KINETIC WAVES & INSTABILITIES
(I) Parallel Alfvénic modes in compensated-current systems

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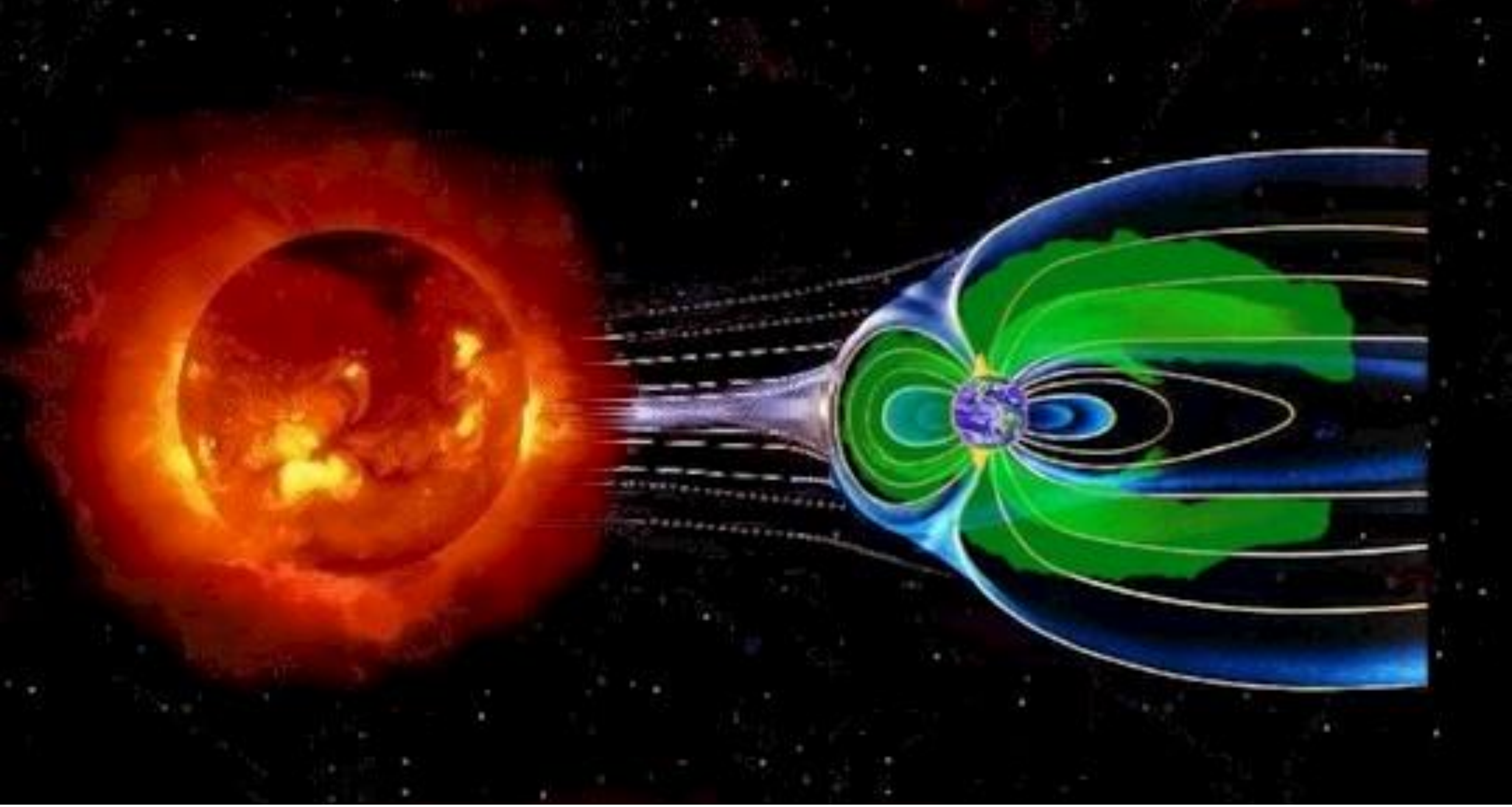
OUTLINE

1. Introduction and motivation
2. Basic algorithm.
3. Plasma model with ion beams
4. Analytical solution for parallel Alfvén modes
5. Kinetic resonant vs reactive instabilities
6. Application to terrestrial foreshock



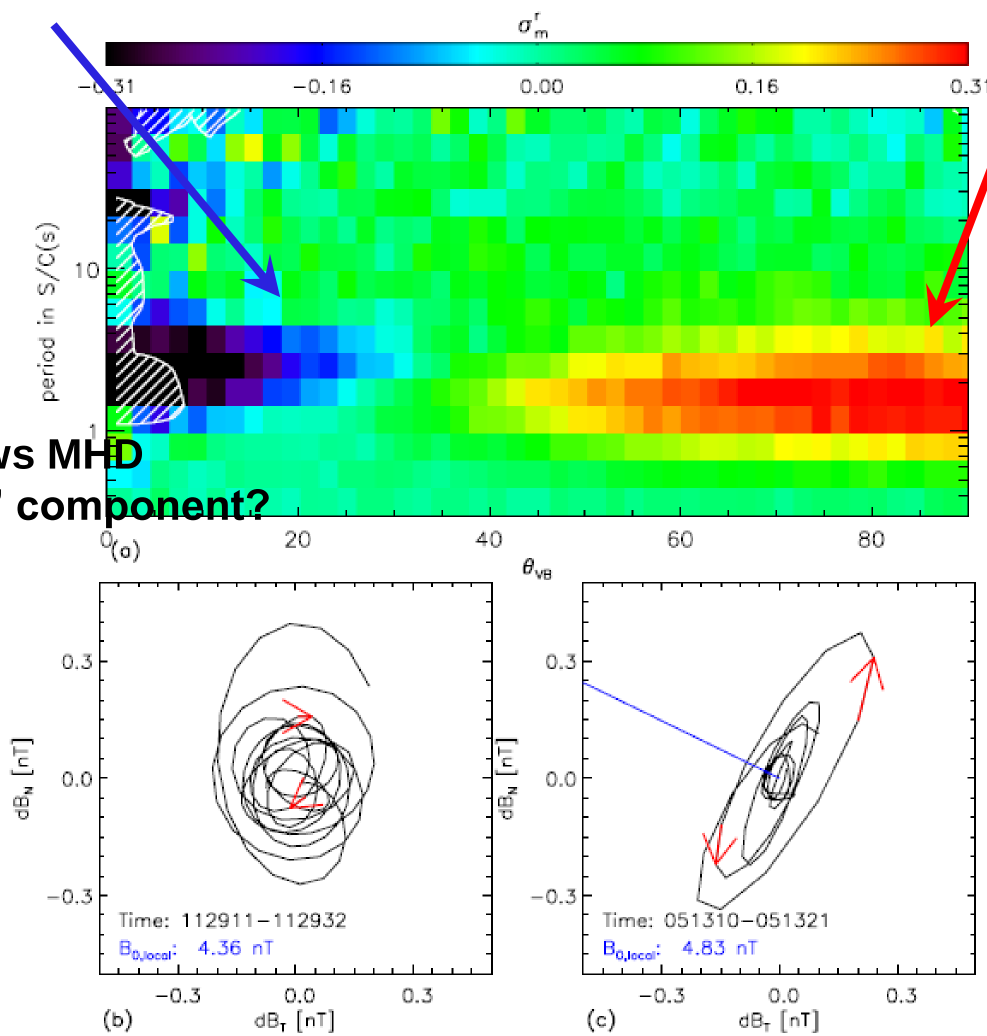
Solar-terrestrial example:
solar magnetic activity -> solar wind -> magnetosphere
-> space weather

WAVES & TURBULENCE !

RECENT OBSERVATIONAL EVIDENCES FOR KAWs

Analysis of polarization (*He et al. 2011,2012; Podesta & Gary 2011*):

TWO ALFVENIC COMPONENTS AT PROTON KINETIC SCALES :
LH QUASI-PARALLEL (15 %) & (DOMINANT) RH QUASI-PERP ALFVEN (85%)



Follows MHD "slab" component?

Follows MHD 2D component?

He et al. (2011,2012)

At small wave lengths we meet natural length scales reflecting plasma microstructure:

- ion gyroradius ρ_i (reflects gyromotion and ion pressure effects);
- ion gyroradius at electron temperature ρ_s (reflects electron pressure effects);
- ion inertial length δ_i (reflects effects due to ion inertia), and
- electron inertial length δ_e (reflects effects due to electron inertia).
- **→ kinetic theory →**
- **kinetic waves and wave-particle interactions**

Vlasov, A. A. – founder of the plasma kinetic theory. Author of concepts "collective interactions – Vlasov equation" and "plasma dispersion equation" (1938)

Landau, L. D. – complemented Vlasov's theory by "collisionless dissipation - Landau damping" (1946)

Alfvén, H. – theoretical prediction of magneto–hydrodynamic waves – MHD Alfvén waves (1946)

Hasegawa, A. – kinetic theory of Alfvén modes at small perpendicular wavelengths – kinetic Alfvén waves (1974-1980)

Vlasov equation (1938):

$$\frac{\partial f_1}{\partial t} + \mathrm{div}_{\mathbf{r}}\, \mathbf{v} f_1 + \frac{e_1}{m_1}\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right) \mathrm{grad}_{\mathbf{v}}\, f_1 = \left[\frac{\partial f_1}{\partial t}\right]_{11}^{\mathrm{st}} + \left[\frac{\partial f_1}{\partial t}\right]_{12}^{\mathrm{st}} + \left[\frac{\partial f_1}{\partial t}\right]_{13}^{\mathrm{st}},$$

$$\frac{\partial f_2}{\partial t} + \mathrm{div}_{\mathbf{r}}\, \mathbf{v} f_2 + \frac{e_2}{m_2}\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right) \mathrm{grad}_{\mathbf{v}}\, f_2 = \left[\frac{\partial f_2}{\partial t}\right]_{21}^{\mathrm{st}} + \left[\frac{\partial f_2}{\partial t}\right]_{22}^{\mathrm{st}} + \left[\frac{\partial f_2}{\partial t}\right]_{23}^{\mathrm{st}},$$

$$\frac{\partial f_3}{\partial t} + \mathrm{div}_{\mathbf{r}}\, \mathbf{v} f_3 = \left[\frac{\partial f_3}{\partial t}\right]_{31}^{\mathrm{st}} + \left[\frac{\partial f_3}{\partial t}\right]_{32}^{\mathrm{st}} + \left[\frac{\partial f_3}{\partial t}\right]_{33}^{\mathrm{st}},$$

$$\mathrm{div}\, \mathbf{E} = 4\pi\rho, \quad \mathrm{rot}\, \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j},$$

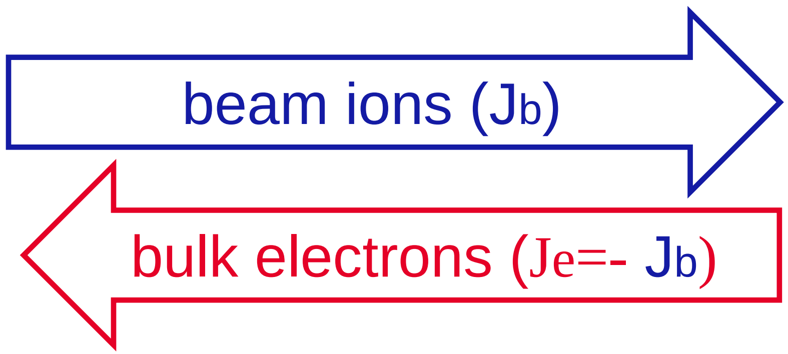
$$\mathrm{div}\, \mathbf{H} = 0, \quad \mathrm{rot}\, \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t},$$

$$\rho = e_1 \int_{-\infty}^{+\infty} f_1\, d\xi\, d\eta\, d\zeta + e_2 \int_{-\infty}^{+\infty} f_2\, d\xi\, d\eta\, d\zeta,$$

$$\mathbf{j} = e_1 \int^{+\infty} \mathbf{v} f_1\, d\xi\, d\eta\, d\zeta + e_2 \int^{+\infty} \mathbf{v} f_2\, d\xi\, d\eta\, d\zeta.$$

COMPENSATED-CURRENT SYSTEMS DUE TO BEAMS

Ion beams injected in a plasma generate there the return electron currents compensating the original beam current:

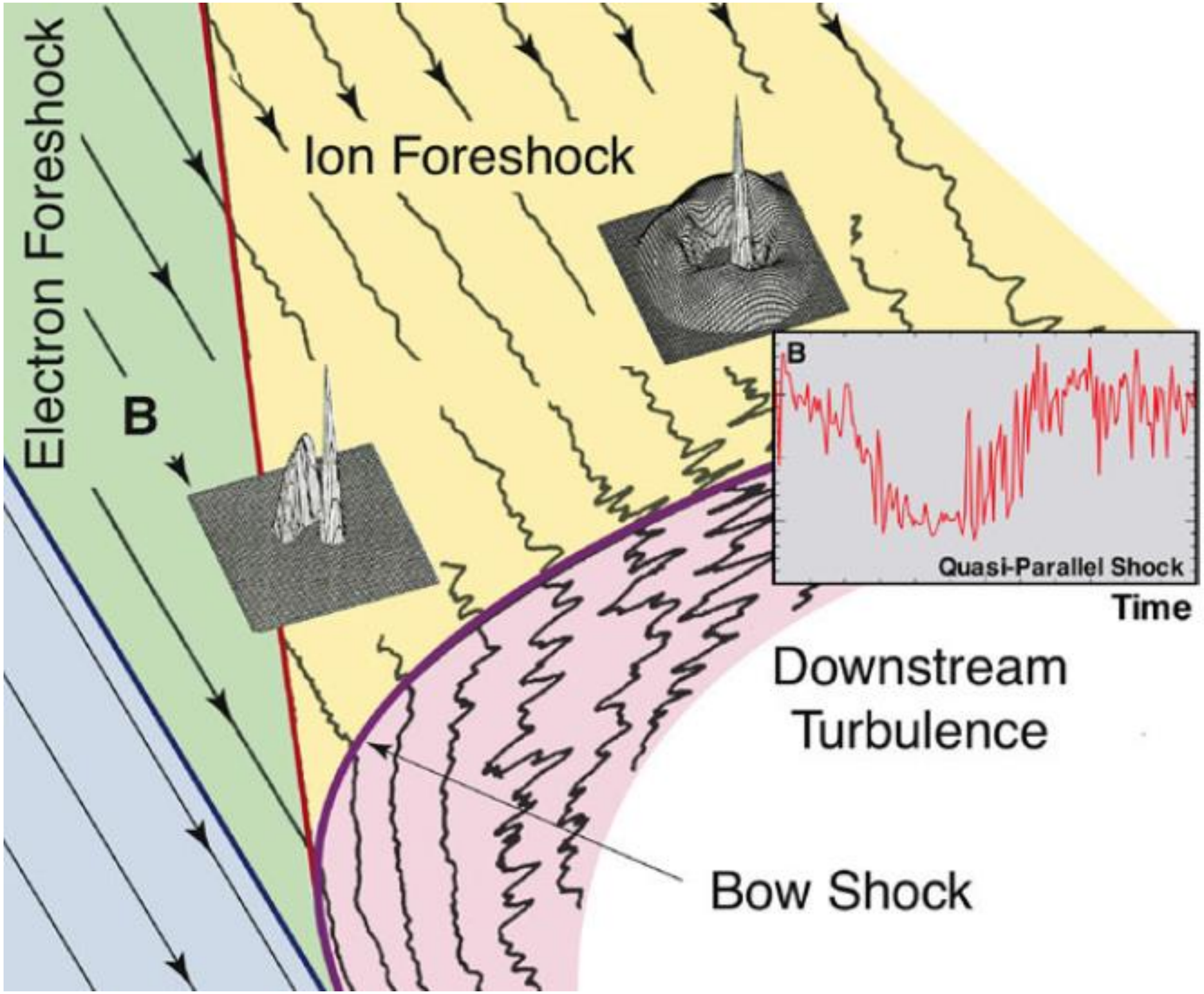


A compensated-current system is formed with zero total: $J_e + J_b = 0$.

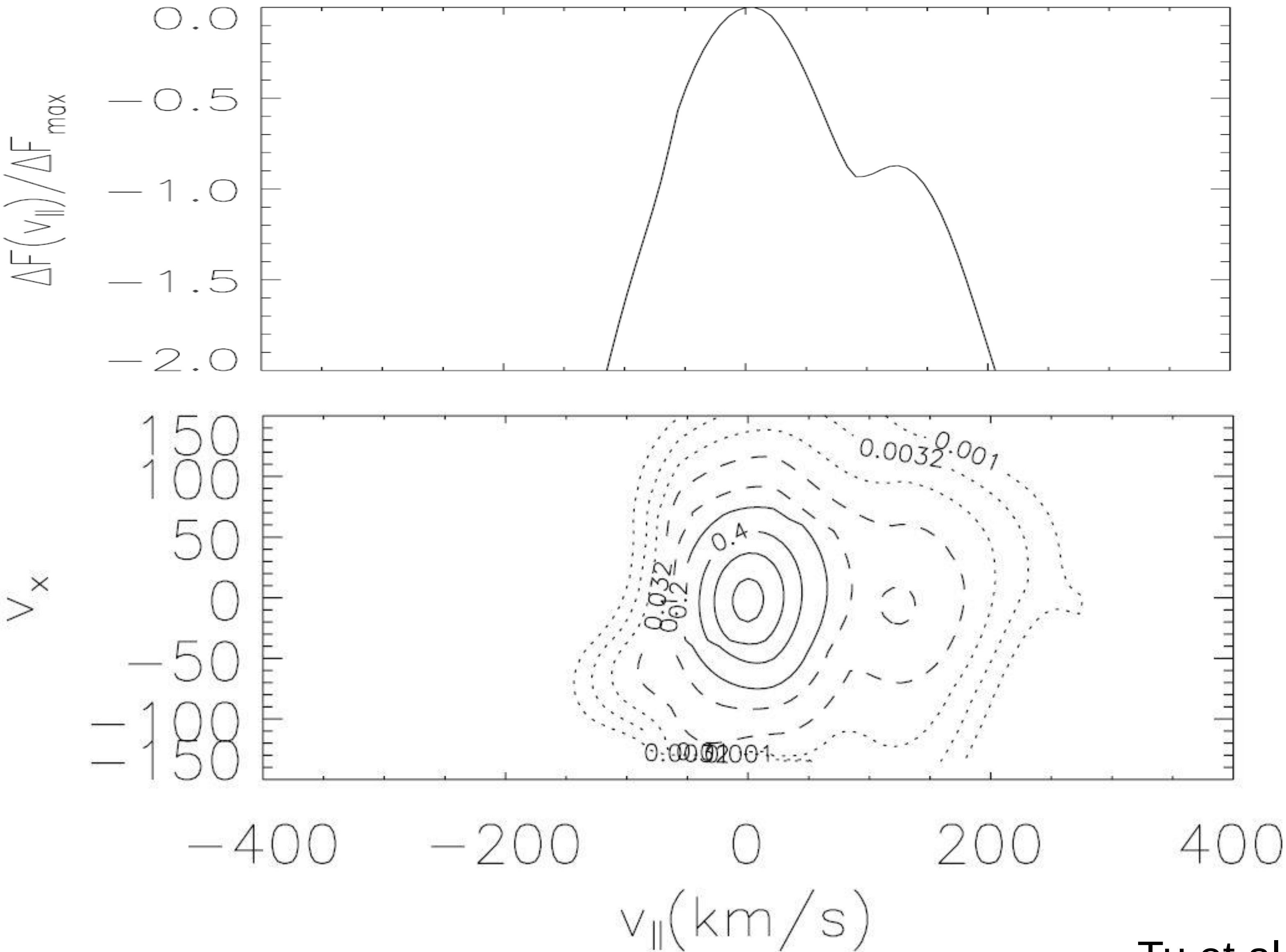
$$n_{\mathrm{e}} u_{z\mathrm{e}} = n_{\mathrm{b}} u_{\mathrm{b}}$$

Typical examples: foreshocks in solar wind and supernova remnants.

ION BEAMS IN TERRESTRIAL FORESHOCK



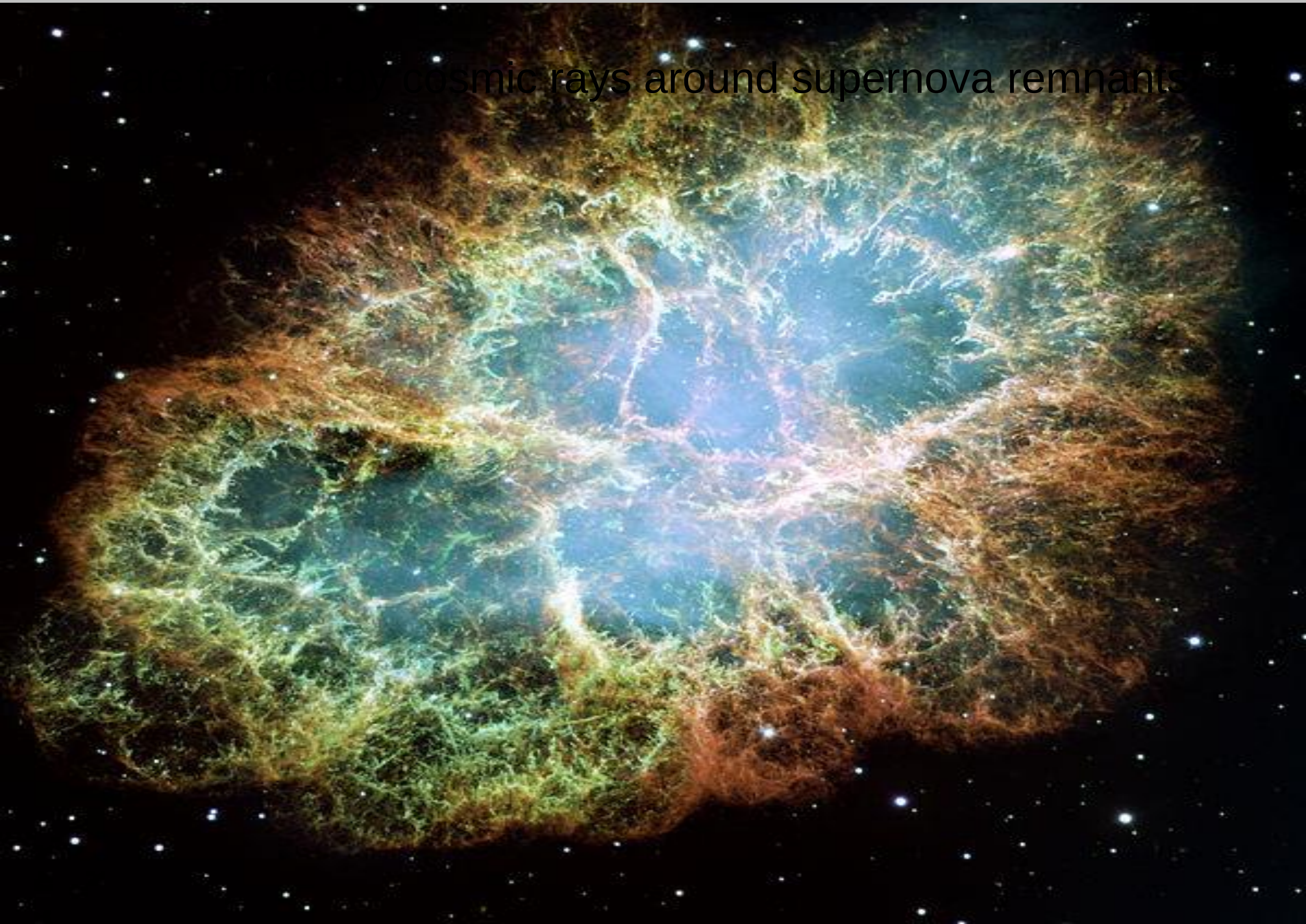
ION BEAMS IN SOLAR WIND



Tu et al. (2004)

ION BEAMS AROUND SUPERNOVA REMNANTS

cosmic rays around supernova remnants



BASIC ALGORITHM

Consider a uniform, fully ionized hydrogen plasma (electrons e and protons i) immersed in the background magnetic field B_0.

Use two fluctuating electromagnetic potentials, scalar $\tilde{\phi}$ and vector $\tilde{\mathbf{A}}$ that obey Ampére law

$$\boxed{\sum_\alpha \tilde{\mathbf{j}}_\alpha = -\frac{c}{4\pi}\nabla^2\tilde{\mathbf{A}},}$$

(A

and Poisson law

$$\boxed{\nabla^2\tilde{\phi} = 4\pi\sum_\alpha e_\alpha\tilde{n}_\alpha.}$$

(P

$\tilde{n}_\alpha = \tilde{n}_\alpha(\tilde{\phi},\tilde{\mathbf{A}})$ and $\tilde{\mathbf{j}}_\alpha = \tilde{\mathbf{j}}_\alpha(\tilde{\phi},\tilde{\mathbf{A}})$ are the number density and current density perturbations induced self-consistently by $\tilde{\phi}$ and $\tilde{\mathbf{A}}$ in the α-th species (e_α is the particle's charge).

BASIC ALGORITHM

Define VDF $F(t,\mathbf{r},\mathbf{V})$ as number of particles in the 6D velocity ($\mathbf{V}$) and space ($\mathbf{r}$) volume d^3rd^3V:

$$\boxed{F(t,\mathbf{r},\mathbf{V})d^3rd^3V = dN}$$

Kinetic theory: $\tilde{n}_\alpha$ and $\tilde{\mathbf{j}}_\alpha$ have to be calculated using fluctuating parts $\tilde{f}_\alpha$ of velocity distributions

$$\boxed{F_\alpha = F^0_\alpha + \tilde{f}_\alpha}$$

$$\boxed{\tilde{\mathbf{j}} = e\int d^3V\mathbf{V}\tilde{f}; \qquad \tilde{n} = \int d^3V\tilde{f}.}$$

(sys2)

Considering wave processes with timescales shorter than the Coulomb collisional timescales, F satisfies the 'collisionless' Vlasov equation

$$\boxed{d_tF = 0,}$$

(vlas)

where d_t, Vlasov operator, is the full time derivative in the space of independent variables $\{\mathbf{X}\}$:

$$\boxed{d_t = \sum_\alpha \frac{\partial X_\alpha}{\partial t}\frac{\partial}{\partial X_\alpha}.}$$

(der)

Coefficients $\partial X_i/\partial t$ obey the standard dynamic equations of particles' motion in el-mag fields.

BASIC ALGORITHM

We solve the Vlasov equation (ref: vlas) by use of perturbation theory, expanding F and d_t as

$$F = F^0 + f^L,$$

$$d_t = d_t^0 + d_t^L,$$

(pertF)

(pertd)

where 0 and L are unperturbed and perturbed (linear in the wave amplitudes) parts.

The linear response has to be found from the linearized Vlasov equation

$$d_t^0 f^L + d_t^L F^0 = 0 \rightarrow f^L = -[d_t^0]^{-1} d_t^L F^0.$$

(lreq)

Evolution of function F^0, affected by the wave-particles collisions, is determined by averaging over fast (wave) time-scales,

$$d_t^0 F^0 + \overline{d_t^L f^L} = 0 \rightarrow F^0 = -[d_t^0]^{-1} \overline{d_t^L f^L}.$$

(qleq)

Depending of $F^0(t = 0)$, waves can be generated or damped, and F^0 is relaxed or modified.

PLASMA MODEL

All plasma components are modeled by the shifted Maxwellian velocity distributions

$$f_{0\alpha} = \frac{n_\alpha}{(2\pi T_\alpha/m_\alpha)^{3/2}} \exp\left(-\frac{m_\alpha v_\perp^2}{2T_\alpha} - \frac{m_\alpha(v_z - u_\alpha)^2}{2T_\alpha}\right), \qquad \#$$

where n_α, T_α, and u_α are the mean number density, temperature, and parallel bulk velocity, respectively, m_α is the particle mass, and $\mathbf{v} = (v_x, v_y, v_z)$ - velocity-space coordinates. The species α can be background ions (i), background electrons (e), and beam ions (b). The beam electrons (be) can be included if they exist. The subscripts z and $\perp$ indicate directions parallel and perpendicular to $\mathbf{B}_0$. The total charge of the plasma is zero.

The current neutrality is assumed to be satisfied,

$$\sum_e n_e u_{ze} = n_b u_b.$$

WAVE EQUATION

The nontrivial solutions to the $\boxed{\text{Maxwell-Vlasov}}$ set of equations in the form

WAVE FORM →

$$\boxed{\delta f_\alpha \sim \exp(\mathbf{k} \cdot \mathbf{r} - \omega t),} \qquad \#$$

exist if the wave frequency ω and the wave vector $\mathbf{k} = (k_x, k_y, k_z)$ satisfy the following dispersion equation (e.g., Alexandrov, Bogdankevich, & Rukhadze, 1984):

WAVE DISPERSION →

$$\left| k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij} \right| = 0, \qquad \#$$

where ε_{ij} is the dielectric tensor, and δ_{ij} is the Kronecker's

For parallel-propagating modes with $\mathbf{k} \parallel \mathbf{B}_0 \parallel \mathbf{z}$

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \sum_{\alpha}\left(\frac{\omega_{P\alpha}}{\omega}\right)^2 \frac{1}{2} \sum_{n=\pm 1}\left(\frac{\xi_{\alpha,0}}{\xi_{\alpha,n}} J_+(\xi_{\alpha,n})\right);$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = i \sum_{\alpha}\left(\frac{\omega_{P\alpha}}{\omega}\right)^2 \frac{1}{2} \sum_{n=\pm 1}\left(n\frac{\xi_{\alpha,0}}{\xi_{\alpha,n}} J_+(\xi_{\alpha,n})\right);$$

$$\varepsilon_{xz} = \varepsilon_{zx} = \varepsilon_{yz} = \varepsilon_{zy} = 0;$$

$$\varepsilon_{zz} = 1 + \sum_{\alpha}\left(\frac{\omega_{P\alpha}}{k_z V_{T\alpha}}\right)^2 [1 - J_+(\xi_{\alpha,0})],$$

Function

$$J_+(x) = x \exp\left(-\frac{x^2}{2}\right) \int_{i\infty}^{x} dt \exp\left(\frac{t^2}{2}\right),$$

has been introduced by Alexandrov et al. (1984).

It has the following asymptotic expansions:

$$J_+(x) = x^2 + O(x^4) - i\sqrt{\frac{\pi}{2}}\, x \exp\left(-\frac{x^2}{2}\right), \qquad |x| \ll 1;$$

$$J_+(x) = 1 + \frac{1}{x^2} + O\left(\frac{1}{x^4}\right) - i\sqrt{\frac{\pi}{2}}\, x \exp\left(-\frac{x^2}{2}\right), \qquad |x| \gg 1.$$

Argument of J_+ is

$$\xi_{\alpha,n} = \frac{\omega - k_z u_\alpha + n\omega_{B\alpha}}{k_z V_{T\alpha}},$$

where $\omega_{P\alpha}$ ($\omega_{B\alpha}$) is the plasma (cyclotron) frequency,

$V_{T\alpha} = \sqrt{T_\alpha / m_\alpha}$ is the thermal velocity.

For parallel propagation, the dispersion equation (determinant) splits into two independent equations

$$\varepsilon_{xx} \pm i\varepsilon_{xy} = \left(\frac{ck_z}{\omega}\right)^2 \qquad \#$$

describing left-hand (sign -) and right-hand (sign +) polarised waves. In what follows we consider the left-hand polarised Alfvén waves destabilized by the compensated currents.

Taking into account quasineutrality $n_i + n_b = n_e$ and current compensation, equation for Alfvén waves can be simplified to

$$\left(\frac{\omega}{\omega_{Bi}}\right)^2 - \frac{n_b}{n_0} A_{k,\omega} \frac{\omega}{\omega_{Bi}} - \left(\frac{k_z v_A}{\omega_{Bi}}\right)^2 + \frac{n_b}{n_0} A_{k,\omega} \frac{k_z u_b}{\omega_{Bi}} = 0$$

where

$$A_{k,\omega} = 1 + \frac{\omega_{Bi}}{k_z v_{Tb}} \frac{k_z v_{Tb}}{\omega - k_z u_b - \omega_{Bi}} J_+\left(\frac{\omega - k_z u_b - \omega_{Bi}}{k_z v_{Tb}}\right).$$

WAVE EQUATION

A_k can be further expanded in small ω/ω_{Bi}: $A_{k,\omega} = A_{k0} + A_{k1}\omega/\omega_{Bi}$, where coefficients

$$A_{k0} = 1 + \frac{\omega_{Bi}}{k_z v_{Tb}} \frac{J_+(\xi_b)}{\xi_b}; \; A_{k1} = \left(\frac{\omega_{Bi}}{k_z v_{Tb}}\right)^2 (1 - J_+(\xi_b));$$

$$\xi_b = \frac{-k_z u_b - \omega_{Bi}}{k_z v_{Tb}},$$

which gives the second-order eigenmode equation Alfvén waves:

$$\boxed{\left(\frac{\omega}{\omega_{Bi}}\right)^2 - \frac{n_b}{n_0}\left(A_{k0} - \frac{k_z u_b}{\omega_{Bi}} A_{k1}\right)\frac{\omega}{\omega_{Bi}} - \left(\frac{k_z v_A}{\omega_{Bi}}\right)^2 + \frac{n_b}{n_0} A_{k0} \frac{k_z u_b}{\omega_{Bi}} = 0.}$$

This is our main equation.

SOLUTION OF WAVE EQUATION

Remember: $A_k = \mathrm{Re}[A_k] + \mathrm{iIm}[A_k]$!

$$\left(\frac{\omega}{\omega_{Bi}}\right)^2 - \frac{n_b}{n_0}\left(\boxed{A_{k0}} - \frac{k_z u_b}{\omega_{Bi}}\boxed{A_{k1}}\right)\frac{\omega}{\omega_{Bi}} - \left(\frac{k_z v_A}{\omega_{Bi}}\right)^2 + \frac{n_b}{n_0}\boxed{A_{k0}}\frac{k_z u_b}{\omega_{Bi}} = 0.$$

The solution is

$$\frac{\omega}{\omega_{Bi}} = \boxed{\frac{n_b}{2n_0}\left(A_{k0} - \frac{k_z u_b}{\omega_{Bi}}A_{k1}\right)} + \sqrt{\boxed{\left[\frac{n_b}{2n_0}\left(A_{k0} + \frac{k_z u_b}{\omega_{Bi}}A_{k1}\right)\right]^2} + \left(\frac{k_z v_A}{\omega_{Bi}}\right)^2 - \boxed{\frac{n_b}{n_0}A_{k0}\frac{k_z u_b}{\omega_{Bi}}}}.$$

This solution describe three unstable regimes:

1. Reactive instability (when contribution of $\mathrm{Im}[A_k]$ is small).
2. Resonant instability (when $\mathrm{Im}[A_k]$ is significant).
3. Mixed resonant/reactive regime.

SOLUTION OF WAVE EQUATION

1. Resonant instability:
(i) driven by resonant particles;
(ii) depends on local gradients ∂f/∂**V** in the velocity space;
(iii) are defined by imaginary part $\mathrm{Im}[\delta f_\alpha]$ of the particles response function;
(iv) well-known example: bump-in-tail instability driven by inverse Landau damping.

2. Reactive instability:
(i) driven by all particles of the particular specie;
(ii) depends on the bulk parameters of the velocity distribution function (bulk velocity, thermal velocity, density);
(iii) are defined by the real part $\mathrm{Re}[\delta f_\alpha]$ of the particles response function;
(iv) well-known example: firehose instability.

HOT ION BEAMS

Under "very hot" beams we mean the beams with velocities ordering

$$v_{Tb}^2 >> u_b^2 > v_A^2.$$

For such beams, and with additional assumption $k_z v_{Tb}/\omega_{Bi} \gg 1$ (it can be checked posteriory), we can use the small argument expansion of $J_+(\xi_b)$. Then the solution can be simplified to

$$\frac{\omega}{\omega_{Bi}} \approx \frac{n_b}{2n_0} + \sqrt{\left(\frac{n_b}{2n_0}\right)^2 + \left(\frac{k_z v_A}{\omega_{Bi}}\right)^2 - \frac{n_b u_b}{n_0 v_A}\left(\frac{k_z v_A}{\omega_{Bi}}\right)}$$

The maximum growth rate

$$\frac{\gamma_{\max}}{\omega_{Bi}} = \frac{1}{2}\frac{n_b}{n_0}\sqrt{\left(\frac{u_b}{v_A}\right)^2 - 1}$$

is achieved at

$$\frac{k_z v_{Tb}}{\omega_{Bi}} = \frac{1}{2}\alpha_b,$$

where we introduce the cumulative destabilizing parameter

$$\boxed{\alpha_b = \frac{n_b}{n_0}\frac{u_b}{v_A}\frac{v_{Tb}}{v_A}.}$$

REACTIVE INSTABILITY

Let us present the growth rate (ref: g) in the following useful form:

$$\frac{\gamma_k}{\omega_{Bi}} = \frac{V_b}{V_{Tb}} k_z \rho_{Tb} \sqrt{1 - \frac{V_A^2}{V_b^2} - \left(1 - \frac{V_A^2}{V_b^2}\frac{\alpha_b}{2}\frac{A_k}{k_z \rho_{Tb}}\right)^2}. \qquad \text{\# (g0)}$$

From (ref: g0), the instability condition is

$$1 - \frac{V_A^2}{V_b^2} > \left(1 - \frac{V_A^2}{V_b^2}\frac{\alpha_b}{2}\frac{A_k}{k_z \rho_{Tb}}\right)^2. \qquad \text{\# (cond)}$$

The maximum of γ_k,

$$\frac{\gamma_{\max}}{\omega_{Bi}} \approx \frac{1}{2}\frac{n_b V_b}{n_0 V_A}\sqrt{\left(1 - \frac{V_A^2}{V_b^2}\right)\left[1 - \left(\frac{\alpha_b^{\mathrm{thr}}}{\alpha_b}\right)^2\right]}, \qquad \text{\# (gm)}$$

occurs at

$$k_z^{\mathrm{m}} \rho_{Tb} \approx \frac{\alpha_b}{2} + \frac{V_b}{V_{Tb}}\frac{2}{\alpha_b}\left(1 - 2\frac{V_A^2}{V_b^2}\right). \qquad \text{\# (km)}$$

REACTIVE INSTABILITY

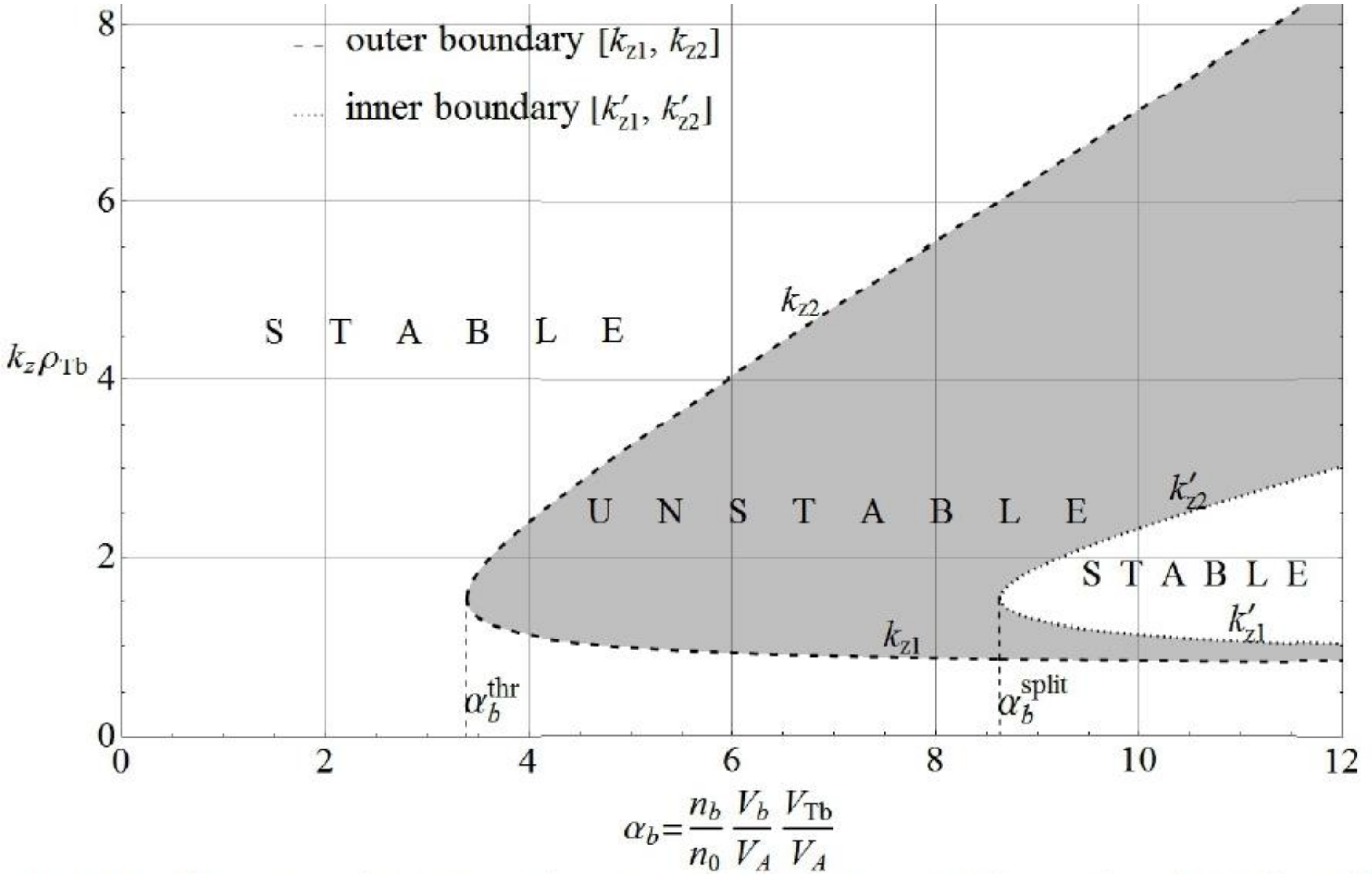


Fig. 4. Unstable wavenumber ranges in the (α_b, k_z) plane for $V_A/V_b = 0.9$. The outer boundary is defined by the left-hand side and the inner boundary by the right-hand side of the condition (17). It is seen that below α_b^{thr} there is no instability, at $\alpha_b^{\mathrm{thr}} < \alpha_b < \alpha_b^{\mathrm{split}}$ there is a single unstable range of k_z, and above $\alpha_b^{\mathrm{split}}$ there are two unstable ranges.

Reactive instability: unstable wavenumber range (splits above some alpha)

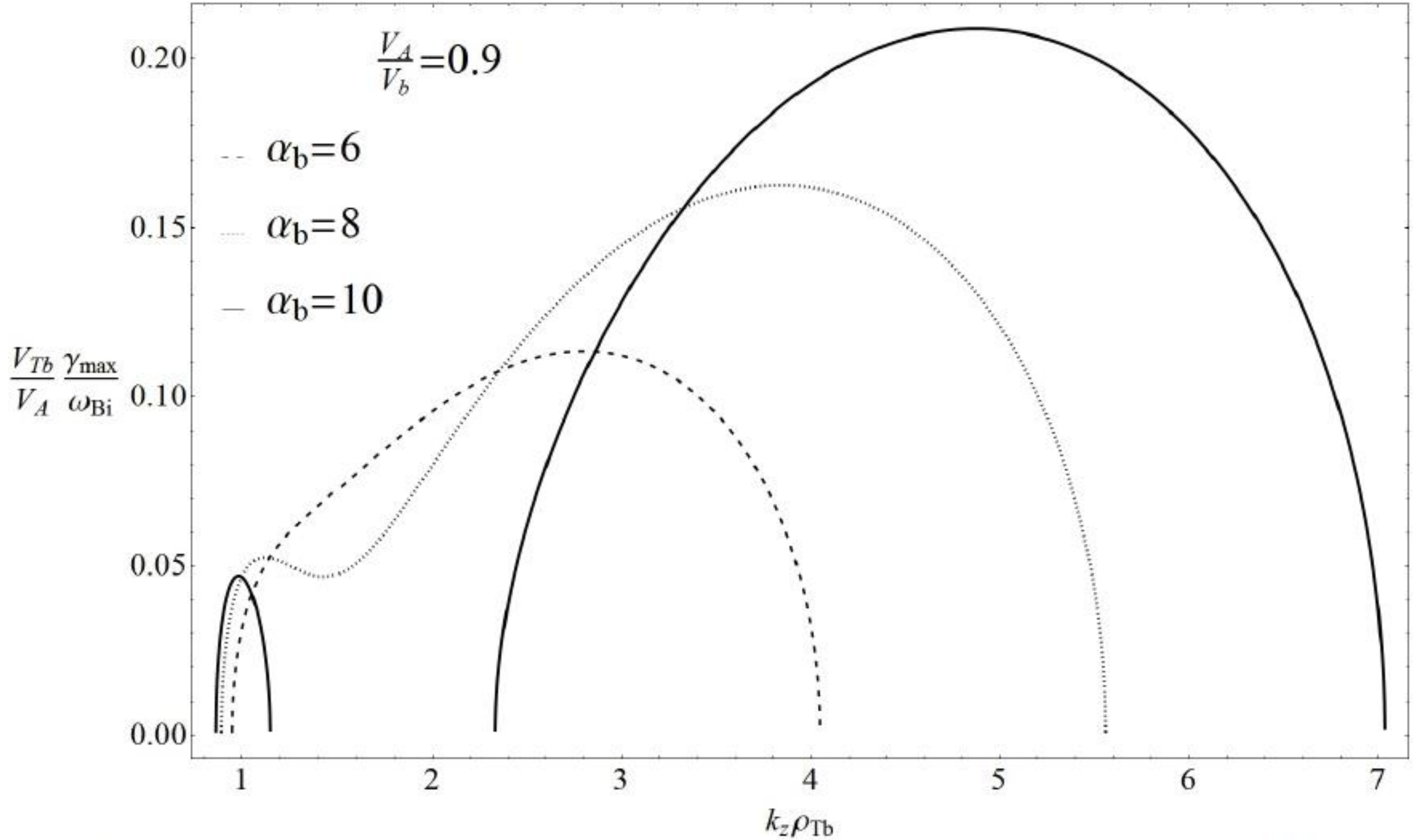


Fig. 5. Wavenumber dependence of the instability growth rate driven by super-Alfvénic ion beams with $V_A/V_b = 0.9$ for three values of α_b: $\alpha_b = 6$, 8, and 10. For larger α_b, the unstable area and the maximum growth rate extend to larger $k_z\rho_{Tb}$.

Reactive instability: growth rate as function of k (two peaks at large alpha)

REACTIVE INSTABILITY

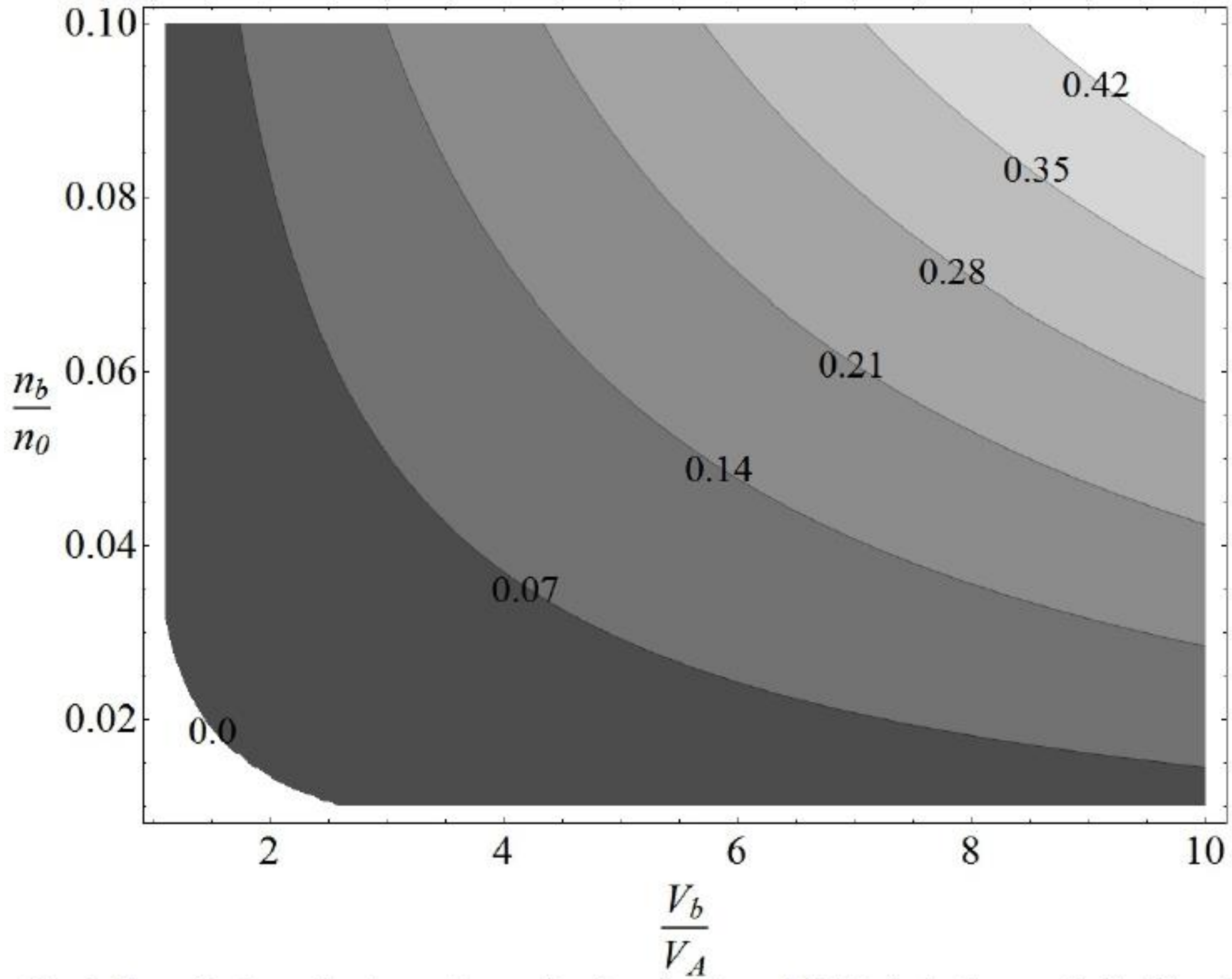


Fig. 6. Normalized growth rate $\gamma_{\max}/\omega_{Bi}$ as function of n_b/n_0 and V_b/V_A for hot beam with $V_{Tb}/V_A = 10^2$. $\gamma_{\max}$ is regularly increasing with both n_b/n_0 and V_b/V_A once the threshold is exceeded.

Reactive instability: maximum growth rate in parameter space

REACTIVE INSTABILITY

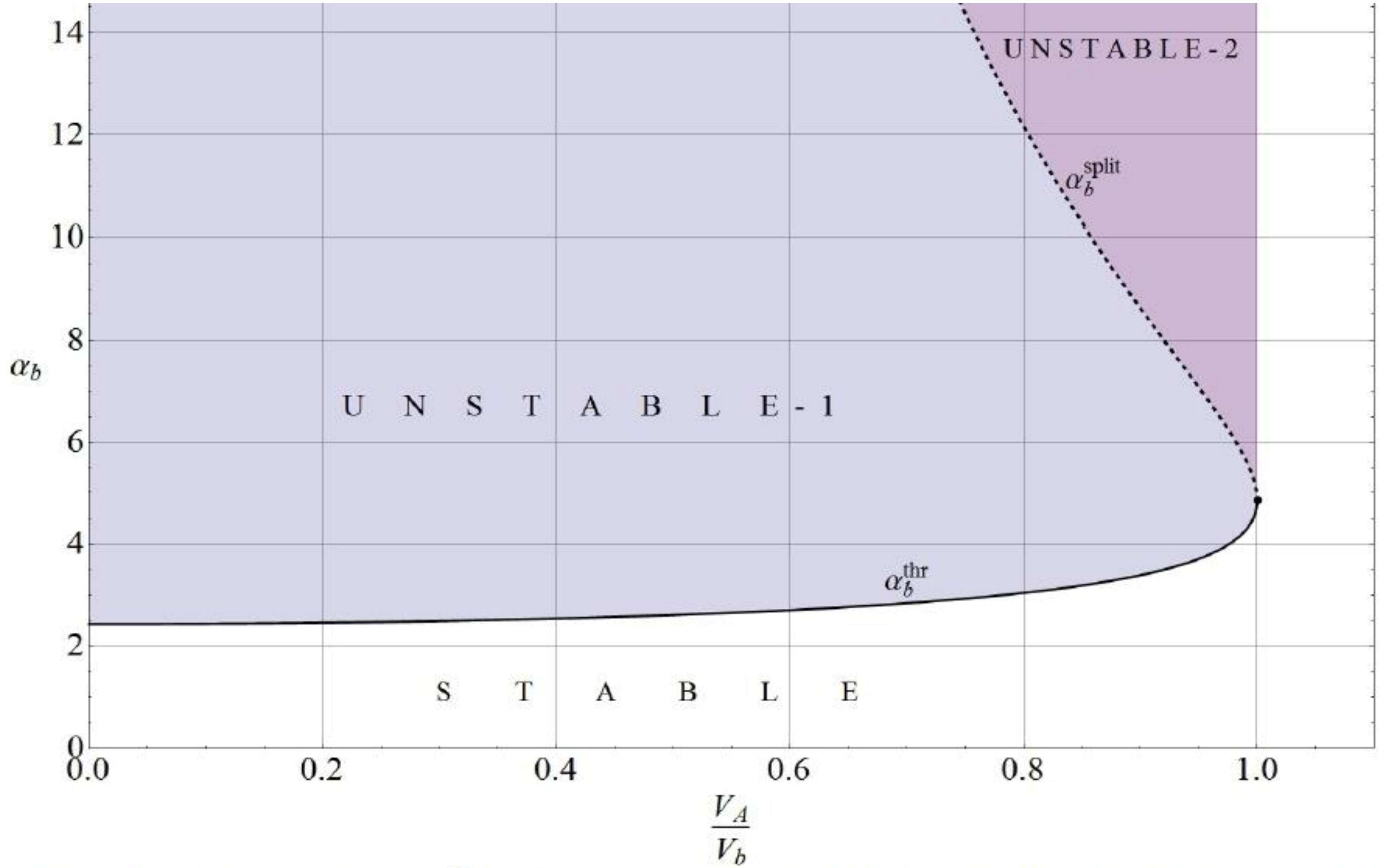


Fig. 1. The instability threshold α_b^{thr} in the parameter space $(\alpha_b, V_A/V_b)$ (solid line); the CCPI develops at all $\alpha_b > \alpha_b^{thr}$. The dashed line shows the split threshold α_{b2}^{thr} above which there are two separate ranges of unstable wavenumbers k_z.

Fig.3a. Reactive instability: unstable range in parameter space

The unstable wavenumber range is

$$\boxed{1 < \frac{k_z v_{Tb}}{\omega_{Bi}} < \alpha_b}$$

The absolute threshold is determined by two conditions:

$$\boxed{\begin{array}{c} u_b \gtrsim v_A; \\ \alpha_b \gtrsim 5. \end{array}}$$

For well super-Alfvén beams, $u_b \geq 2v_A$, $\alpha_b > 2.5$.

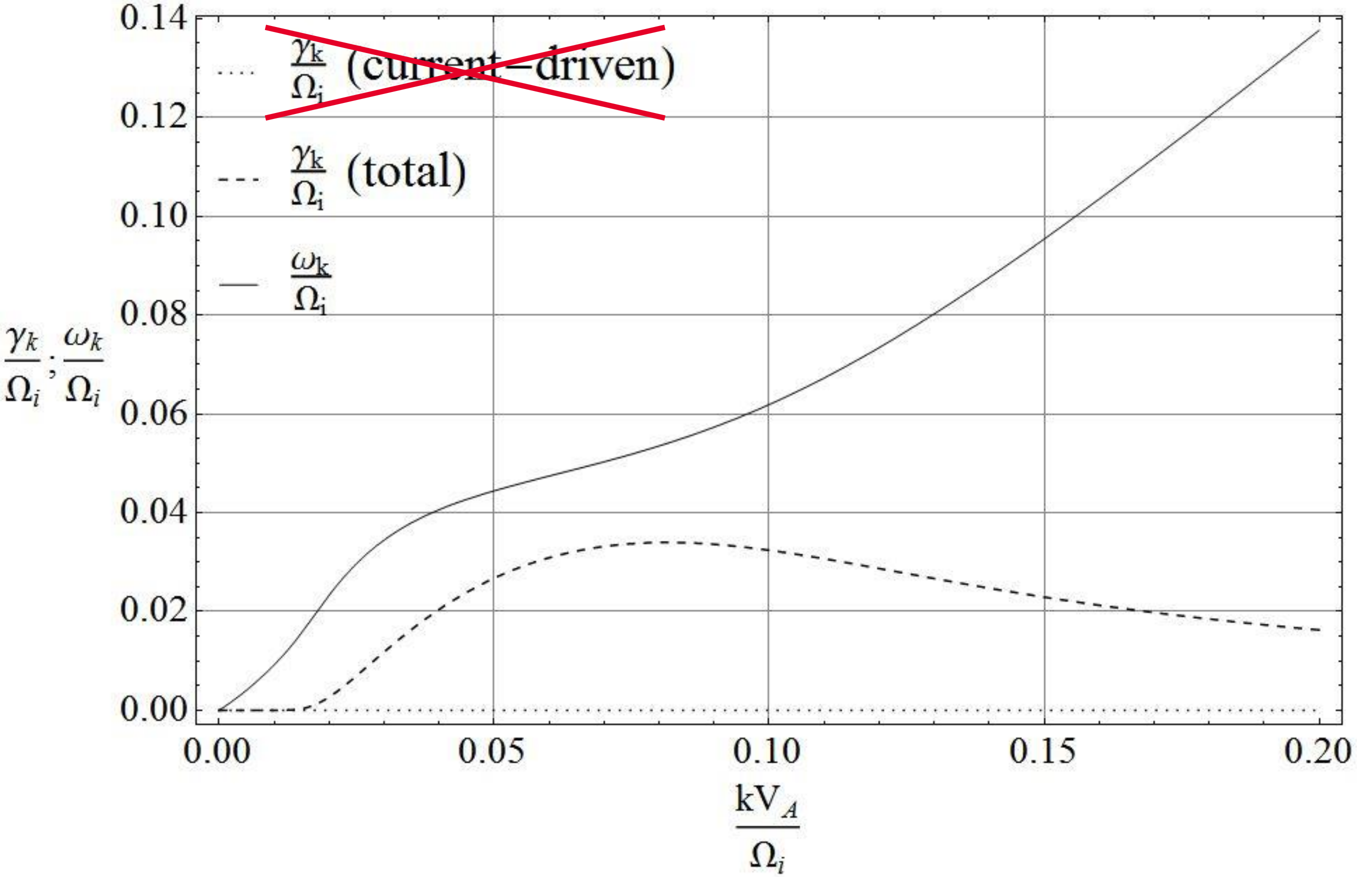


Fig.4a. Growth rate from our analytical solution

$$\frac{\omega}{\omega_{Bi}} = \frac{n_b}{2n_0}\left(A_{k0} - \frac{k_z u_b}{\omega_{Bi}} A_{k1}\right) + \sqrt{\left[\frac{n_b}{2n_0}\left(A_{k0} + \frac{k_z u_b}{\omega_{Bi}} A_{k1}\right)\right]^2 + \left(\frac{k_z v_A}{\omega_{Bi}}\right)^2 - \frac{n_b}{n_0} A_{k0} \frac{k_z u_b}{\omega_{Bi}}}.$$

Fig.4. Growth rate from numerical solutions by Gary (1985)

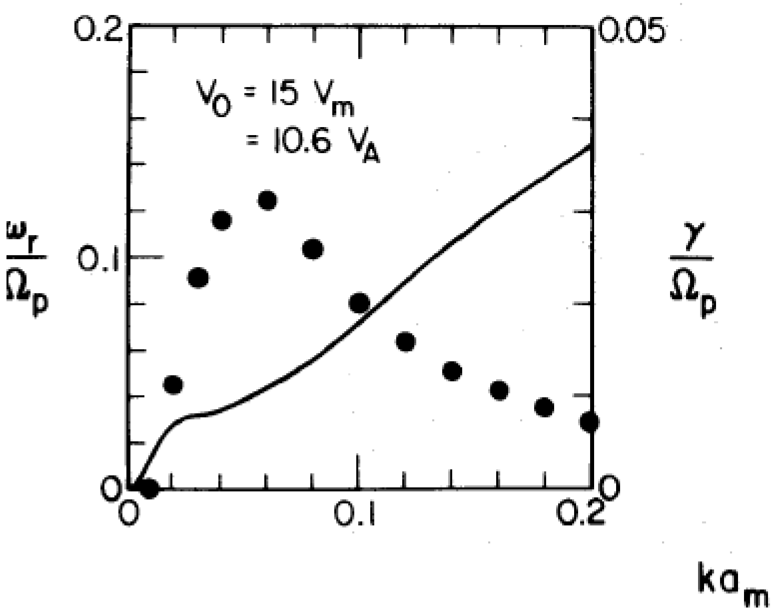
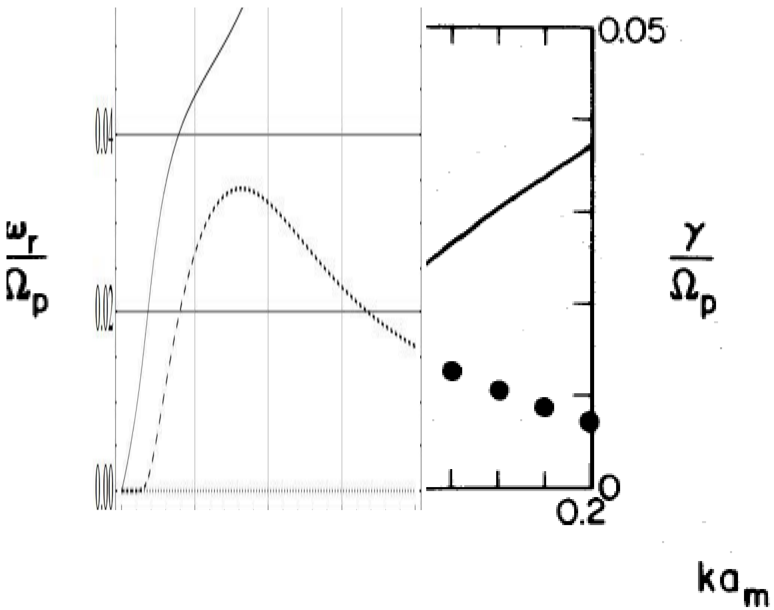


Fig.4b. Growth rate of kinetic instability: numerical calculations by Gary (1984) and our analytical expression fit each other



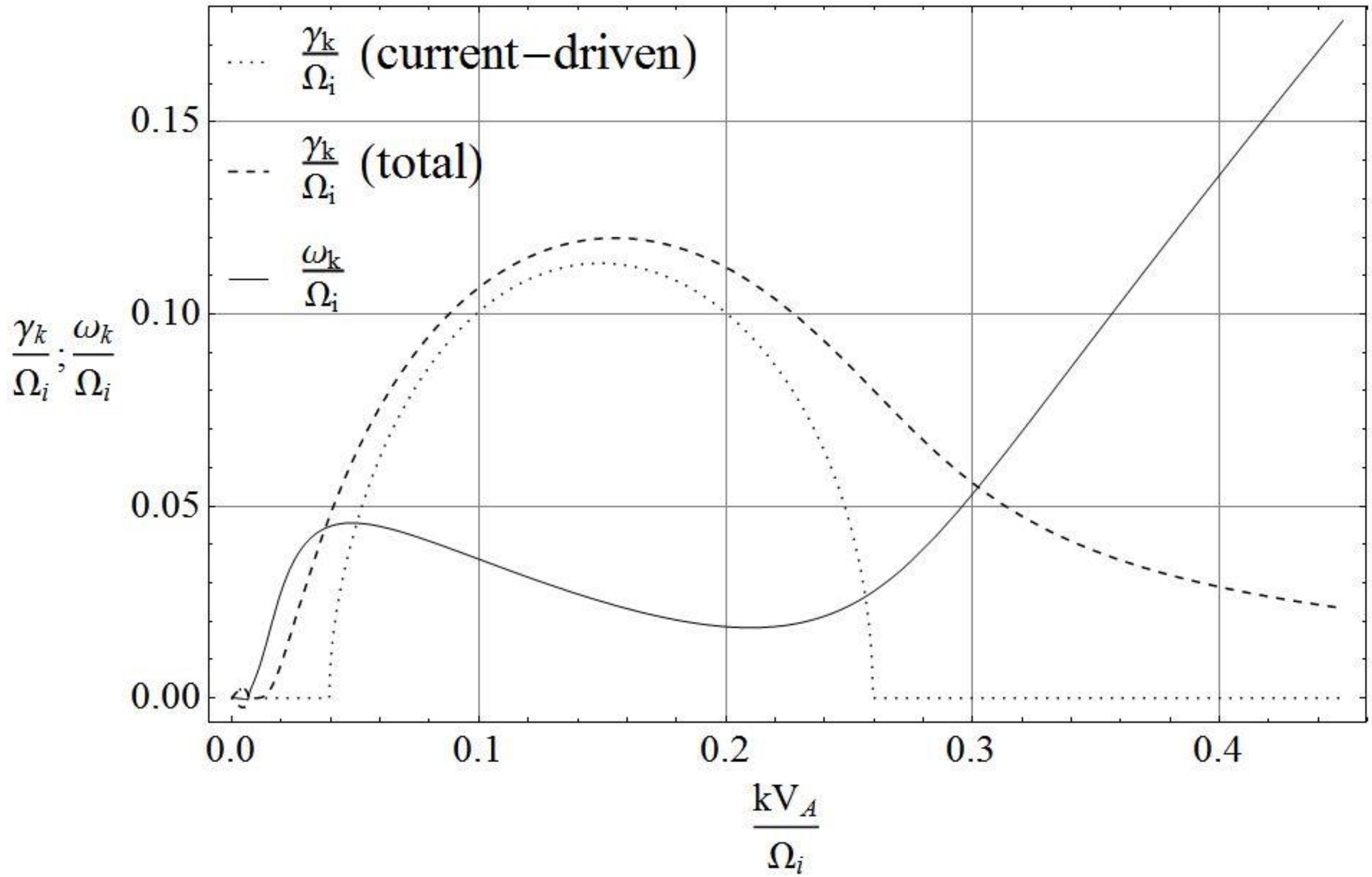


Fig.5. Contribution of the reactive (current-driven) instability to the total growth rate: analytical solution

SUMMARY: THEORY

We have a concise analytical solution describing parallel-propagating Alfvénic modes in the compensated-current systems:

$$\frac{\omega}{\omega_{Bi}} = \frac{n_b}{2n_0}\left(A_{k0} - \frac{k_z u_b}{\omega_{Bi}}A_{k1}\right) + \sqrt{\left[\frac{n_b}{2n_0}\left(A_{k0} + \frac{k_z u_b}{\omega_{Bi}}A_{k1}\right)\right]^2 + \left(\frac{k_z v_A}{\omega_{Bi}}\right)^2 - \frac{n_b}{n_0}A_{k0}\frac{k_z u_b}{\omega_{Bi}}}.$$

Resonant instability has lower threshold for drifting-Maxwellian velocity distributions

BUT

reactive (current-driven) instability becomes dominant as soon as its threshold is exceeded.

RELATION TO THE BELL INSTABILITY

Our analytical solution is universal. It describes all regimes of the parallel-propagating Alfvénic mode and its instabilities in compensated-current systems.

In the asymptotic over-threshold regime our solution becomes reactive (current-driven) and identical to the instability discovered by Bell (2004).

As the system relaxes back to the near-threshold regime, its nature becomes mixed, reactive-kinetic. In this regime formalism by Bell (2004) is inapplicable.

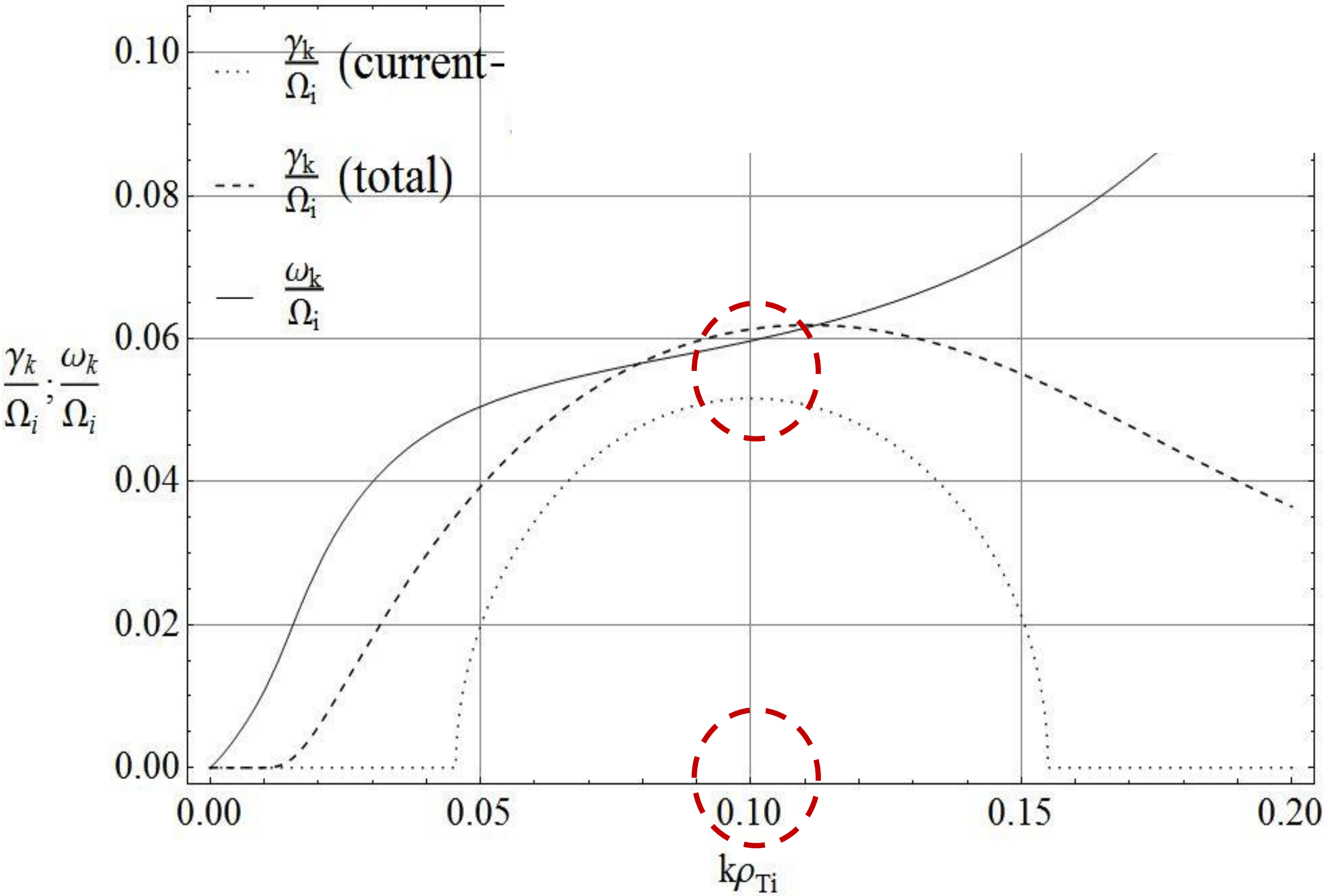
APPLICATION: QUASI-PARALLEL FORESHOCK

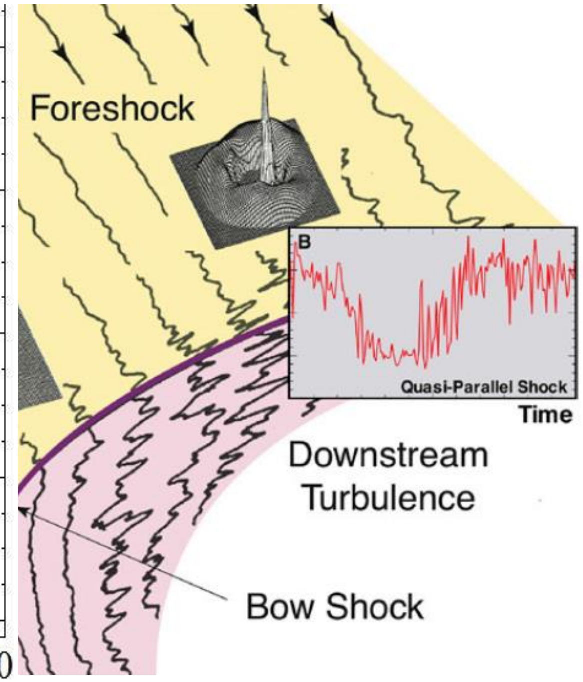
Diffusive ion beams in quasi-parallel foreshock region:

$$\frac{T_b}{T_i} = 500; \quad \frac{V_b}{V_A} = 4; \quad \frac{V_{Tb}}{V_A} = 25; \quad \frac{n_b}{n_0} = 0.04.$$

For these beam parameters,

$$\alpha_b = \frac{n_b}{n_0}\frac{u_b}{v_A}\frac{v_{Tb}}{v_A} = 4,$$





Quasi-parallel wave propagation $\theta_{kB} < 45^{\circ}$ in the terrestrial foreshock (Narita et al 2006)

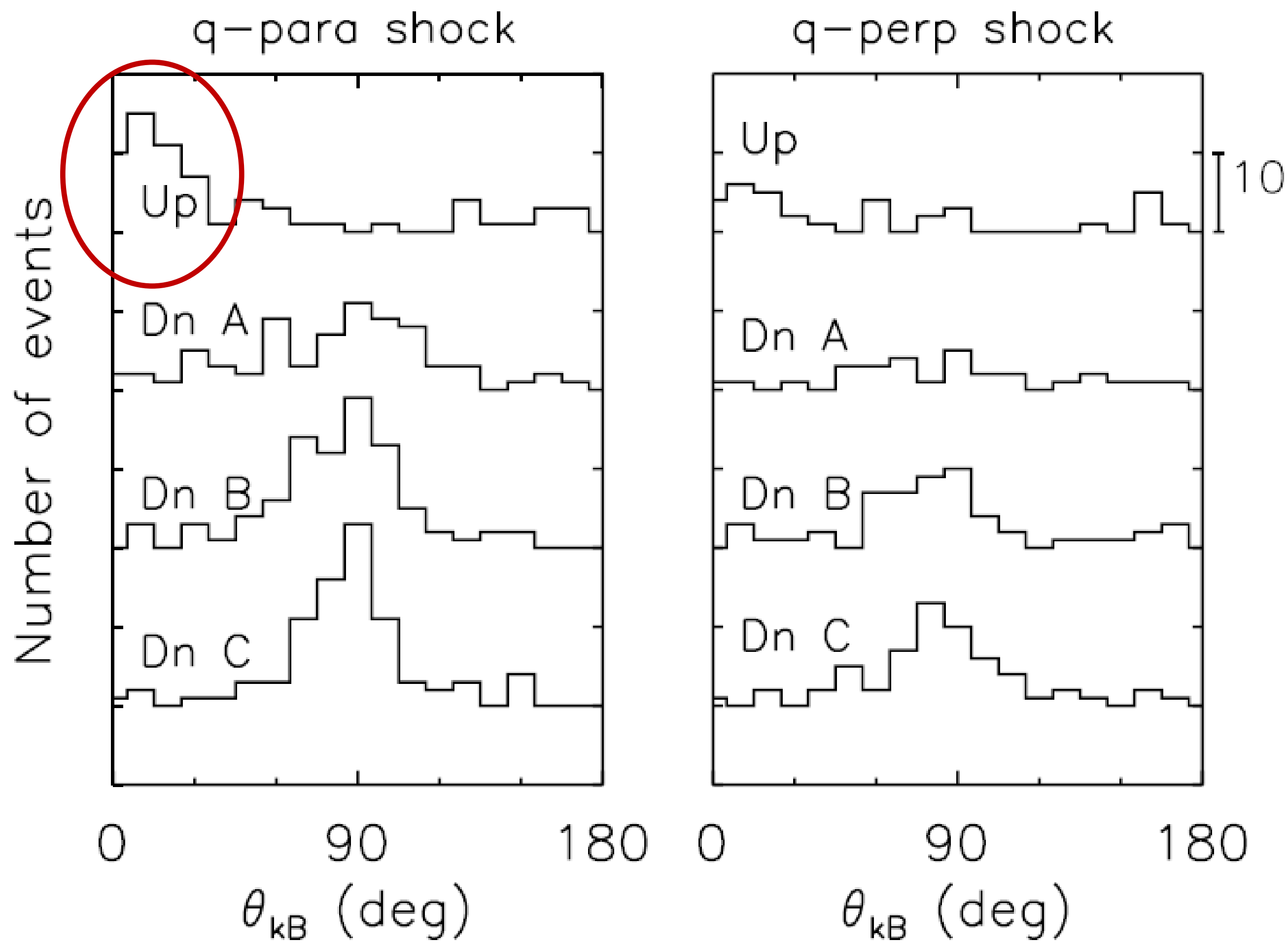


Figure 13. Histogram of angles between the wave vector and the background magnetic field. Panel format is same as

Bi-modal wave spectrum in the terrestrial foreshock (Narita et al 2006)

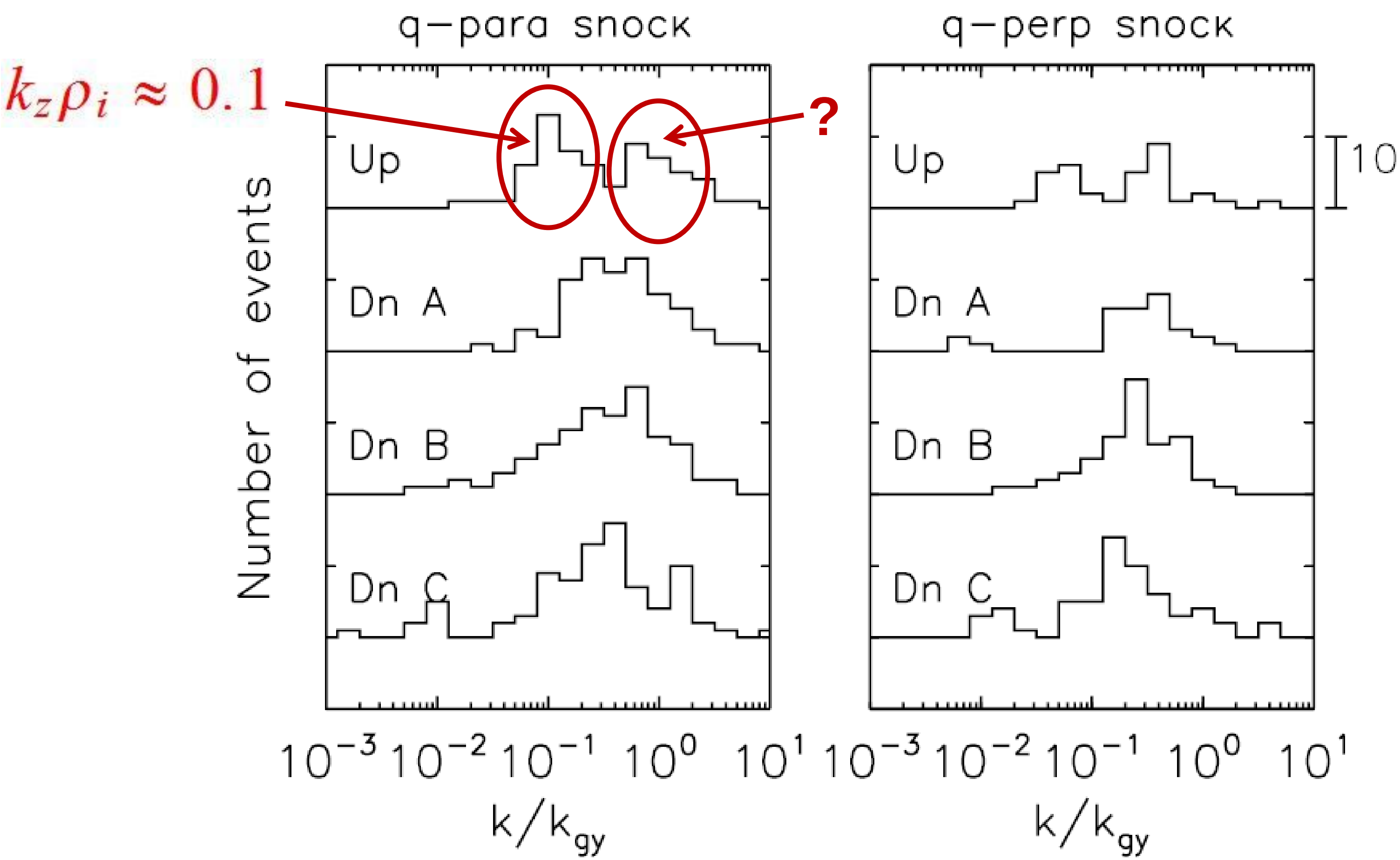


Figure 9. Histogram of wave numbers divided by the one for thermal ion gyroradius. Panel format is same as Figure 7.