LECTURE 1

COSMIC RAY ACCELERATION

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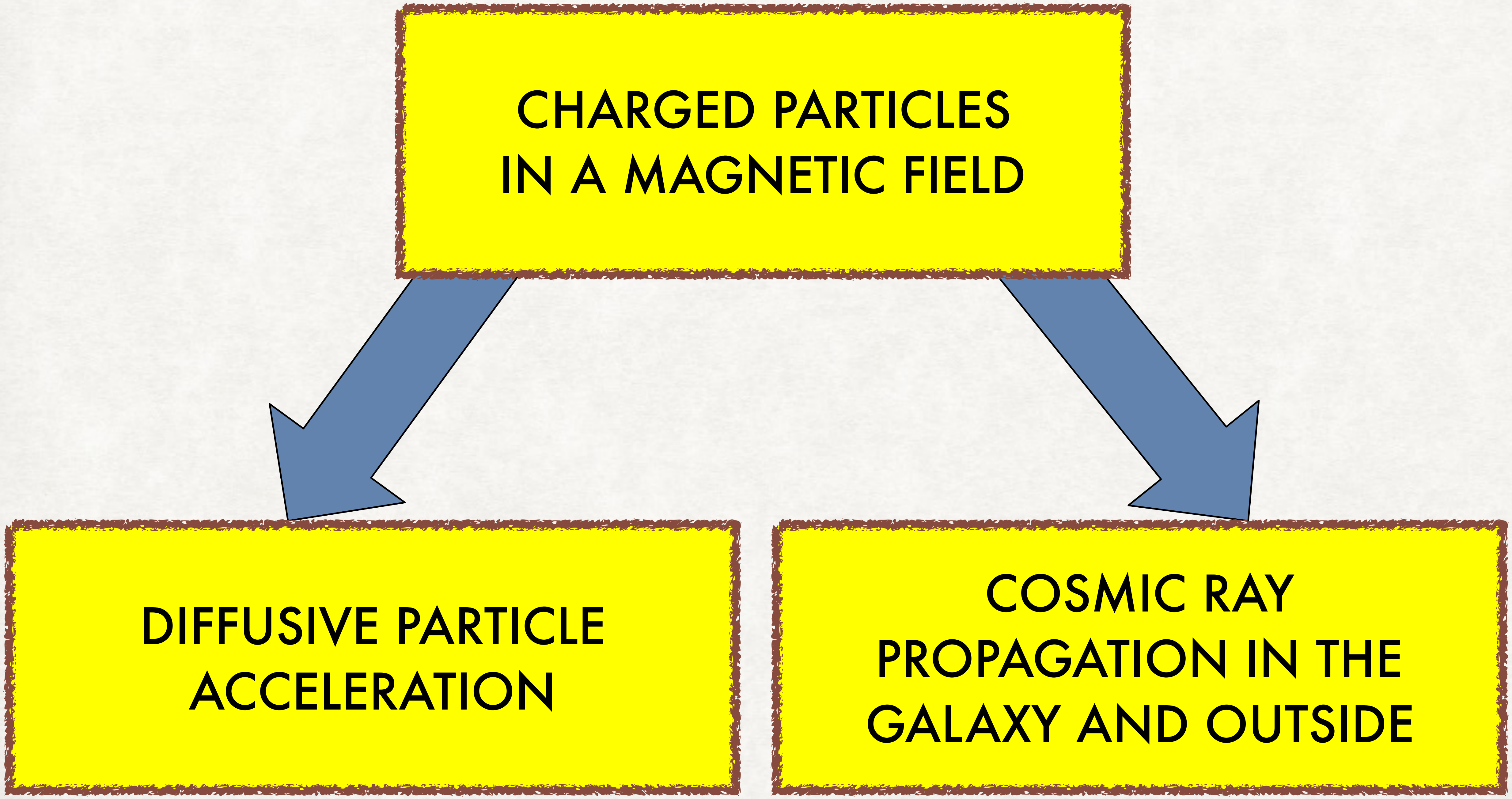
OUTLINE OF THE MINI-COURSE

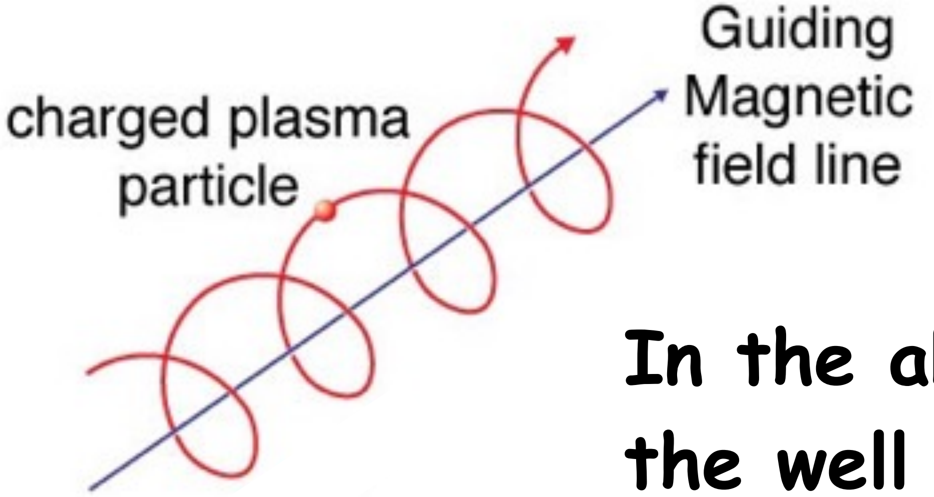
- **First Lecture**
  - *Principles of CR transport*
  - *Second Order Fermi Acceleration*
  - *Diffusive Shock Acceleration: test particle theory*
  - *Diffusive Shock Acceleration: modern theory including non linear aspects*

- **Second Lecture**
  - *Propagation of CR in the Galaxy: classical theory*
  - *Non linear propagation of CR in the Galaxy*
  - *Contact with observables - spectra and mass composition*

COSMIC RAY TRANSPORT



CHARGED PARTICLES IN A REGULAR B FIELD



$$\frac{d\vec{p}}{dt} = q\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right]$$

In the absence of an electric field one obtains the well known solution:

$$p_z = \text{Constant}$$

$$v_x = V_0 \cos[\Omega\, t]$$

$$v_y = V_0 \sin[\Omega\, t]$$

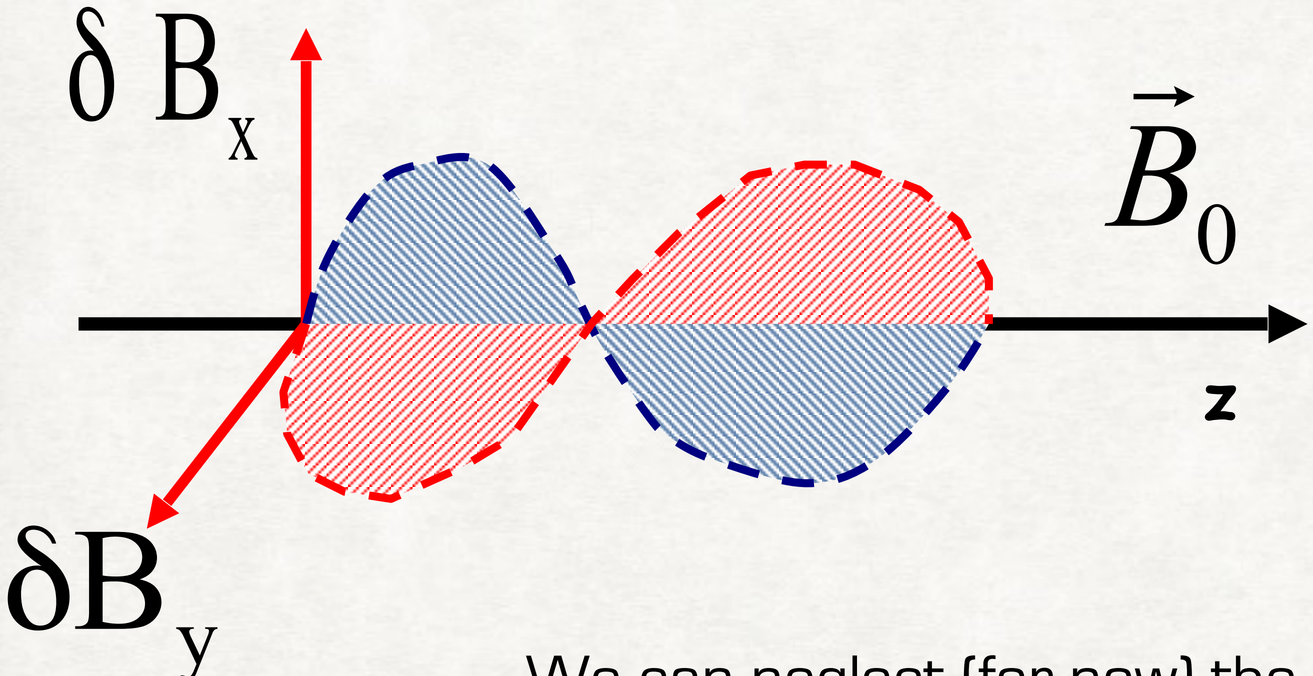
LARMOR FREQUENCY

$$\Omega = \frac{q\, B_0}{m\, c\, \gamma}$$

A FEW THINGS TO KEEP IN MIND

- THE MAGNETIC FIELD DOES NOT CHANGE PARTICLE ENERGY -> NO ACCELERATION BY B FIELDS
- A RELATIVISTIC PARTICLE MOVES IN THE Z DIRECTION ON AVERAGE AT C/3

MOTION OF A PARTICLE IN A WAVY FIELD



Let us consider an Alfven wave propagating in the z direction:

$$\delta B << B_0 \qquad \delta\vec{B} \perp \vec{B}_0$$

We can neglect (for now) the electric field associated with the wave, or in other words we can sit in the reference frame of the wave:

$$\frac{d\vec{p}}{dt} = q\,\frac{\vec{v}}{c} \times (\vec{B}_0 + \delta\vec{B})$$

THIS CHANGES ONLY THE X AND Y COMPONENTS OF THE MOMENTUM

THIS TERM CHANGES ONLY THE DIRECTION OF $P_Z = P\mu$

Remember that the wave typically moves with the Alfven speed:

$$v_a = \frac{B}{(4\pi\rho)^{1/2}} = 2 \times 10^6 B_\mu n_1^{-1/2} \ cm/s$$

Alfven waves have frequencies << ion gyration frequency $\Omega_p = qB/m_p c$

It is therefore clear that for a relativistic particle these waves, in first approximation, look like static waves.

The equation of motion can be written as:

$$\frac{d\vec{p}}{dt} = \frac{q}{c}\vec{v} \times (\vec{B}_0 + \vec{\delta}B)$$

If to split the momentum in parallel and perpendicular, the perpendicular component cannot change in modulus, while the parallel momentum is described by

$$\frac{dp \parallel}{dt} = \frac{q}{c}|\vec{v}_\perp \times \delta\vec{B}| \qquad p_\parallel = p \ \mu$$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos(\Omega t - kx + \psi)$$

Wave form of the magnetic field with
a random phase and frequency

$$\Omega = qB_0/mc\gamma$$ Larmor frequency

In the frame in which the wave is at rest we can write $x = v\mu t$

$$\frac{d\mu}{dt} = \frac{q}{pc} v(1-\mu^2)^{1/2} \delta B cos\left[(\Omega - kv\mu)t + \psi\right]$$

It is clear that the mean value of the pitch angle variation over a long enough time vanishes

$$\langle \Delta\mu \rangle_t = 0$$

We want to see now what happens to $\langle \Delta\mu\Delta\mu \rangle$

Let us first average upon the random phase of the waves:

$$\langle \Delta\mu(t')\Delta\mu(t")\rangle_\psi = \frac{q^2 v^2 (1-\mu^2)\delta B^2}{2c^2p^2} cos\left[(\Omega - kv\mu)(t' - t")\right]$$

And integrating over time:

$$\langle \Delta\mu\Delta\mu\rangle_t = \frac{q^2 v^2 (1-\mu^2)\delta B^2}{2c^2p^2} \int dt' \int dt" \, cos\left[(\Omega - kv\mu)(t' - t")\right]$$

$$= \frac{q^2 v (1-\mu^2)\delta B^2}{c^2p^2\mu} \delta(k - \Omega/v\mu)\Delta t$$

RESONANCE

Many waves

IN GENERAL ONE DOES NOT HAVE A SINGLE WAVE BUT RATHER A POWER SPECTRUM:

$$P(k) = B_k^2/4\pi$$

THEREFORE INTEGRATING OVER ALL OF THEM:

$$\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \rangle = \frac{q^2(1-\mu^2)\pi}{m^2c^2\gamma^2} \frac{1}{v\mu} 4\pi \int dk \frac{\delta B(k)^2}{4\pi} \delta(k - \Omega/v\mu)$$

OR IN A MORE IMMEDIATE FORMALISM:

$$\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle = \frac{\pi}{2} \Omega (1 - \mu^2) k_{res} F(k_{res})$$

$$k_{res} = \frac{\Omega}{v\mu}$$

RESONANCE!!!

DIFFUSION COEFFICIENT

THE RANDOM CHANGE OF THE PITCH ANGLE IS DESCRIBED BY A DIFFUSION COEFFICIENT

$$D_{\mu\mu} = \left\langle \frac{\Delta\theta\Delta\theta}{\Delta t} \right\rangle = \frac{\pi}{4}\Omega k_{res} F(k_{res})$$

FRACTIONAL POWER $(\delta B/B_0)^2$ $=G(k_{res})$

THE DEFLECTION ANGLE CHANGES BY ORDER UNITY IN A TIME:

$$\tau \approx \frac{1}{\Omega\, G(k_{res})}$$

PATHLENGTH FOR DIFFUSION ~ $v\tau$

$$\left\langle \frac{\Delta z\Delta z}{\Delta t} \right\rangle \approx v^2\, \tau = \frac{v^2}{\Omega\, G(k_{res})}$$

SPATIAL DIFFUSION COEFF.

PARTICLE SCATTERING

- EACH TIME THAT A RESONANCE OCCURS THE PARTICLE CHANGES PITCH ANGLE BY $\Delta\theta \sim \delta B/B$ WITH A RANDOM SIGN

- THE RESONANCE OCCURS ONLY FOR RIGHT HAND POLARIZED WAVES IF THE PARTICLES MOVES TO THE RIGHT (AND VICEVERSA)

- THE RESONANCE CONDITION TELLS US THAT 1) IF $k \ll 1/r_L$ PARTICLES SURF ADIABATICALLY AND 2) IF $k \gg 1/r_L$ PARTICLES HARDLY FEEL THE WAVES

A RATHER GENERAL EQUATION

$$\boxed{\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} + Q(x,p,t)}$$

Time Dep. Diffusion Advection compression Injection

THIS EQUATION, THOUGH IN ONE DIMENSION, CONTAINS ALL THE MAIN EFFECTS DESCRIBED BY MORE COMPLEX TREATMENTS

1. *TIME DEPENDENCE*
2. *DIFFUSION (EVEN SPACE AND MOMENTUM DEPENDENCE)*
3. *ADVECTION (EVEN WITH A SPACE DEPENDENT VELOCITY)*
4. *COMPRESSION AND DECOMPRESSION*
5. *INJECTION*

IT DOES NOT INCLUDE 2nd ORDER AND SPALLATION, BUT EASY TO INCLUDE

IT APPLIES EQUALLY WELL TO TRANSPORT OF CR IN THE GALAXY OR TO CR ACCELERATION AT A SUPERNOVA SHOCK

ACCELERATION OF NONTHERMAL PARTICLES

The presence of non-thermal particles is ubiquitous in the Universe (solar wind, Active galaxies, supernova remnants, gamma ray bursts, Pulsars, micro-quasars)

WHEREVER THERE ARE MAGNETIZED PLASMAS THERE ARE NON-THERMAL PARTICLES

↓

PARTICLE ACCELERATION

BUT THERMAL PARTICLES ARE USUALLY DOMINANT, SO WHAT DETERMINES THE DISCRIMINATION BETWEEN THERMAL AND ACCELERATED PARTICLES?

INJECTION

ALL ACCELERATION MECHANISMS ARE ELECTROMAGNETIC IN NATURE

MAGNETIC FIELD CANNOT MAKE WORK ON CHARGED PARTICLES THEREFORE ELECTRIC FIELDS ARE NEEDED FOR ACCELERATION TO OCCUR

REGULAR ACCELERATION

THE ELECTRIC FIELD IS LARGE SCALE:

$$\langle \vec{E} \rangle \neq 0$$

STOCHASTIC ACCELERATION

THE ELECTRIC FIELD IS SMALL SCALE:

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

STOCHASTIC ACCELERATION

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

Most acceleration mechanisms that are operational in astrophysical environments are of this type. We have seen that the action of random magnetic fluctuations is that of scattering particles when the resonance is achieved. In other words, the particle distribution is isotropized in the reference frame of the wave.

Although in the reference frame of the waves the momentum is conserved (B does not make work) in the lab frame the particle momentum changes by

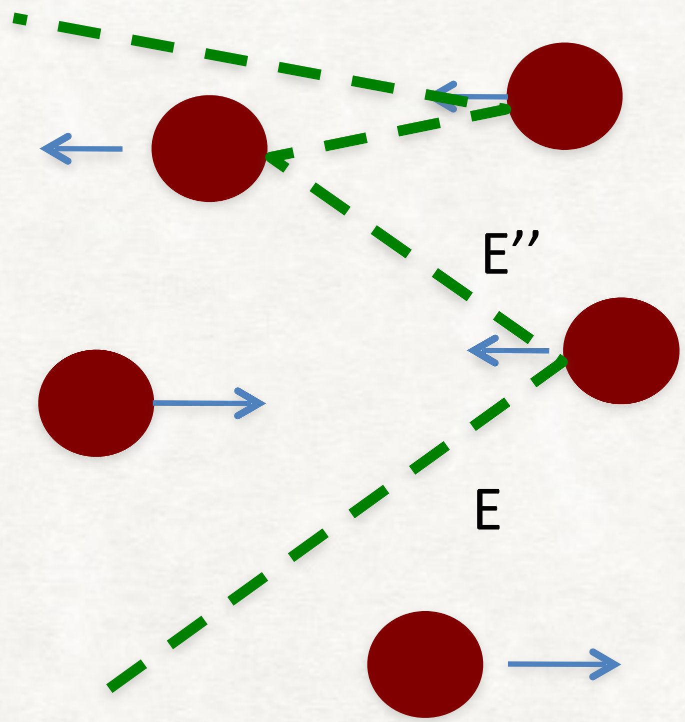
$$\Delta p \sim p \frac{v_A}{c}$$

In a time T which is the diffusion time as found in the last lecture. It follows that

$$D_{pp} = \langle \frac{\Delta P \Delta p}{\Delta t} \rangle \sim p^2 \frac{1}{T} \left(\frac{v_A}{c} \right)^2 \rightarrow \tau_{pp} = \frac{p^2}{D_{pp}} T \left(\frac{c}{v_A} \right)^2 \gg T$$

THE MOMENTUM CHANGE IS A SECOND ORDER PHENOMENON !!!

SECOND ORDER FERMI ACCELERATION



We inject a particle with energy E. In the reference frame of a cloud moving with speed b the particle energy is:

$$E' = \gamma E + \beta\gamma p\mu$$

and the momentum along x is:

$$p'_x = \beta\gamma E + \gamma p\mu$$

Assuming that the cloud is very massive compared with the particle, we can assume that the cloud is unaffected by the scattering, therefore the particle energy in the cloud frame does not change and the momentum along x is simply inverted, so that after 'scattering' $p'_x \rightarrow -p'_x$. The final energy in the Lab frame is therefore:

$$E'' = \gamma E' + \beta\gamma p'_x =$$

$$\gamma^2 E\left(1 + \beta^2 + 2\beta\mu\frac{p}{E}\right)$$

$$\frac{p}{E} = \frac{mv\gamma}{m\gamma} = v$$

Where v is now the dimensionless Particle velocity

It follows that:

$$E" = \gamma^2 E \left(1 + \beta^2 + 2\beta\mu v\right)$$

and:

$$\frac{E" - E}{E} = \gamma^2 \left(1 + 2\beta v\mu + \beta^2\right) - 1$$

and finally, taking the limit of non-relativistic clouds g→1:

$$\frac{E" - E}{E} \approx 2\beta^2 + 2\beta v\mu$$

We can see that the fractional energy change can be both positive or negative, which means that particles can either gain or lose energy, depending on whether the particle-cloud scattering is head-on or tail-on.

We need to calculate the probability that a scattering occurs head-on or Tail-on. The scattering probability along direction m **is proportional to the Relative velocity in that direction:**

$$P(\mu) = Av_{rel} = A\frac{\beta\mu + v}{1 + v\beta\mu} \rightarrow_{v\rightarrow 1} \approx A(1+\beta\mu)$$

The condition of normalization to unity:

$$\int_{-1}^{1} P(\mu)d\mu = 1$$

leads to A=1/2. It follows that the mean fractional energy change is:

$$\langle\frac{\Delta E}{E}\rangle = \int_{-1}^{1} d\mu P(\mu)\left(2\beta^2 + 2\beta\mu\right) = \frac{8}{3}\beta^2$$

NOTE THAT IF WE DID NOT ASSUME RIGID REFLECTION AT EACH CLOUD BUT RATHER ISOTROPIZATION OF THE PITCH ANGLE IN EACH CLOUD, THEN WE WOULD HAVE OBTAINED (4/3) b^2 **INSTEAD OF (8/3)** b^2

THE FRACTIONAL CHANGE IS A SECOND ORDER QUANTITY IN $\beta<<1$. This is the reason for the name SECOND ORDER FERMI ACCELERATION

The acceleration process can in fact be shown to become more Important in the relativistic regime where $\beta \to 1$

THE PHYSICAL ESSENCE CONTAINED IN THIS SECOND ORDER DEPENDENCE IS THAT IN EACH PARTICLE-CLOUD SCATTERING THE ENERGY OF THE PARTICLE CAN EITHER INCREASE OR DECREASE → WE ARE LOOKING AT A PROCESS OF DIFFUSION IN MOMENTUM SPACE

THE REASON WHY ON AVERAGE THE MEAN ENERGY INCREASES IS THAT HEAD-ON COLLISIONS ARE MORE PROBABLE THAN TAIL-ON COLLISIONS

WHAT IS DOING THE WORK?

We just found that particles propagating in a magnetic field can change their momentum (in modulus and direction)...

BUT MAGNETIC FIELDS CANNOT CHANGE THE MOMENTUM MODULUS... ONLY ELECTRIC FIELDS CAN

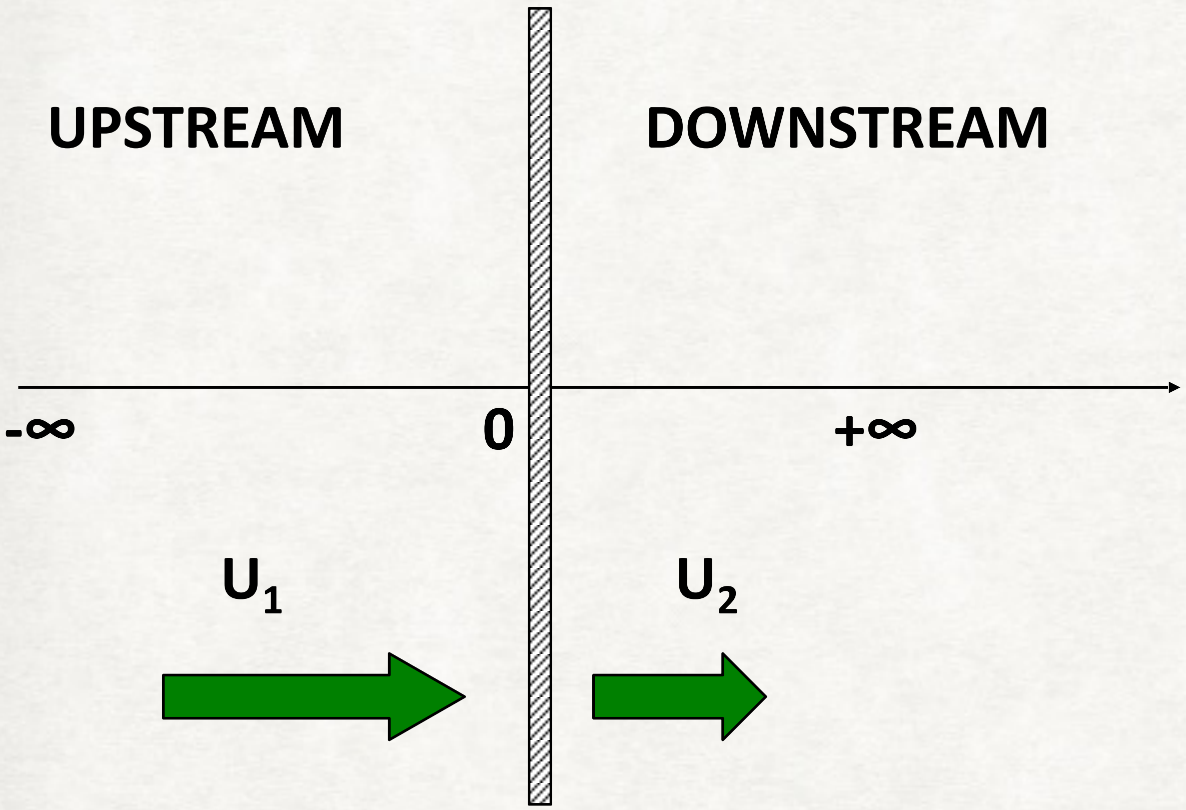
WHAT IS THE SOURCE OF THE ELECTRIC FIELDS???
Moving Magnetic Fields

The induced electric field is responsible for this first instance of particle acceleration

The scattering leads to momentum transfer, but to WHAT?

Recall that particles isotropize in the reference frame of the waves…

SHOCK SOLUTIONS



Let us sit in the reference frame in which the shock is at rest and look for stationary solutions

$$\frac{\partial}{\partial x}(\rho u) = 0$$

$$\frac{\partial}{\partial x}(\rho u^2 + P) = 0$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma - 1}uP\right) = 0$$

It is easy to show that aside from the trivial solution in which all quantities remain spatially constant, there is a discontinuous solution:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}$$

M_1 is the upstream Fluid Mach number

$$\frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - \gamma(\gamma-1)][(\gamma-1)M_1^2 + 2]}{(\gamma+1)^2 M_1^2}$$

STRONG SHOCKS M_1>>1

In the limit of strong shock fronts these expressions get substantially simpler and one has:

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{\gamma+1}{\gamma-1}$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2}{\gamma+1}$$

$$\frac{T_2}{T_1} = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_1^2, \quad T_2 = 2\frac{\gamma-1}{(\gamma+1)^2} m u_1^2$$

ONE CAN SEE THAT SHOCKS BEHAVE AS VERY EFFICENT HEATING MACHINES IN THAT A LARGE FRACTION OF THE INCOMING RAM PRESSURE IS CONVERTED TO INTERNAL ENERGY OF THE GAS BEHIND THE SHOCK FRONT...

COLLISIONLESS SHOCKS

While shocks in the terrestrial environment are mediated by particle-particle collisions, astrophysical shocks are almost always of a different nature. The pathlength for ionized plasmas is of the order of:

$$\lambda \simeq \frac{1}{n\sigma} = 3.2Mpc\ n_1^{-1} \left(\frac{\sigma}{10^{-25}cm^2}\right)^{-1}$$

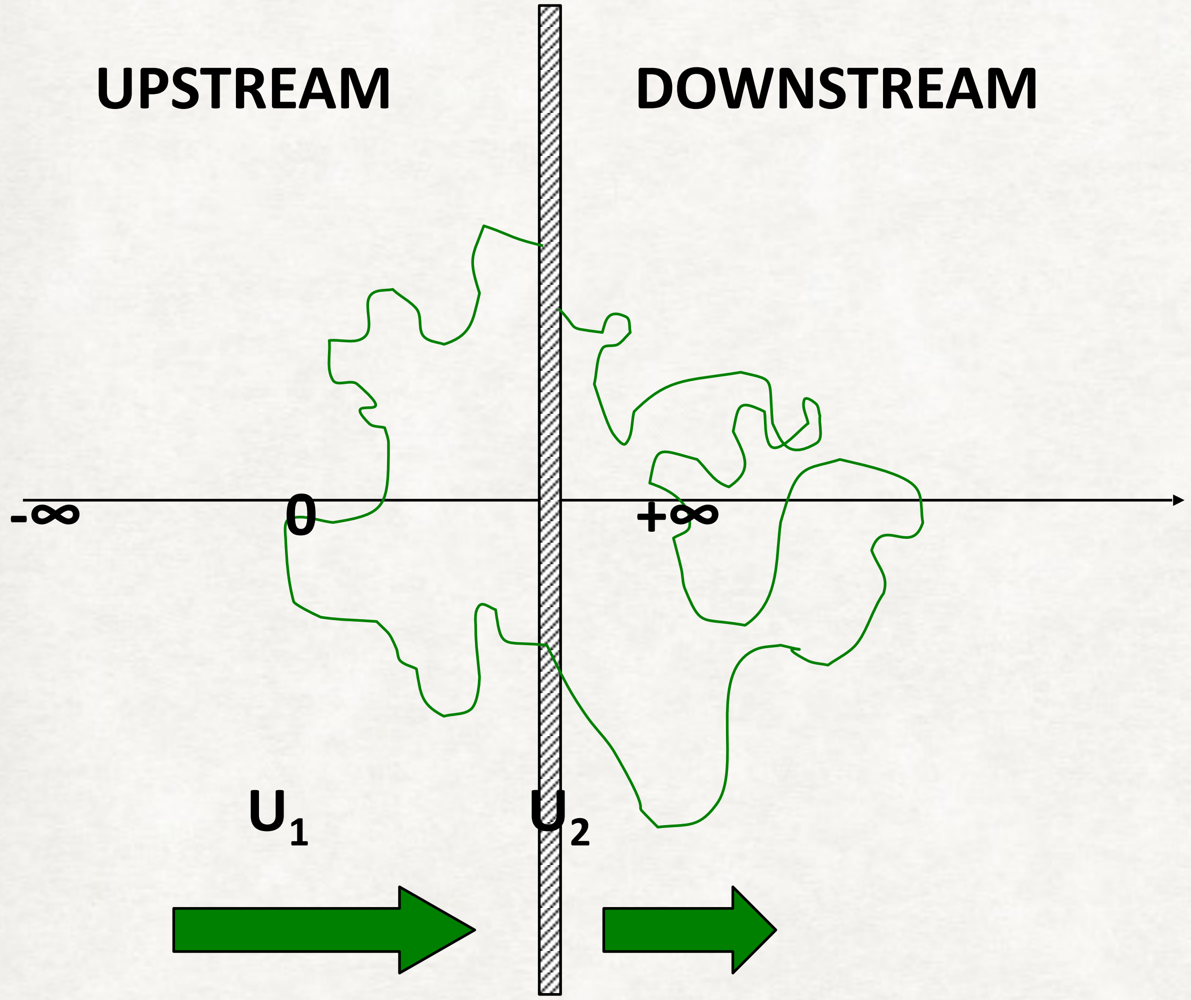
Absurdly large compared with any reasonable length scale. It follows that astrophysical shocks can hardly form because of particle-particle scattering but REQUIRE the mediation of magnetic fields. In the downstream gas the Larmor radius of particles is:

$$r_{L,th} \approx 10^{10} B_\mu T_8^{1/2}\ cm$$

The slowing down of the incoming flow and its isotropization (thermalization) is due to the action of magnetic fields in the shock region (COLLISIONLESS SHOCKS)

DIFFUSIVE SHOCK ACCELERATION
OR
FIRST ORDER FERMI ACCELERATION

BOUNCING BETWEEN APPROACHING MAGNETIC MIRRORS



Let us take a relativistic particle with energy E~p upstream of the shock. In the downstream frame:

$$E_d = \gamma E(1 + \beta\mu) \quad 0 \leq \mu \leq 1$$

where β = u_1-u_2>0. In the downstream frame the direction of motion of the particle is isotropized and reapproaches the shock with the same energy but pitch angle μ'

$$E_u = \gamma E_d - \beta E_d \gamma \mu' = \gamma^2 E(1 + \beta\mu)(1 - \beta\mu')$$

$$-1 \leq \mu' \leq 0$$

In the non-relativistic case the particle distribution is, at zeroth order, isotropic Therefore:

TOTAL FLUX

$$J = \int_0^1 d\Omega \frac{N}{4\pi} v\mu = \frac{Nv}{4} \quad \Rightarrow \quad P(\mu)d\mu = \frac{ANv\mu}{\frac{Nv}{4}} d\mu = 2\mu d\mu$$

The mean value of the energy change is therefore:

$$\langle \frac{E_u - E}{E} \rangle = -\int_0^1 d\mu 2\mu \int_{-1}^0 d\mu' 2\mu' \left[\gamma^2(1+\beta\mu)(1-\beta\mu') - 1\right] \approx \frac{4}{3}\beta = \frac{4}{3}(u_1 - u_2)$$

A FEW IMPORTANT POINTS:

I. There are no configurations that lead to losses

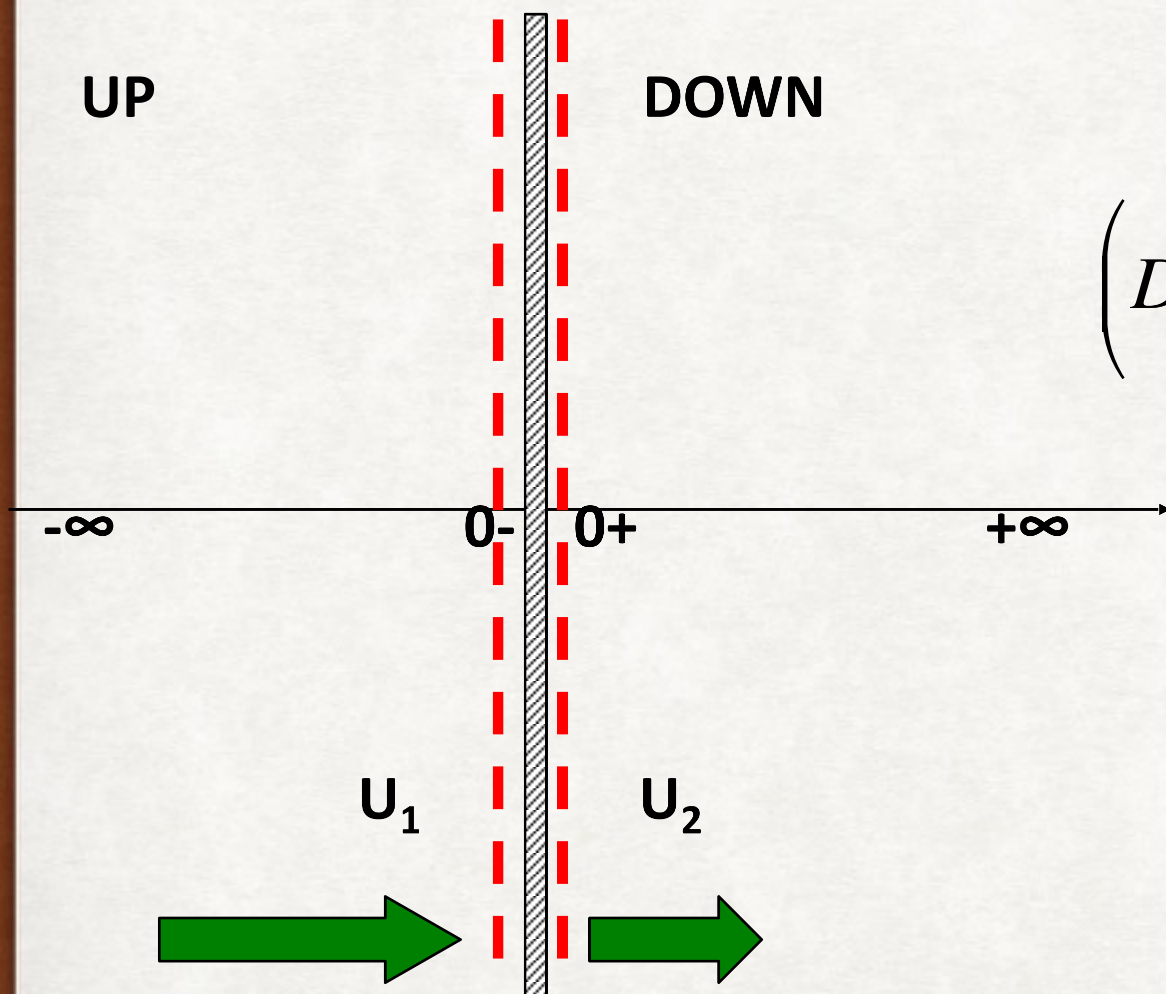
II. The mean energy gain is now first order in β

III. The energy gain is basically independent of any detail on how particles scatter back and forth!

THE TRANSPORT EQUATION APPROACH

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left[D\frac{\partial f}{\partial x}\right] - u\frac{\partial f}{\partial x} + \frac{1}{3}\frac{du}{dx}p\frac{\partial f}{\partial p} + Q(x,p,t)$$

DIFFUSION ADVECTION COMPRESSION INJECTION



Integrating around the shock:

$$\left(D\frac{\partial f}{\partial x}\right)_2 - \left(D\frac{\partial f}{\partial x}\right)_1 + \frac{1}{3}(u_2 - u_1)p\frac{df_0(p)}{dp} + Q_0(p) = 0$$

Integrating from upstr. infinity to 0-:

$$\left(D\frac{\partial f}{\partial x}\right)_1 = u_1 f_0$$

and requiring homogeneity downstream:

$$p\frac{df_0}{dp} = \frac{3}{u_2 - u_1}(u_1 f_0 - Q_0)$$

THE TRANSPORT EQUATION APPROACH

INTEGRATION OF THIS SIMPLE EQUATION GIVES:

$$f_0(p) = \frac{3u_1}{u_1 - u_2} \frac{N_{inj}}{4\pi p_{inj}^2} \left(\frac{p}{p_{inj}}\right)^{\frac{-3u_1}{u_1 - u_2}}$$

DEFINE THE COMPRESSION FACTOR $r = u_1/u_2 \rightarrow 4$ (strong shock)

THE SLOPE OF THE SPECTRUM IS

$$\frac{3u_1}{u_1 - u_2} = \frac{3}{1 - 1/r} \rightarrow 4 \quad \text{if r} \rightarrow 4$$

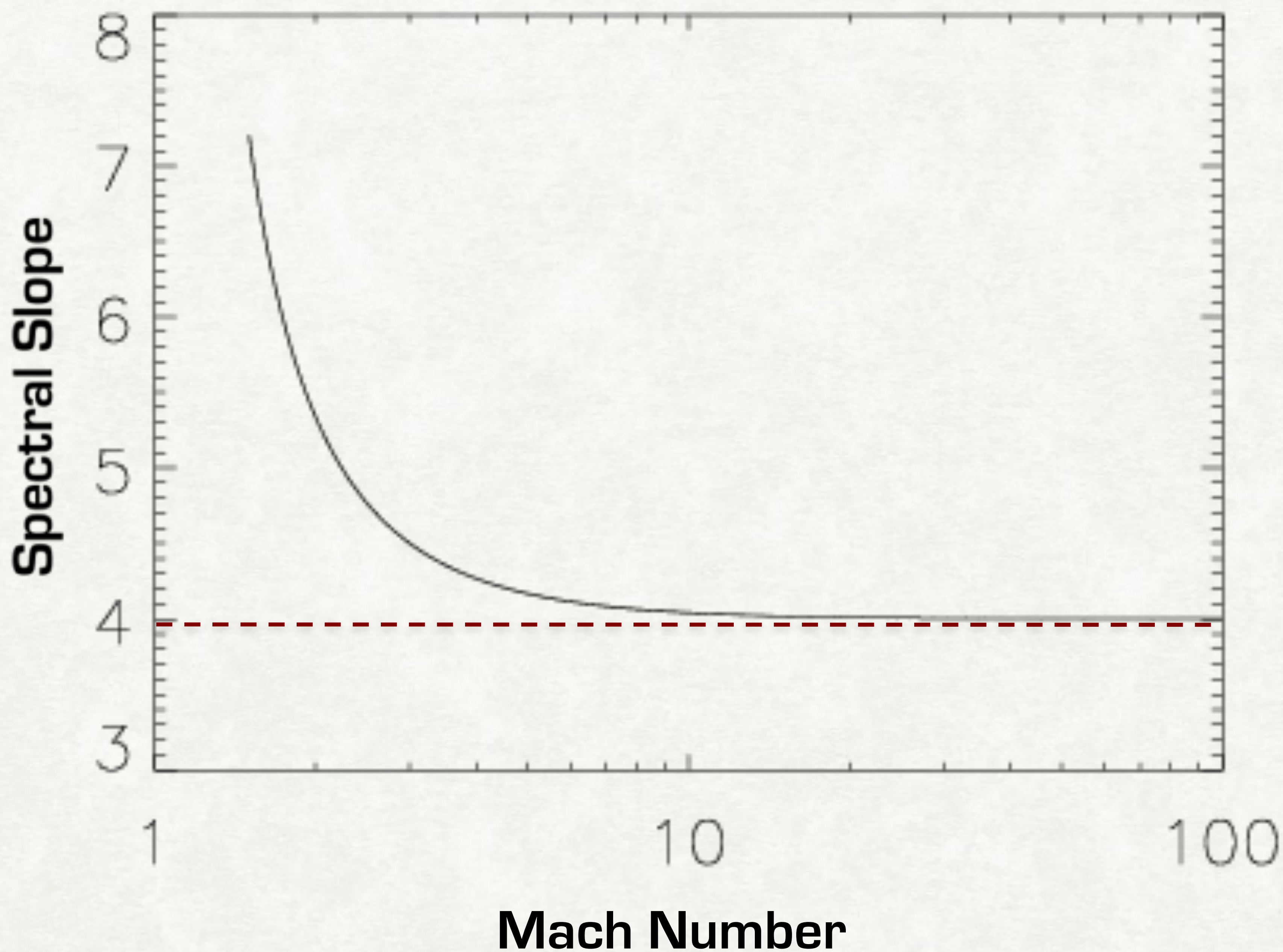
NOTICE THAT: $$N(p)dp = 4\pi p^2 f(p) dp \rightarrow N(p) \propto p^{-2}$$

1. **THE SPECTRUM OF ACCELERATED PARTICLES IS A POWER LAW IN MOMENTUM EXTENDING TO INFINITE MOMENTA**

2. **THE SLOPE DEPENDS UNIQUELY ON THE COMPRESSION FACTOR AND IS INDEPENDENT OF THE DIFFUSION PROPERTIES**

3. **INJECTION IS TREATED AS A FREE PARAMETER WHICH DETERMINES THE NORMALIZATION**

TEST PARTICLE SPECTRUM



SOME IMPORTANT COMMENTS

- THE STATIONARY PROBLEM DOES NOT ALLOW TO HAVE A MAX MOMENTUM!
- THE NORMALIZATION IS ARBITRARY THEREFORE THERE IS NO CONTROL ON THE AMOUNT OF ENERGY IN CR
- AND YET IT HAS BEEN OBTAINED IN THE TEST PARTICLE APPROXIMATION
- THE SOLUTION DOES NOT DEPEND ON WHAT IS THE MECHANISM THAT CAUSES PARTICLES TO BOUNCE BACK AND FORTH
- FOR STRONG SHOCKS THE SPECTRUM IS UNIVERSAL AND CLOSE TO E^{-2}
- IT HAS BEEN IMPLICITELY ASSUMED THAT WHATEVER SCATTERS THE PARTICLES IS AT REST (OR SLOW) IN THE FLUID FRAME

MAXIMUM ENERGY

The maximum energy in an accelerator is determined by either the age of the accelerator compared with the acceleration time or the size of the system compared with the diffusion length D(E)/u. The hardest condition is the one that dominates.

Using the diffusion coefficient in the ISM derived from the B/C ratio:

$$D(E) \approx 3 \times 10^{28} E_{GeV}^{1/3} cm^2/s$$

and the velocity of a SNR shock as u=5000 km/s one sees that:

$$t_{acc} \sim D(E)/u^2 \sim 4 \times 10^3 E_{GeV}^{1/3} \; years$$

Too long for any useful acceleration → NEED FOR ADDITIONAL TURBULENCE

$$t_{acc}(p) = \langle t \rangle = \frac{3}{u_1 - u_2} \int_{p_0}^{p} \frac{dp'}{p'} \left[\frac{D_1(p')}{u_1} + \frac{D_2(p')}{u_2} \right]$$

ENERGY LOSSES AND ELECTRONS

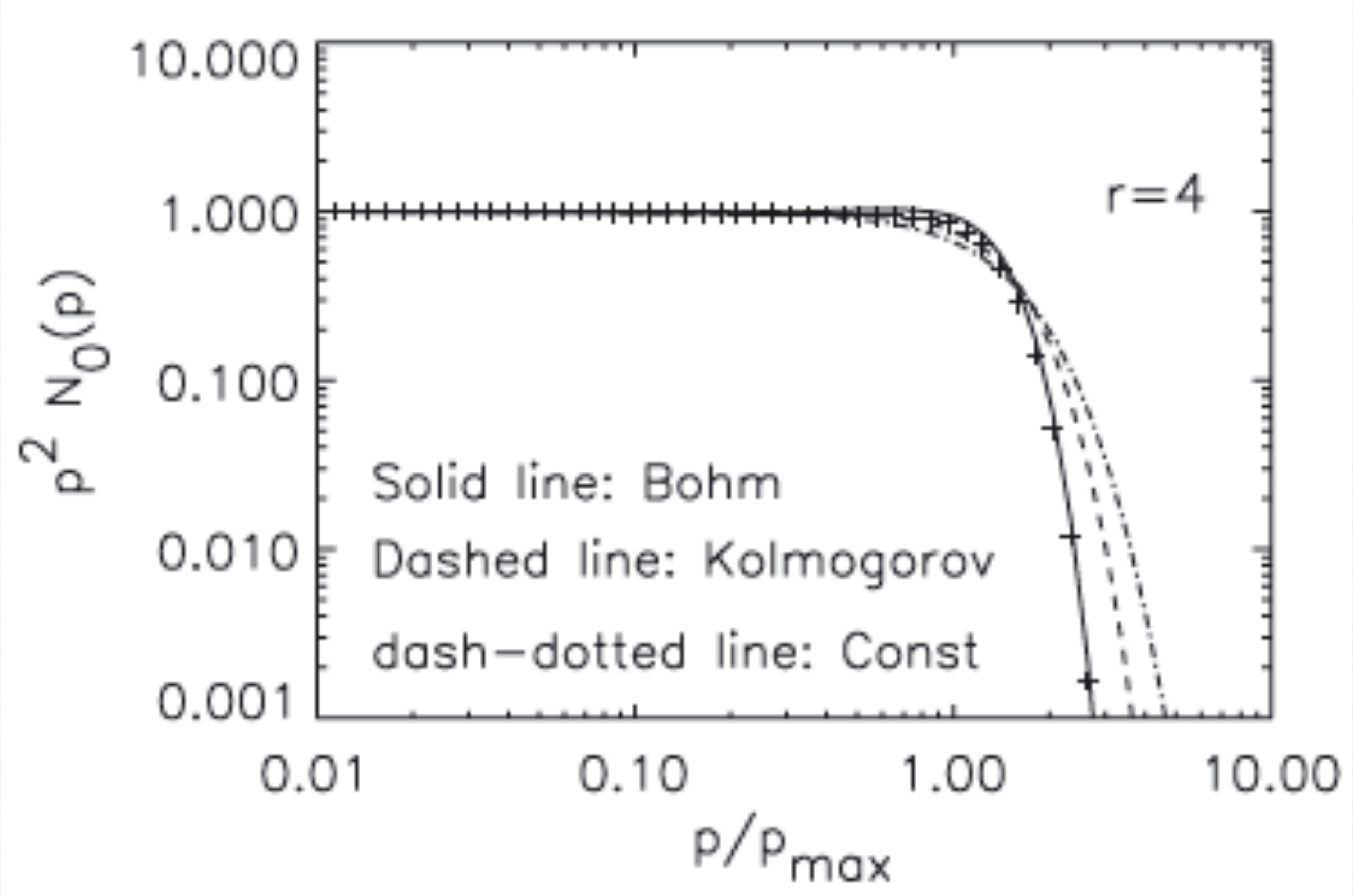
For electrons, energy losses make acceleration even harder.

The maximum energy of electrons is determined by the condition:

$$t_{acc} \leq Min\left[Age, \tau_{loss}\right]$$

Where the losses are mainly due to synchrotron and inverse Compton Scattering.

ELECTRONS IN ONE SLIDE



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NON LINEAR THEORY OF DSA

WHY DO WE NEED A NON LINEAR THEORY?

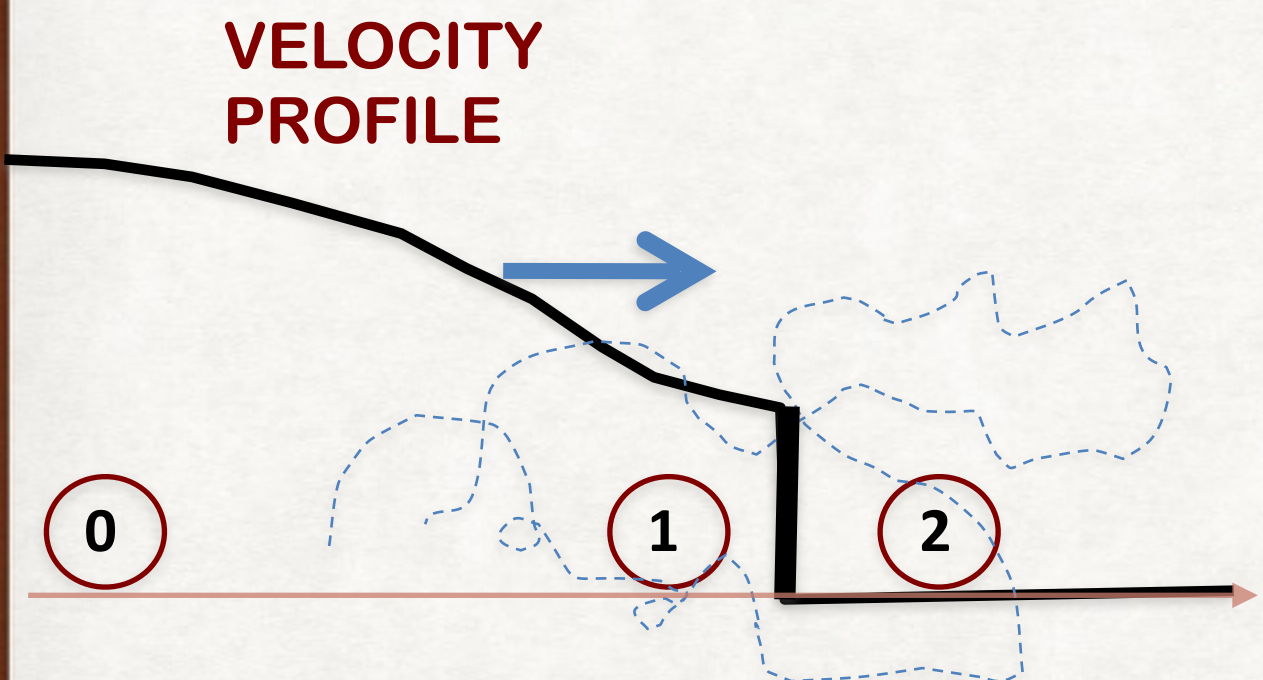
TEST PARTICLE THEORY PREDICTS ENERGY DIVERGENT SPECTRA

THE TYPICAL EFFICIENCY EXPECTED OF A SNR (~10%) IS SUCH THAT TEST PARTICLE THEORY IS A BAD APPROXIMATION

THE MAX MOMENTUM CAN ONLY BE INTRODUCED BY HAND IN TEST PARTICLE THEORY

SIMPLE ESTIMATES SHOW THAT EMAX IS VERY LOW UNLESS CR TAKE PART IN THE ACCELERATION PROCESS, BY AFFECTING THEIR OWN SCATTERING

DYNAMICAL REACTION OF ACCELERATED PARTICLES



Particle transport is described by using the usual transport equation including diffusion and advection

But now dynamics is important too:

$$\rho_0 u_0 = \rho_1 u_1$$

Conservation of Mass

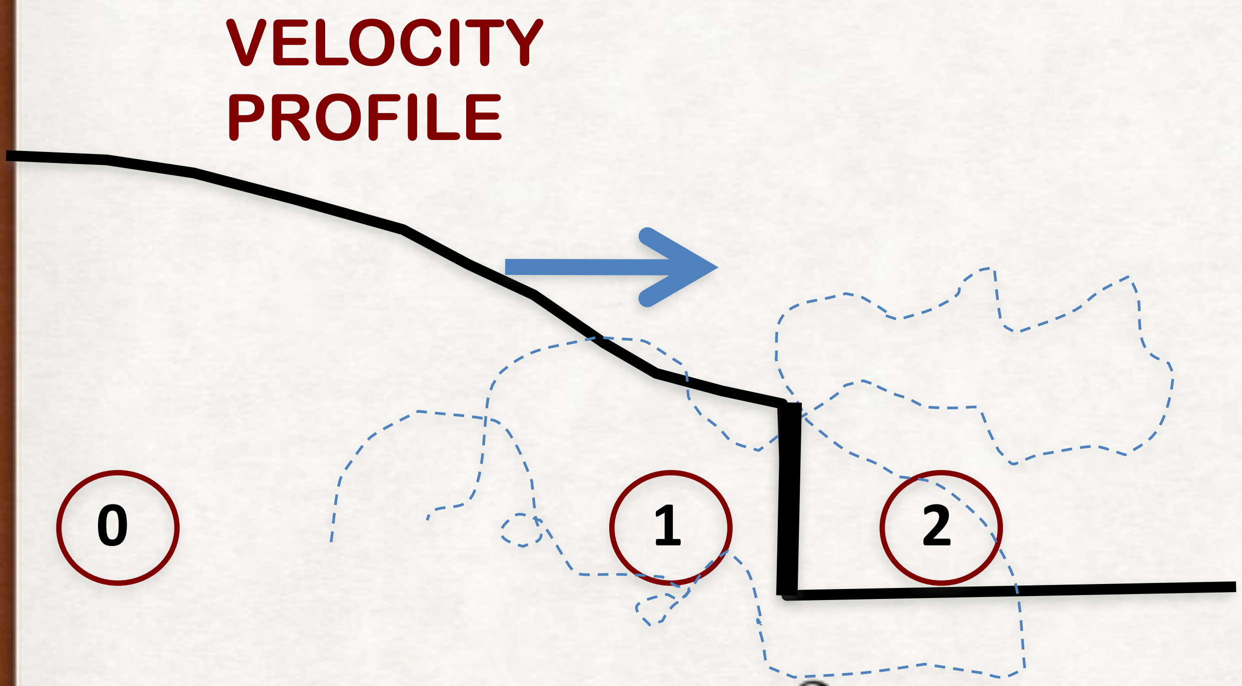
$$\rho_0 u_0^2 + P_{g,0} = \rho_1 u_1^2 + P_{g,1} + P_{c,1}$$

Conservation of Momentum

Conservation of Energy

$$\frac{1}{2}\rho_0 u_0^3 + \frac{P_{g,0} u_0 \gamma_g}{\gamma_g - 1} - F_{esc} = \frac{1}{2}\rho_1 u_1^3 + \frac{P_{g,1} u_1 \gamma_g}{\gamma_g - 1} + \frac{P_{c,1} u_1 \gamma_c}{\gamma_c - 1}$$

FORMATION OF A PRECURSOR - SIMPLIFIED



$$\frac{\partial}{\partial x}[\rho u] = 0 \rightarrow \rho(x)u(x) = \rho_0 u_0$$

$$\frac{\partial}{\partial x}\left[P_g + \rho u^2 + P_{CR}\right] = 0$$

$$P_g(x) + \rho u^2 + P_{CR} = P_{g,0} + \rho_0 u_0^2$$

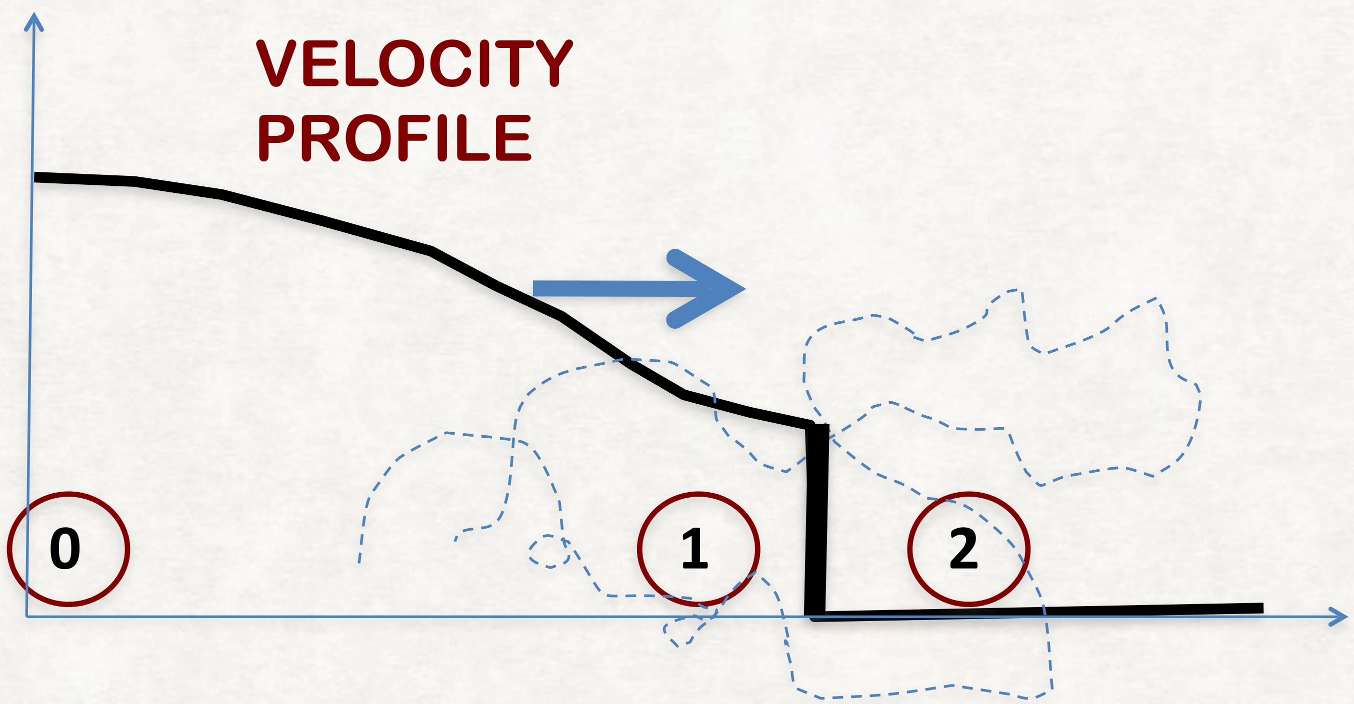
AND DIVIDING BY THE RAM PRESSURE AT UPSTREAM INFINITY:

$$\frac{P_g}{\rho_0 u_0^2} + \frac{u}{u_0} + \frac{P_{CR}}{\rho_0 u_0^2} = \frac{P_{g,0}}{\rho_0 u_0^2} + 1 \Longrightarrow \frac{u}{u_0} \approx 1 - \xi_{CR}(x)$$

WHERE WE NEGLECTED TERMS OF ORDER 1/M²

$$\xi_{CR}(x) = \frac{P_{CR}(x)}{\rho_0 u_0^2}$$

BASIC PREDICTIONS OF NON LINEAR THEORY



COMPRESSION FACTOR BECOMES FUNCTION OF ENERGY

SPECTRA ARE NOT PERFECT POWER LAWS (CONCAVE)

GAS BEHIND THE SHOCK IS COOLER FOR EFFICIENT SHOCK ACCELERATION

SYSTEM SELF REGULATED

EFFICIENT GROWTH OF B-FIELD IF ACCELERATION EFFICIENT

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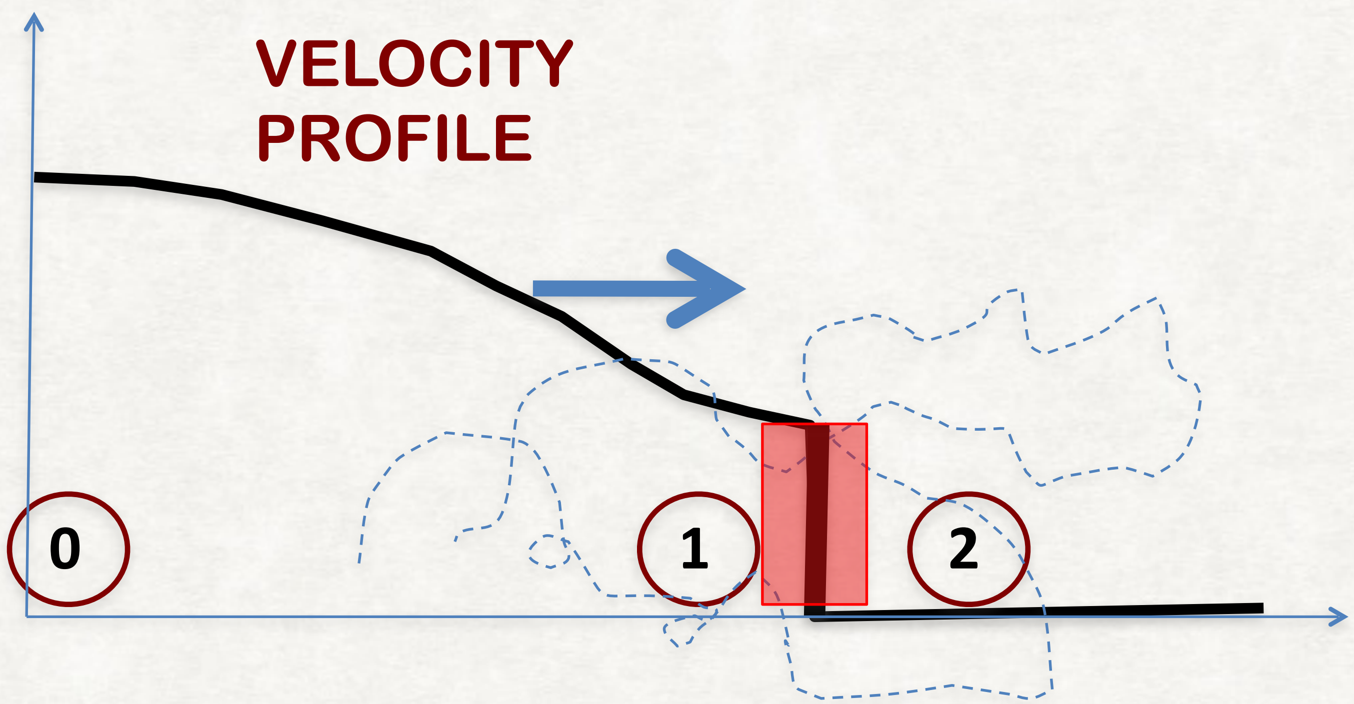
BASIC PREDICTIONS OF NON LINEAR THEORY



PB+2010

COMPRESSION FACTOR BECOMES FUNCTION OF ENERGY

SPECTRA ARE NOT PERFECT POWER LAWS (CONCAVE)

GAS BEHIND THE SHOCK IS COOLER FOR EFFICIENT SHOCK ACCELERATION

SYSTEM SELF REGULATED

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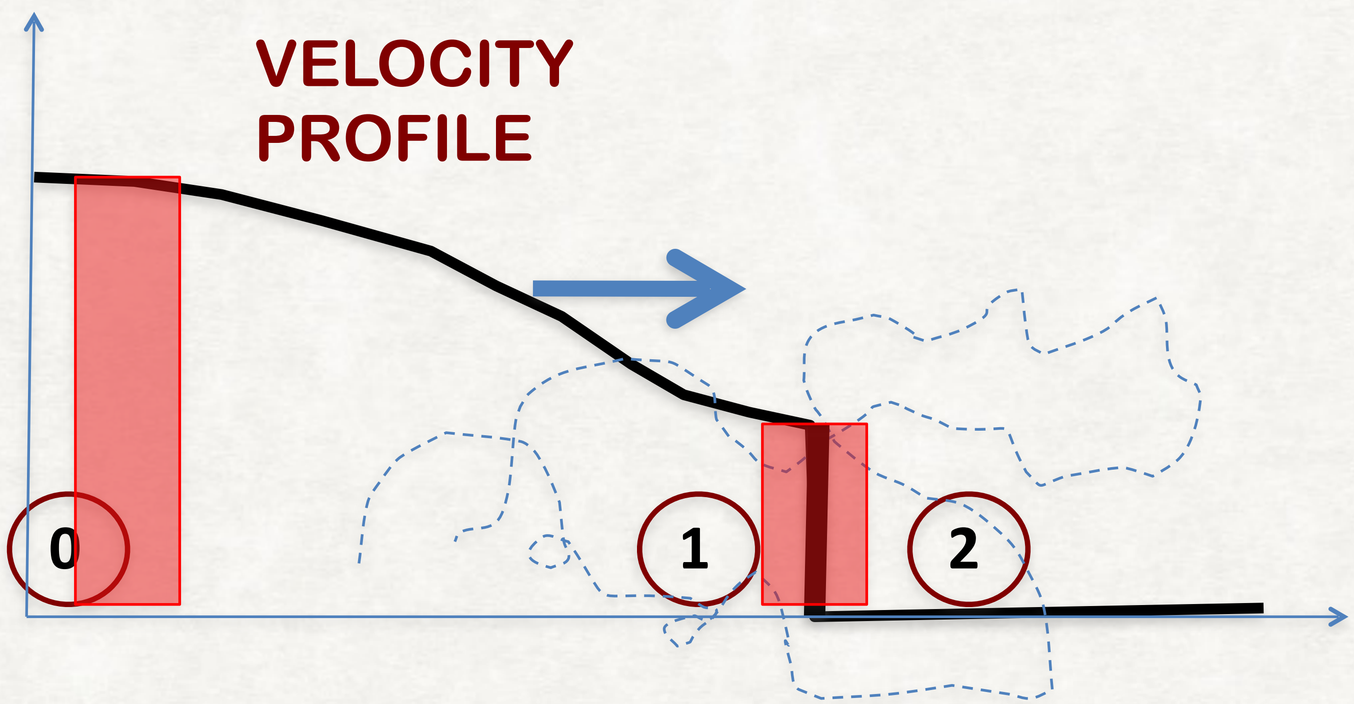

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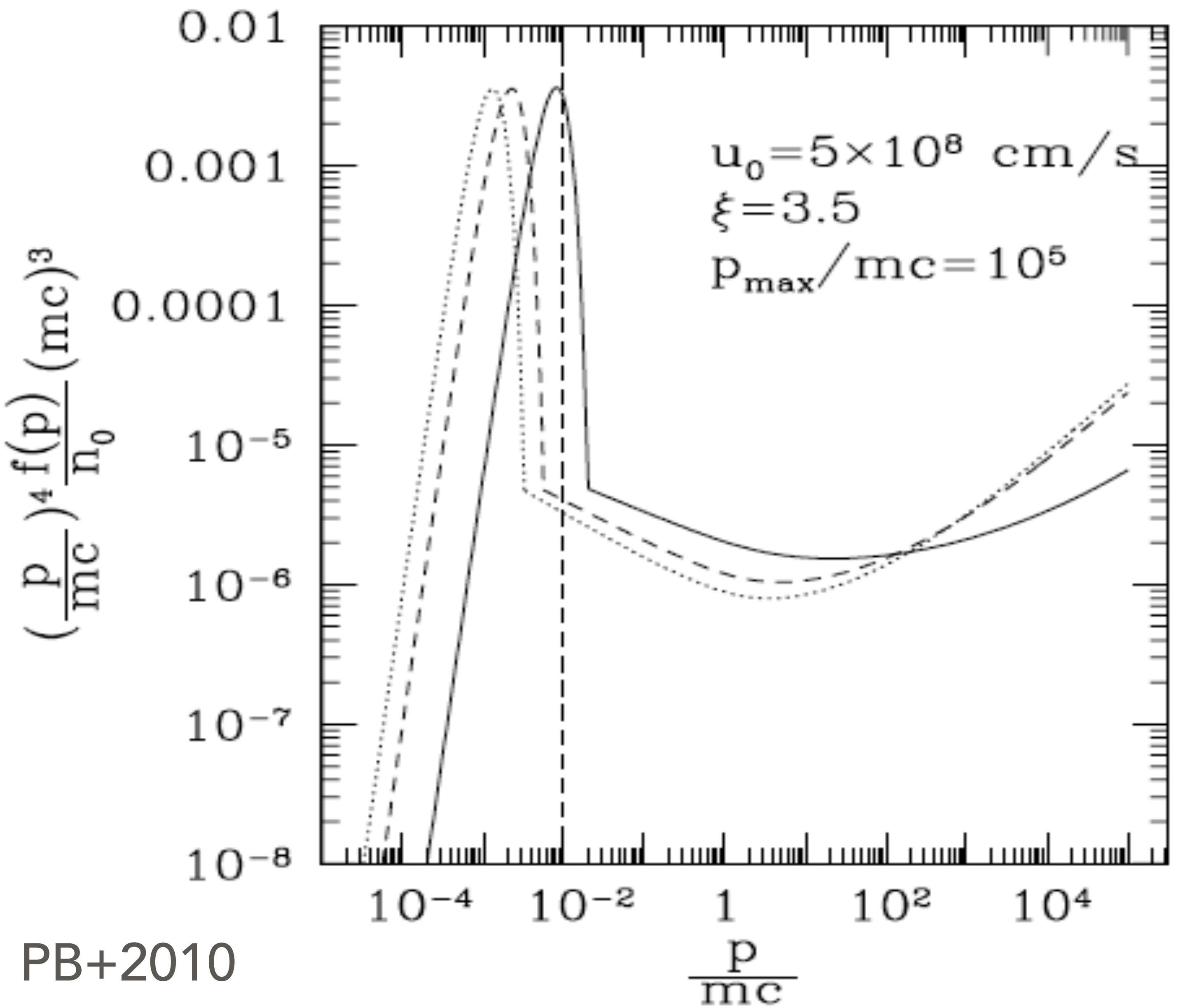

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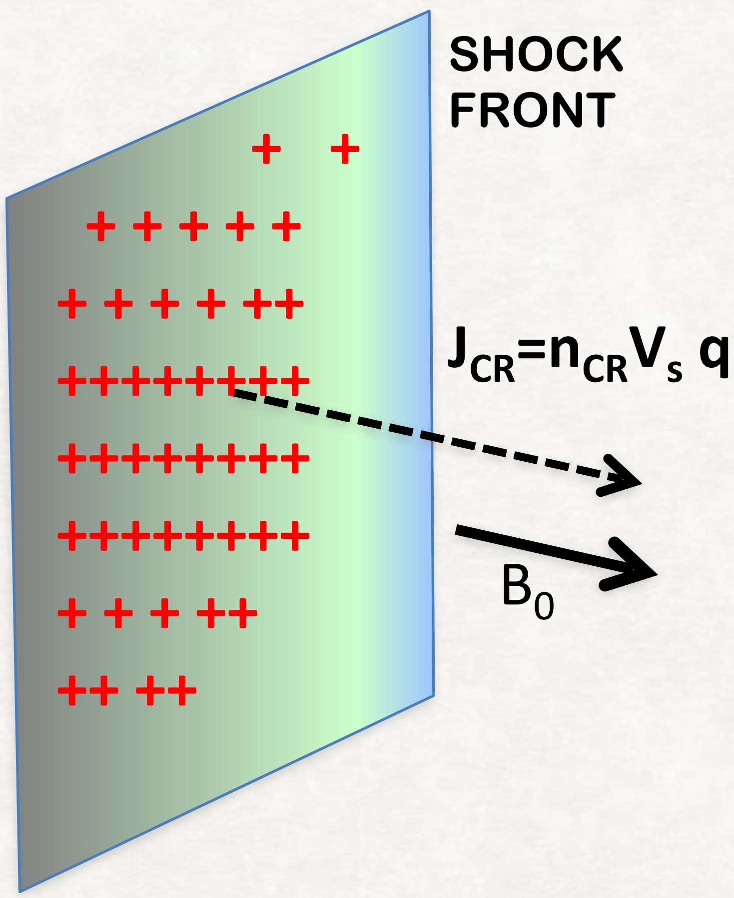
BASICS OF CR STREAMING INSTABILITY



THE UPSTREAM PLASMA REACTS TO THE UPCOMING CR CURRENT BY CREATING A RETURN CURRENT TO COMPENSATE THE POSITIVE CR CHARGE

THE SMALL INDUCED PERTURBATIONS ARE UNSTABLE (ACHTERBERG 1983, ZWEIBEL 1978, BELL 1978, BELL 2004, AMATO & PB 2009)

CR MOVE WITH THE SHOCK SPEED (>> V_A). THIS UNSTABLE SITUATION LEADS THE PLASMA TO REACT IN ORDER TO SLOW DOWN CR TO <V_A BY SCATTERING PARTICLES IN THE PERP DIRECTION (B-FIELD GROWTH)

STREAMING INSTABILITY - THE SIMPLE VIEW

CR streaming with the shock leads to growth of waves. The general idea is simple to explain:

$$n_{CR} m v_D \rightarrow n_{CR} m V_A \Rightarrow \frac{dP_{CR}}{dt} = \frac{n_{CR} m (v_D - V_A)}{\tau} \qquad \frac{dP_w}{dt} = \gamma_W \frac{\delta B^2}{8\pi} \frac{1}{V_A}$$

and assuming equilibrium:

$$\gamma_W = \sqrt{2}\,\frac{n_{CR}}{n_{gas}} \frac{v_D - V_A}{V_A} \Omega_{cyc}$$

And for parameters typical of SNR shocks:

$$\gamma_W \simeq \sqrt{2}\,\xi_{CR} \left(\frac{V_s}{c}\right)^2 \frac{V_s}{V_A} \Omega_{cyc} \sim \mathcal{O}(10^{-4}\ \ seconds^{-1})$$

BRANCHES OF THE CR INDUCED STREAMING INSTABILITY

A CAREFUL ANALYSIS OF THE INSTABILITY REVEALS THAT THERE ARE TWO BRANCHES

RESONANT

MAX GROWTH AT
K=1/LARMOR

NON RESONANT

MAX GROWTH AT
K>>1/LARMOR

THE MAX GROWTH CAN ALWAYS BE WRITTEN IN THE FORM

$$\gamma_{max} = k_{max} v_A$$

WHERE THE WAVENUMBER IS DETERMINED BY THE TENSION CONDITION:

$$k_{max} B_0 \approx \frac{4\pi}{c} J_{CR} \rightarrow k_{max} \approx \frac{4\pi}{c B_0} J_{CR}$$

THE SEPARATION BETWEEN THE TWO REGIMES IS AT $k_{MAX}\, r_L=1$

IF WE WRITE THE CR CURRENT AS $J_{CR} = n_{CR}(> E) e v_D$

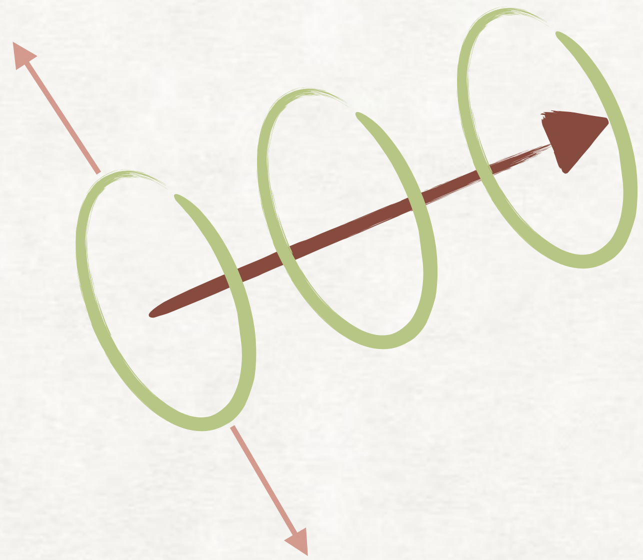
WHERE E IS THE ENERGY OF THE PARTICLES DOMINATING THE CR CURRENT, WE CAN WRITE THE CONDITION ABOVE AS

$$\boxed{\frac{U_{CR}}{U_B} = \frac{c}{v_D}} \qquad U_{CR} = n_{CR}(> E)E \qquad U_B = \frac{B^2}{4\pi}$$

IN CASE OF SHOCKS **VD=SHOCK VELOCITY** AND THE CONDITION SAYS THAT THE NON-RESONANT MODES DOMINATED WHEN THE SHOCK IS VERY FAST AND ACCELERATION IS EFFICIENT —- *FOR TYPICAL CASES THIS IS ALWAYS THE CASE*

BUT RECALL! THE WAVES THAT GROW HAVE K MUCH LARGER THAN THE LARMOR RADIUS OF THE PARTICLES IN THE CURRENT —> NO SCATTERING BECAUSE EFFICIENT SCATTERING REQUIRES RESONANCE!!!

THE EASY WAY TO SATURATION OF GROWTH



The current exerts a force of the background plasma

$$\rho \frac{dv}{dt} \sim \frac{1}{c} J_{CR} \delta B$$

which translates into a plasma displacement:

$$\Delta x \sim \frac{J_{CR}}{c\rho} \frac{\delta B(0)}{\gamma_{max}^2} exp(\gamma_{max} t)$$

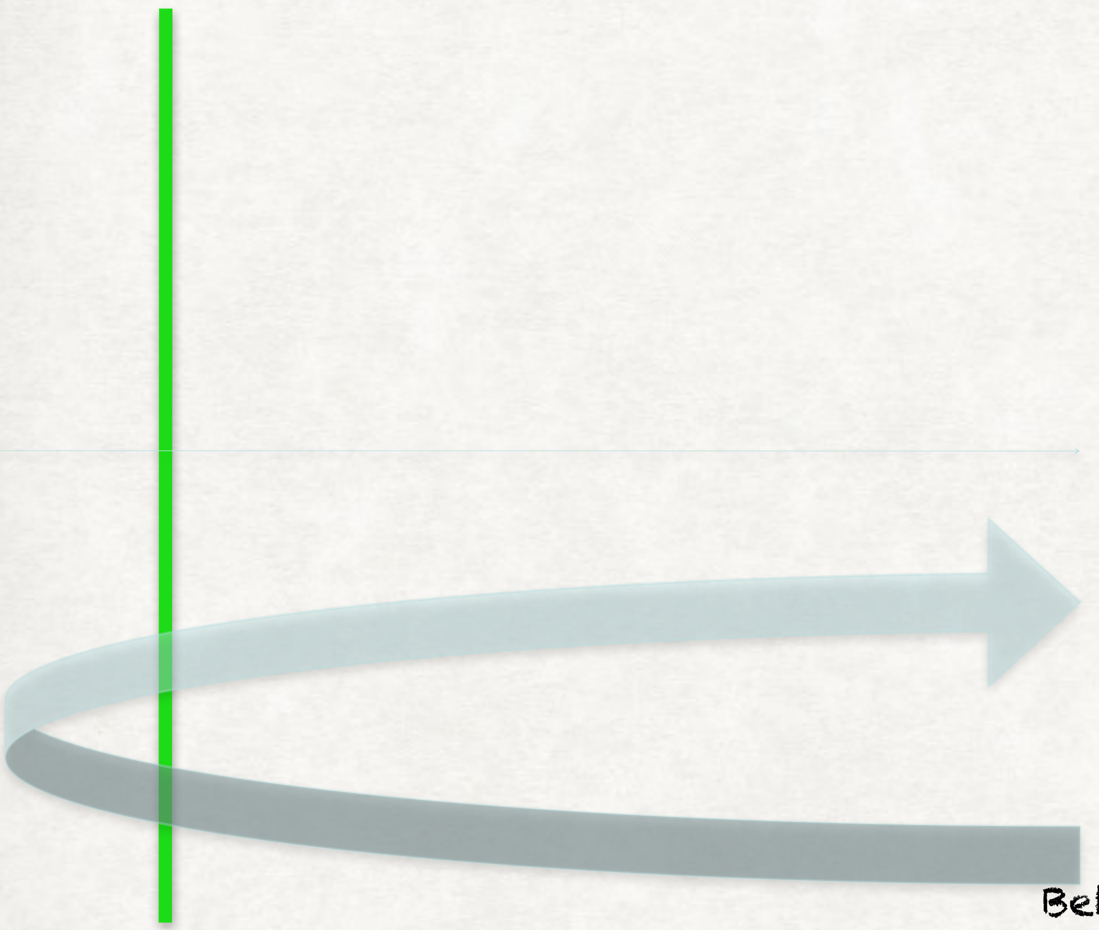
which stretches the magnetic field line by the same amount…

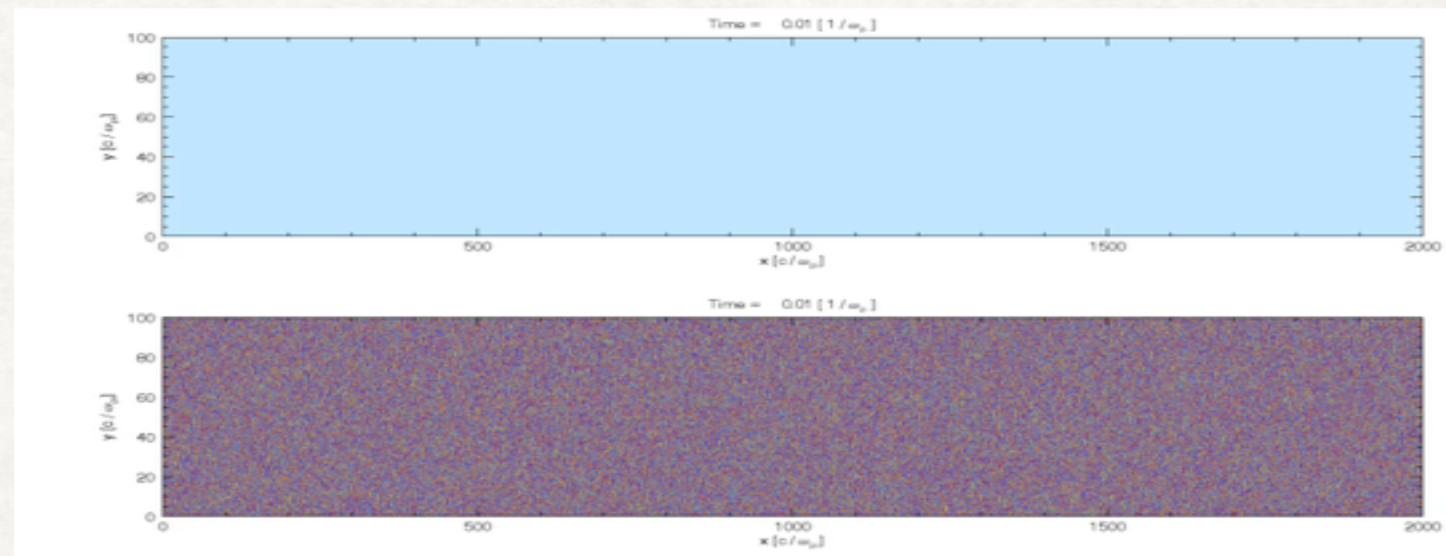
The saturation takes place when the displacement equals the Larmor radius of the particles in the field **δ**B … imposing this condition leads to:

$$\frac{\delta B^2}{4\pi} = \frac{\xi_{CR}}{\Lambda} \rho v_s^2 \frac{v_s}{c} \qquad \Lambda = \ln(E_{max}/E_{min})$$

specialized to a shock and a spectrum E^{-2}

A QUALITATIVE PICTURE OF ACCELERATION





Caprioli & Spitkovsky 2013

Bell & Schure 2013
Cardillo, Amato & PB 2015

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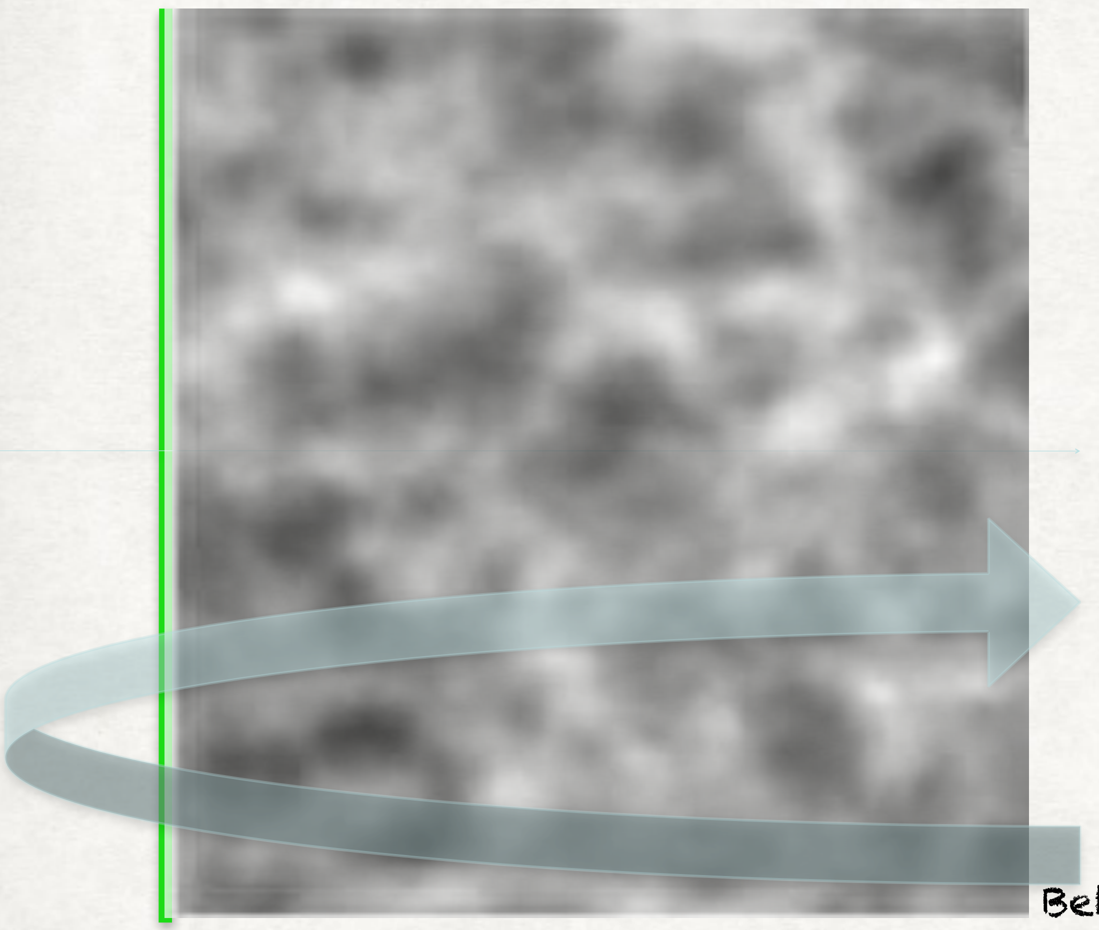

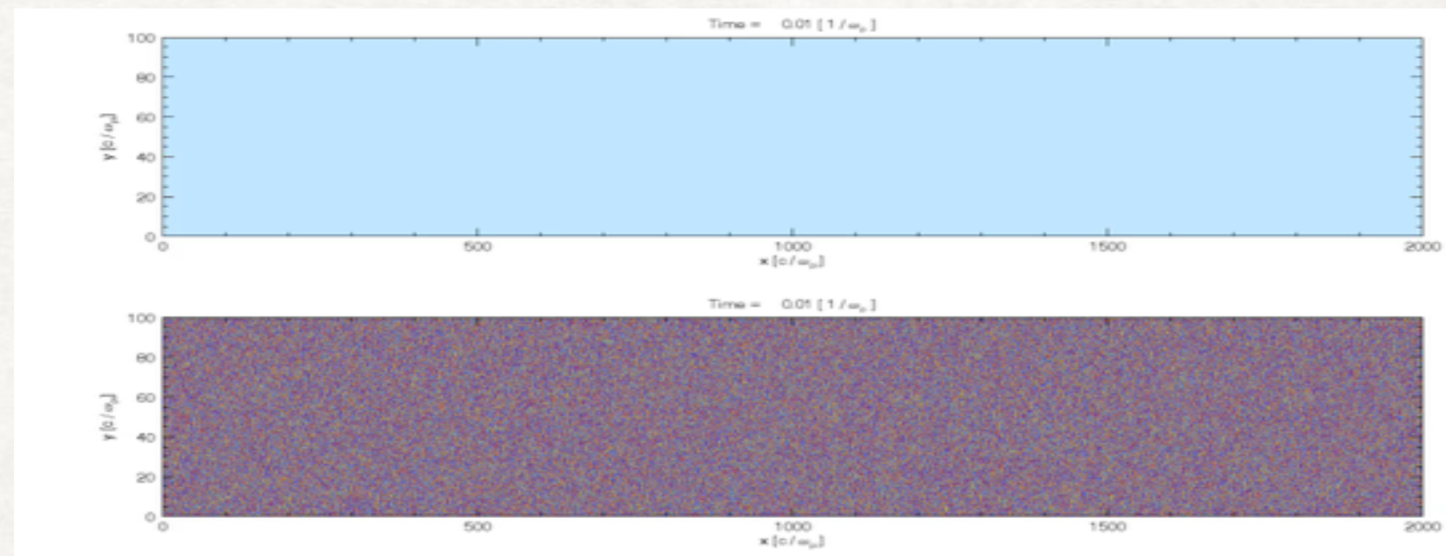

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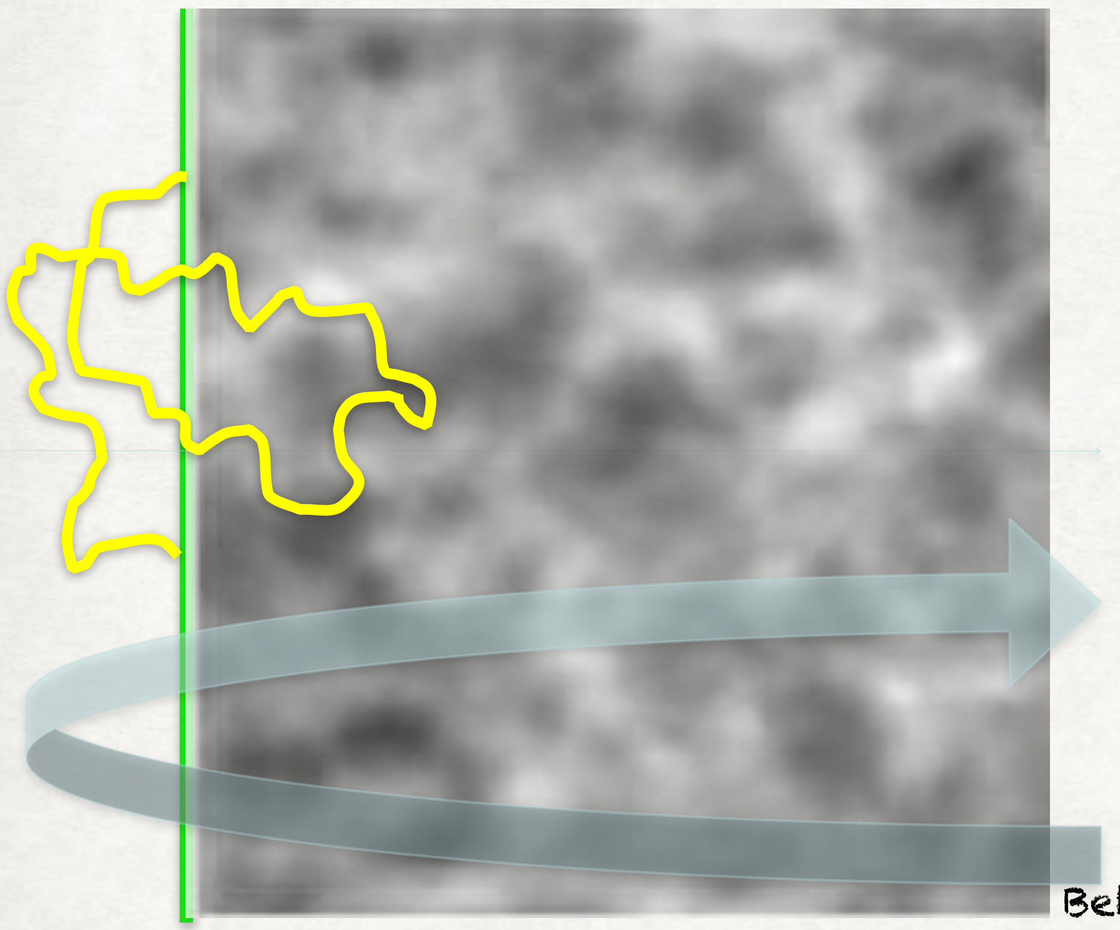

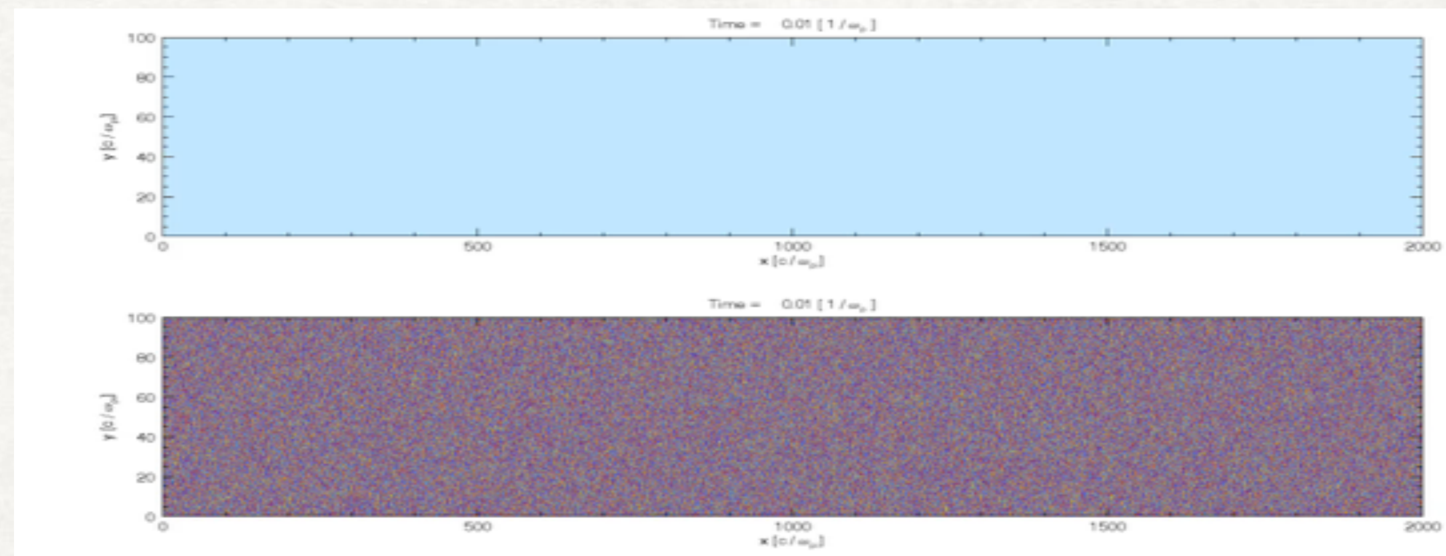

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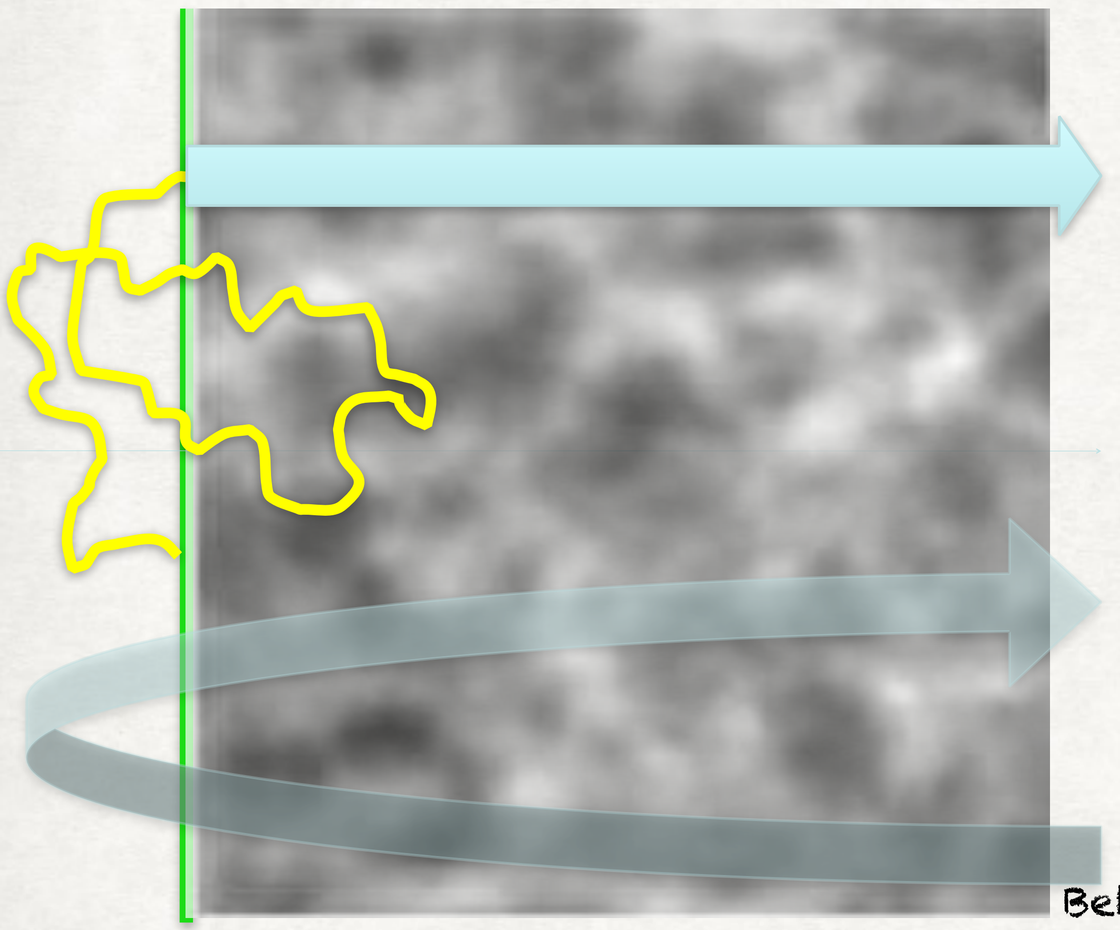

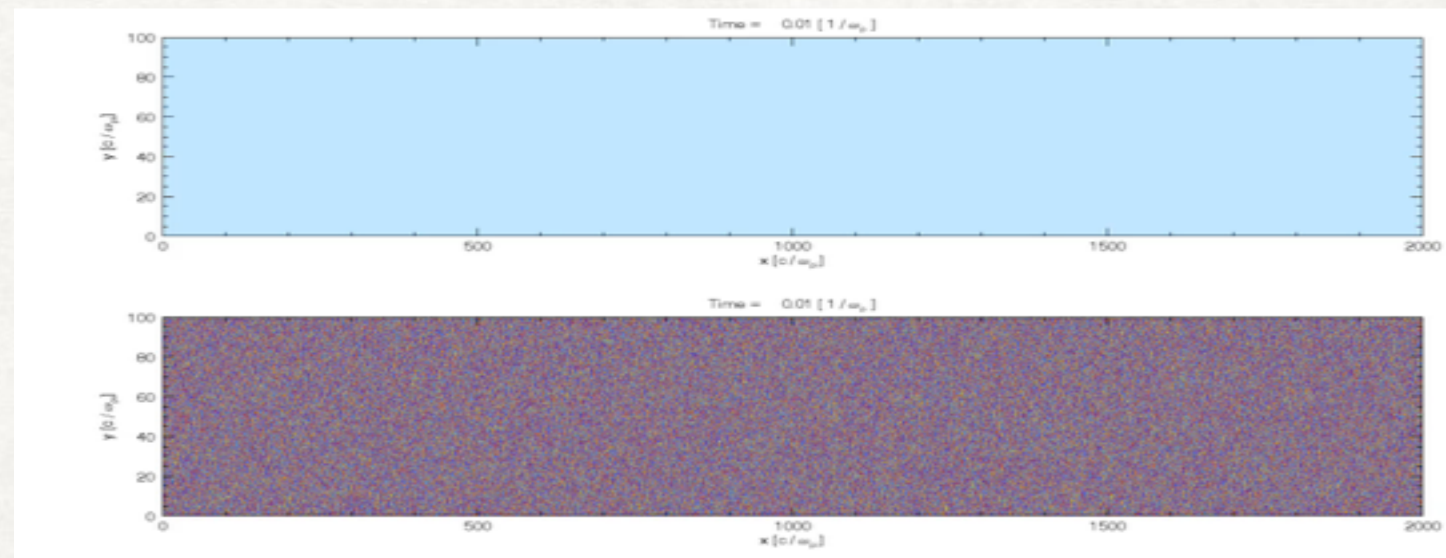

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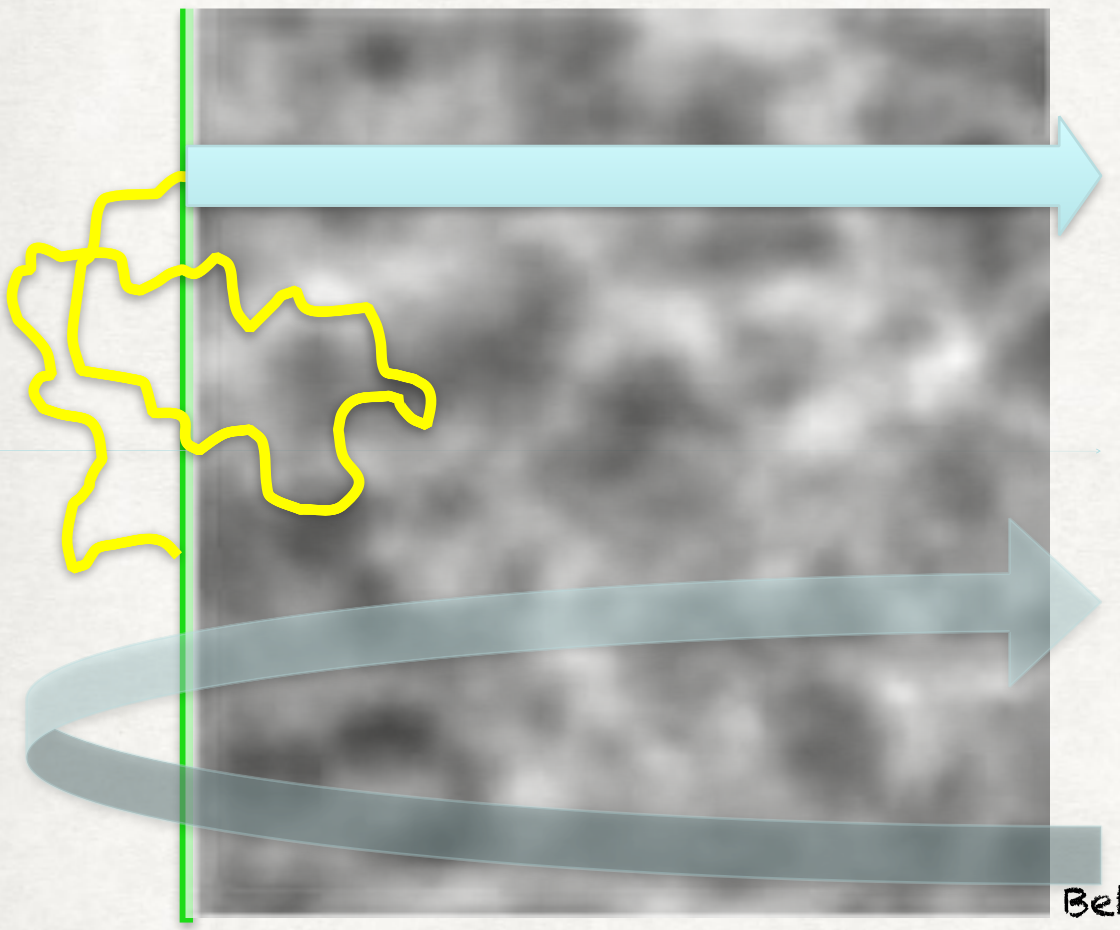

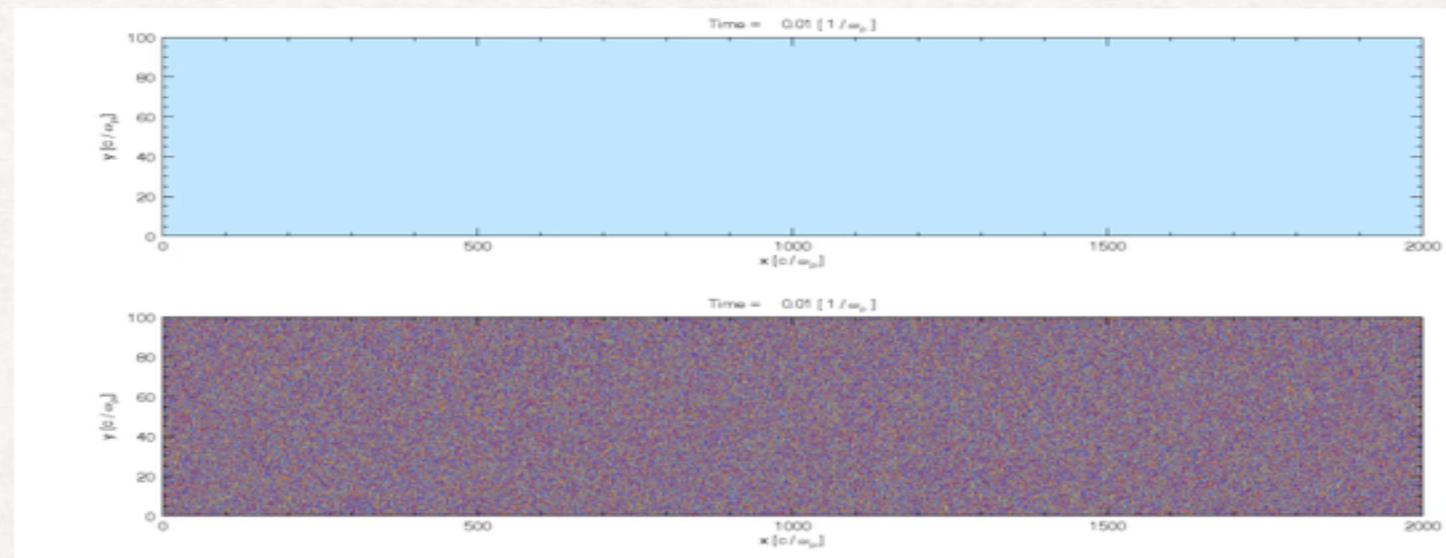

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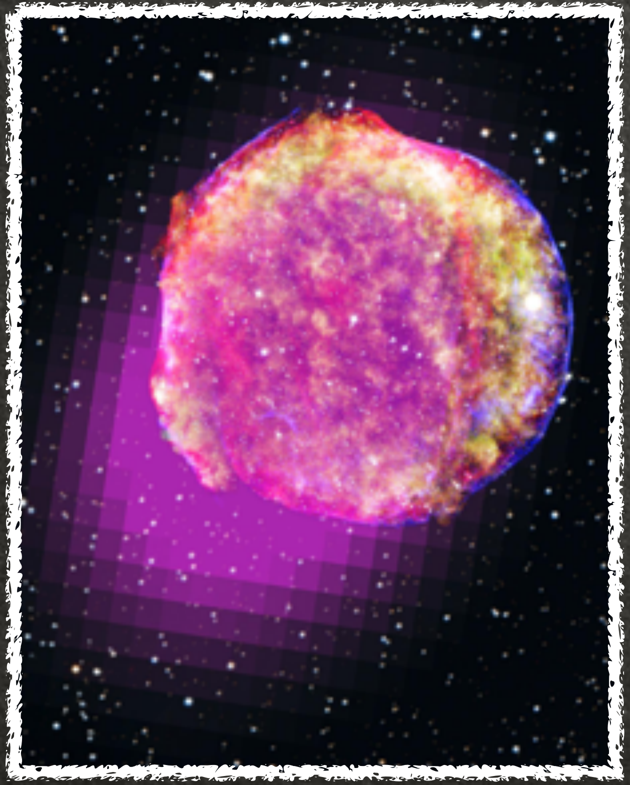
IMPLICATIONS FOR MAXIMUM ENERGY

Supernovae of type Ia

Explosion takes place in the ISM with spatially constant density

$$E_{max} \approx 130\ TeV \left(\frac{\xi_{CR}}{0.1}\right)\left(\frac{M_{ej}}{M_\odot}\right)^{-2/3}\left(\frac{E_{SN}}{10^{51} erg}\right)\left(\frac{n_{ISM}}{cm^{-3}}\right)^{1/6}$$



Supernovae of type II

IMPLICATIONS FOR MAXIMUM ENERGY

Supernovae of type Ia

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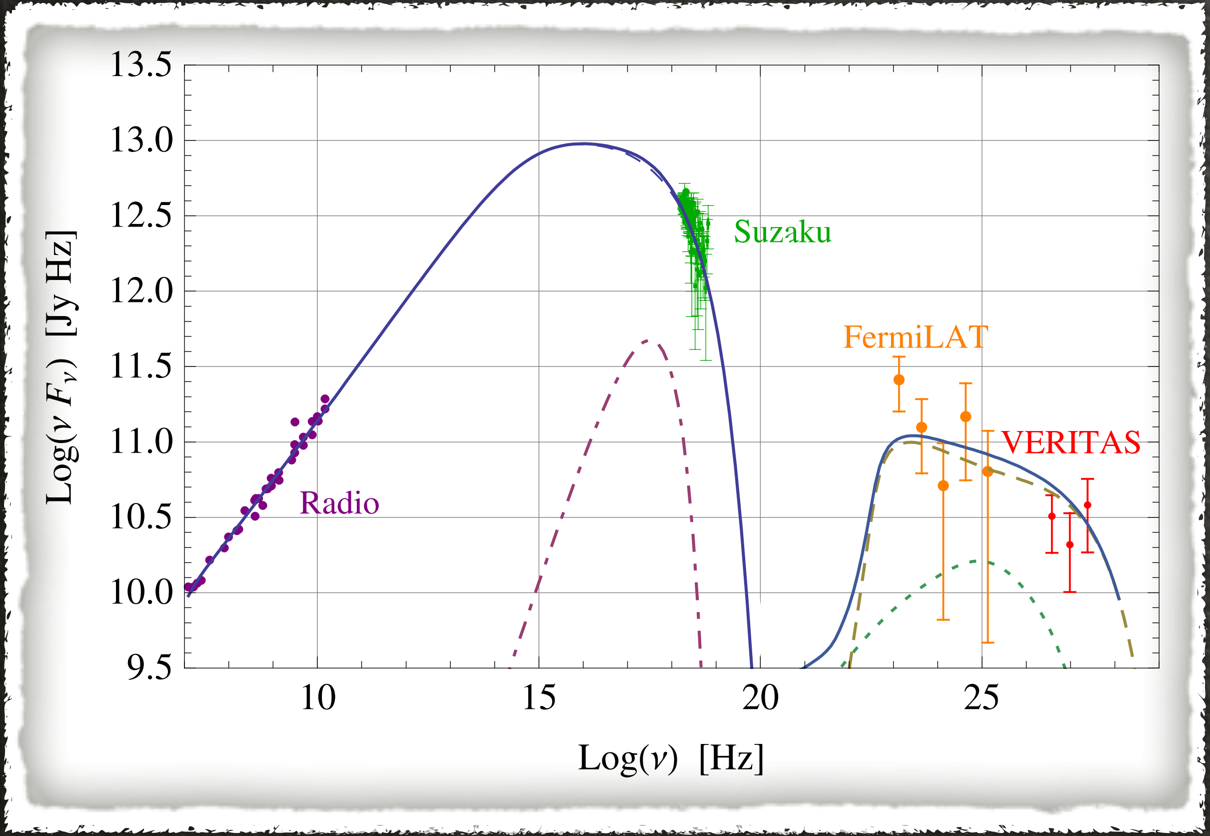

Supernovae of type II

IMPLICATIONS FOR MAXIMUM ENERGY

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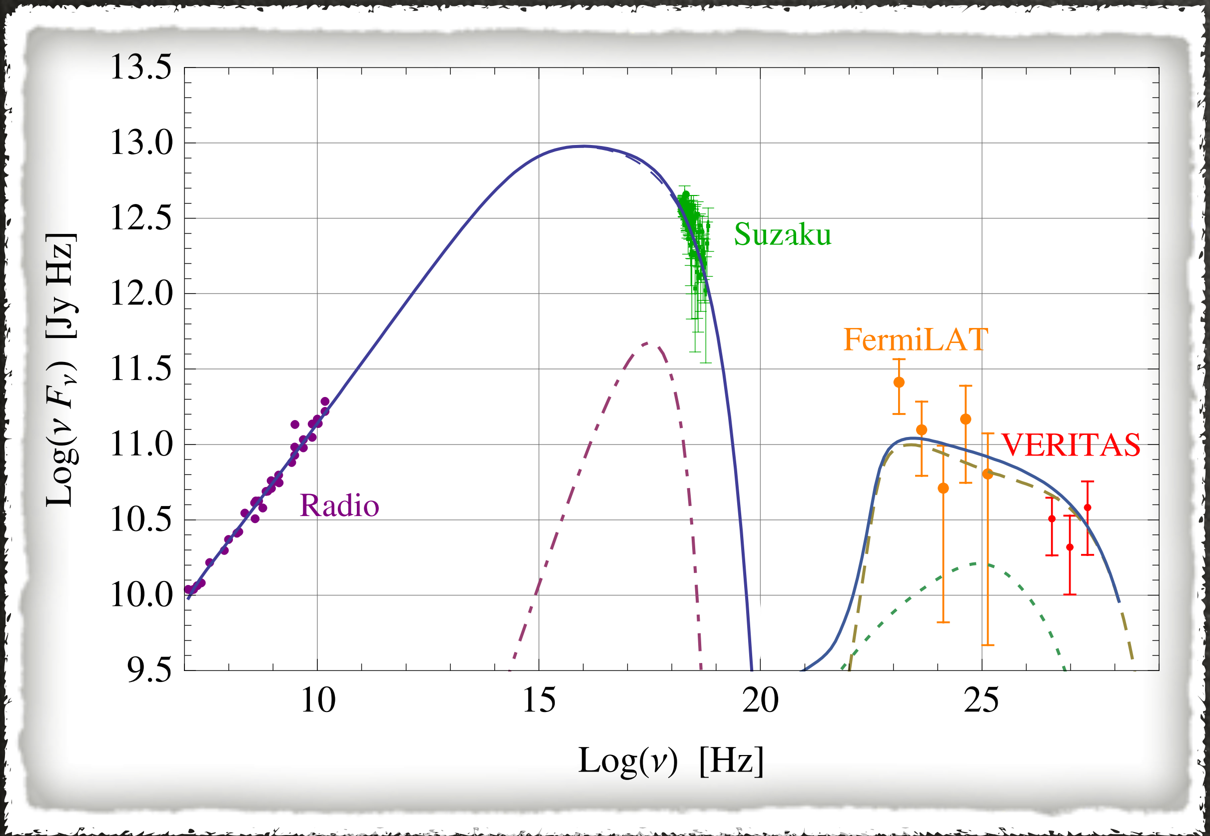


Supernovae of type II

In most cases the explosion takes place in the dense wind of the red super-giant progenitor

$$\rho(r) = \frac{\dot{M}}{4\pi r^2 v_W}$$

The Sedov phase reached while the shock expands inside the wind

$$R = M_{ej} v_W / \dot{M}$$

This corresponds to typical times of few tens of years after the SN explosion !!!

$$E_{max} \approx 1\ PeV \left(\frac{\xi_{CR}}{0.1}\right)\left(\frac{M_{ej}}{M_\odot}\right)^{-1}\left(\frac{E_{SN}}{10^{51}erg}\right)\times \left(\frac{\dot{M}}{10^{-5}M_\odot yr^{-1}}\right)^{1/2}\left(\frac{v_{wind}}{10km/s}\right)^{-1/2}$$

X-ray rims and B-field amplification

TYPICAL THICKNESS OF FILAMENTS: ~ 10^{-2} pc

The synchrotron limited thickness is:

$$\Delta x \approx \sqrt{D(E_{max})\tau_{loss}(E_{max})} \approx 0.04\ B_{100}^{-3/2}\ \mathrm{pc}$$

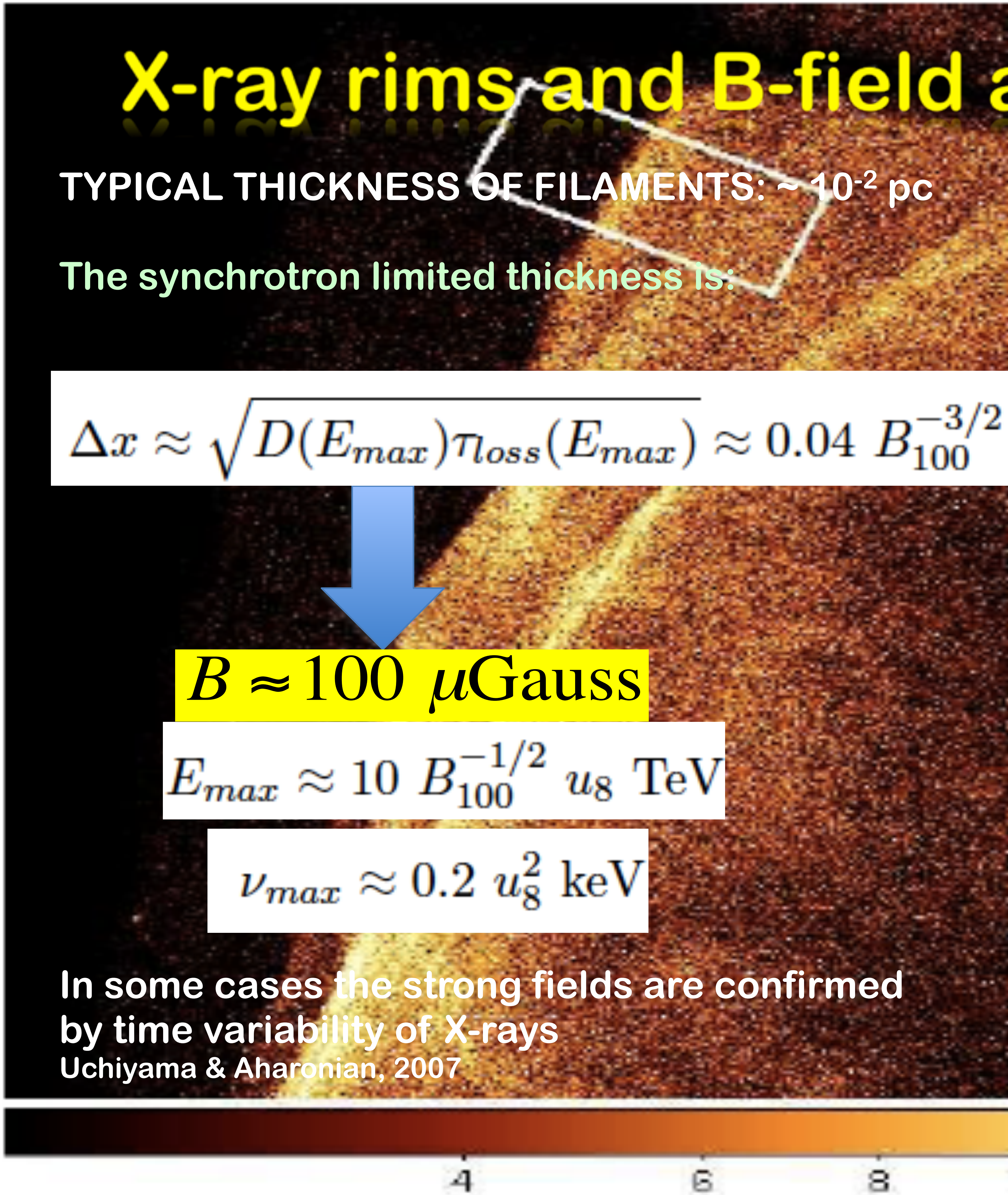
$$B \approx 100\ \mu\mathrm{Gauss}$$

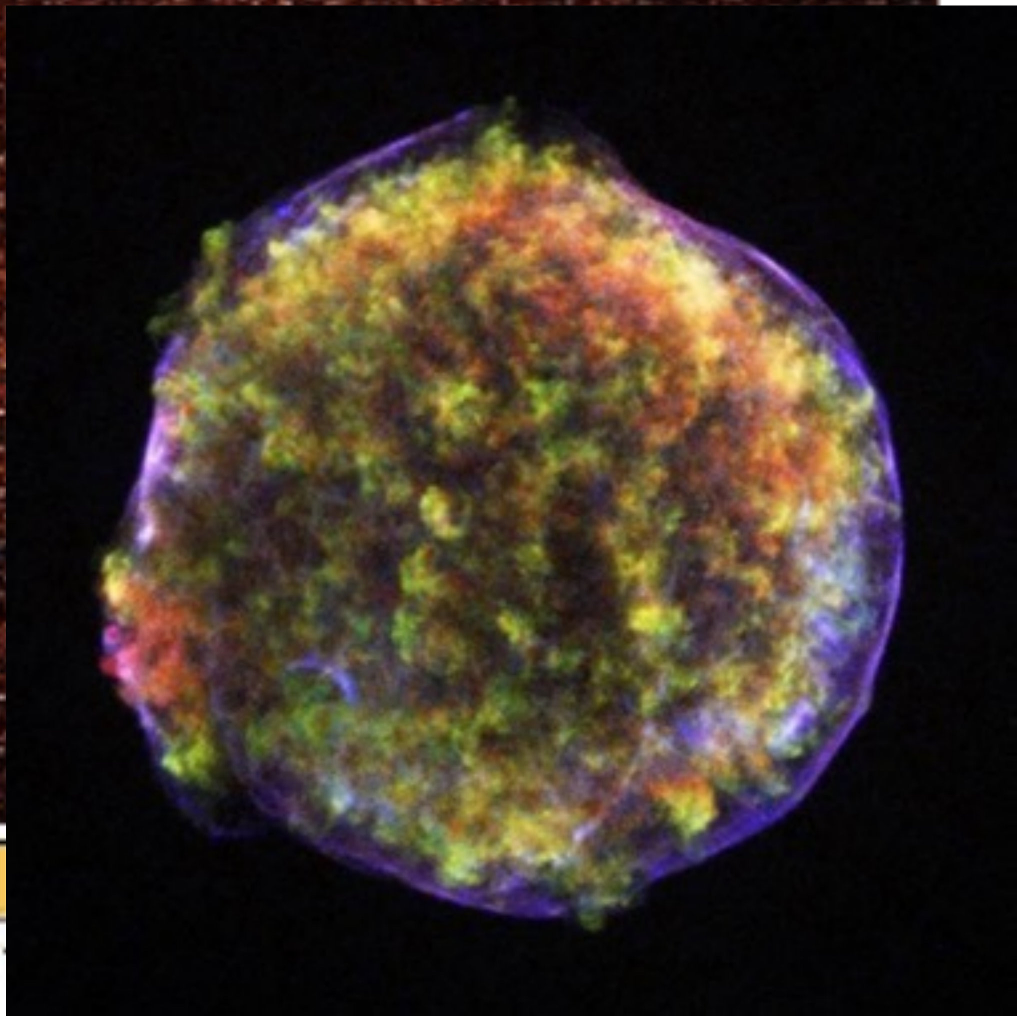
$$E_{max} \approx 10\ B_{100}^{-1/2}\ u_8\ \mathrm{TeV}$$

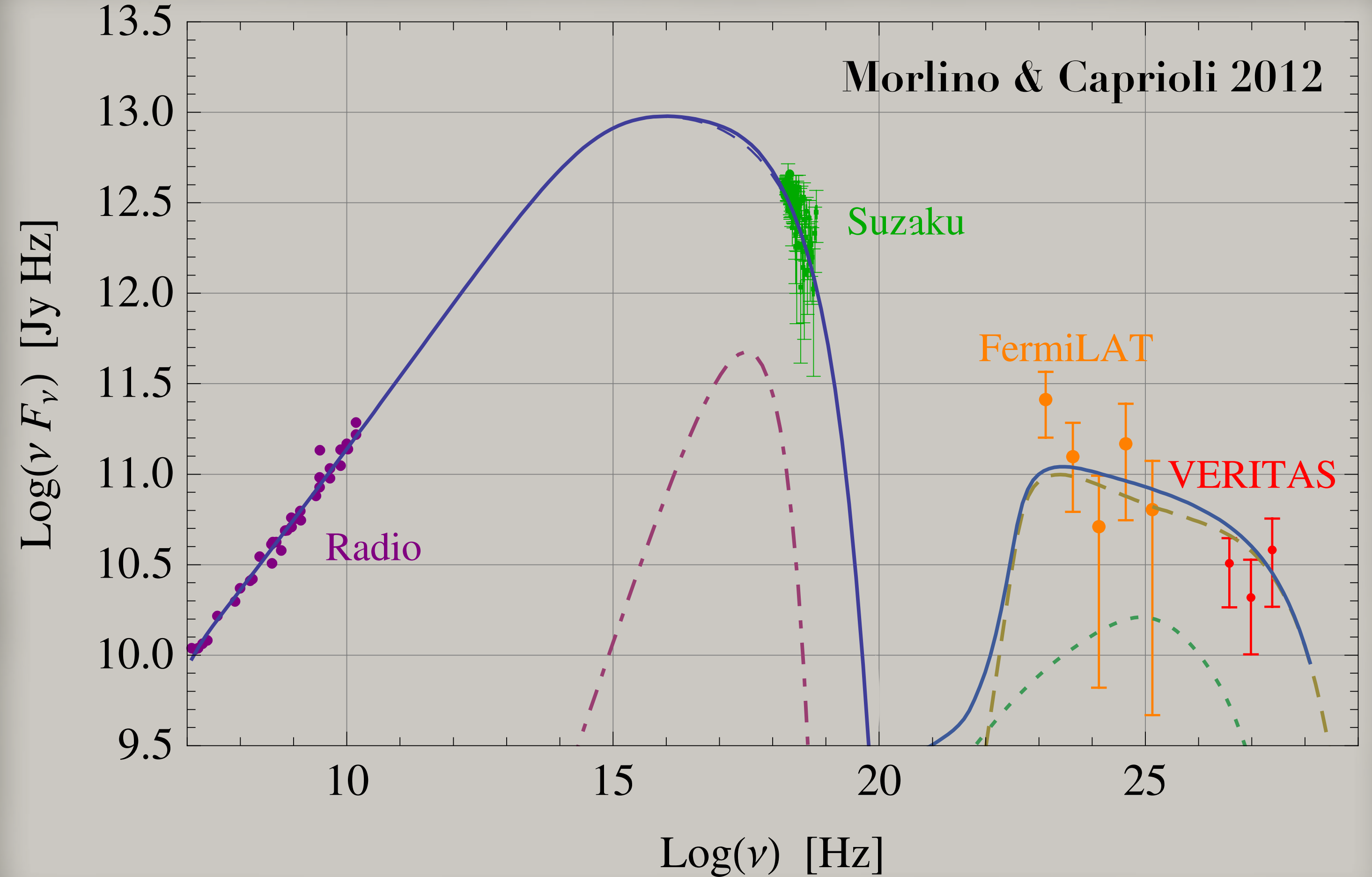
$$\nu_{max} \approx 0.2\ u_8^2\ \mathrm{keV}$$

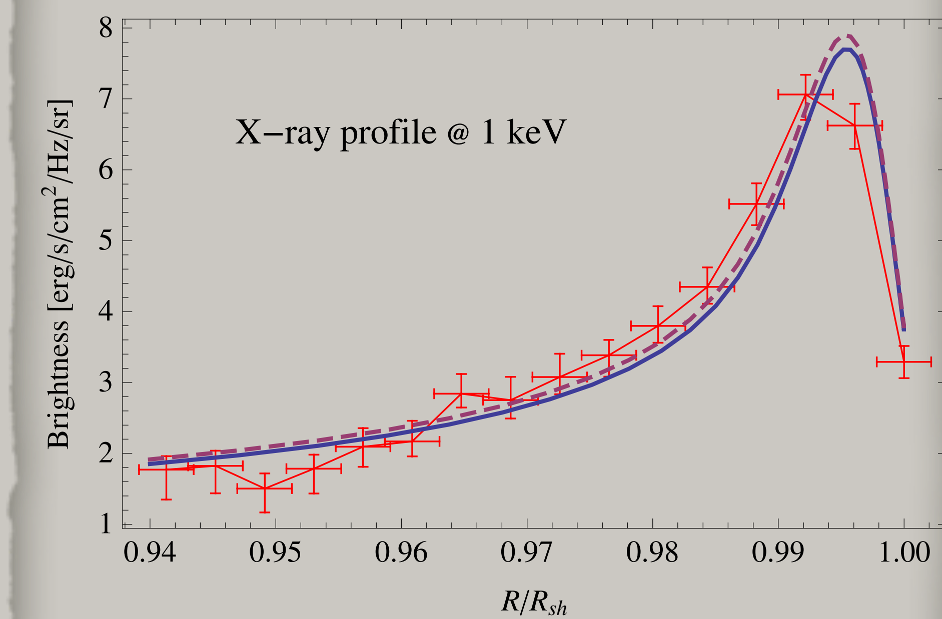
In some cases the strong fields are confirmed by time variability of X-rays

Uchiyama & Aharonian, 2007



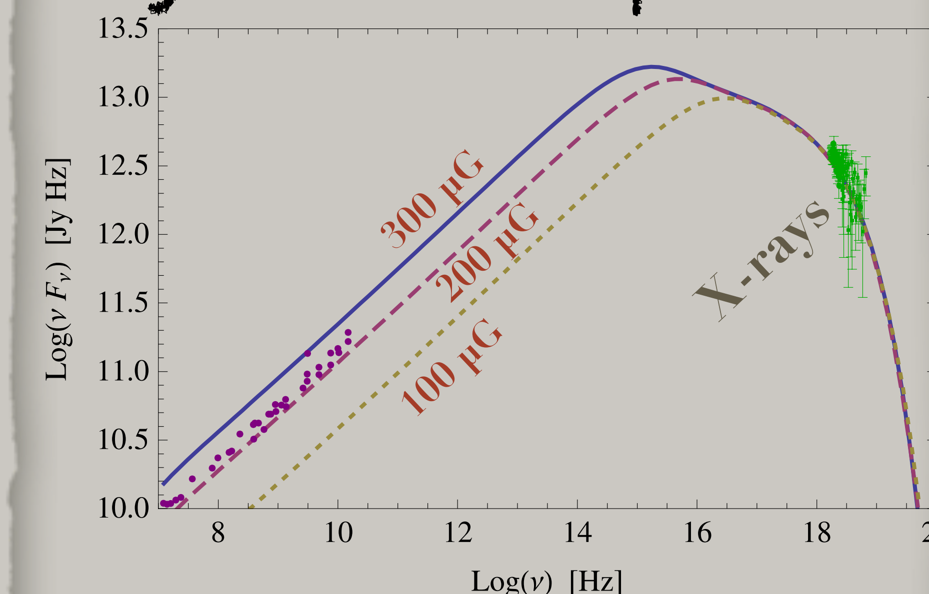






X-Ray Morphology

Synchrotron spectra



SPECTRUM AND MORPHOLOGY APPEAR TO BE WELL DESCRIBED BY EFFICIENT CR ACCELERATION

THE MAXIMUM ENERGY IS IN THE RANGE OF A FEW HUNDRED TeV