Calculation of load Love numbers and static stresses for the interior structure model of Mars with an elastic mantle

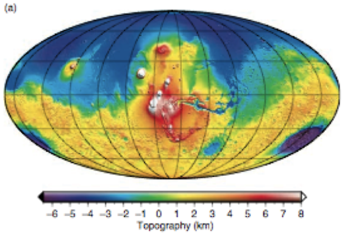
A.V. Batov[1], T.V. Gudkova[2], V.N. Zharkov[2]

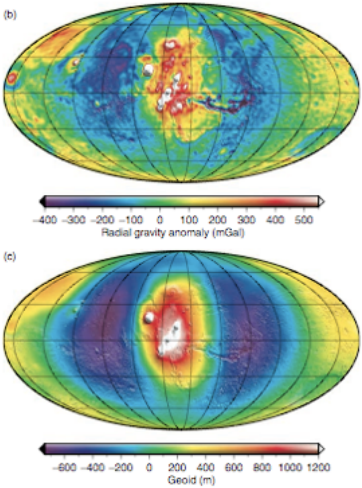
[1]V.A. Trapeznikov Institute of Control Sciences RAS, Moscow, Russia (batov@ipu.ru)

[2]Schmidt Institute of Physics of the Earth RAS, Moscow, Russia

L'Aquila, 2016

Topography, geoid, and gravity anomalies





a Topography data *(Mars Global Surveyor) (Smith et al., 2001)*

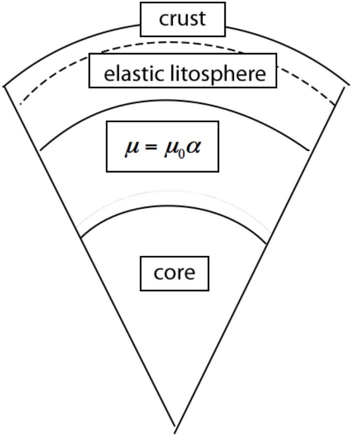
b Gravity data *(MGS, Mars Odyssey Mars Reconnaissance Orbiter)*

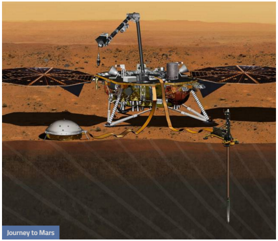
Model MRO120D *(Konopliv et al., 2016)*

Model GMM3120 *(Genova et al., 2016)*

Investigating Mars

Our goal is to identify areas of high values of shear and tension-compression stresses as possible marsquakes focuses





NASA Targets May 2018 Launch of Mars InSight Mission

- completely elastic models of Mars
- models with an elastic lithosphere and much weaker astenosphere
  (a relaxed shear modulus under elastic lithosphere is reduced in comparison with an elastic value and in extreme case is approaching to zero)
  α **changes from 1 to** 10^{-5}**, 0**

Combined interpretation of the anomalous gravitational field and topography

Data on the gravitational field and topography are the boundary conditions for calculations

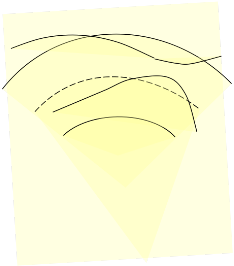
- **Steady-state approach**
  - Mars is modeled as an elastic, self-gravitating body
  - Calculations are carried out by means of the Greens function technique *(Marchenkov et al., 1984, Zharkov et al., 1986, 1991)*
- **Dynamic approach**
  - Viscous convective flows are assumed to be the sources of nonhydrostatic stresses
  - The solution is sought based on the equations for a viscous fluid *(Ricard et al., 1984; 1989; Hager et al., 1985, see also Dehant et al., 2000; Van Hoolst et al., 2000; Defraigne et al.,2001)*

Mars has a thick elastic lithosphere (about 300 km) (Phillips et. al., 2008).
It is capable of elastically supporting nonhydrostatic loads over the duration of geologic times. $\Longrightarrow$ *If thermal convection exists under the lithosphere of Mars, it is of a secondary order Stress state is mainly caused by elastic deformations of subsurface layers.*

Steady-state approach

The source of the anomalous gravitational field: the topographic loading and anomalies of density in the interiors

For an interpretation of these data the anomalous density field is expanded in terms of spherical harmonics



$$\delta\rho^0(r,\theta,\lambda) = \sum_{i,n,m} \delta\rho^0_{i,n,m}(r) Y_{i,n,m}(\theta,\lambda)$$

$$\delta\rho^1(r,\theta,\lambda) = \sum_{i,n,m} \delta\rho^1_{i,n,m}(r) Y_{i,n,m}(\theta,\lambda)$$

The amplitudes $\delta\rho_{i,n,m}(r)$ are selected so that the anomalous gravitational field can be reproduced

The topographic loading as an equivalent infinitly weighted thin layer is at the reference surface (R_0), while the rest of the anomalous mass as an equivalent infinitly weighted thin layer is at the crust-mantle boundary (R_1)

Load Love numbers

Gravity field of such layer at the surface

$$\Delta V = 4\pi GR \sum_{i,n,m} \left(\frac{r}{R}\right)^{n+2} \frac{R_{i,n,m}(r)}{2n+1} Y_{i,n,m}(\theta, \varphi)$$

The anomalous density acts as a load, to account the deformation

$$\Delta V = 4\pi GR \sum_{i,n,m} \left(\frac{r}{R}\right)^{n+2} \frac{R_{i,n,m}(r)(1 + k_{nj}(r))}{2n+1} Y_{i,n,m}(\theta, \varphi)$$

k_{nj} is the load number for the n-th harmonic of the anomalous density wave located at depth x_j

$(1 + k_{nj})$ defines the total change in the gravitational potential on the Mars's surface

Load Love numbers, real expansion coefficients

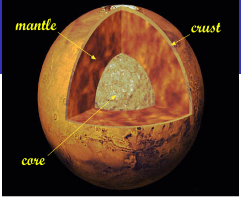
Spherical expansion coefficients of the anomalous density waves on the surface $R^1_{i,n,m}(\theta,\varphi)$ and at the crust-mantle boundary $R^2_{i,n,m}(\theta,\varphi)$ are related to expansion coefficients C_{ginm} of the anomalous gravitational field and expansion coefficients C_{ginm} of the Mars topography by equations:

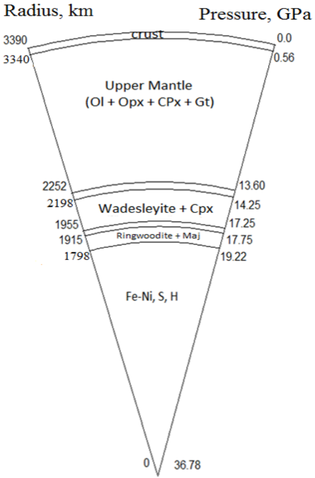
$$C_{ginm} = \frac{R^1_{inm}}{R\rho_0}\frac{3(1+k_n(R))}{(2n+1)} + \frac{R^2_{inm}}{R\rho_0}\frac{3(1+k_n(R_1))}{(2n+1)}\left(\frac{R_1}{R}\right)^{n+2}$$

$$C_{tinm} = \frac{R^1_{inm}}{R\rho_c} + \frac{R^1_{inm}}{R\rho_0}\frac{3(1+h_n(R))}{(2n+1)} + \frac{R^2_{inm}}{R\rho_0}\frac{3(1+h_n(R_1))}{(2n+1)}\left(\frac{R_1}{R}\right)^{n+2},$$

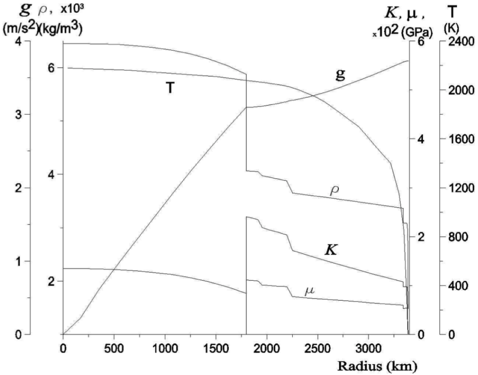
where ρ_0 – mean density of Mars, ρ_c – density at the core-mantle boundary R – Mars radius, R_1 – radius of core-mantle boundary.

Model M7 (Zharkov, Gudkova, 2016)





Density ρ, gravity g, temperature T, bulk modulus K and shear modulus μ as a function of radius



Mars departs from hydrostatic equilibrium to significant extent

It is seen if we compare the first even gravitational moments for a set of trial Martian models in hydrostatic equilibrium
$J_2^0 \approx (1.790 \div 1.835) \times 10^{-3}$; $J_4^0 \approx -(7.62 \div 7.88) \times 10^{-6}$
with gravitational moments of the planet
$J_2 = 1.956609 \times 10^{-3}$; $J_4 = -15.387292 \times 10^{-6}$

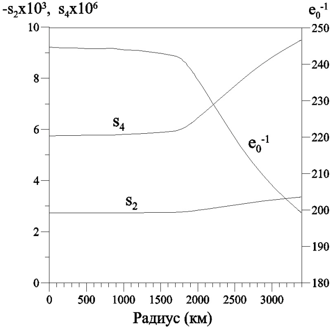
(model igmro110c, Konopliv et al., 2011)

To avoid uncontrollable stresses and deformations in the mantle of the planet due significant deviation of Mars from hydrostatic state, an outer surface of hydrostatical model is taken as outer reference surface

$$r(s,\theta) = s\{1 + s_0(s) + s_2(s)P_2(t) + s_4(s)P_4(t) + \ldots\},$$

where $s = R_0$ is the mean radius (the radius of an equivolume sphere), $t = \cos\theta$ is the polar distance, $P_2(t)$ and $P_4(t)$ are the first even ordinary Legendre polynomials

Mars departs from hydrostatic equilibrium to significant extent



For test model:

$$s_2 = -3.342 \times 10^{-3} \quad J_2^0 = 1.804 \times 10^{-3}$$
$$s_4 = 9.402 \times 10^{-6} \quad J_4^0 = -7.673 \times 10^{-6}$$

The distribution of the parameters of the equilibrium figure of Mars s_2, s_4 and e_0^{-1} along the planetary radius for a trial model.

Topography and gravity field of Mars are defined with respect to the reference surface – equilibrium spheroid

$$R_{relief}(r,\theta,\varphi) = R + \sum_{n=1}^{90}\sum_{m=0}^{n}[C_{tnm}\cos m\varphi + S_{tnm}\sin m\varphi]P_{nm}(\cos\theta)$$

Topography is defined as: $h_{Mars}(r,\theta,\varphi) = R_{relief}(r,\theta,\varphi) - r(s,\theta)$

$$V(r,\theta,\varphi) = \frac{GM}{r}\left[1 + \sum_{n=2}^{110}\sum_{m=0}^{n}\left(\frac{R_e}{r}\right)^n (C_{gnm}\cos m\varphi + S_{gnm}\sin m\varphi)P_{nm}(\cos\theta)\right]$$

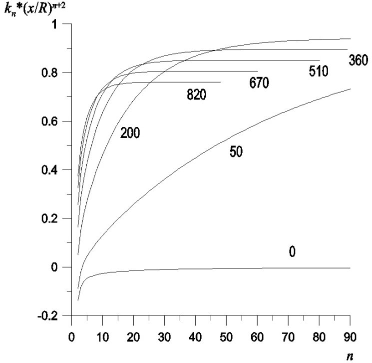
The outer gravitational potential of the equilibrium planet $V^0(r,t)$ also contains only even harmonics

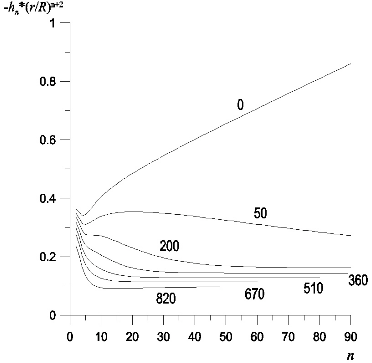
$$V^0(r,\theta) = \frac{GM}{r}\left[1 + \left(\frac{R_e}{r}\right)^2 C_{g20}P_{20}(\cos\theta) + T\left(\frac{R_e}{r}\right)^4 C_{g40}P_{40}(\cos\theta)\right]$$

The nonequilibrium part of the gravitational potential (the deviation from normal potential): $T_g(r,\theta,\varphi) = V(r,\theta,\varphi) - V^0(r,\theta)$

Loading coefficients for deeply buried density anomalies k_n and h_n of n-th harmonic of ADW located at depth r_j

They define the total change in the gravitational potential on the surface of Mars and deformation of the planets surface under the action of load

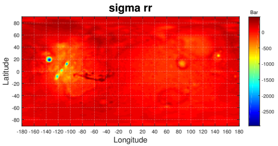


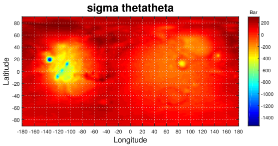


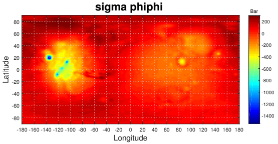
Load numbers k_n and h_n for various depths of the density anomaly as a function of the order n. Values correspond the depths (in km) of loading.

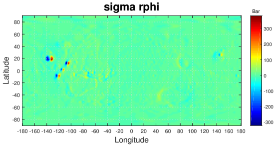
Stress tensor components

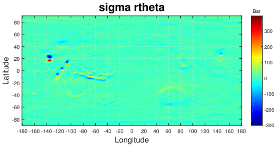
$$\sigma = \sigma_{ij} = \begin{bmatrix} \sigma_{rr} & \sigma_{r\theta} & \sigma_{r\varphi} \\ \sigma_{\theta r} & \sigma_{\theta\theta} & \sigma_{\theta\varphi} \\ \sigma_{\varphi r} & \sigma_{\varphi\theta} & \sigma_{\varphi\varphi} \end{bmatrix}; \quad \begin{aligned} \sigma_{rr} &= \lambda\Delta + 2\mu\varepsilon_{rr}; & \sigma_{r\theta} &= 2\mu\varepsilon_{r\theta}; \\ \sigma_{\theta\theta} &= \lambda\Delta + 2\mu\varepsilon_{\theta\theta}; & \sigma_{r\varphi} &= 2\mu\varepsilon_{r\varphi}; \\ \sigma_{\varphi\varphi} &= \lambda\Delta + 2\mu\varepsilon_{\varphi\varphi}; & \sigma_{\theta\phi} &= 2\mu\varepsilon_{\theta\phi}; \end{aligned}$$

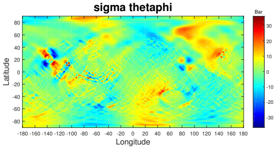












Shear stresses and tension-compression stresses

beneath the 50km-crust

Tensor is reduced to diagonal form with the principal stresses $\sigma_3 \leq \sigma_2 \leq \sigma_1$ compression-tension stresses and maximum shear stresses are calculated as $(\sigma_1 + \sigma_2 + \sigma_3)/3$ and $(\sigma_1 - \sigma_2)/2$, respectively.