Tidal heating and orbital evolution of terrestrial exoplanets

Michaela Walterová and Marie Běhounková

Faculty of Mathematics and Physics

Charles University

numerical modelling of tidal effects (tidal deformation, heating, orbital and rotational evolution) on rocky exoplanets without atmosphere

Maxwell or Andrade viscoelastic rheology

parametric studies and qualitative comparison with recent analytical models (Correia et al., 2014)

Observational motivation

tidally evolved moons in the Solar system

increasing number of detected exoplanets (preferentially close-in)

increasing number of detected <u>terrestrial</u> exoplanets

Theoretical motivation

analytical models describe main features of orbital evolution

internal structure and more complex rheologies can be taken into account by numerical models

possible coupling of orbital and internal evolution

Model and methods

governing equations for the planetary mantle are discretized using **spherical harmonic decomposition** in the lateral directions and **staggered finite difference scheme** in the radial direction

computation in the time domain

tidal deformation → tidal heating and boundary deflections → disturbing potential and disturbing force → orbital evolution (Gauss's planetary equations)

Tidal heating and orbital evolution of terrestrial exoplanets

Michaela Walterová*, Marie Běhounková

Charles University, Faculty of Mathematics and Physics, Department of Geophysics, Czech Republic

(*e-mail: kanova@karel.troja.mff.cuni.cz)

I. Introduction

Tides play a major role in the long-term evolution of close-in exoplanets and planetary satellites. Tidal effects include enhanced **heating** of the planetary interior, secular **despinning** with eventual locking into a spin-orbit resonance, and secular **changes in orbital elements**. Traditionally, the response of a planetary body to the tidal potential is modeled using simplified rheological assumptions that are, however, valid only in special cases (e.g. Kaula, 1964; Mignard, 1979). A more general description of tides in terrestrial planets and moons, taking into account viscoelastic effects, has been developed only recently by several authors (e.g. Henning et al., 2009; Makarov and Efroimsky, 2013; Ferraz-Mello, 2013; Correia et al., 2014).

Aforementioned studies present analytical methods of addressing the main features of tides on a homogeneous planetary body. Alternatively, the tidal interaction can be described numerically. A numerical model (such as Kelvin-Voigt mass-spring model of Frouard et al., 2016) would enable implementation of more complex rheologies and internal structures and description of phaenomena that are not easy to approach analytically. Here, we present a **numerical model** based on the direct evaluation of tidal deformation of a planetary mantle governed either by **the Maxwell or the Andrade rheology**. The model is then utilized for the evaluation of tidal heating, tidal deformation and the long-term orbital evolution.

II. Model and Methods

a) Tidal deformation

The planet is modeled as a spherical body composed of a solid inner core, liquid outer core and a viscoelastic mantle. We compute the tidal deformation and dissipation solely in the mantle, using a **spherical harmonic decomposition** in the lateral directions and a **staggered finite difference scheme** in the radial direction (Tobie et al., 2008). The effects of self-gravity, pressure within the liquid core and centrifugal force are included in the model.

b) Tidal heating

The average power associated with the tidal heating of the mantle is computed as

$$P_t = \int_{V_M} \frac{1}{T_t} \int_{t}^{t+T_t} \dot{\varepsilon}(\tau) : \mathbf{D}(\tau) \, \mathrm{d}\tau \, \mathrm{d}V$$

where $\mathbf{D}$ symbolizes the deviatoric part of the Cauchy stress tensor and $\dot{\varepsilon}$ is the strain rate tensor. We evaluate the heating over one orbital period and investigate its parameter dependence. In the case of the Maxwell rheology we compute the tidal heating for effective (tidal) viscosities, which are much lower than standard viscosities used in mantle convection models (e.g. Běhounková et al., 2013).

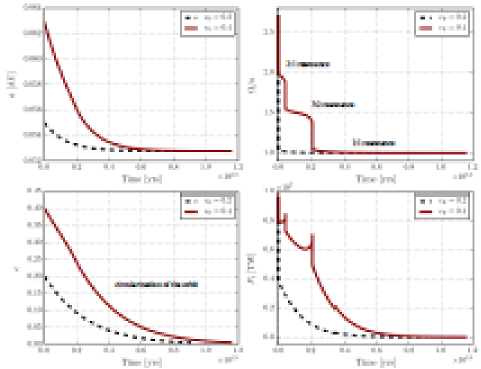
c) Orbital evolution

Once we have computed the tidal deformation inside of the planet, it is possible to evaluate the disturbing potential and the disturbing force associated with boundary deflections. While the disturbing potential at the surface enables us to compute the **potential Love number** k, the disturbing force at the host star's position may be inserted into the **Gauss's planetary equations** for the semi-major axis and the eccentricity, supplemented with an additional evolution equation for the spin rate (the total angular momentum has to be conserved). The obliquity of the planet is assumed to be zero. We compute the set of evolution equations explicitly on two time-scales: on the shorter time-scale we seek the equilibrium spin rate corresponding to the actual orbital parameters, on the longer time scale we take a step in the semi-major axis and in the eccentricity.

d) Model planet

Exo-Earth orbiting a host star with 0.1 solar masses.

Semi-major axis for the tidal heating and Love number computation: 0.1 AU.



Secular tidal evolution of the semi-major axis (top left), eccentricity of the orbit (bottom left), spin-orbit ratio (top right) and tidal heating (bottom right). Initial eccentricity of the model planet, governed by the Maxwell rheology, was either $e_0 = 0.2$ (black dashed line) or $e_0 = 0.4$ (red solid line). Effective tidal viscosity of the planet is 10^{16} Pa s.

III. Results and Conclusions

a) Love numbers

Potential Love numbers of viscoelastic bodies are **frequency-dependent quantities**. As the tidal loading of planets on eccentric orbits operates on multiple frequencies, the Love numbers, as well as the phase lag between the tidal and the disturbing potential, vary during the orbit. Similarly, if we keep a constant frequency, the Love numbers vary with increasing viscosity of the mantle, as is shown on the figures below (cf. analytical results of Correia et al., 2014).

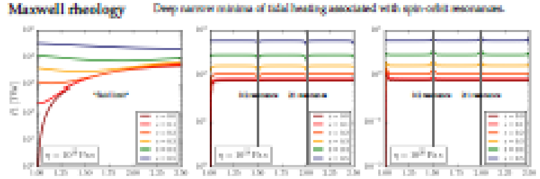
b) Tidal heating

We evaluated the average tidal heating as a function of the spin-orbit ratio and the eccentricity. Due to different frequency dependencies of dissipation in the Maxwell and the Andrade model, the tidal heating in the first case achieves deep and narrow local minima, while in the latter case the local minima are broad and shallow.

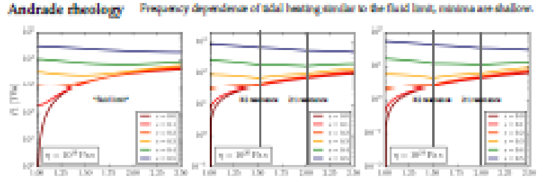
c) Orbital evolution

Tidal evolution of the planet leads to the circularization of its orbit. As the eccentricity gradually decays, the planet achieves several equilibrium spin states: the **spin-orbit resonances** (cf. Correia et al., 2014). The final spin-state, stable for very low eccentricities, is a 1:1 resonance, or the exact synchronization. Each transition between two spin-orbit resonances is preceded by an era of **pronounced tidal heating**. This may have consequences for the internal evolution, including the type of mantle convection and possible volcanism on the planet.

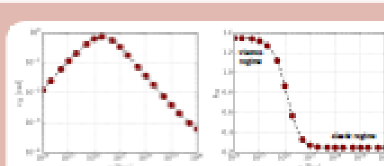
Maxwell rheology Deep narrow minima of tidal heating associated with spin-orbit resonances.



Andrade rheology Frequency dependence of tidal heating similar to the fluid limit, minima are shallow.



Average tidal heating of the mantle as a function of the spin-orbit ratio Ω/n and the eccentricity e for both rheologies and three different viscosities. Prominent features - local minima of tidal heating - are associated with **spin-orbit resonances**, where the rotational frequency equals integer or half-integer multiple of the orbital frequency. Relative depth of distinct minima depends on the eccentricity.



Tidal lag (left) and **Love number** k_{22} (right) of a Maxwellian body as a function of effective tidal viscosity. The spin-orbit ratio of the model planet with $e = 0$ was set constant, $\Omega/n = 2.1$.

References

Běhounková, M., Tobie, G., Choblet, G., Čadek, O. (2013). Impact of tidal heating on the onset of convection in Enceladus's ice shell. Icarus 226.

Correia, A.C.M., Boué, G., Laskar, J. and Rodríguez, A. (2014). Deformation and tidal evolution of close-in planets and satellites using a Maxwell viscoelastic rheology. A&A 571(A50).

Ferraz-Mello, S. (2013). Tidal synchronization of close-in satellites and exoplanets. A rheophysical approach. Celest. Mech. Dyn. Astron. 116(2).

Frouard, J., Quillen, A.C., Efroimsky, M. and Giannella, D. (2016) Numerical Simulation of Tidal Evolution of a Viscoelastic Body Modelled with a Mass-Spring Network. MNRAS 458(3).

Henning, W.G., O'Connell, R.J., Sasselov, D.D. (2009). Tidally Heated Terrestrial Exoplanets: Viscoelastic Response Models. ApJ 707(2).

Kaula, W. M. (1964). Tidal dissipation by solid friction and the resulting orbital evolution. Rev. Geophys. 2(4).

Makarov, V.V. and Efroimsky, M. (2013). No Pseudosynchronous Rotation for Terrestrial Planets and Moons. ApJ 764(1).

Mignard, F. (1979). The Evolution of the Lunar Orbit Revisited. I. Moon and Planets 20.

Tobie, G., Čadek, O. and Sotin, C. (2008). Solid tidal friction above a liquid water reservoir as the origin of the south pole hotspot on Enceladus. Icarus 196.

Acknowledgements

This work was supported by the Grant Agency of Charles University (project No. 338216) and the Czech Science Foundation (project No. 14-04145S).