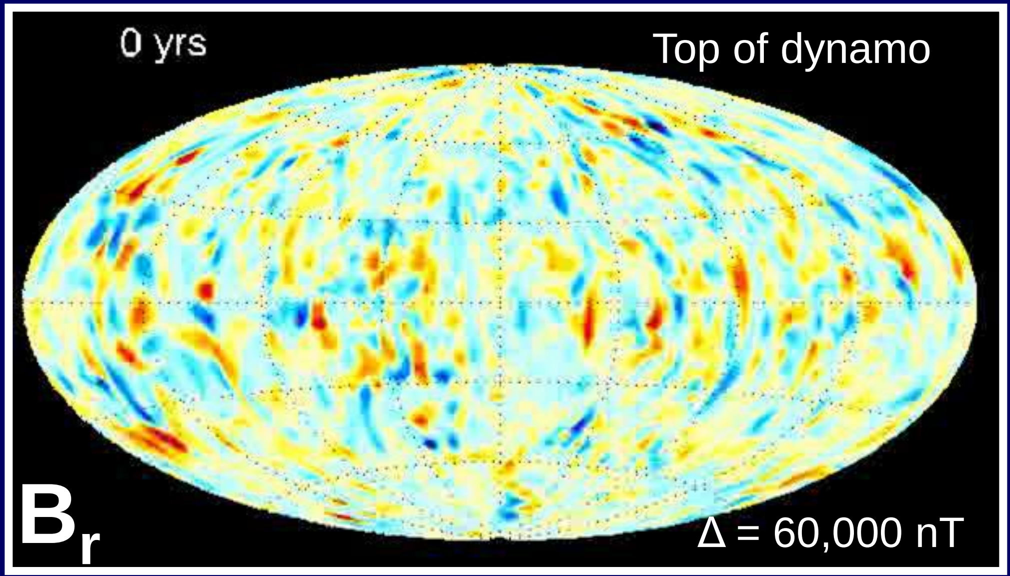


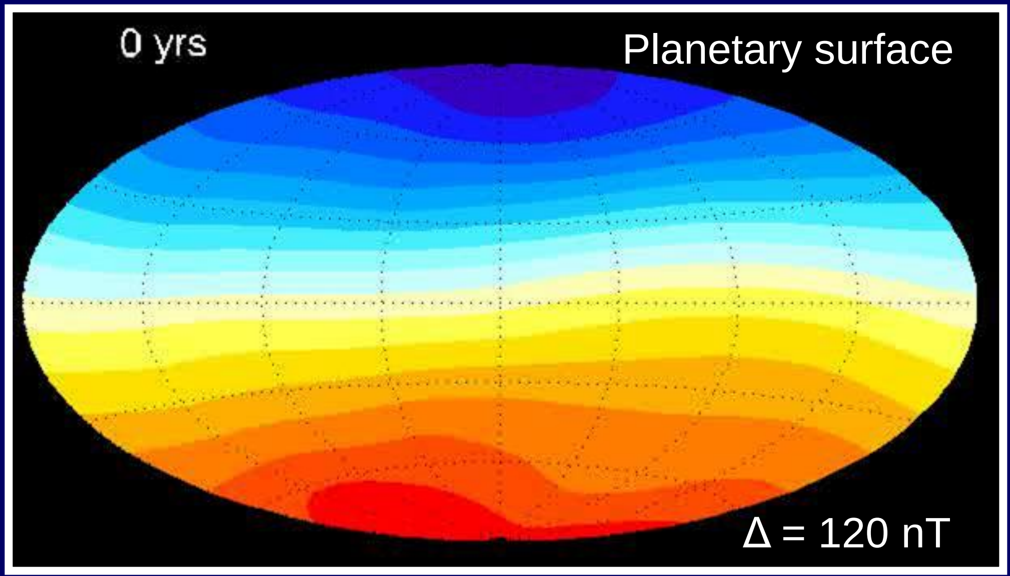
- **Mercury is slow rotator (59 days)**
- **Field dipole-dominated but weak (g_{10}=190 nT)**
- **Dipole tilt wrt rotation axis small (< 0.8°)**
- **Relatively large axial quadrupole (g_{20}/g_{10} = 0.39)**

(Anderson et al., 2011, 2012)

Dynamo below stable fluid layer

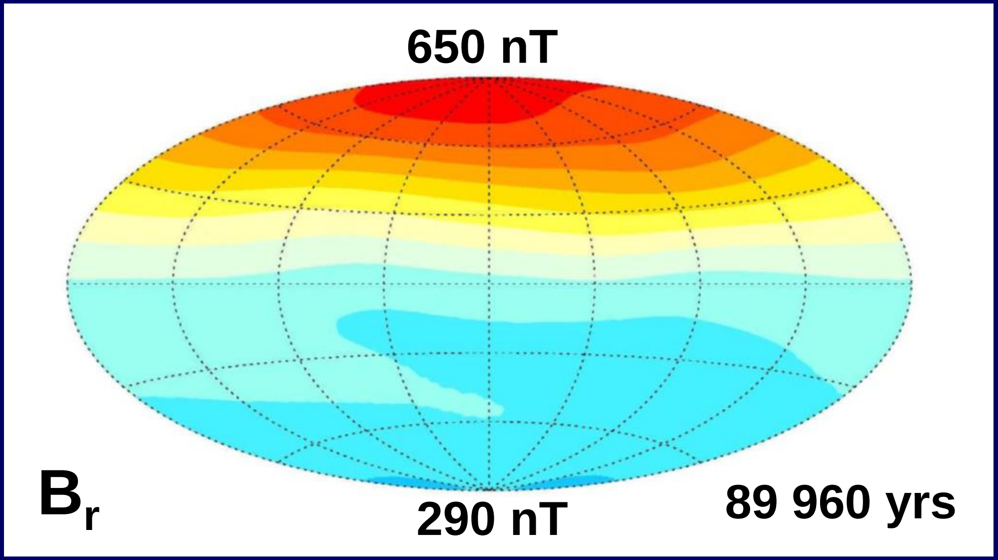


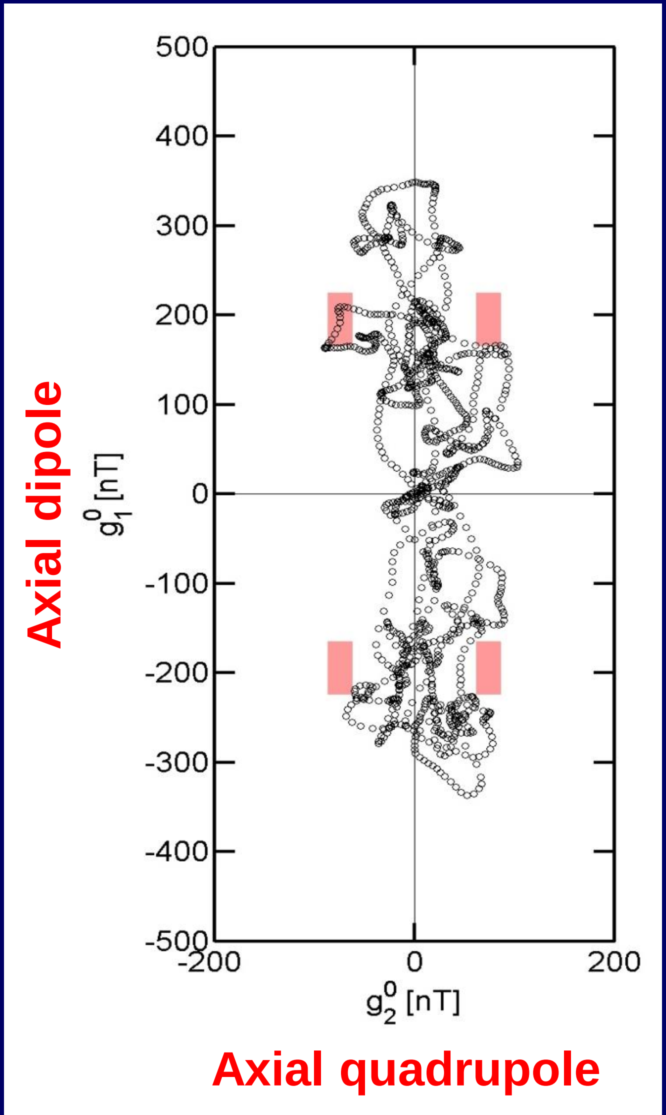




- **Internal field strong & small-scale**
- **Surface field weak & large-scale**

Mercury's quadrupole / dipole ratio



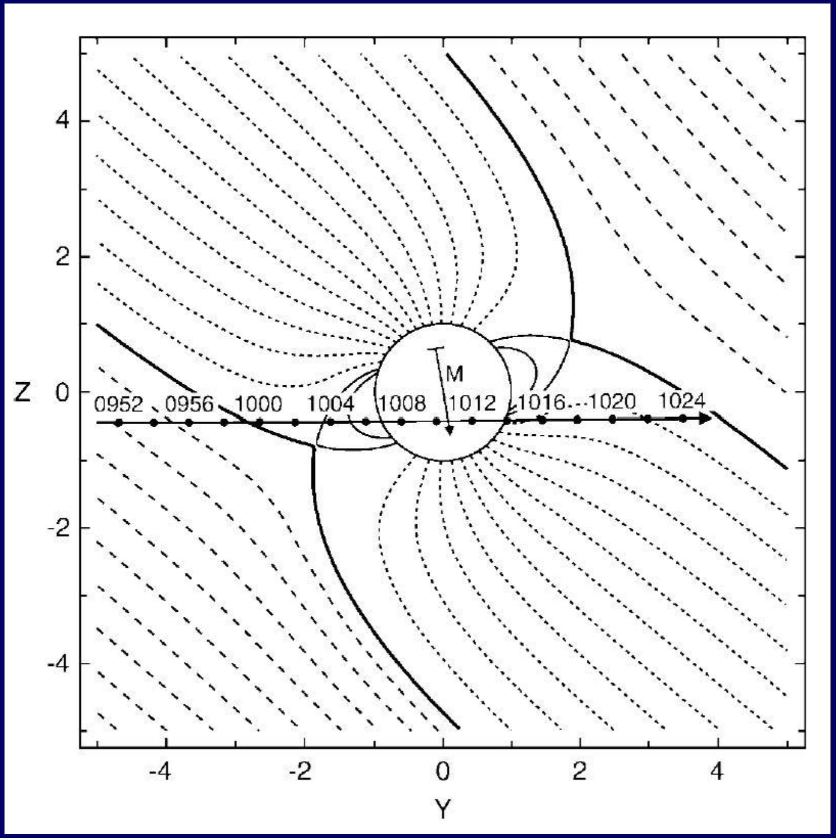


Snapshot fitting the present-day field of Mercury in terms of field strength and g_{20}/g_{10} ratio

One case re-analysed from Christensen & Wicht, Icarus, 2008

Ganymede‘s magnetic field

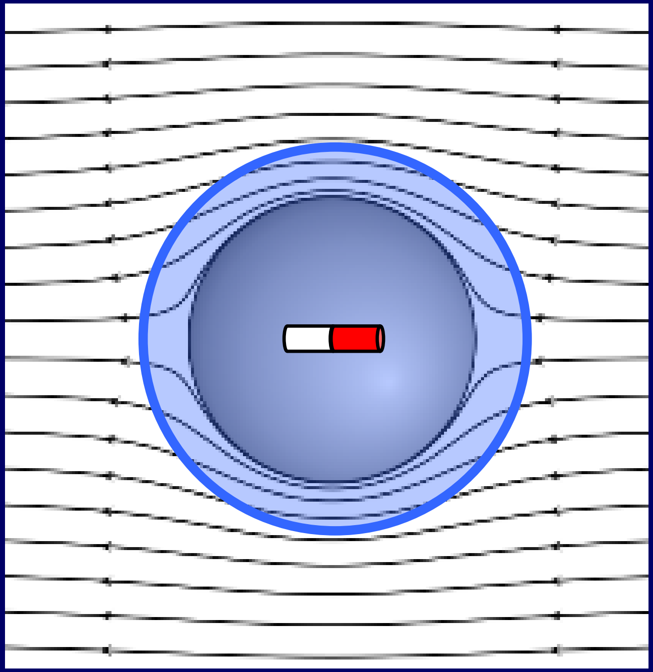
- **Galileo mission detected intrinsic magnetic field**
- **g_{10}=711 nT   Dipole tilt 4°**
- **Two field models equally consistent with data:**
  (1) Dipole + quadrupole
  (2) Dipole + field induced in ocean, no quadrupole



Kivelson et al., Icarus, 2002

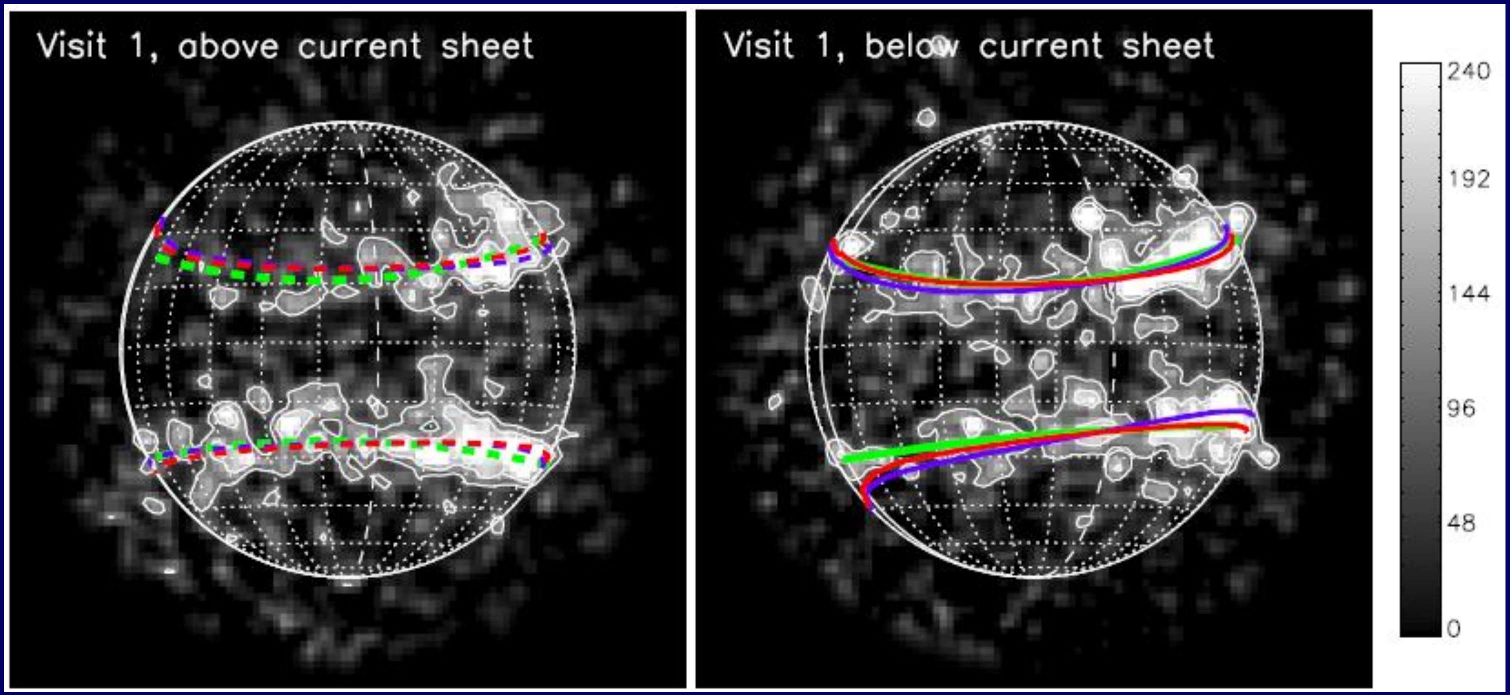
Induced magnetic fields

- The Galilean moons orbit in Jupiter‘s magnetosphere
- Because Jupiter‘s dipole is tilted by ~10º, the moons are exposed to a variable field component, changing with Jupiter‘s synodic rotation period
- In a conducting interior, currents are induced whose field counteracts the imposed field change (Lenz‘ rule)



Planet with nearly perfectly con-ducting core in a time-variable uniform field. At the surface of the conductor, the radial field must vanish. This can be described by an internal dipole aligned opposite to the external field.

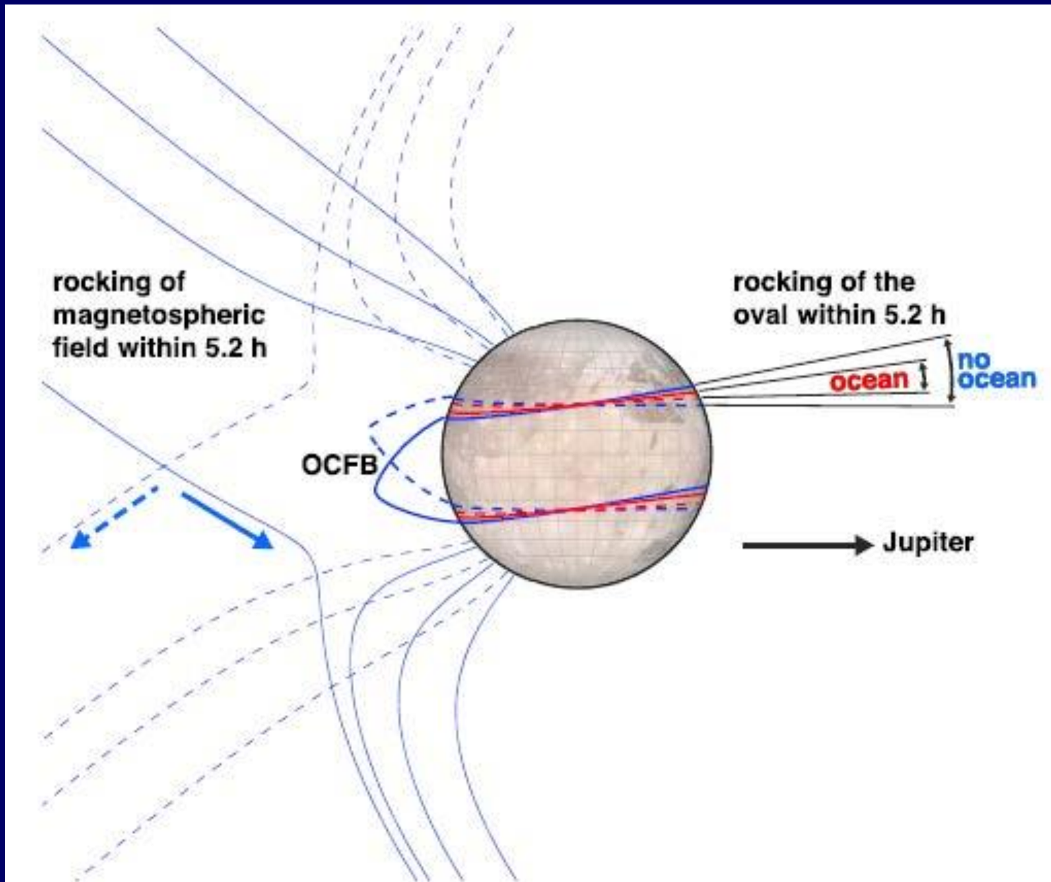
Aurorae at Ganymede



Aurorae observed on Ganymede by HST

Energetic particles precipitate along open-closed fieldline boundary and interact with Ganymede's tenuous atmosphere

Aurorae: evidence for induced field



Because Jupiter's tilted dipole rotates, the location of the aurorae rocks up and down with a 10.4h period.

With induction in a salty water ocean, rocking amplitude is less than without.

Observed rocking amplitude requires induced signal.

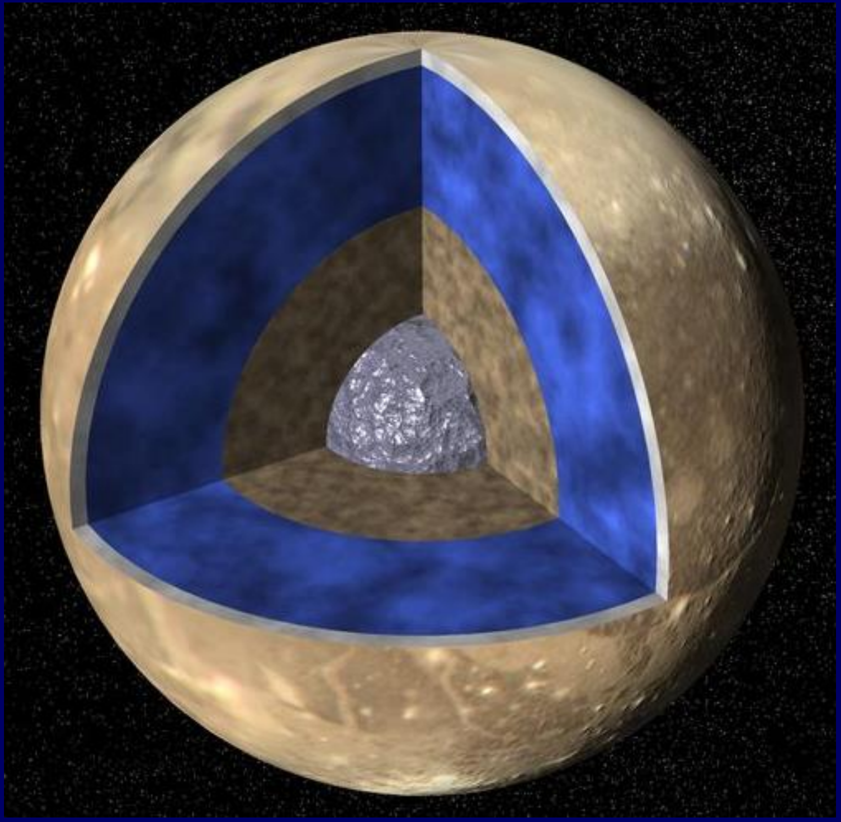
Saur et al., JGR, 2015

Origin of Ganymede's field

- Internal field generated by dynamo in metallic core
- Induced field superimposed
- Core radius r_c is 1/4 – 1/3 of planetary radius r_G
- Quadrupole untypically small at CMB

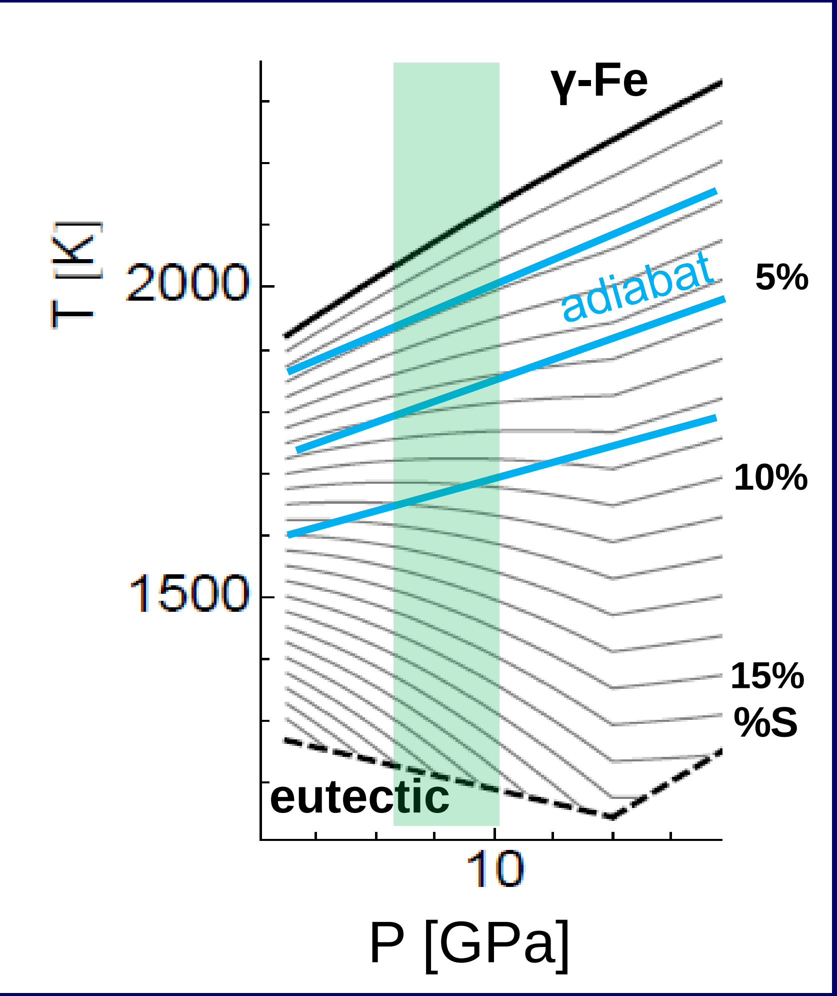
$$R_2 / R_1 < 0.04$$

(Earth CMB 2010: $R_2/R_1 = 0.14$)



Fe – FeS melting curve

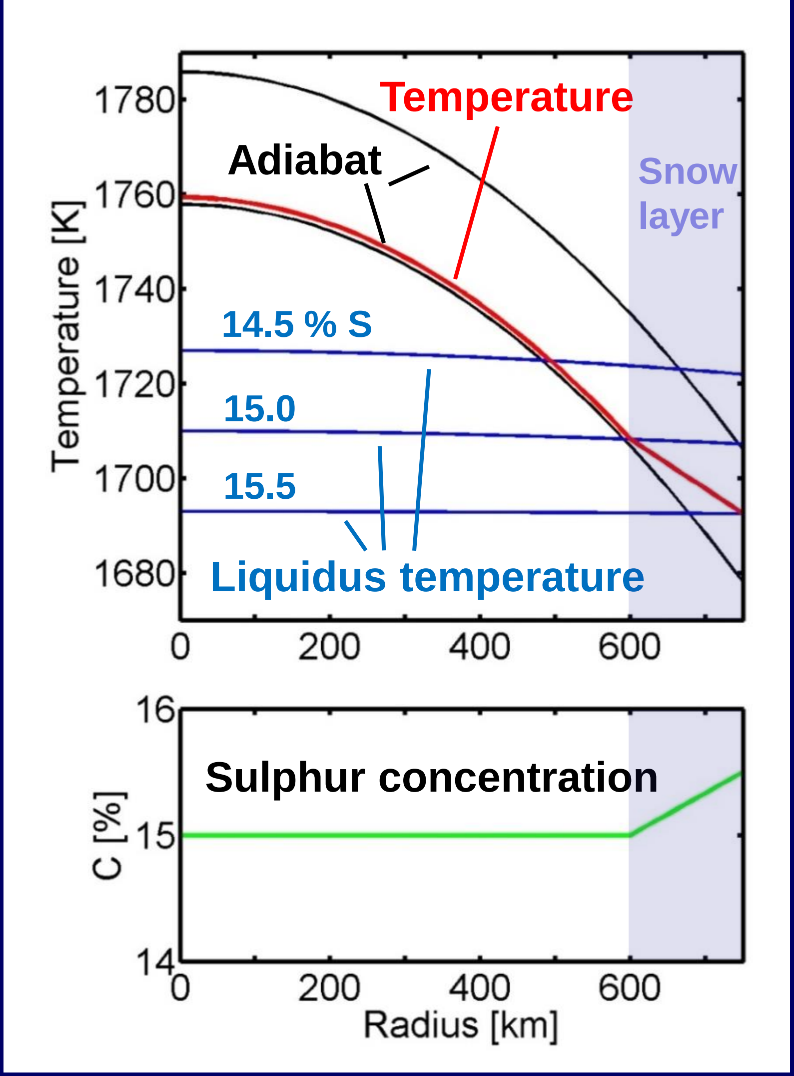
- **Sulphur likely abundant in outer solar system**
- **S reduces melting temperature T_M and gradient dT_M/dP**
- **In 5-10 GPa pressure range $dT_M/dP < 0$ for >10% S**
- **For more than few % S, dT_M/dP less steep than adiabatic T-gradient**
- ➢ **Crystallization of iron proceeds top-down**



Hauck et al., 2006; Chen et al., 2008; Williams, 2009; Buono & Walker, 2010

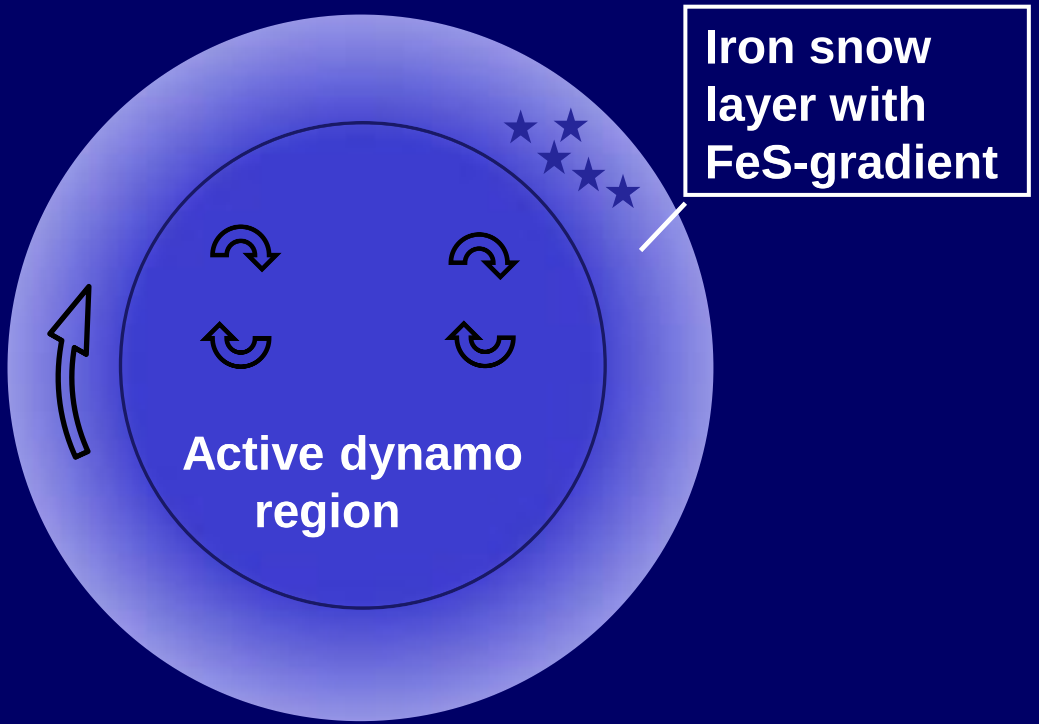
Top-down crystallization

- Adiabat steeper than melting point gradient
- Iron snow forms in top layer. Sinks and dissolves at bottom of this layer
- Sulphur-enrichment with stable compositional gradient in snow-forming layer. Here temperature everywhere at liquidus



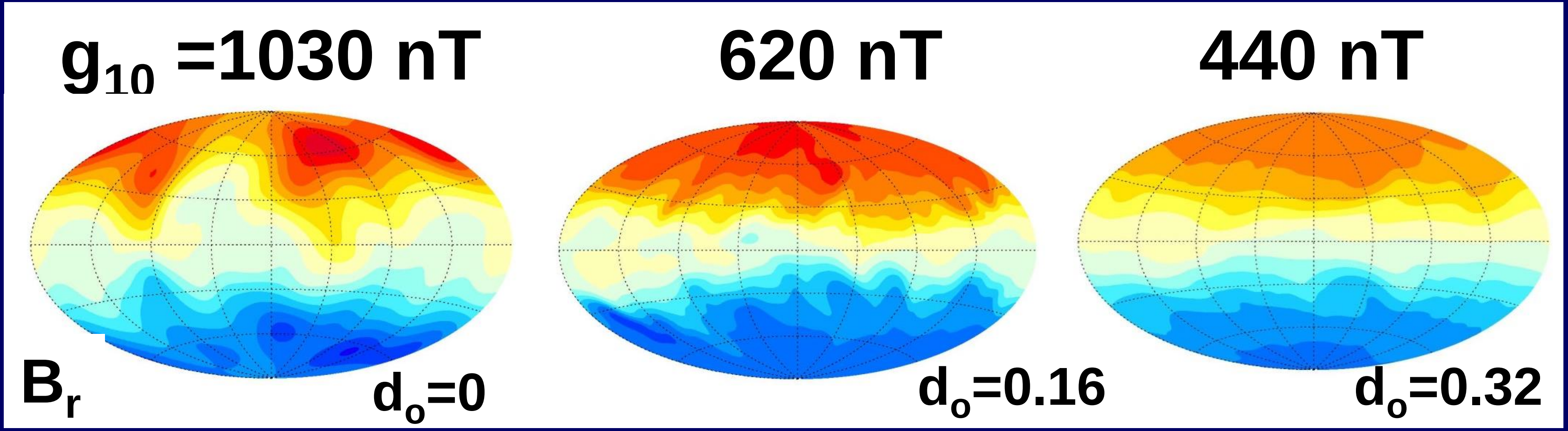
Hauck et al., 2006

Dynamo below a snow layer



- Iron snow sinks from top layer that becomes enriched in FeS
- Fe-enrichment by melting of snow at top of interior region drives compositional convection
- Stable $\Delta\rho$ in snow layer > 10^5 times larger than typical $\Delta\rho$ of convection $\Rightarrow$ horizontal flow but no radial overturn

Models with different thicknesses of stable layer

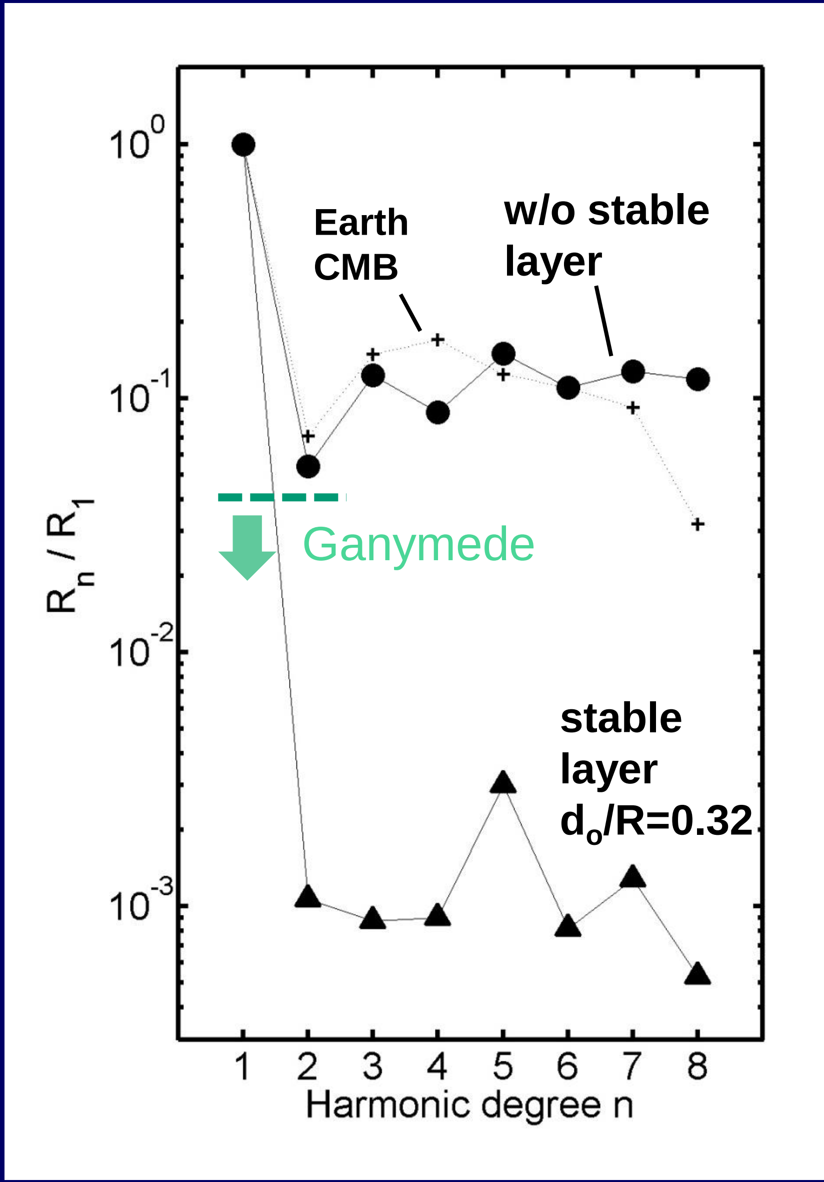


(Observed g_{10} = 711 nT)

With increasing thickness of stable layer, dipole moment drops, higher multipole moments drop even more

Magnetic power spectra

- At top of the core, time-averaged, normalized with dipole
- Reference model w/o stable layer similar to geodynamo spectrum
- With thick snow layer drop in multipoles
- $R_2 / R_1 \sim 0.001$



Perspectives

- Determine fine structure of magnetic fields of Jupiter (JUNO 2016/17), Saturn (CASSINI Grand Finale 2017), Ganymede (JUICE 2032/33). Improve on low-degree structure of Mercury‘s field (BepiColombo 2025)
- Find non-rotationally symmetric field component of Saturn or lower the upper bound on their magnitude
- Determine secular variation (or tighten upper bounds)
- Separate induced, intrinsic and external field components at Ganymede and determine induction response at different frequencies
- Possibly the detection of magnetic fields at extrasolar planets by their radioemissions