Introduction to

Space Plasma Complexity

Tom Chang
Kavli Institute for Astrophysics and Space Research
Massachusetts Institute of Technology
Cambridge, MA USA

COMPLEXITY

IS

Everywhere

What is COMPLEXITY?

CRITICAL PHENOMENON

EXAMPLES:

Binary Mixtures

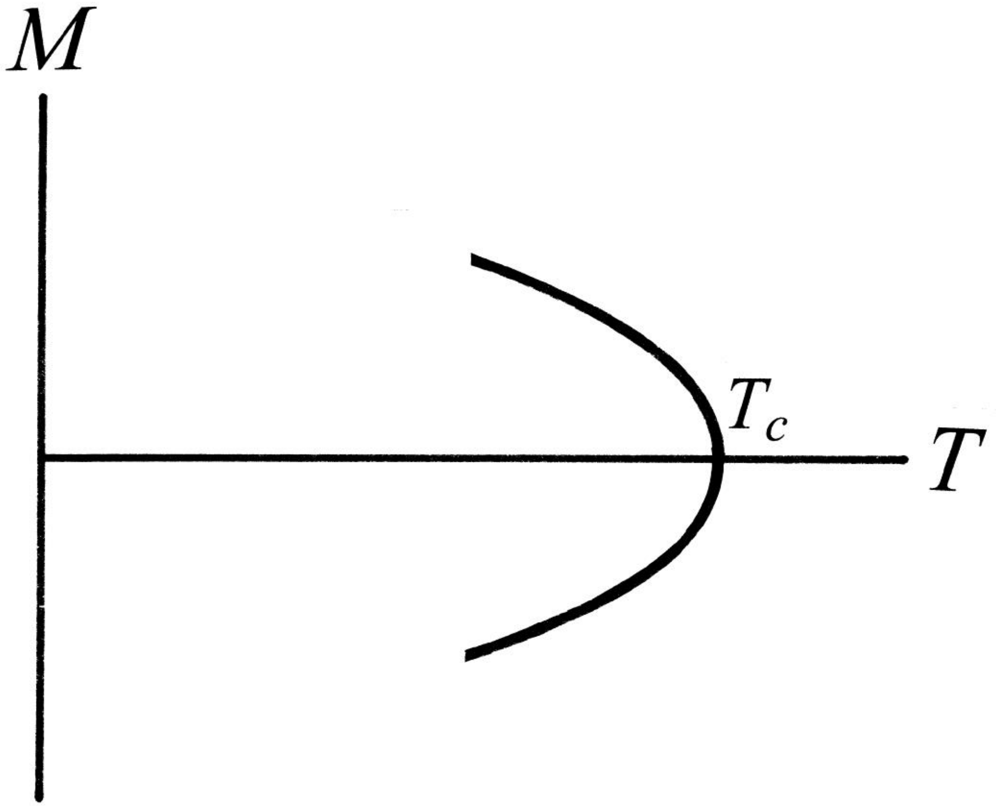
Ferromagnets

Partition Function

$$Z \equiv \sum_{a.p.c.} \exp\left\{-H(s)/k\,T\right\}$$

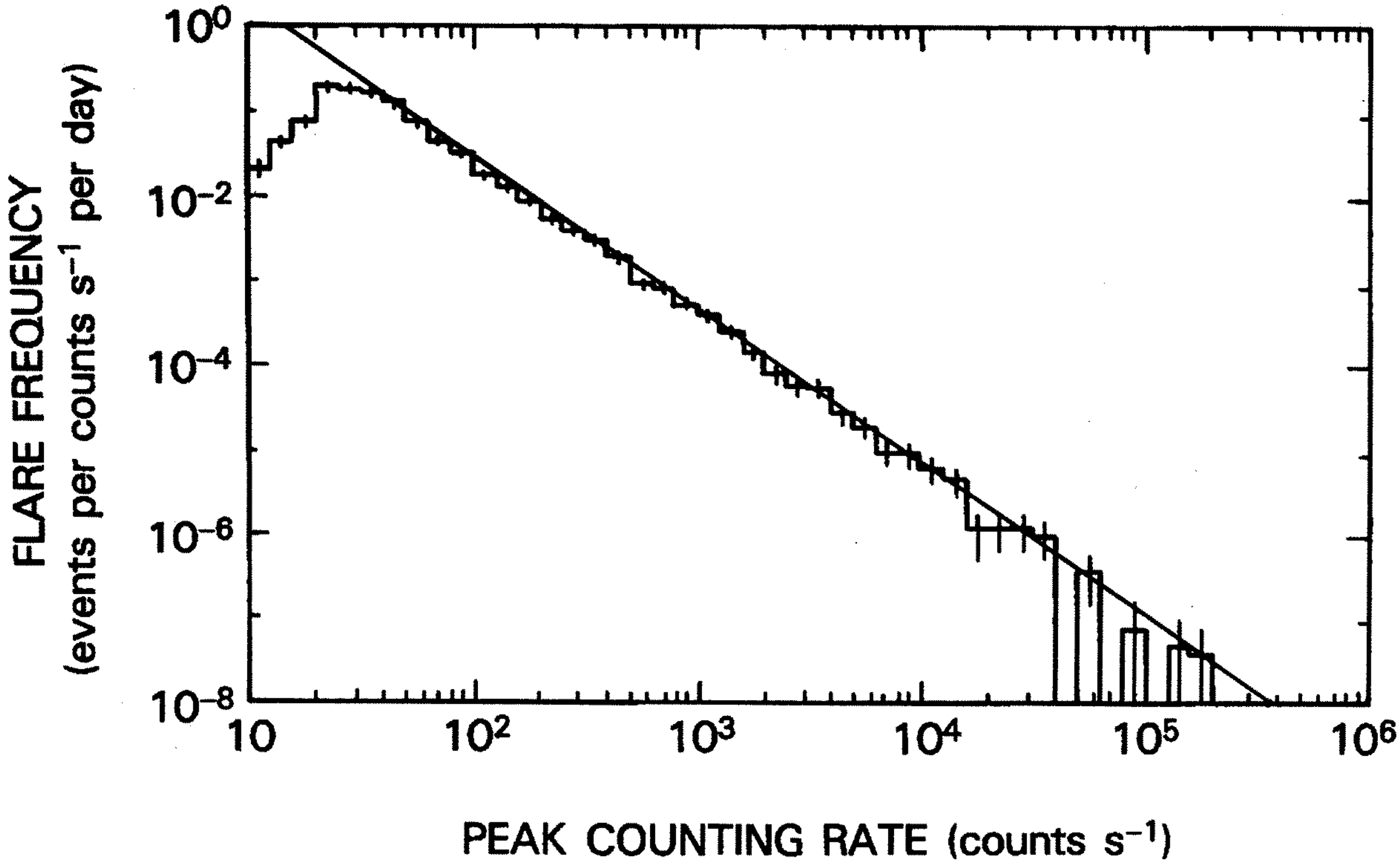
$$H(s) = J \sum_{\widehat{i,j}} s_i\, s_{\widehat{j}}$$

MAGNETIZATION vs. TEMPERATURE



Self-Organized Criticality

Bak, Tang, Wiesenfeld, 1987



FLARE FREQUENCY
(events per counts s^{-1} per day)
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10
10^2
10^3
10^4
10^5
10^6
PEAK COUNTING RATE (counts s^{-1})

Dennis, 1985

SPACE PLASMAS

MHD Equations

Fluctuations

Alfven Waves

Resonances

Coherent Structures

- Flux Tubes
- Solitary Structures

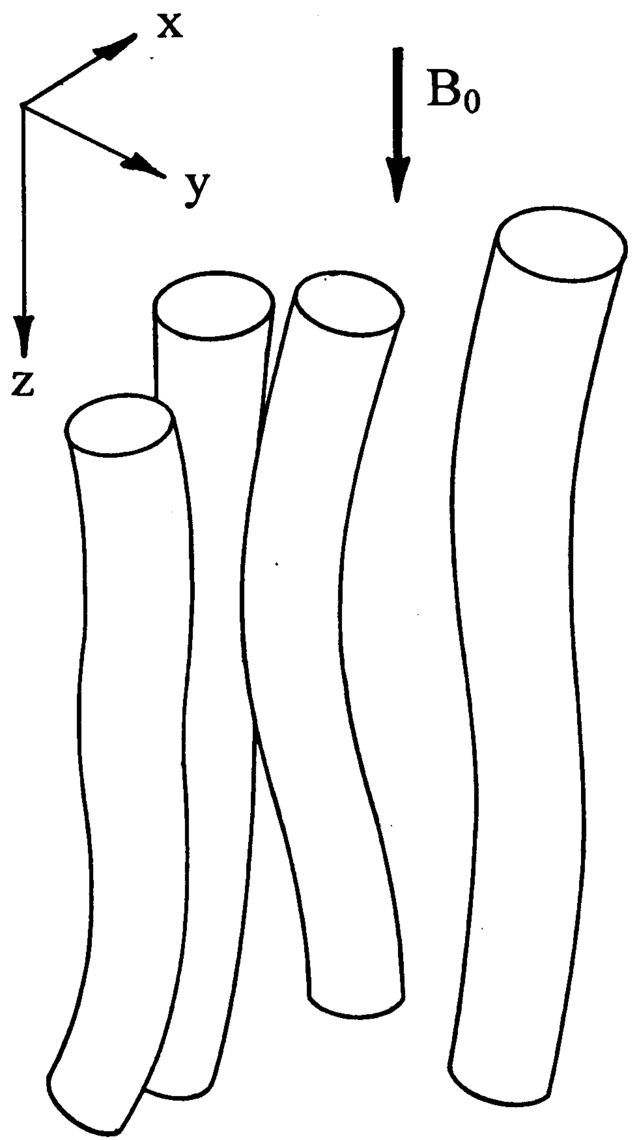
ALFVÉN WAVES

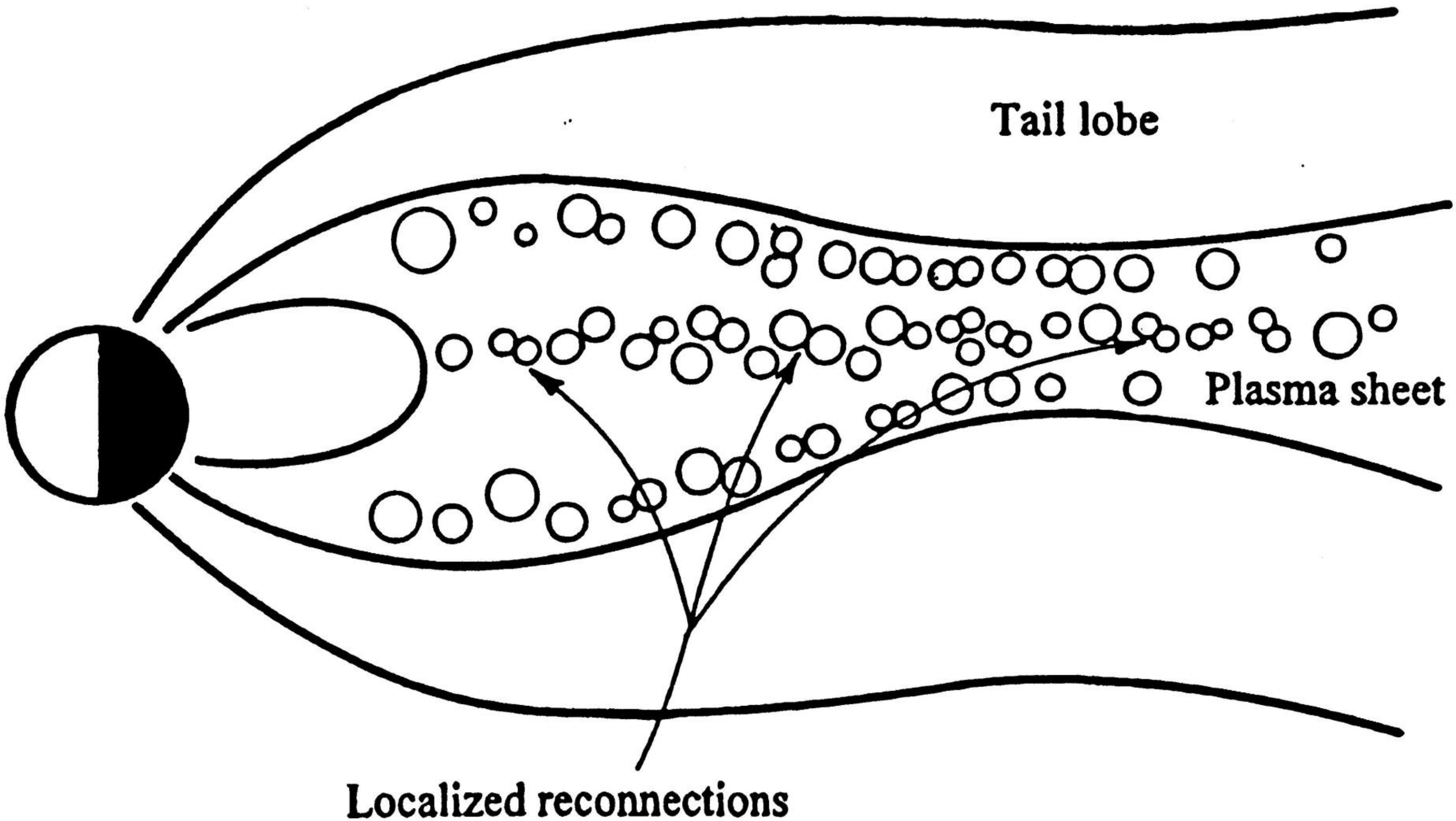
$$\frac{d\mathbf{B}}{dt} = (\mathbf{B} \bullet \nabla)\mathbf{V}$$

$$\rho \frac{d\mathbf{V}}{dt} = (\mathbf{B} \bullet \nabla)\mathbf{B} + \cdots$$

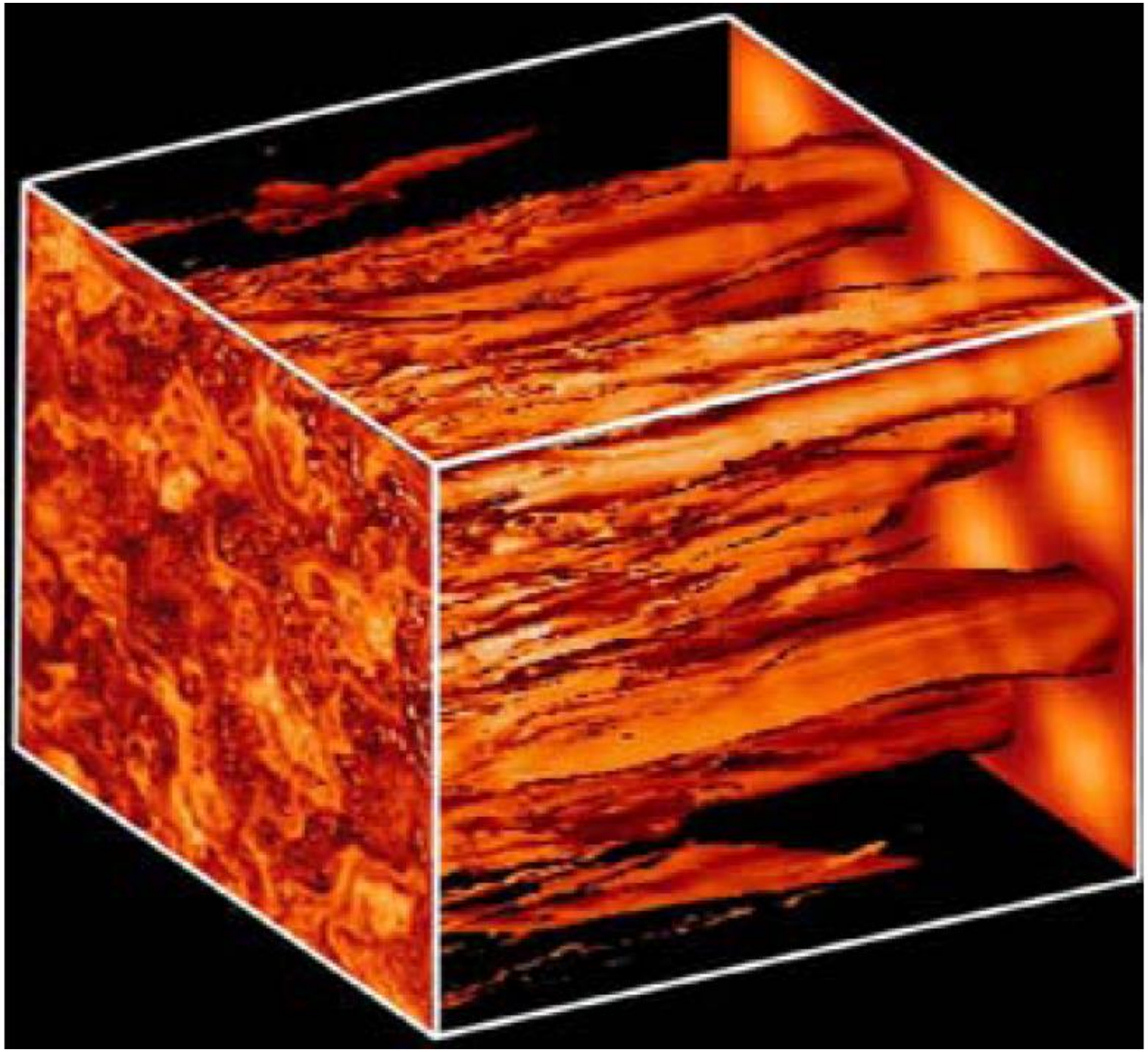
ALFVÉN RESONANCES

$$\mathbf{B}\bullet\nabla \rightarrow i\mathbf{k}\bullet\mathbf{B}=0$$

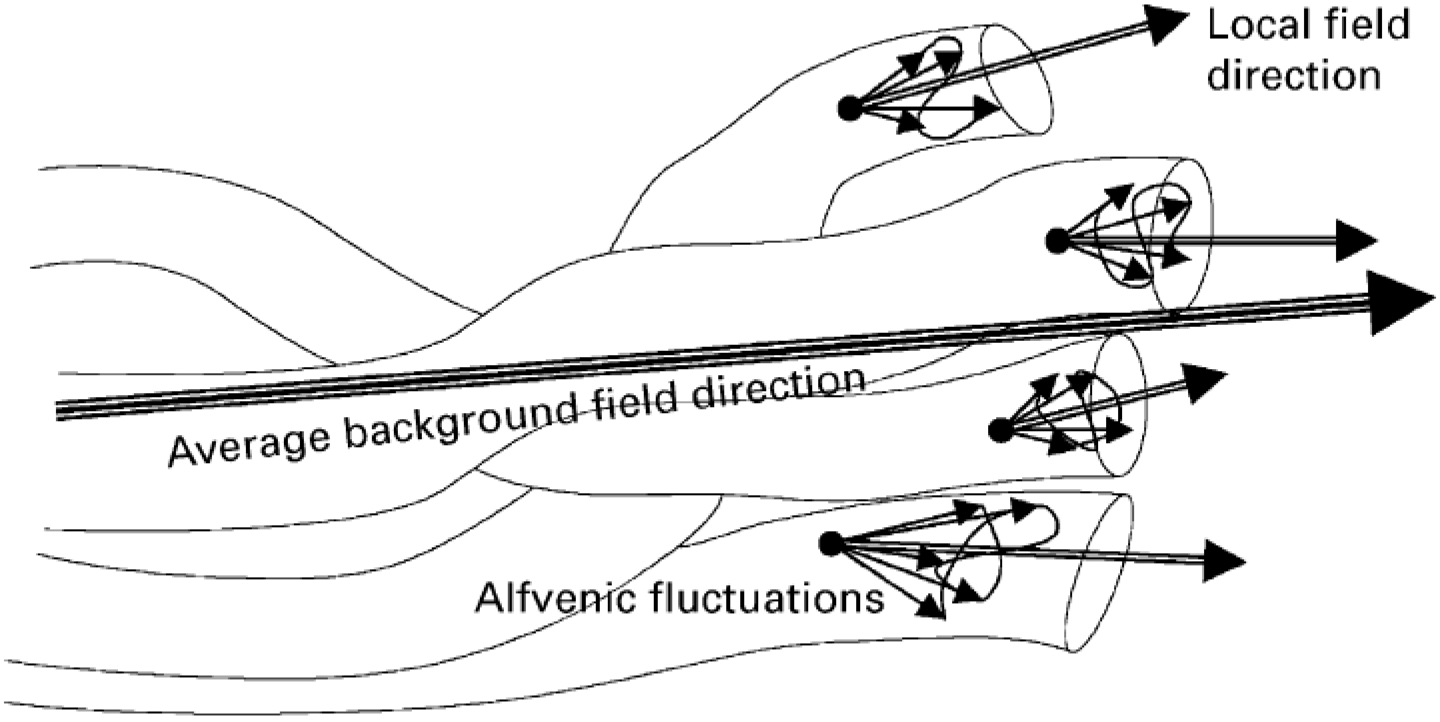

x
y
z
B_0



CHANG, 1998

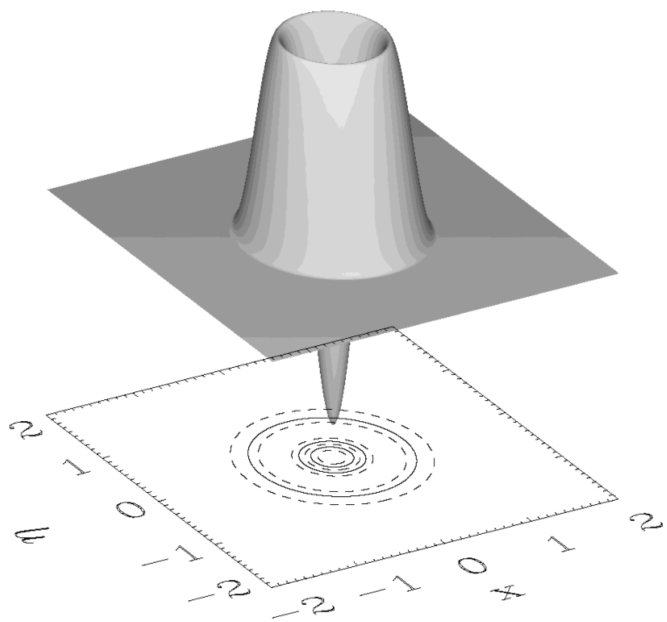


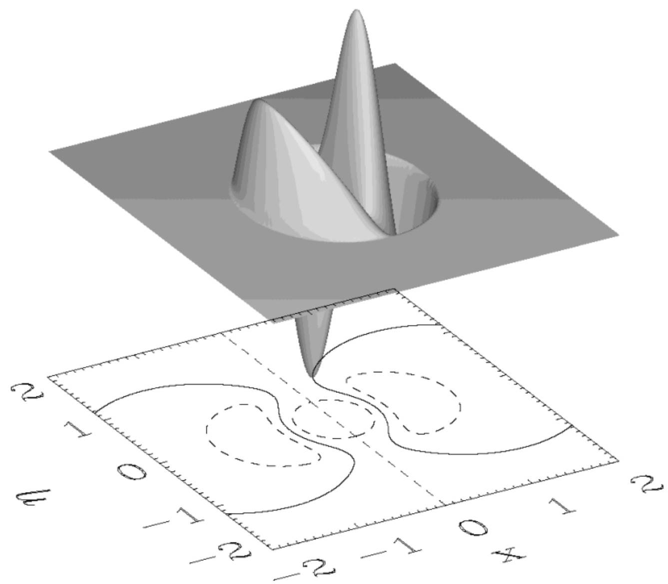
Matthaeus, Qin, Bieber, and Zank, *Astrophys. J. Lett.* 2003

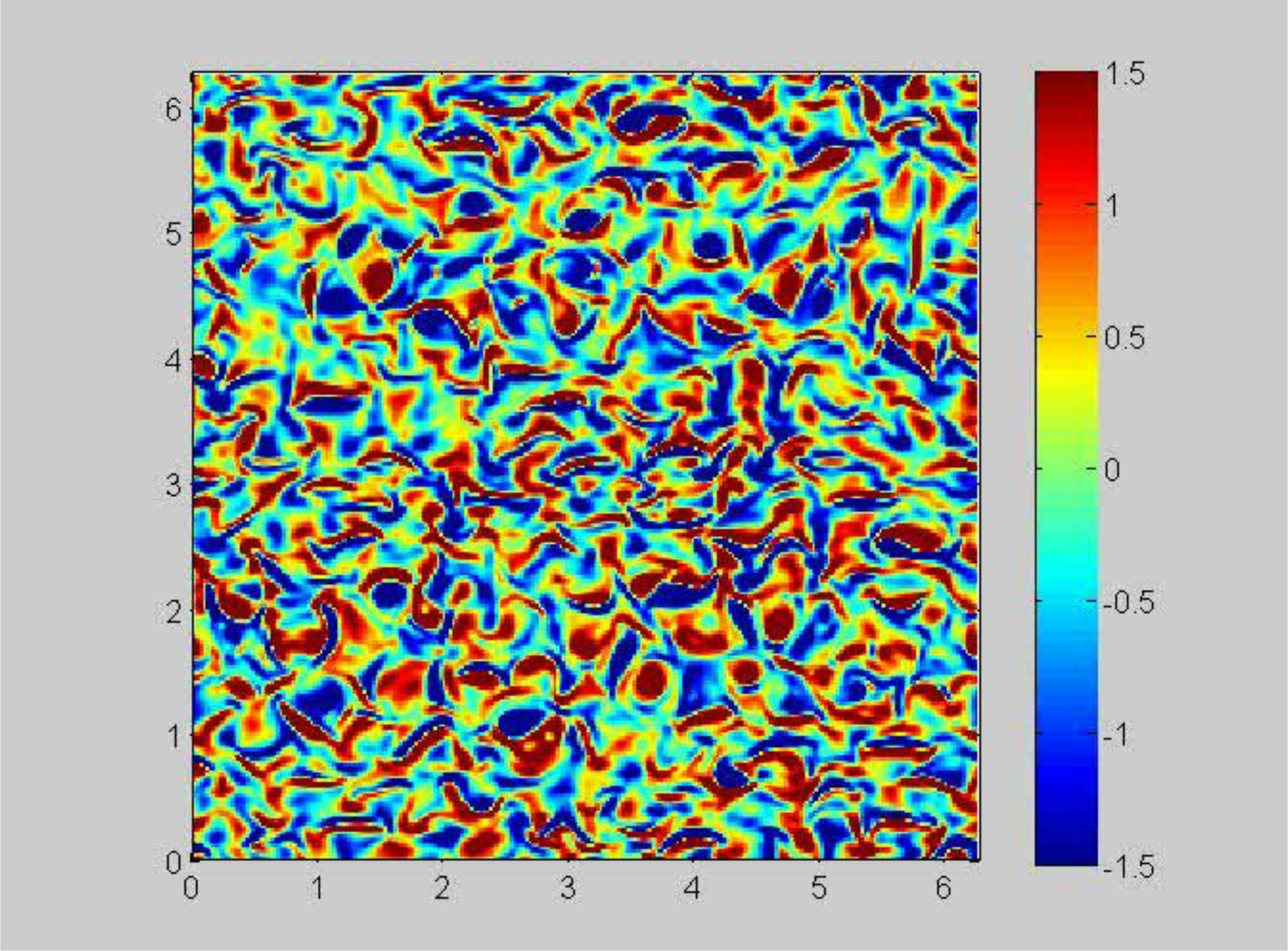


Bruno et al., 2001

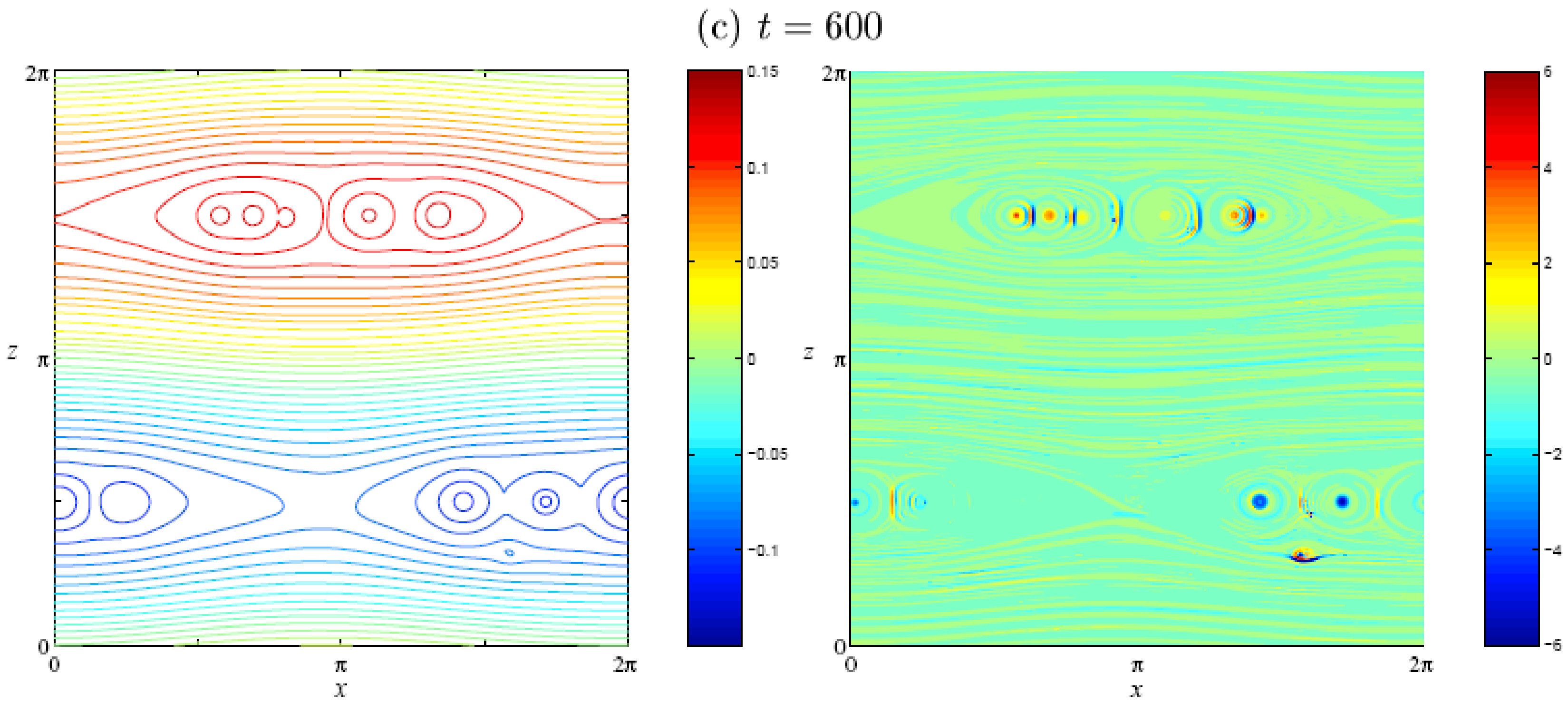
ALEXANDROVA 2008







WU & CHANG, 2006

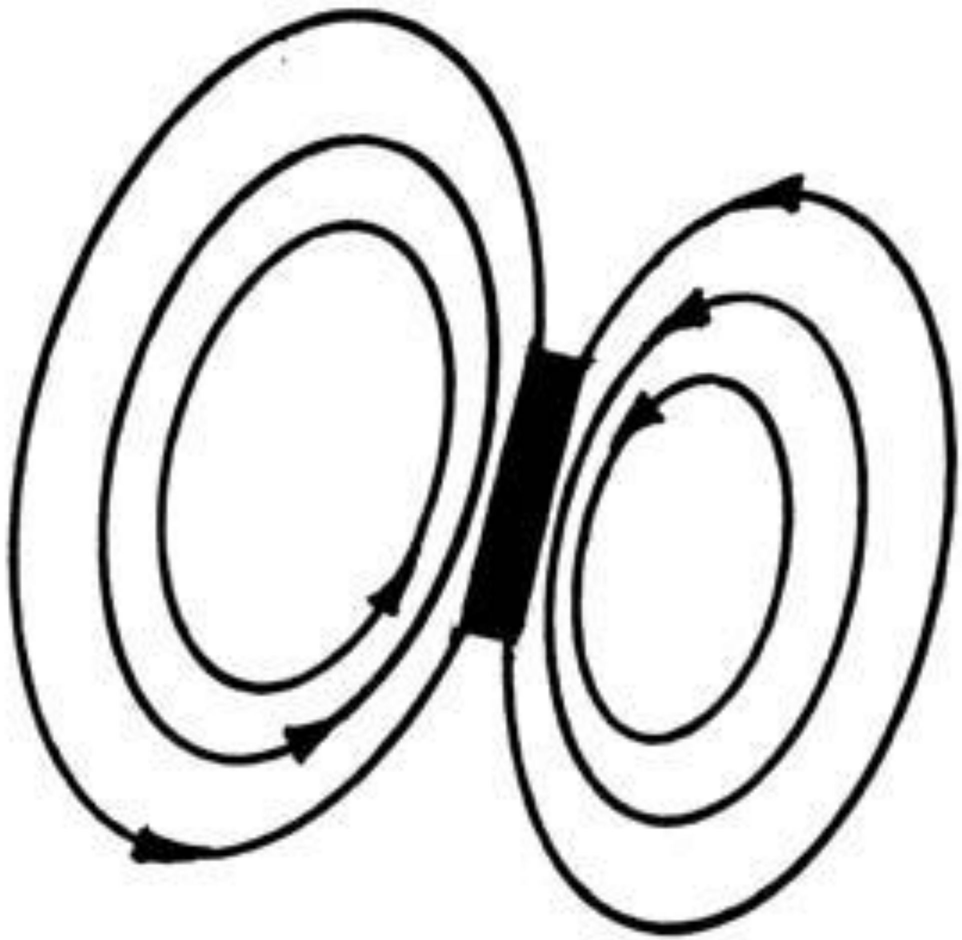


Coarse-Grained Dissipation

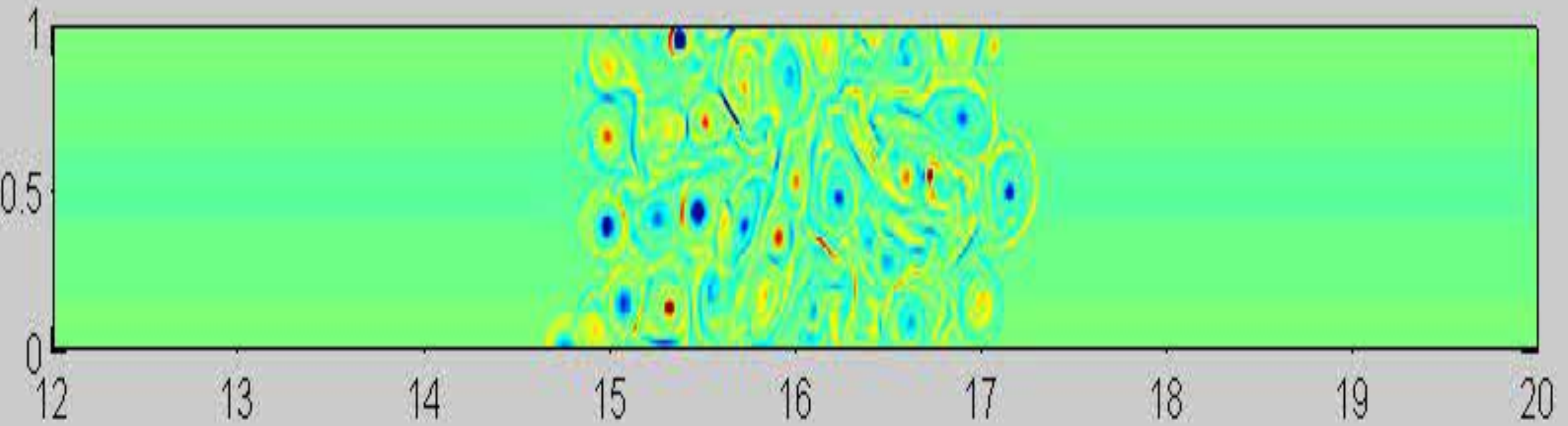
And

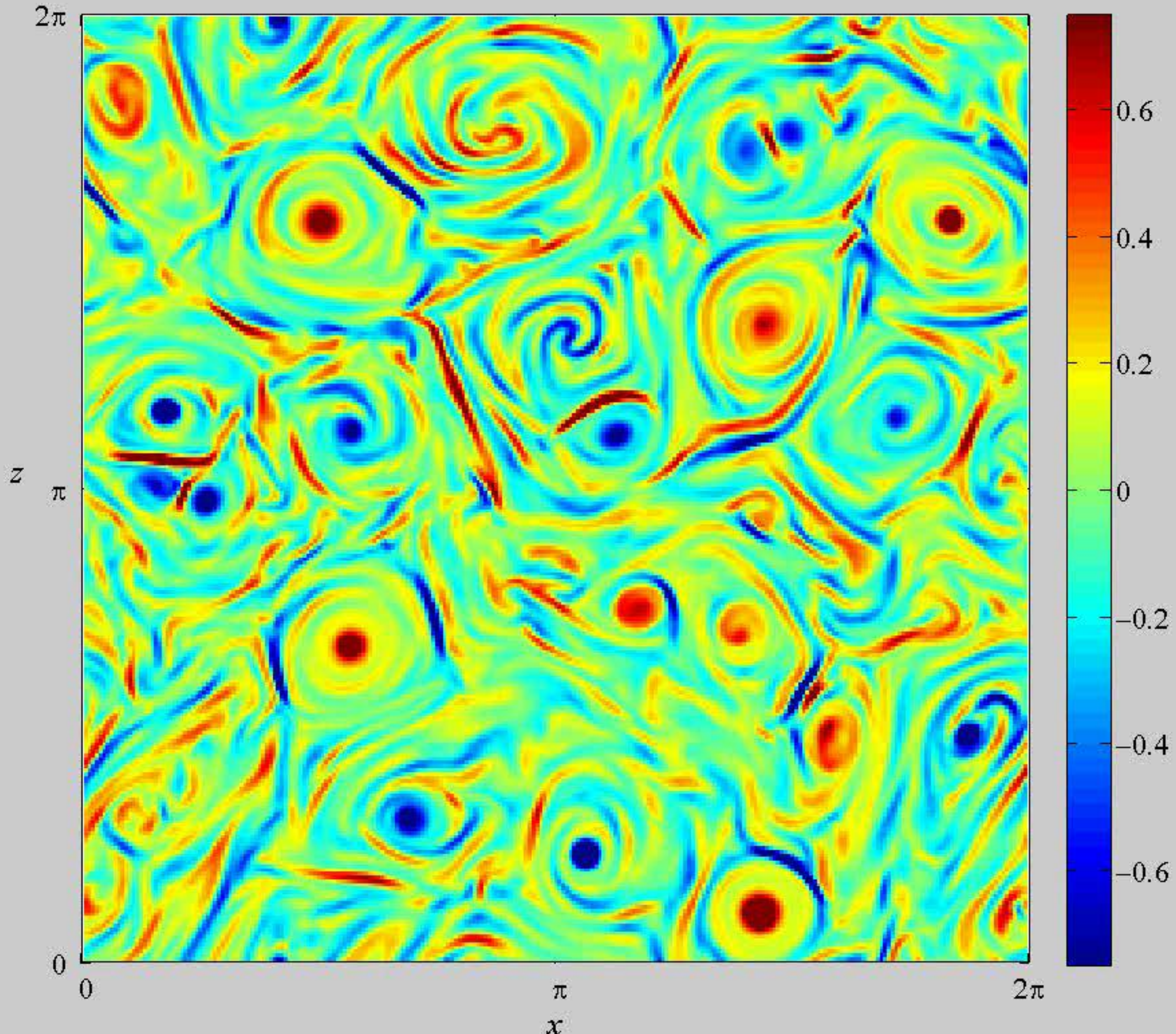
Magnetic Reconfiguration

WU & CHANG, 2001

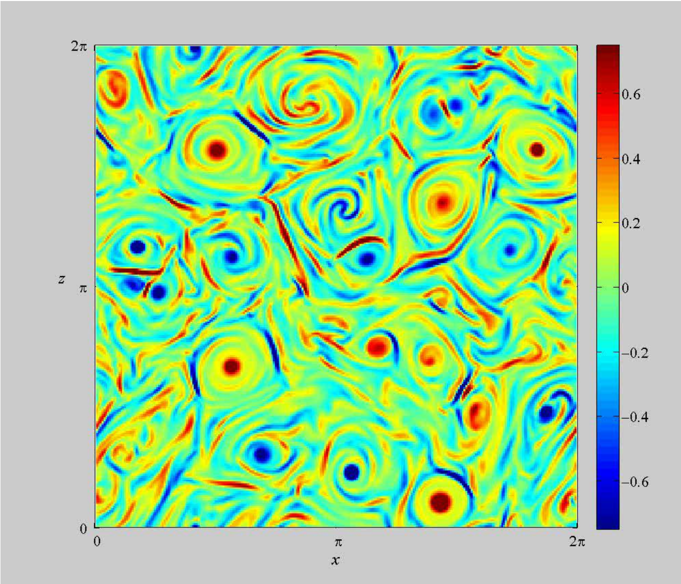


COARSE-GRAINED DISSIPATION

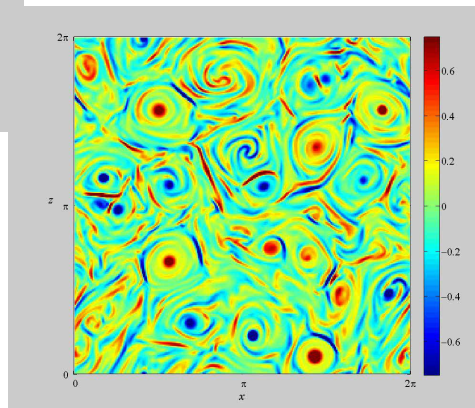




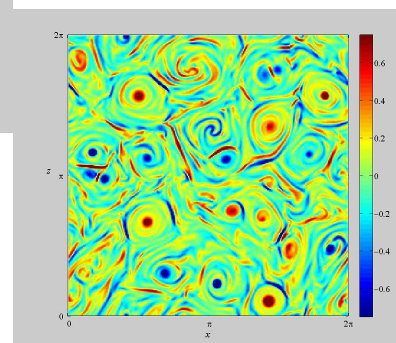
2π
π
0
z
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6



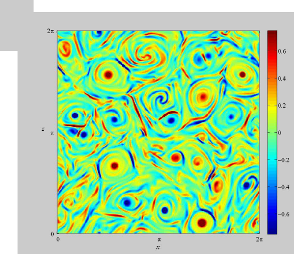
2π
π
0
z
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6



2π
π
0
z
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6



2π
π
0
z
0
π
2π
x
0.6
0.4
0.2
0
−0.2
−0.4
−0.6



TYPES OF CHANGES:

Physical

Dynamical

Topological

EFFECTIVE ACTION:

$$A\{P_n\}$$

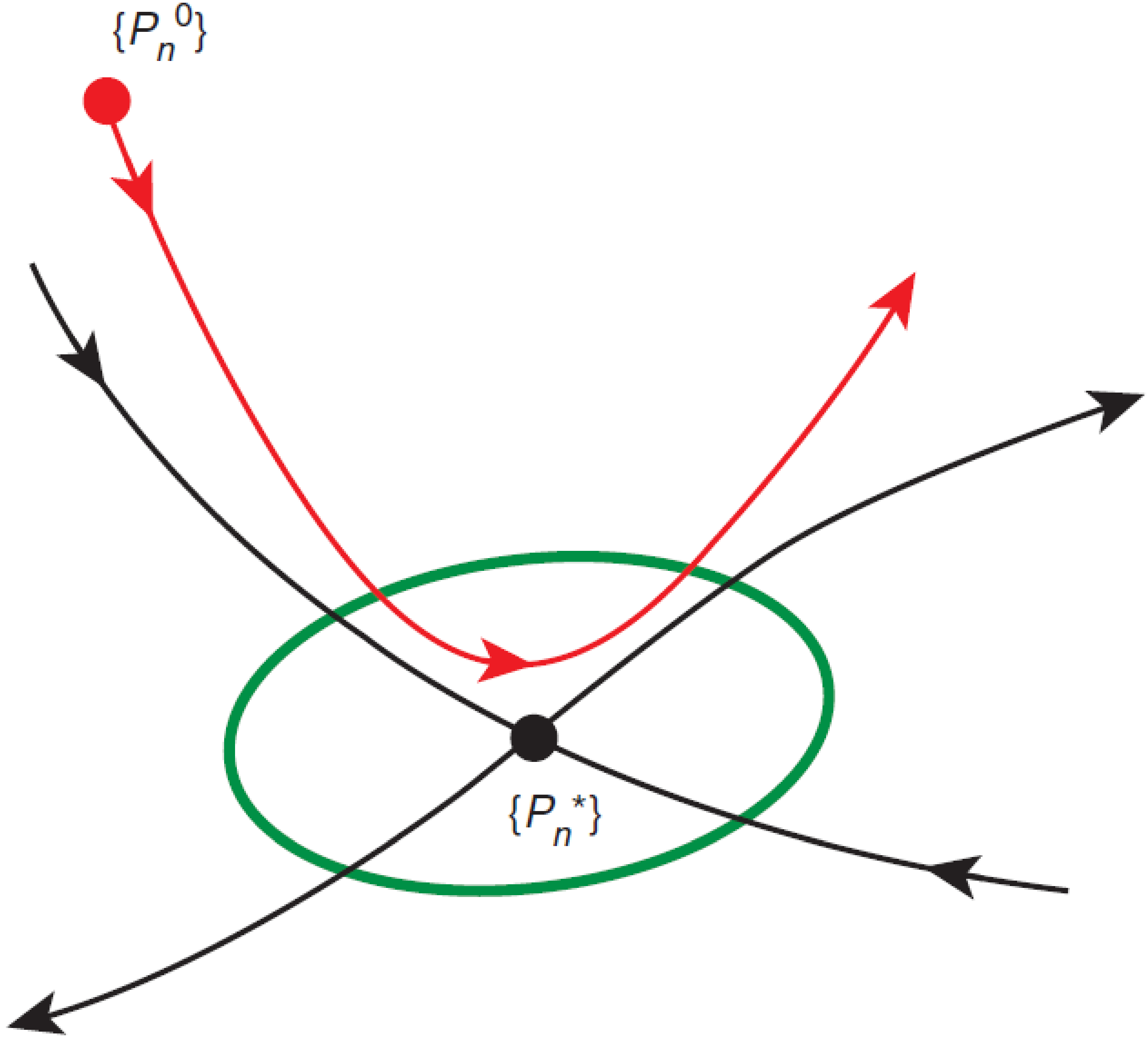
PARAMETER SET: $\{P\}$

DRG TRANSFORMATION:

$$d\{P\}/dl = \mathcal{R}\{P\}$$

Procedure:

Search for Invariants



{P_n^0}
{P_n^*}

LINEAR TRANSFORMATIONS

$\rightarrow$ POWER LAW INVARIANTS.

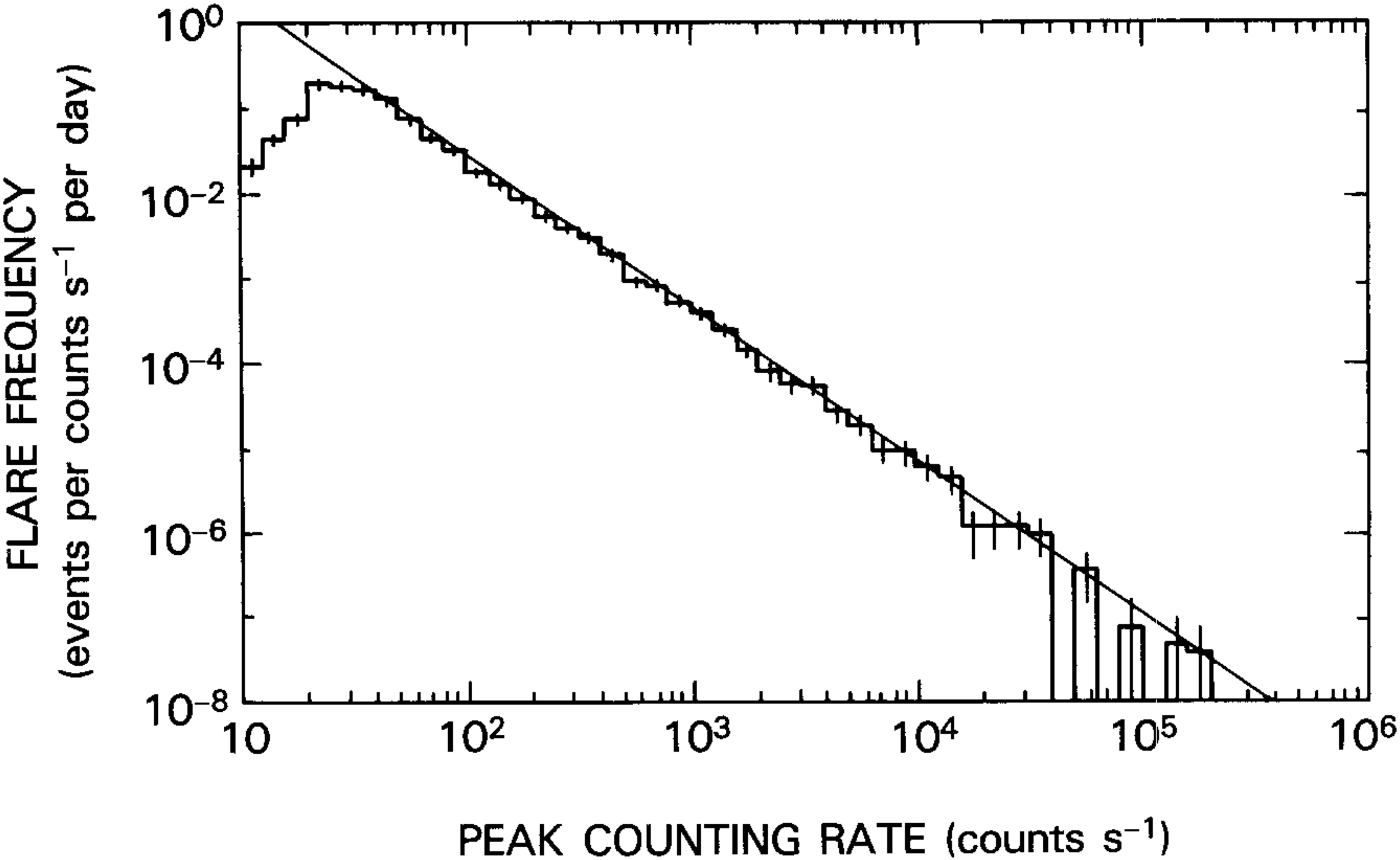
****

__EXAMPLE:__

$P \rightarrow e^{a_P} P, \quad E \rightarrow e^{a_E} E$ *LEADS TO*

$P / E^{a_P / a_E}$ AS AN INVARIANT.

Dennis, 1985



FLARE FREQUENCY
(events per counts s⁻¹ per day)
10⁰
10⁻²
10⁻⁴
10⁻⁶
10⁻⁸
10
10²
10³
10⁴
10⁵
10⁶
PEAK COUNTING RATE (counts s⁻¹)

$$P(E,L)$$

$$P/E^a = I$$

$$E/L^b = J$$

$$\rightarrow I = F(J)$$

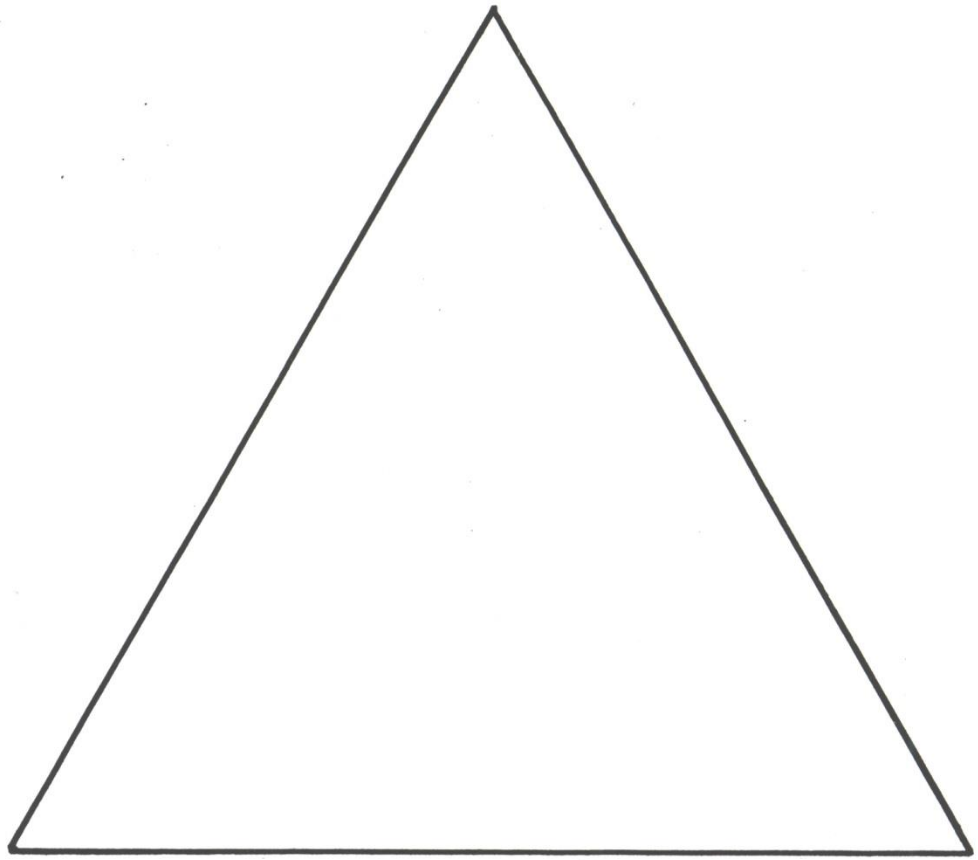
Finite Size Scaling

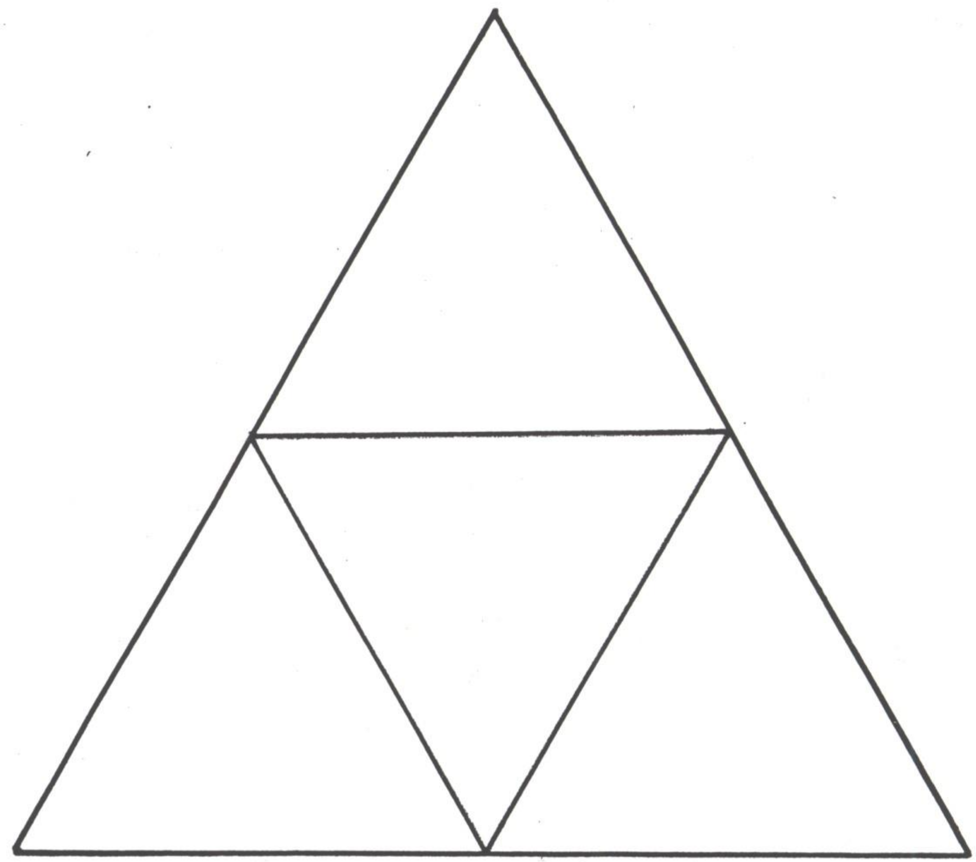
(FSS)

FRACTALS

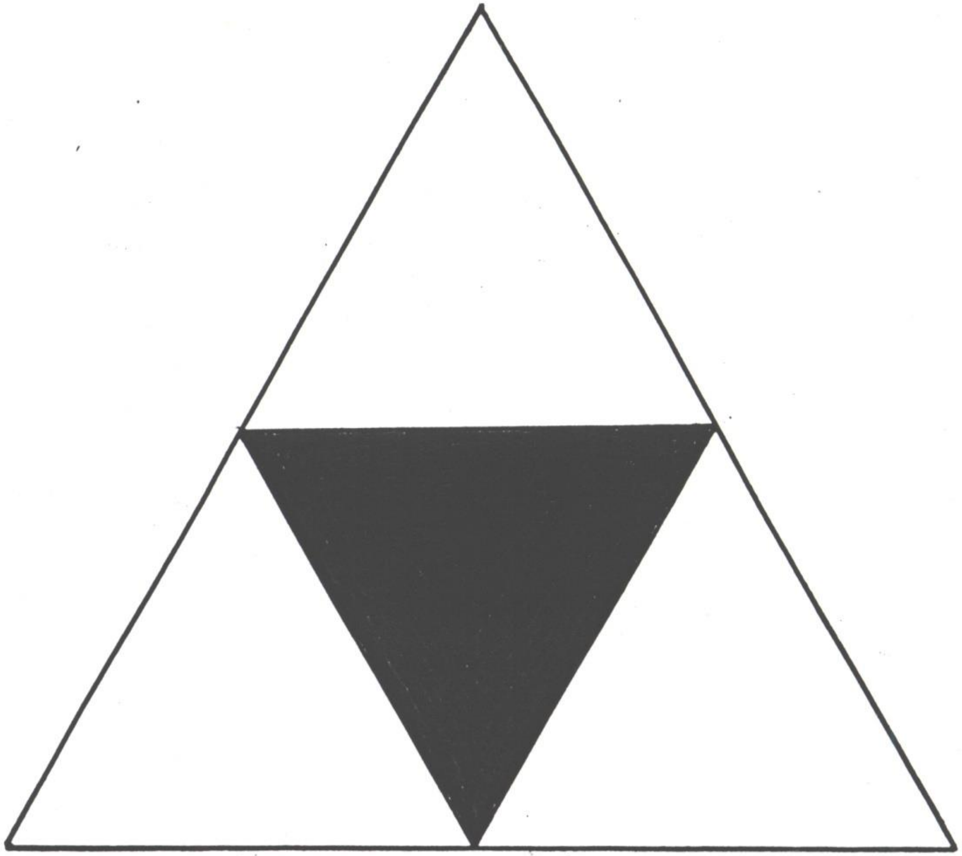
Mandelbrot, 1977

HOW DO WE DEFINE *DIMENSIONS?*

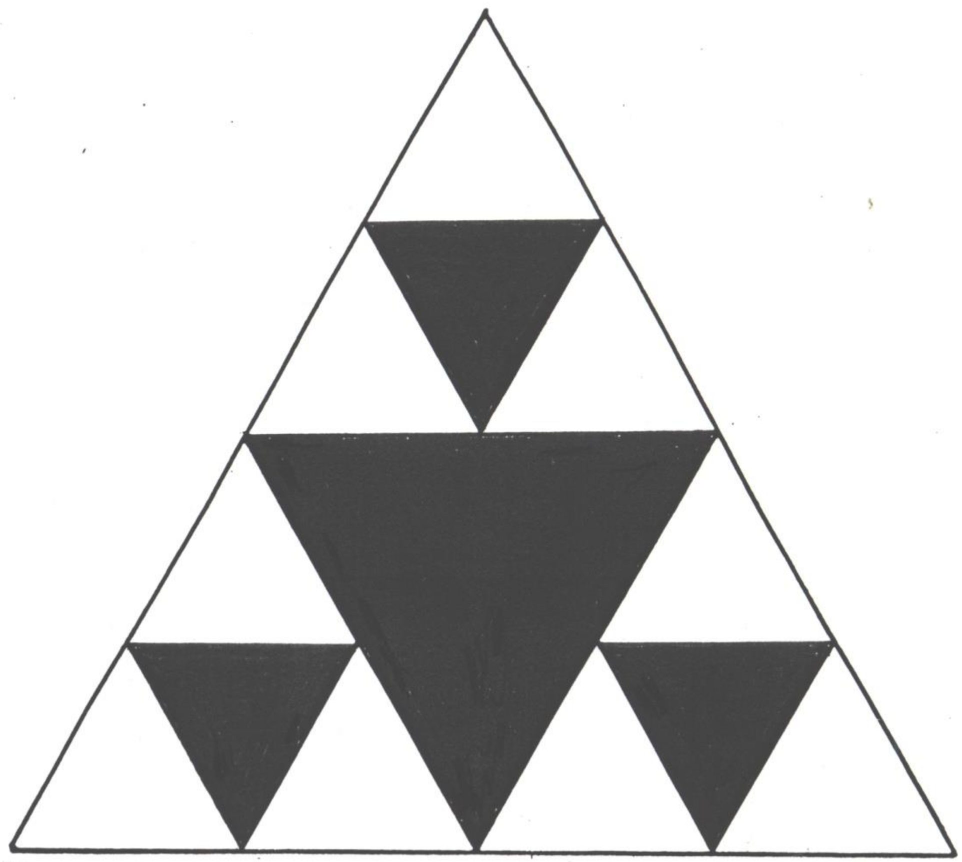




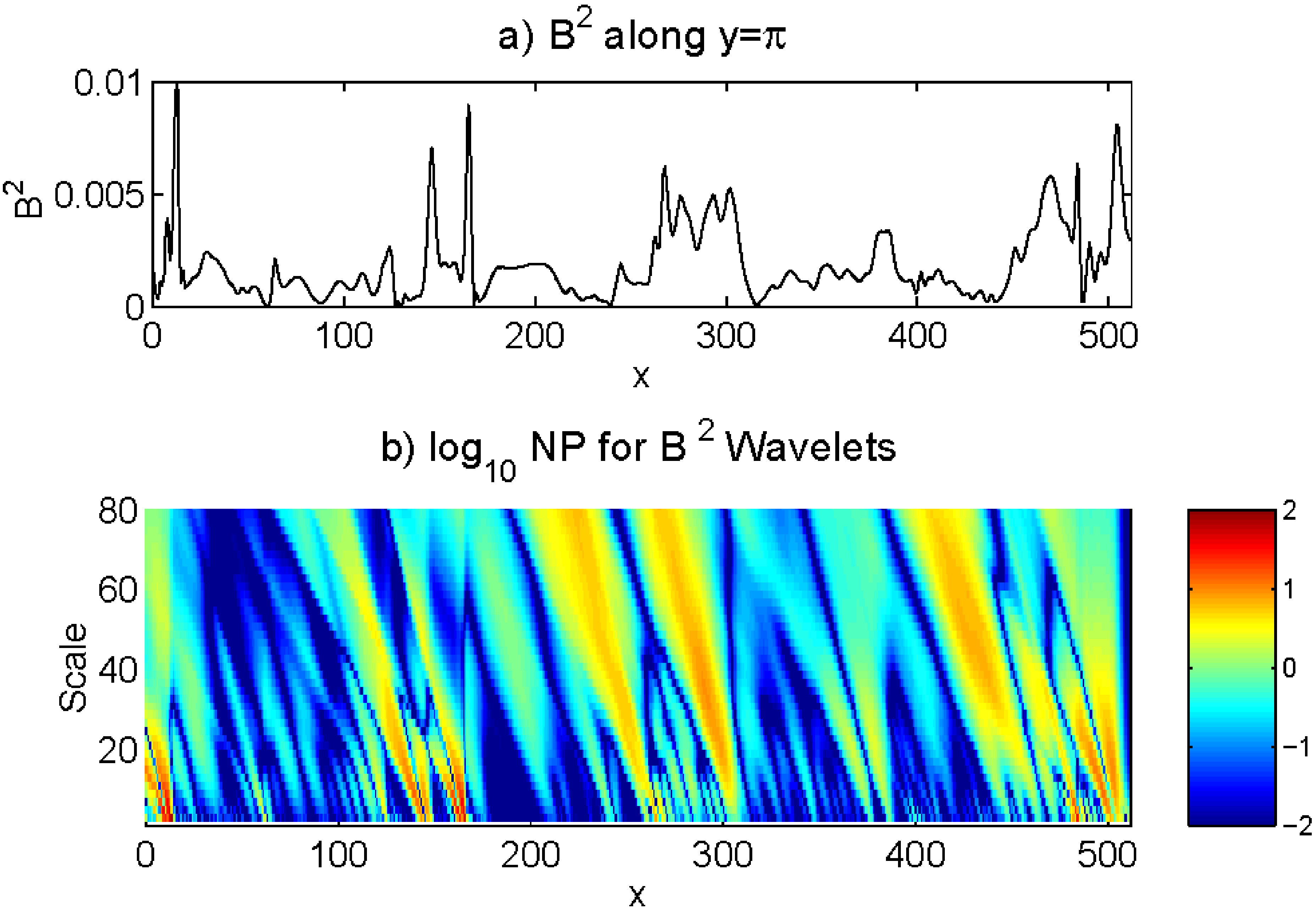
$d = \log 4 / \log 2 = 2$



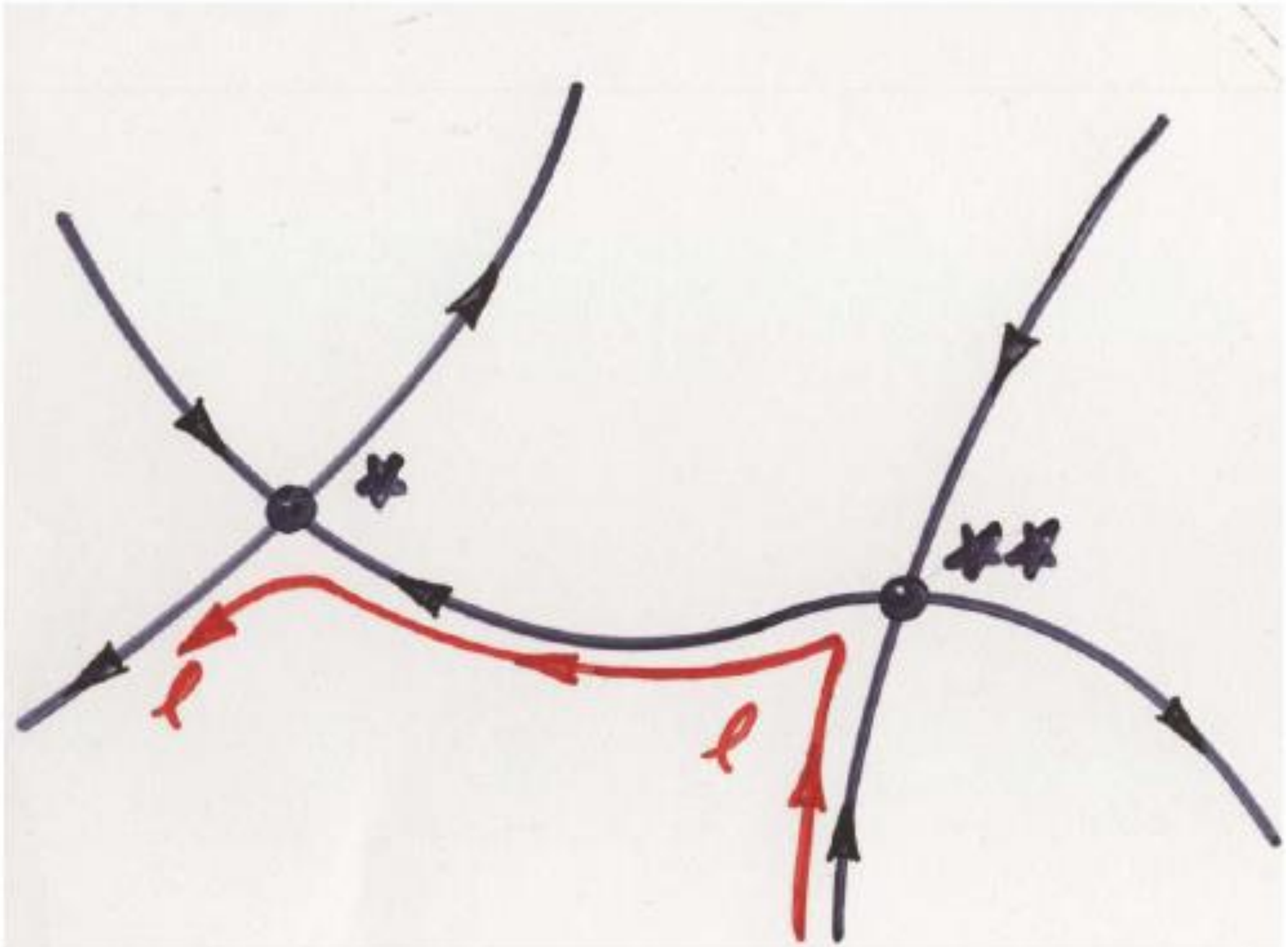
SIERPINSKI TRIANGLE (1916)



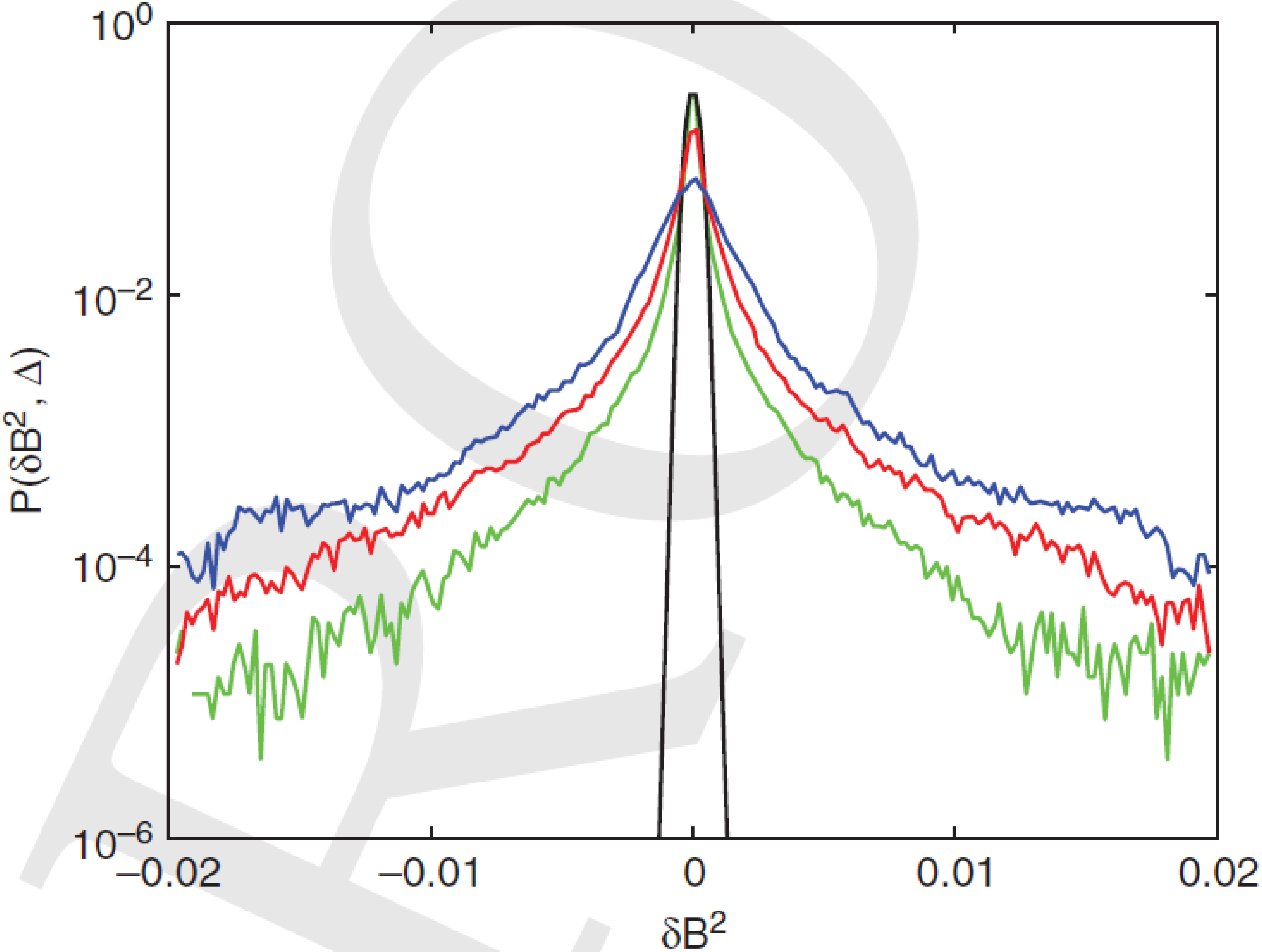
$d = \log 3 / \log 2 \approx 1.58$


a) B2 along y=π
B2
0.01
0.005
0
0
100
200
300
400
500
x
b) log10 NP for B 2 Wavelets
Scale
80
60
40
20
0
100
200
300
400
500
x
2
1
0
−1
−2

$\{P_N\}$



Fixed Points: $\frac{d\{P_N\}}{d\ell}=0$



10^0
10^{-2}
10^{-4}
10^{-6}
P(δB², Δ)
−0.02
−0.01
0
0.01
0.02
δB²

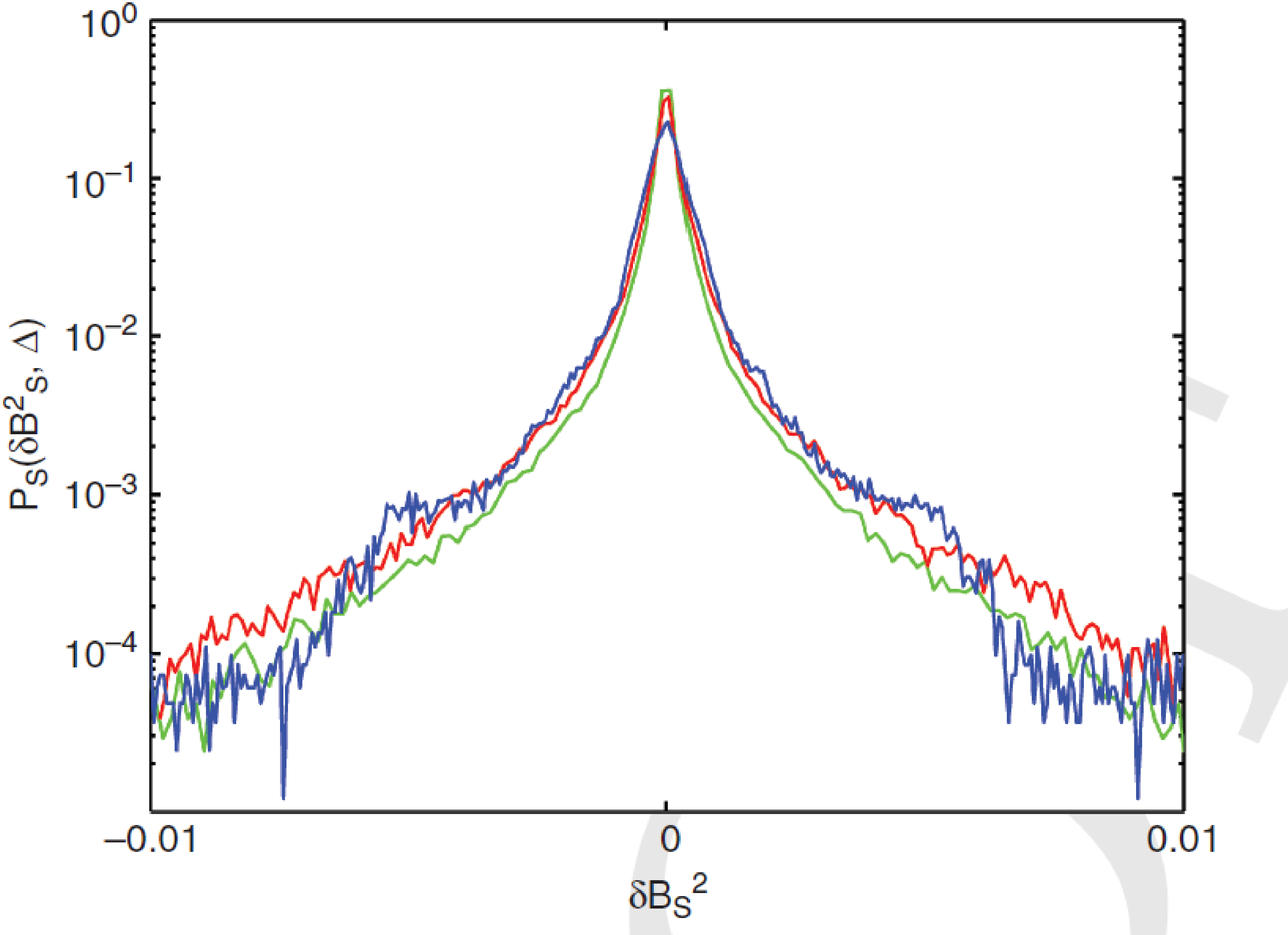
PROBABILITY DISTRIBUTION FUNCTION OF INCREMENTAL FLUCTUATIONS

$$P(\delta B^2, \Delta)$$

SCALING NEAR DYNAMIC CRITICALITY

$$P(\delta B^2, \Delta)\Delta^{-s} = P_s(\delta B^2{}_s)$$

$$\delta B^2{}_s = \delta B^2 / \Delta^s$$

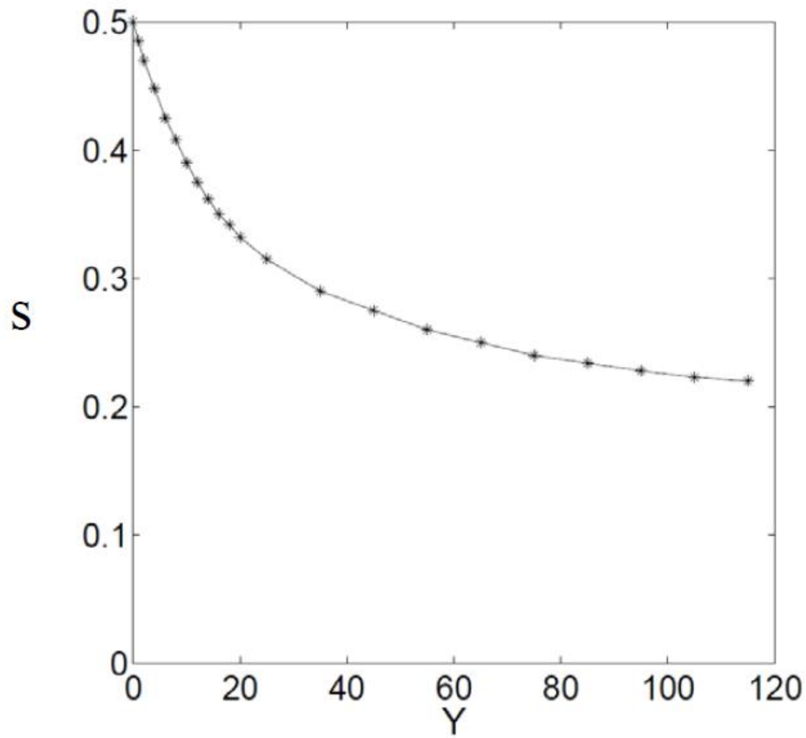


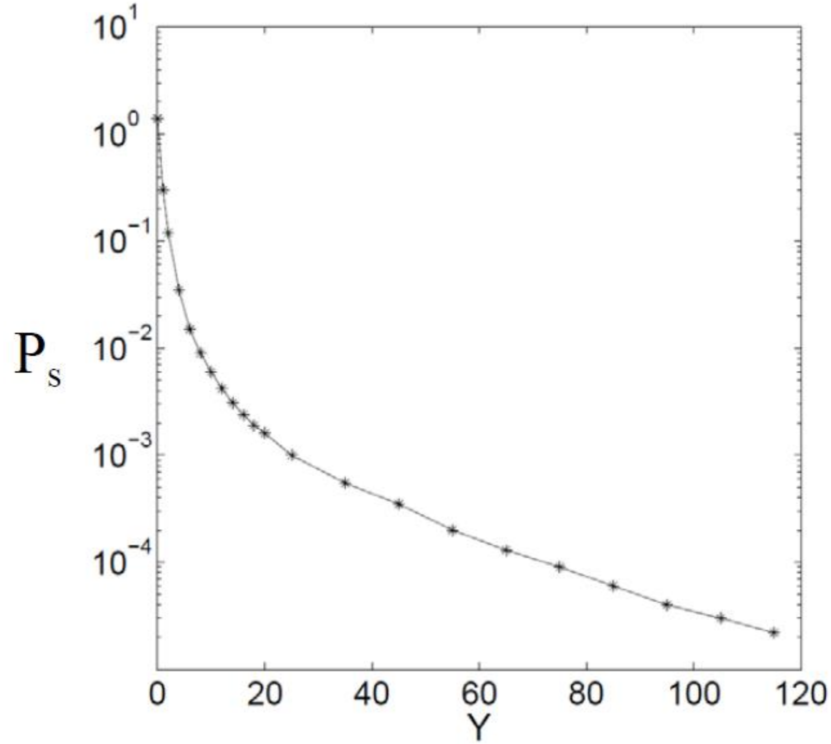
Hnat et al., 2002 (Solar Wind)

Chang et al., 2004

$$P(\delta B^2,\Delta)\Delta^{-s} = P_s(\delta B^2{}_S)$$

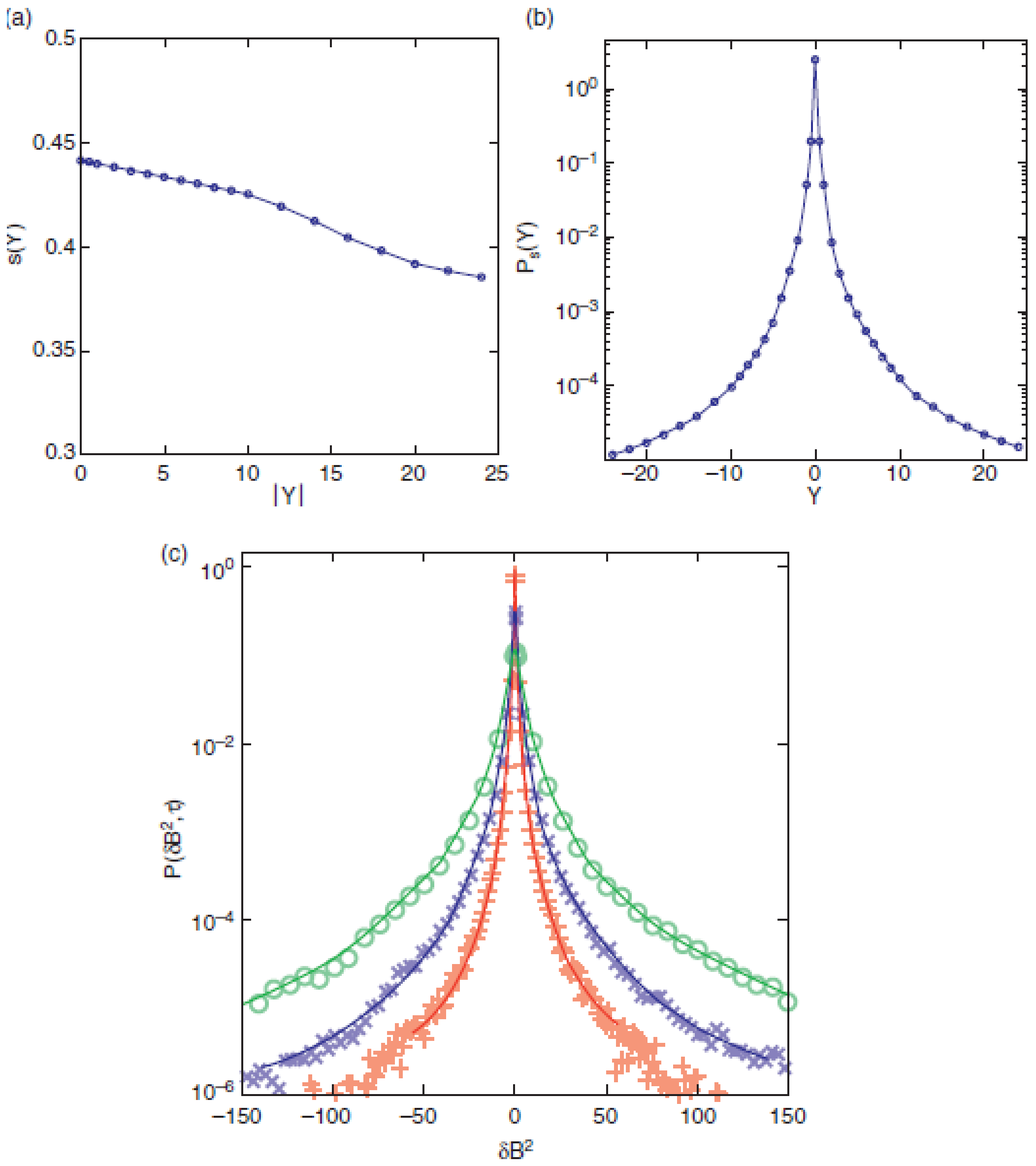
$$Y = \delta B^2{}_S = \delta B^2 / \Delta^s \qquad S=S(Y)$$



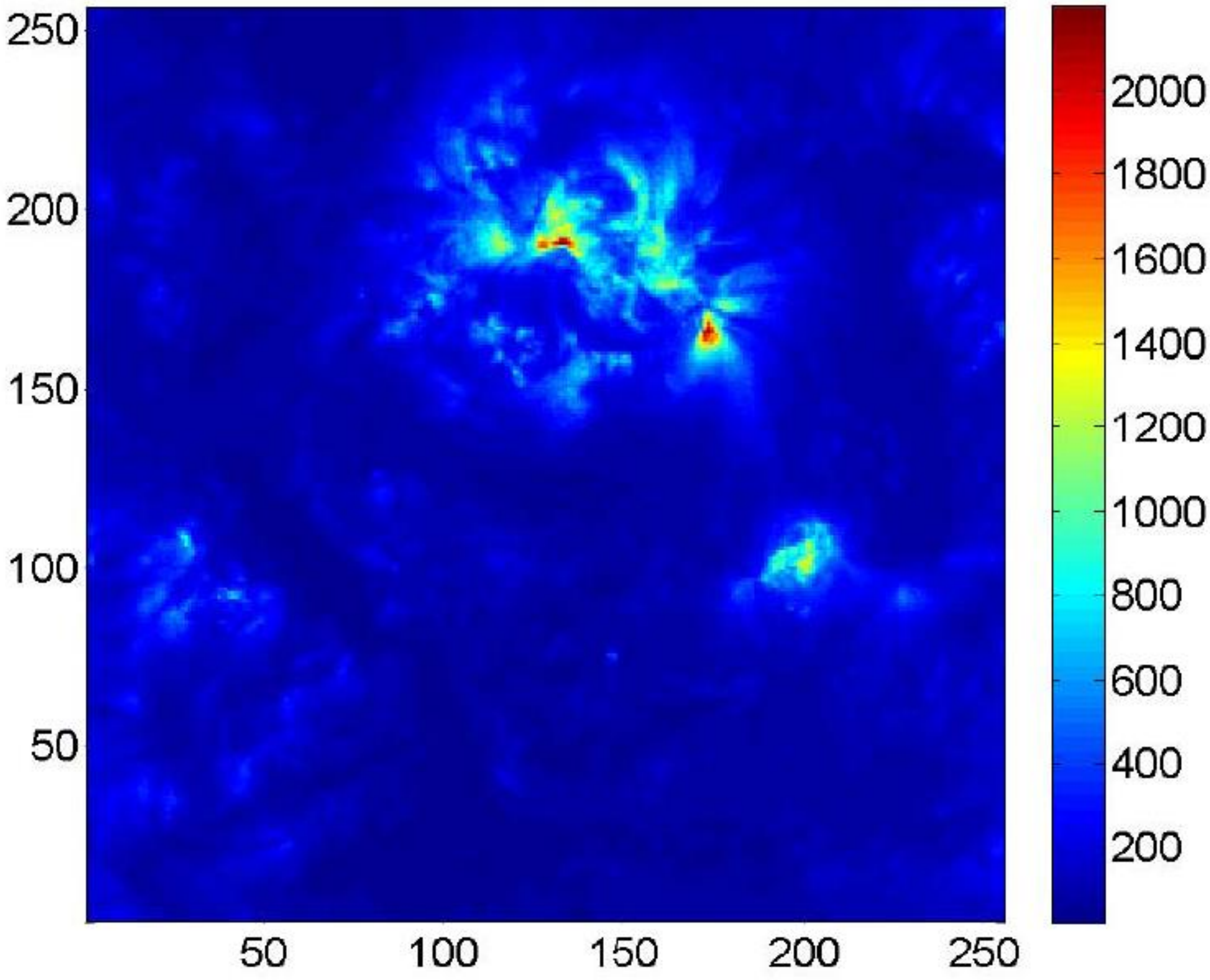


ROMA, Chang and Wu 2008

SOLAR WIND, Podesta, 2010

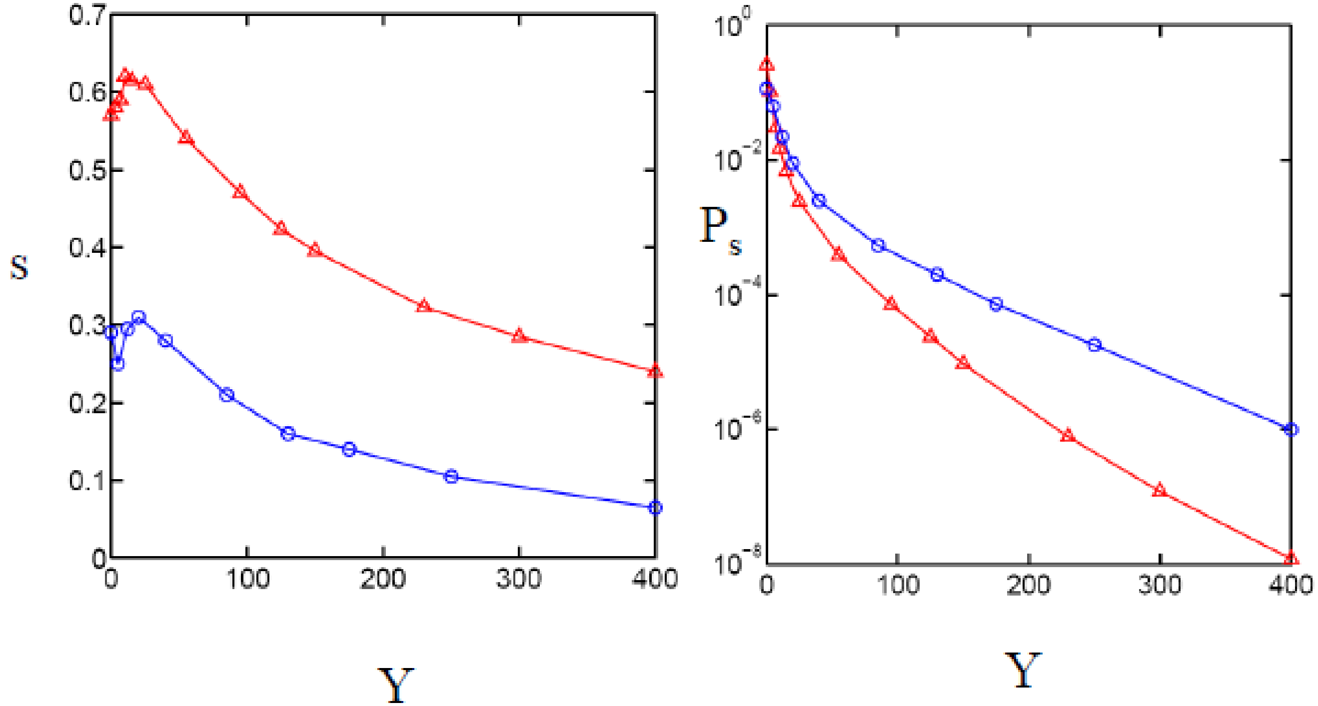


Corona EUV Emissions – SOHO (Wu et al., 2011



Wu, Chang, Uritsky; 2011

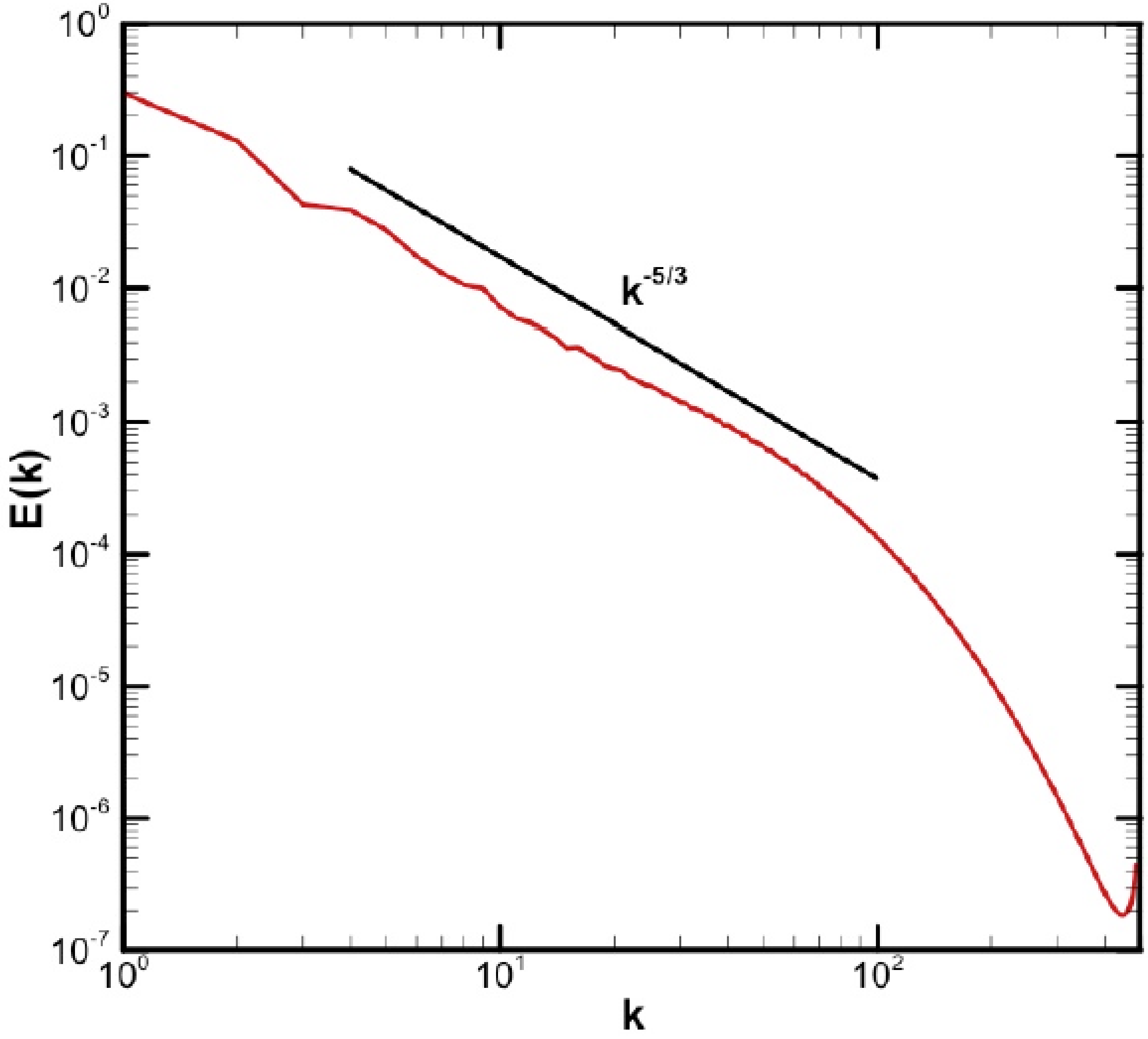
ROMA analysis of probability distributions for the SOHO EIT data II

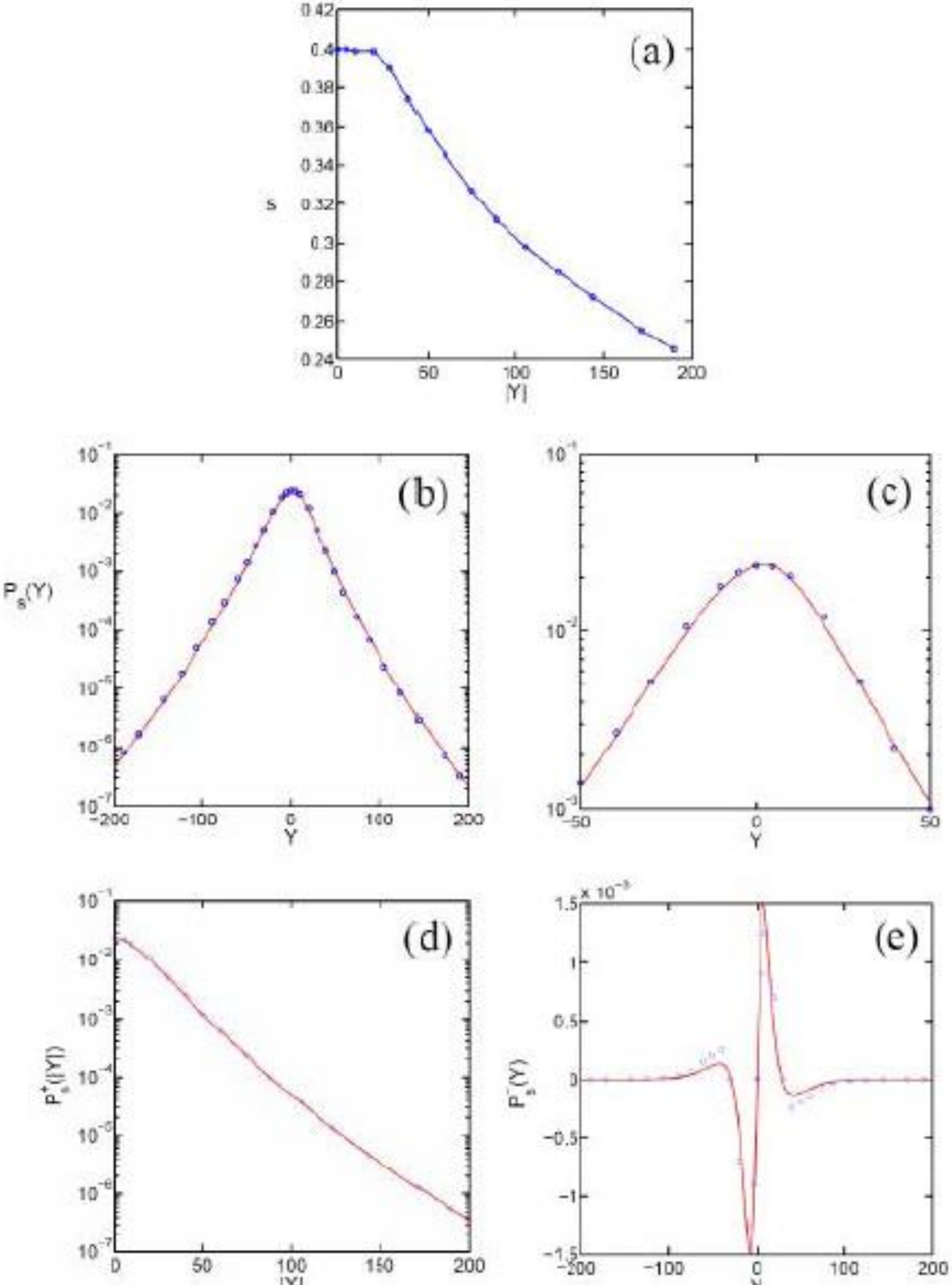


Red triangles: δ from 2 to 8; blue circles: δ from 20 to 80.

In this study, the SOHO EIT data set is shown to include two multifractal rank-ordered regimes, one for δ from 2 to 8 and the other for δ from 20 to 80.

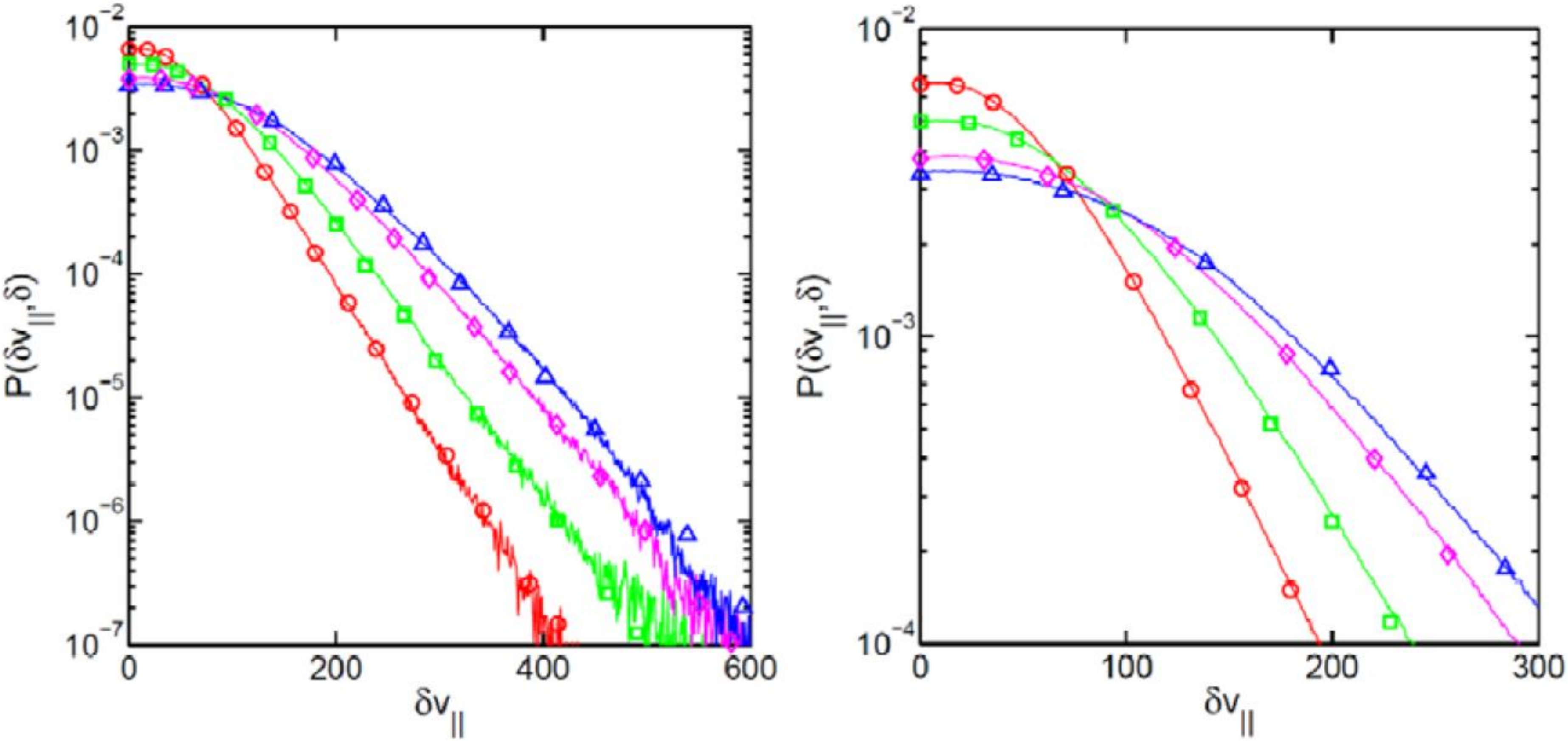
ISOTROPIC 3D FLUID TURBULENCE
--jhu turbulence data base --




(a)
s
|Y|
0.42
0.4
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0
50
100
150
200
(b)
P_s(Y)
Y
−200
−100
0
100
200
(c)
Y
−50
0
50
(d)
P_s^+(|Y|)
|Y|
0
50
100
150
200
(e)
P_s^-(Y)
Y
1.5
1
0.5
0
−0.5
−1
−1.5
−200
−100
0
100
200

PROCEDURE IS REVERSIBLE

Result is Unique



APPLICATIONS TO SPACE PLASMAS

- Auroral electric field fluctuations (Tam et al.)
- Cusp and magnetosheath magnetic field fluctuations (Echim and Lamy)
- AE index (Consolini and De Michelis)
- EUV solar emissions (Wu and Uritsky)
- Solar wind (Podesta)
- Isotropic hydrodynamic turbulence (Wu)
- Cosmic Web (Chang et al.)

[1] Tom Tien Sun Chang, *An Introduction to Space Plasma Complexity*, Cambridge University Press (New York, 2015).

[2] Tom Chang, Cheng-Chin Wu, Marius Echim, Herve Lamy, Mark Vogelsberger, Lars Hernquist, and Debora Sijacki, *Complexity Phenomena and ROMA of the Earth's Magnetospheric Cusp, Hydrodynamic Turbulence, and the Cosmic Web*, Pure and Applied Geophysics, DOI 10.1007/s00024-014-0784-z, 2014.

[3] Tom T.S. Chang, *Complexity Induced Lifshitz Ordering with Multifractal Antiscreening/Screening (CILOMAS),* Physics Letters, <u>A380</u>, 1566-1569, 2016.