Space plasma complexity *(from time series analysis)*: approaches and methods

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Outlook of the lectures

1. **Space plasma complexity: approaches and methods – I**

    1. **Introduction**
    2. **Complexity: definition(s) and a conceptual brief overview**
    3. **Space Plasma Complexity: short review of data analysis methods**
    4. **Approaches and Methods to assess the Power Spectral Density from time series analysis (mostly practicalities for space plasma data analysis)**
    5. **Approaches and Methods to assess the Probability Distribution Functions and their moments from time series analysis**

1. **Space plasma complexity: approaches and methods – II**

    1. **Approaches and Methods for Dynamical analysis with Wavelets applied on time series analysis**
    2. **Approaches and Methods adapted for the Multifractal Analysis of time series**
    3. **Brief Introduction to the Interactive Nonlinear Analysis (INA) library**
    4. **Summary of the Lectures; homework**

Terminology

- **Turbulence:** irregular, efficient transfer of energy over a broad range of scales and emergence of « active » processes that enable this transfer (e.g. eddies, waves, coherent structures)
- **Criticality**: physical equivalence of all temporal and spatial scales (*Consolini and Chang, 2001*), « a phenomenon of true infinity » characterized by **power law singularities**
  - **Self Organized Criticality**: ``spontaneous emergence of criticality in complex dissipative systems'' (*Bak et al., 1987*); F/SOC magnetosphere driven by the solar wind (*Chang, 1992*).
- **Intermittency**: dynamical emergence of burstiness followed/preceeded by quiteness, sporadic and localized interactions (*Chang et al., 2004*)
- **Complexity: all/nothing** from above and more

T. Chang, Space Plasmas, Dynamical Complexity in,
Encyclopedia of Complexity and Systems Science, Springer-Verlag, 2009

Complexity: definition(s)

- **Complexity:**

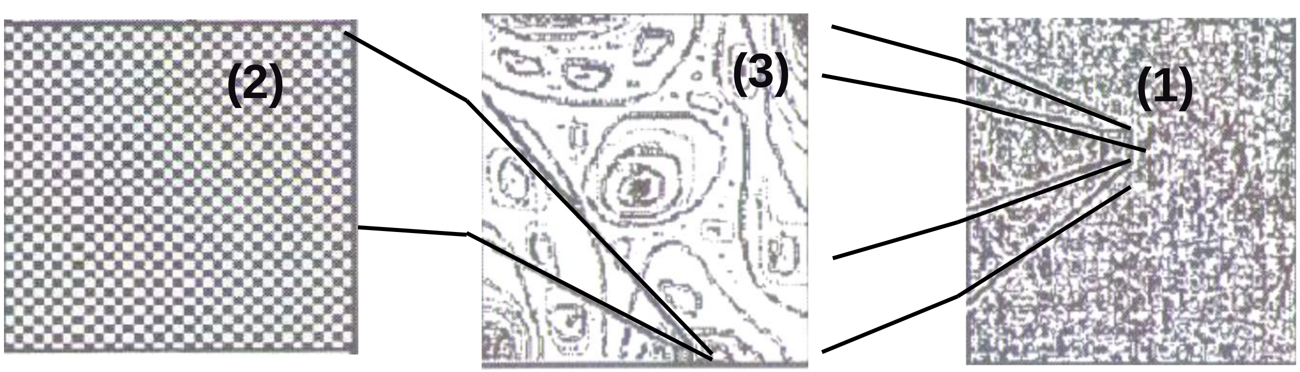
➢ ***in space plasma*** : the simultaneous manifestation of mutually nonlinearly interacting dynamical phenomena/structures (waves, coherent structures, convective forms, nonlinear solitary structures, pseudo-equilibrium configurations, etc.);

- **Levels of Complexity in the Universe** (from D.M. Keirsey – the structure of Existence in « The Evolution of Complexity », VUB Press, 1999)

Complexity Level	Species Complexity	Major species	MacroSystem lineage
1	Quanta	Photon	Universe
2	Hadron	Proton	Galaxy System
3	Atom	Hydrogen	Star System
4	Molecule	Water	Planetary System
5	Cell	Cyanobacteria	Biological System (Earth)
6	Organism	Plankton	Organism System
7	Family	Ant Colonies	Family System (Earth)
8	Society	Human Nations	Societal System (Earth)
9	Cyber Societies	Electronic Civilization	CyberSocietal System (Earth)

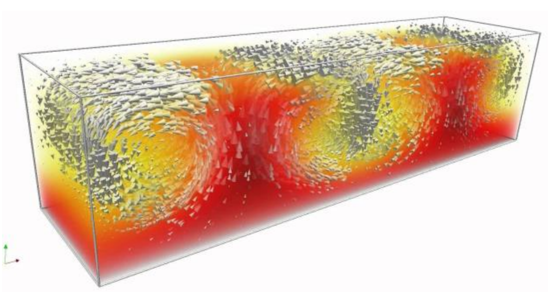
Dynamical Complexity

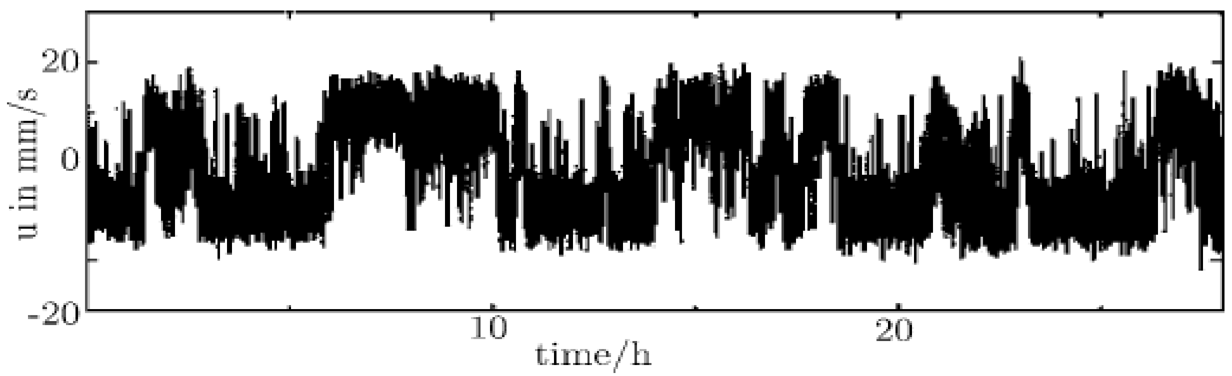
- **<u>Definition:</u>**
  - is a property of systems to manifest complex interactions and exchange between a large variety of component parts that render « the system to behave diffferently than the sum of its components » (e.g. neurons, stock market, neural networks, plasmas, universe)
  - in space plasma : the simultaneous manifestation of mutually nonlinearly interacting dynamical phenomena/structures (waves, coherent structures, convective forms, nonlinear solitary structures, pseudo-equilibrium configurations, etc., *Chang, Introduction to Space Plasma Complexity, 2015*);
- **<u>Two components:</u>**
  - the system "atomic" parts (e.g. the electrons and ions in a plasma) leads to **<u>distinction</u>** and *chaos and disorder* (e.g. the brownian motion in a gas)
  - the connections between parts (e.g. long range correlations in a plasma) leads to **<u>variety</u>** leads to *order* (e.g. a crystal)
  - Complexity exists only when the two are simultaneously present
- **<u>Three regimes:</u>**
  - **(1) Chaos –** dominated by the "atomic" parts
  - **(2) Order -** dominated by the connections between parts
  - **(3) Edge to chaos: #COMPLEXITY** (*B. Edmonds, What is Complexity*, in The Evolution of Complexity1999)



Dynamical Complexity

- **Symmetries:** invariance of a pattern under a group of transformations
  - **Ordered systems** manifest (spatial) symmetries under a large group of transformations
  - **Chaotic systems** manifest symmetry of ***probabilities***
  - ***Scale*** may be seen as a dimension subject to symmetry -> « *scale-thin* » systems (« *its distinguishable structure is seen over one or few scales »,* e.g. a cube*, (Heylighen, The growth of complexity, in The Evolution of Complexity, 1999*)
- **Complexity in time:** non-regular time behaviour of a describing variable $U \in R^n$; the time evolution may be achieved by differential equations or by discrete dynamics *(Friedrich et al., Springer Encyclopedia of Complexity, 2009)*
- **Complexity in space:** is characterized by spatial disorder that can be described by a scale dependent quantity/measure, w(L,x). Power law behavior of the moments of w(L,x), $U\langle w(L,x)^q \rangle \propto L^{\zeta(q)}$, is ascribed to fractal complexity (e.g. turbulent fields, financial market) *(Friedrich et al., Springer Encyclopedia of Complexity, 2009)*
- **Complexity in scale**: the relations and dependencies between different levels change with scale



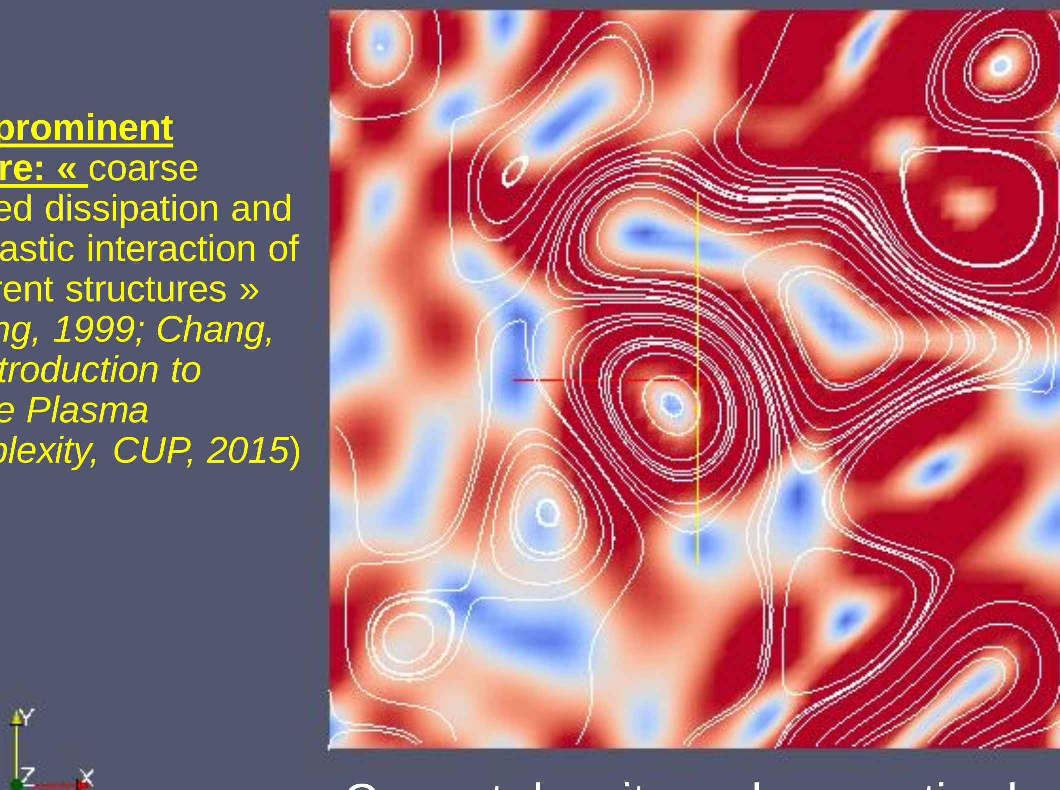


Stochastic time series measured in a Rayleigh – Benard experiment

Plasma complexity: Coherent structures interactions, turbulence

- **Plasma complexity:** the simultaneous manifestation of mutually nonlinearly interacting dynamical phenomena/structures (waves, coherent structures, convective forms, nonlinear solitary structures, pseudo-equilibrium configurations, etc.);

One prominent feature: « coarse grained dissipation and stochastic interaction of coherent structures » (*Chang, 1999; Chang, An Introduction to Space Plasma Complexity, CUP, 2015*)



Resonant fluctuations broaden the region of high magnetic shear at the interface between two aligned structures; Non resonant fluctuations propagate away as Alfven waves ➔ Interaction and merging (*Chang , 1999, Consolini and Chang, 2002*)

Current density and magnetic shear (PIC simulations, Lapenta, 2013)

Magnetosheath turbulence/complexity: example, Cluster 1 data

Ion spectra

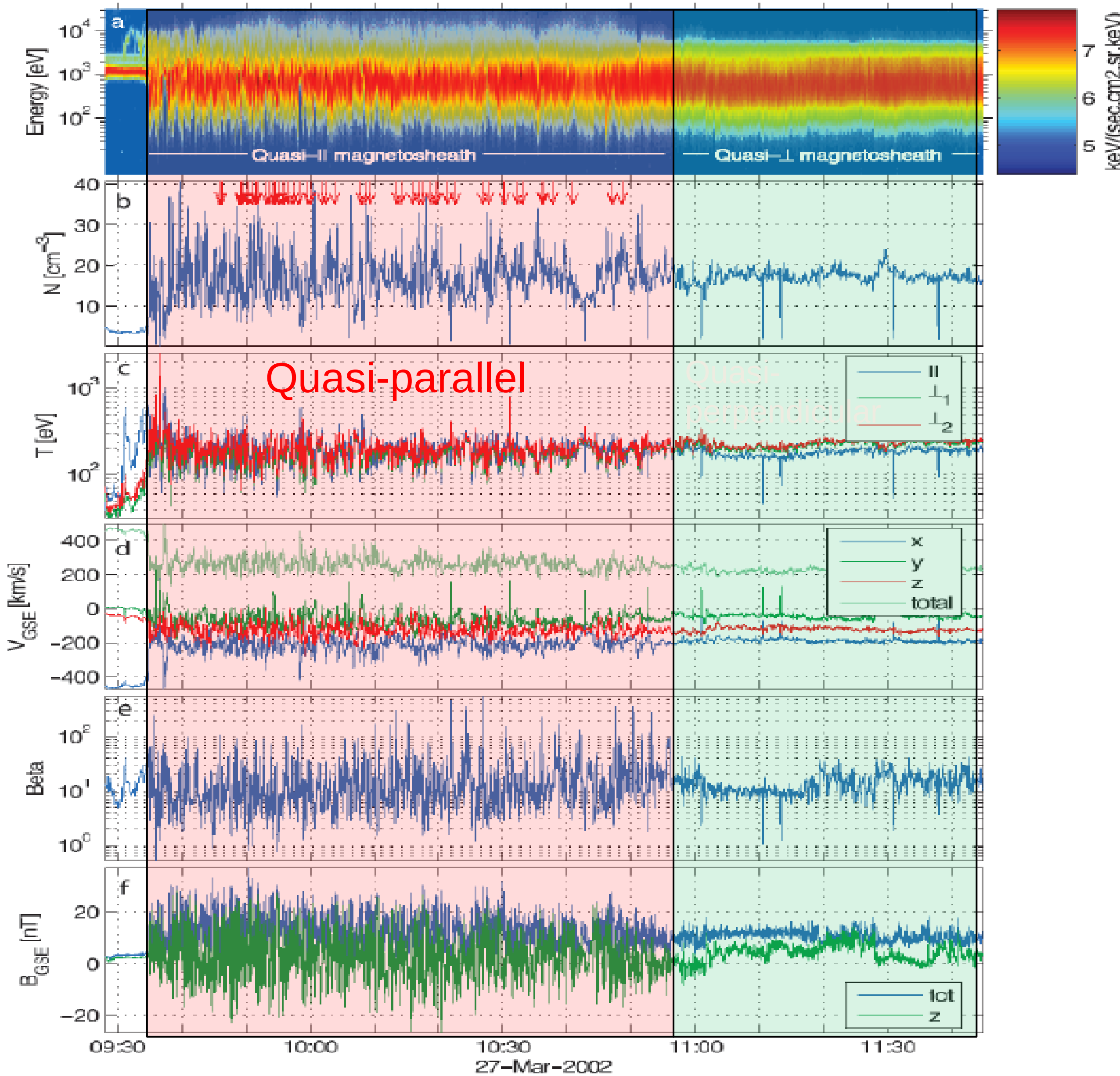
Density

Temperature

Bulk velocity

Electric field

Magnetic field



(Sundkvist et al., 2007)

Approaches/methods to investigate complexity from time series analysis (an incomplete list)

- [*stationary*] **Power Spectral Density:** the distribution of energy in Fourier/frequency space

- [*fractal/multifractal*] **Probability distribution functions (PDFs) and their moments:** collect statistics of fluctuations at a range of scales and describe variability in terms of probabilities; scaling and rescaling

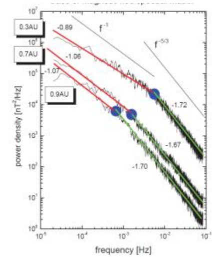
- [non-*stationary fractal*] **Space-frequency analysis with wavelets :** moves beyond the « Fourier » world and provides an image of « active » structures and their spatio-temporal localization; provides a quantitative measure of intermittency (LIM)

- [non-*stationary fractal*] **Higher Order(structural) analysis/Structure Functions**: (« poor's man wavelets », Bacry et al., 1993) provides scaling exponents through ensemble averages of various orders for different scales

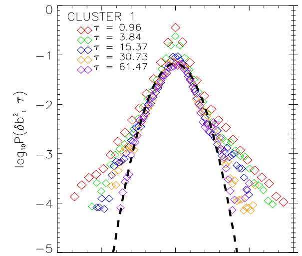
- [non-*stationary fractal*] **Fractal/Multifractal analysis**: provides a topological description in terms of geometrical analogs; provides singularity measures, i.e. the relative prevalence of power law behaviour in the configuration space

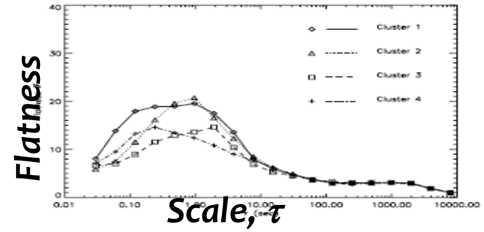
Approaches/methods to investigate complexity

- Power Spectral Density (PSD)

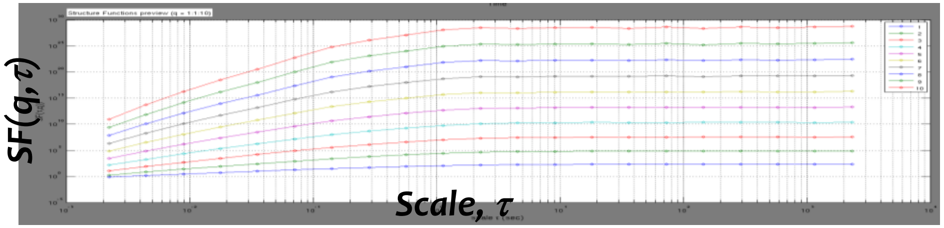


- Probability distribution functions (PDFs)

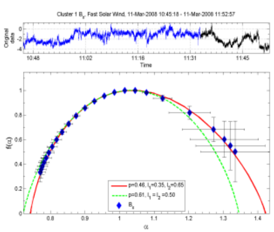


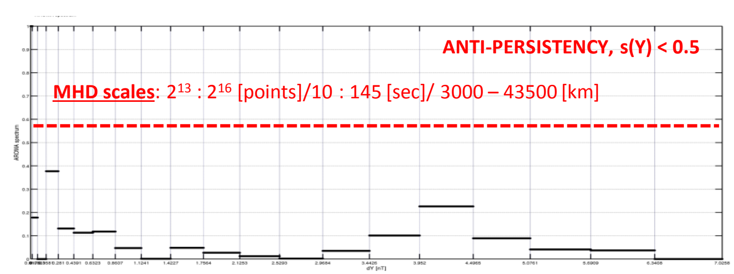


- Higher Order Analysis

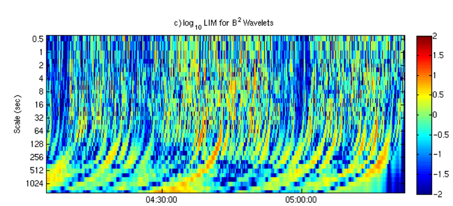


- Multifractals





- Wavelet Analysis



POWER SPECTRAL DENSITY

- Spectral leakage, windowing
- MHD complexity/turbulence: Reduced spectrum, Trace spectrum
- Treatment of gaps

PSD 1. The power spectral density (PSD)

The power spectral density gives a measure of the power content (variance/mean square value) per frequency unit [2]

➢ Definitions:

$$G^{(1)}{}_{PSD}(f) = 2\int_{-\infty}^{\infty} R_x(\tau)\cdot e^{-j\omega\tau}\,d\tau$$

(G.1) (Wiener-Khincin)

$$R_x(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)x(t+\tau)dt$$

Rx= the time-autocorrelation function

$$G^{(2)}{}_{PSD}(f) = 2\lim_{T\to\infty}\frac{1}{T}\left|\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cdot e^{-j\omega t}dt\right|^2$$

(G.2)

Three alternative estimators :

- « Standard » (Blackman-Tukey):

$$R_r = \frac{1}{N-r}\sum_{i=1}^{N-r} x_i x_{i+r} \qquad G_k^{(1)} = 2\Delta t\left(R_0 + 2\sum_{r=1}^{m-1} R_r\cos\left(\frac{\pi k r}{m}\right) + R_m\cos(\pi k)\right)$$

- Fourier (Periodogram):

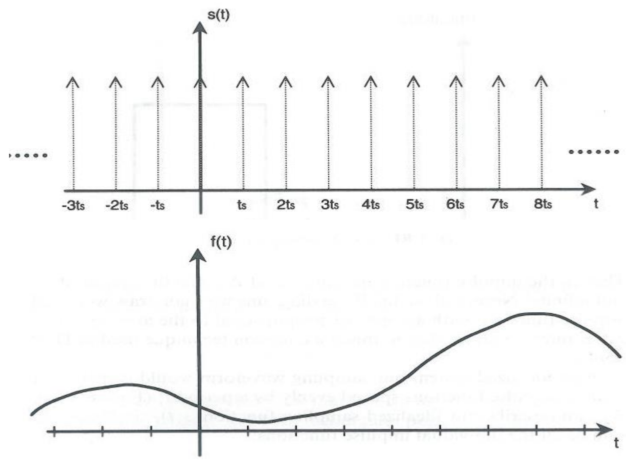
$$X_N(k) = \sum_{n=0}^{N-1} x(n)\cdot e^{-j\frac{2\pi nk}{N}} \qquad G_k^{(2)} = \frac{2\Delta t}{N}\left|x_k\right|^2 \quad k = 0,\ldots,N-1$$

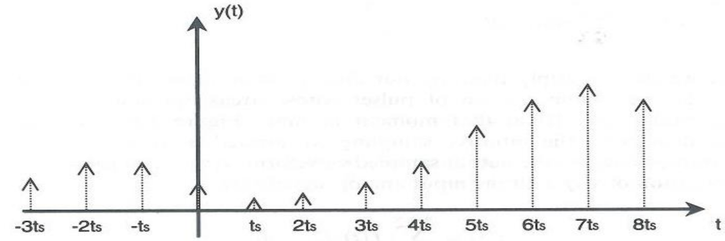
- Band pass filtering:

$$y_i^k = \sum_{l=1}^{M} h_l^{(k)} y_{i-l}^{(k)} + \sum_{l=0}^{M} g_l^{(k)} x_{i-l} \qquad G_k^{(3)} = \frac{1}{B_k N}\sum_{i=1}^{N}\left[y_i^{(k)}\right]^2$$

PSD 2. Sampling, aliasing, harmonic spectral components

- Mathematical representation of sampling [1]





$$y(t) = \sum_{n=-\infty}^{n=\infty} f(t)\cdot\delta(t - nt_s)$$

y(t)=discrete sampling of continuous f(t); $\delta(t\text{-}nt_S)$ is the (Dirac) impulse signal

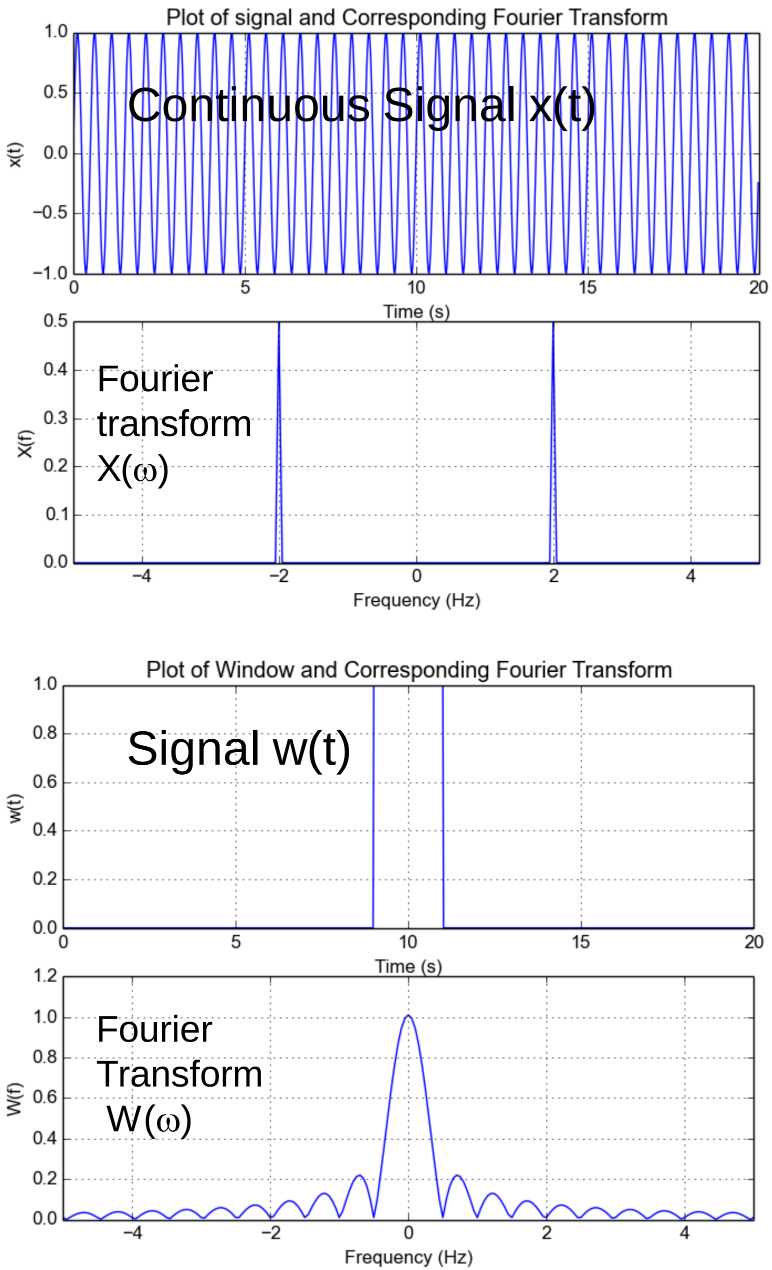
Continuous and Discrete Fourier spectrum [1]

Continuous $X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{j(-\omega t)}dt \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\cdot e^{j(\omega t)}d\omega$

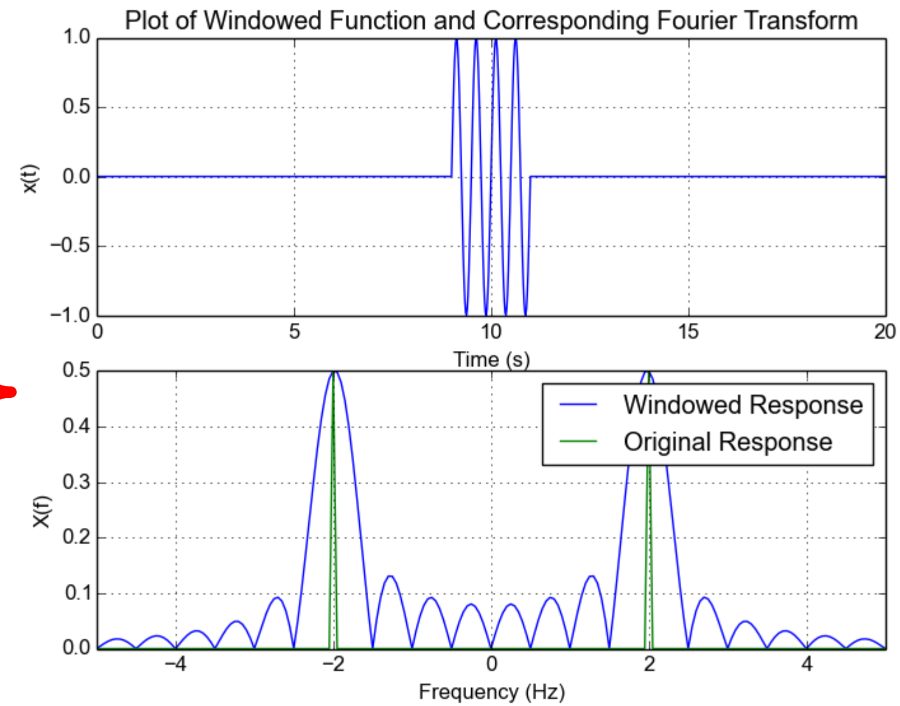
Discrete $X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\cdot e^{j(-\omega T_s n)} \quad x(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\cdot e^{j(\omega T_s n)}d(\omega T_s)$

FFT $X_N(k) = \sum_{n=0}^{N-1} x(n)\cdot(W_N)^{kn} \quad x(n) = \frac{1}{N}\sum_{n=0}^{N-1} X_N(k)\cdot(W_N)^{-kn} \qquad W_N = e^{-j\left(\frac{2\pi}{N}\right)}$

PSD 3. Time limited signal, Spectral leakage

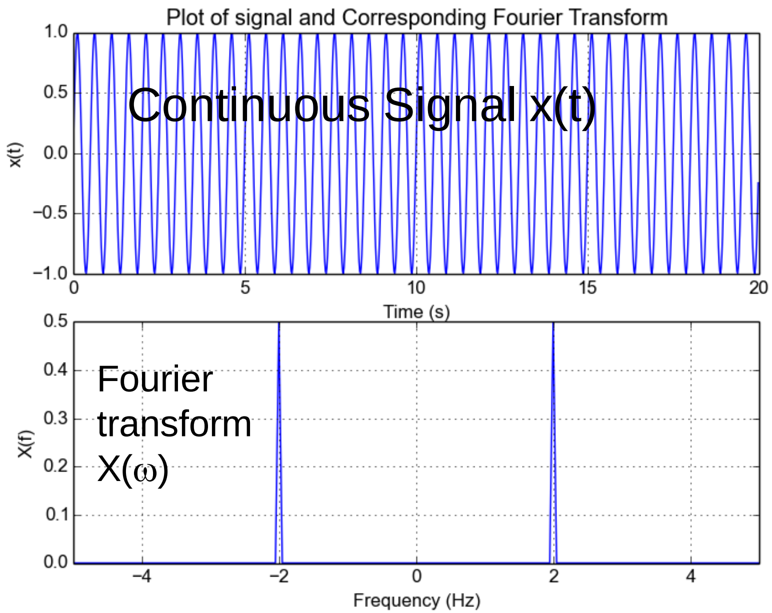


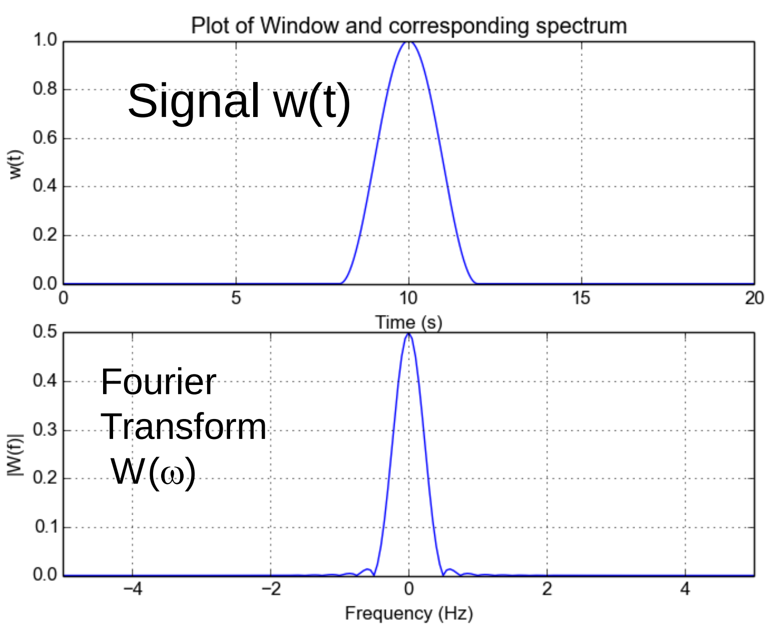
$$X(\omega) = \int_{-\infty}^{\infty} [x(t) \cdot W(t)] e^{j(-\omega t)} dt$$



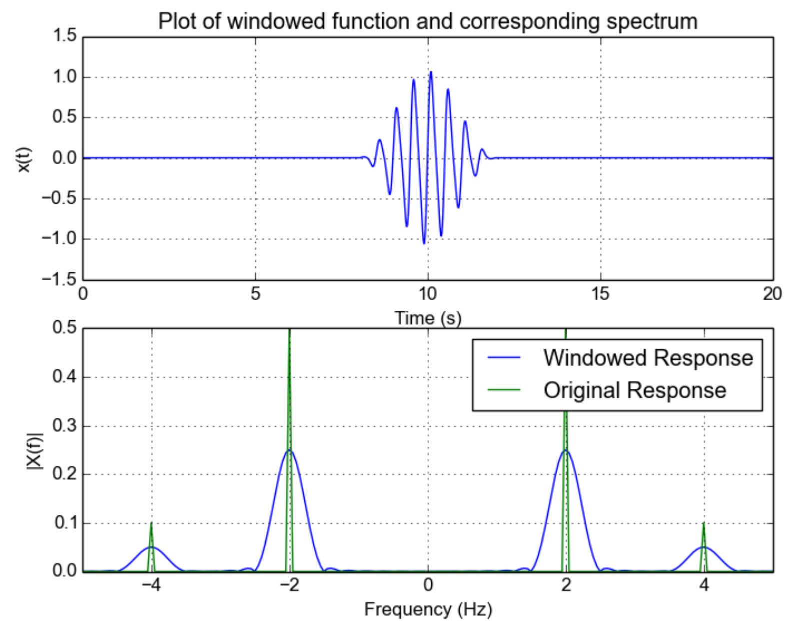
Continuous tranform of a limited time function: the result is a convolution between the Fourier transform of the original signal (upper left panel) and the Fourier transform of the box-car function – a ***sinc*** function. The result show a spectral leakage to the lobes of the real frequencies. The width of the lobes scales as 1/N.

PSD 4. Spectral leakage reduction, windowing



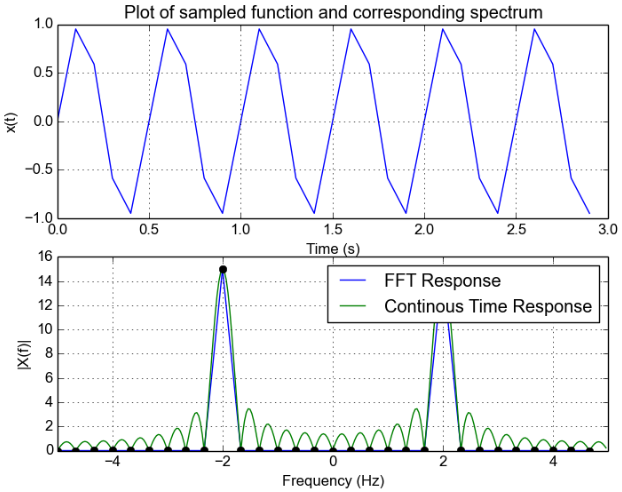


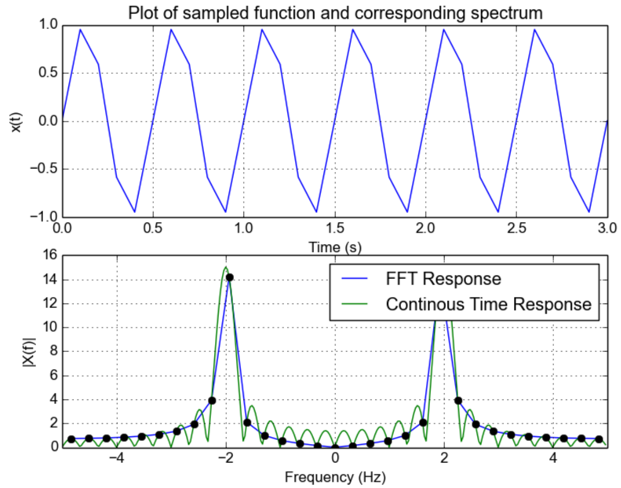
$$X(\omega) = \int_{-\infty}^{\infty} [x(t) \cdot W(t)] e^{j(-\omega t)} dt$$

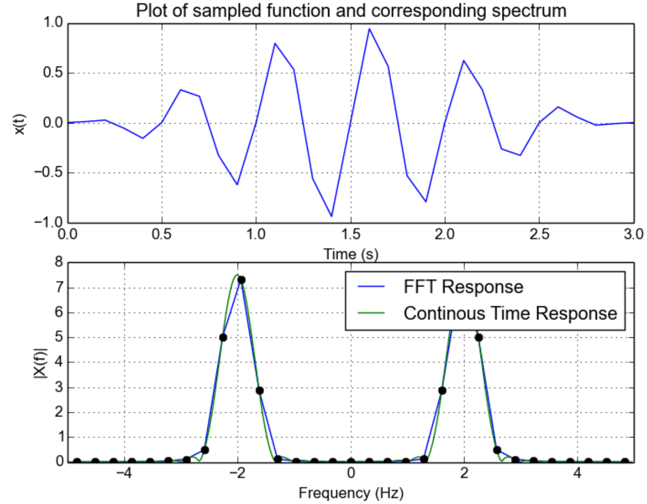


Continuous tranform of a limited time function pre-processed with a Hanning window whose Fourier transform is a Gauss function. The result shows a reduction of the spectral leakage compared to the box-car window.

PSD 5. Discrete limited time signals, spectral leakage, windowing





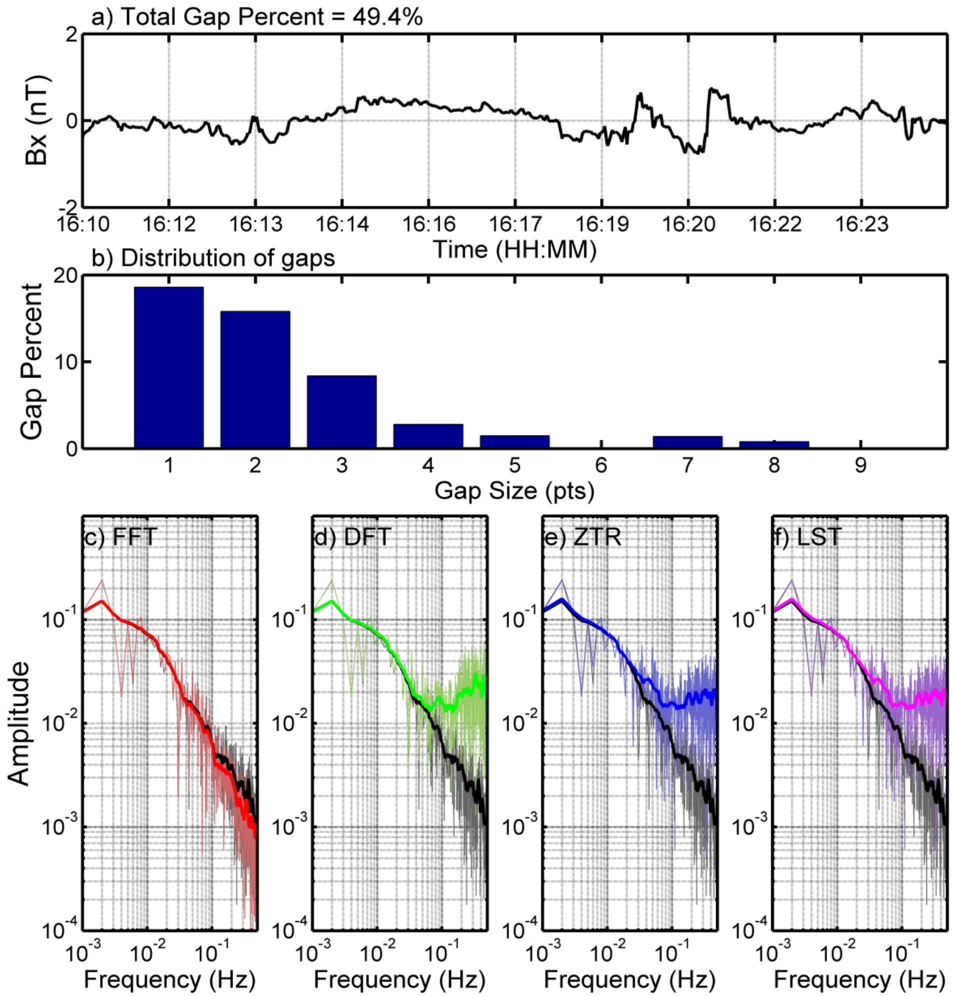


Discrete transform (DFT) of a limited time discrete function (box-car windowed) where the number of frequency samples is precisely equal to the number of time samples. In this case the frequencu is sampled precisely in the zeros of the sinc function.

Discrete transform (DFT) of a limited time discrete function (box-car windowed) where the number of frequency samples is larger than the time samples (zero padding). In this case the DFT exhibit spectral leakage similar to the continous one.

Discrete transform (DFT) of a limited time discrete function Hanning windowed for a number of frequency samples larger than the number of time samples. The leakage is reduced as in the case of the continuous transform.

PSD 6. Discrete limited time signals, gaps/non-uniform sampling



FFT :

$$X_N(k) = \sum_{n=0}^{N-1} x(n)\cdot (W_N)^{kn} \qquad W_N = e^{-j\left(\frac{2\pi}{N}\right)}$$

DFT:

$$y(\omega) = \sum_{j=1}^{n} x(t_j) e^{-i\omega t_j} \frac{\Delta t_j}{2}$$

Z-transform :

$$y(\omega) = \sum_{j=1}^{n} x(t_j) e^{-i\omega t_j}$$

Lomb-Scargle (least squares fit:)

$$P_x(\omega) = \frac{1}{2}\left(\frac{\left[\sum_j X_j \cos\omega(t_j-\tau)\right]^2}{\sum_j \cos^2\omega(t_j-\tau)} + \frac{\left[\sum_j X_j \sin\omega(t_j-\tau)\right]^2}{\sum_j \sin^2\omega(t_j-\tau)} \right)$$

(from *Munteanu et al., 2016*) Spectral analysis of a signal with (artificially produced) gaps interpolated linearly. Four different spectral methods are applied: FFT, DFT, Z-transform, Lombe-Scargle Black profile is the PSD obtained for the original signal, without gaps.

PSD 7. Space borne measurements, Taylor hypothesis, reduced spectrum

In the most general case turbulence/complexity is a three dimensional , time dependent process.

The rest frame Fourier spectrum of the variable $x(\vec{r},t)$:

$$G^{(1)}{}_{PSD}(\vec{L},\omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R(\vec{L},\tau)e^{-j(\vec{k}\cdot\vec{L}-\omega\tau)}d\tau dL_x dL_y dL_z \qquad R(\vec{L},\tau)=\lim_{V,T\to\infty}\left\{\frac{1}{V}\int_V\left[\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}x(\vec{r}+\vec{L},t+\tau)x(\vec{r},t)dt\right]d^3r\right\}$$

The field x(r,t) is sampled by a moving platform in translation with velocity $\vec{V}$ with respect to the rest frame; the autocovariance R is computed essentialy for $\vec{L}=\vec{V}\tau$. The Fourier transform of the on-board signal, S(Ω) is related to the rest frame Fourier transform, $\Phi(\vec{L},\Omega)$ through (Fredricks, Coroniti 1976) :

$$S(\Omega)=\lim_{V,T\to\infty}\left\{\frac{1}{2VT}\int_V\left[\Phi(\vec{k},\Omega+\vec{k}\cdot\vec{V})\Phi^*(\vec{k},\Omega+\vec{k}\cdot\vec{V})+\Phi(\vec{k},\Omega-\vec{k}\cdot\vec{V})\Phi^*(\vec{k},\Omega-\vec{k}\cdot\vec{V})\right]d^3r\right\}$$

Additional assumptions on the turbulent field, $x(\vec{r},t)$

- **Time independence**
- **The medium is itself in motion (solar wind) and carries the the fluctuations over the spacecraft much faster than the waves speed (Taylor hypothesis)**

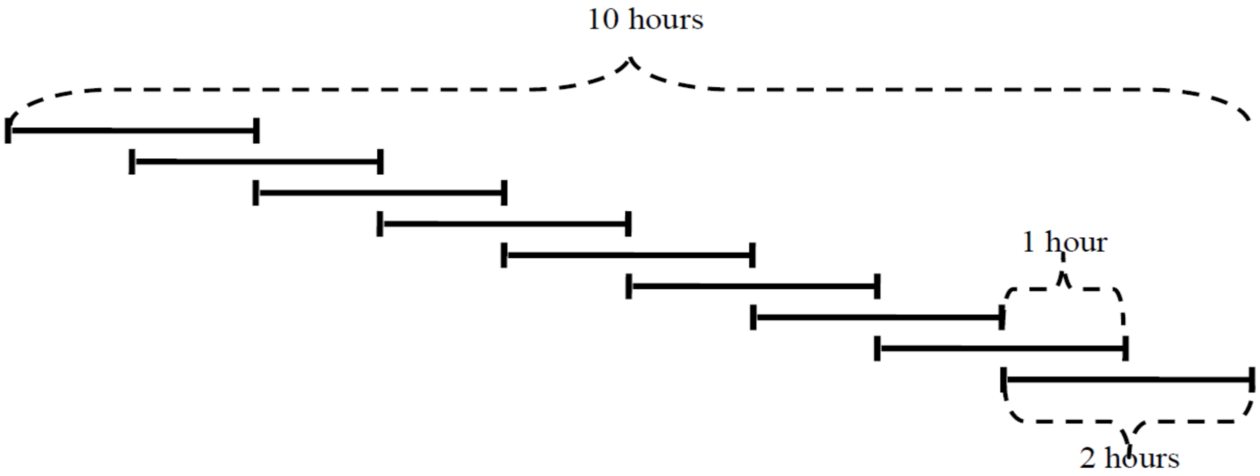
Reduced spectrum (Forman et al, 2011)

$$P(f,\theta_B)=\int\int\int d^3\mathbf{k}P(\mathbf{k})\delta(2\pi f-\mathbf{k}\bullet\mathbf{V})$$
$$=\frac{1}{V}\int\int\int d^3\mathbf{k}P(\mathbf{k})\delta\left(\frac{2\pi f}{V}-k_x\sin\theta_B-k_z\cos\theta_B\right)$$

PSD 8. A practical approach: estimation of PSD with the Welch algorithm based on periodogram

- **How does it work** *(Vitale et al., 2006)*

1. The algorithm is based on the Periodogram PSD estimate, $\boxed{G_k^{(2)}}$
2. The signal x(t) is first low pass filtered to avoid aliasing
3. The signal x(t) is then split up into N_w overlapping segments of length M, overlapping by D points.
4. Each overlapping segment is windowed in the time domain. Overlapping is thus justified by windowing that alters data at the edges
5. After doing the above, the periodogram is calculated by computing the discrete Fourier transform, and then computing the squared magnitude of the result.
6. The individual periodograms are then time-averaged, which reduces the variance of the individual power measurements. The end result is an array of power measurements vs. frequency "bin".



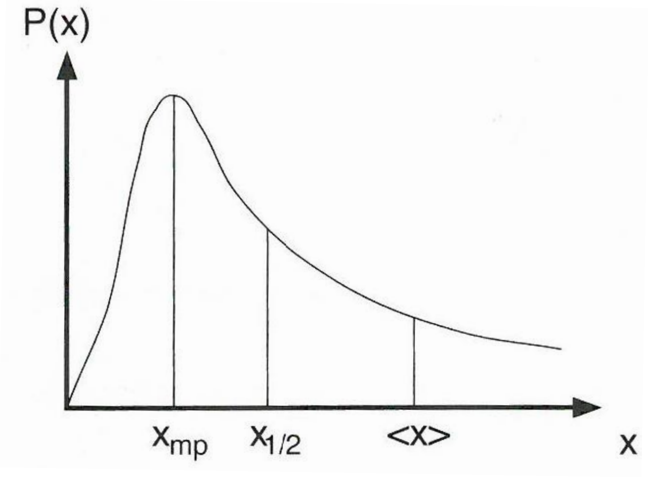
PROBABILITY DENSITY FUNCTIONS

- Definition, characteristics
- Central Limit Theorem – relevance of Gaussian PDF as hallmark of uncorrelated processes
- Histogram based, Improved (kernel methods)
- Moments, Flatness
- Rescaling, Finite Size effects (SOC)
- Examples from space plasmas
- Stochastic versus deterministic
- Joint probabilities versus one scale probabilities

PDF 1. DEFINITION(S)

- For a stochastic or deterministic process, X, whose outcome is a real number the **Probability Density Function (PDF, *distribution*)** ***P(x)*** of X is defined such that the probability that X is found in a small interval Δx around *x* is *P(x)Δx* (Sornette, 2000). The probability **P** that X is between a and b is given by the integral of *P(x)* between *a* and *b*:

$$\mathrm{P}(a < X < b) = \int_a^b P(x)dx \qquad \int_{x_{\min}}^{x_{\max}} P(x)dx = 1$$



- Cumulative distribution, $\mathrm{P}_{\leq}(x)$:

$$\mathrm{P}_{\leq}(x) = \mathrm{P}_{\leq}(X \leq x) = \int_{-\infty}^{x} P(y)dy$$

- Measures of central tendency and variation

$$\langle x \rangle = \int_{-\infty}^{+\infty} xP(x)dx \qquad \sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = \int_{-\infty}^{+\infty} \left(x - \langle x \rangle^2\right) P(x)dx$$

- Moments: $m_n = \langle x^n \rangle = \int_{-\infty}^{+\infty} x^n P(x)dx \qquad \sigma^2 = m_2 - m_1^2$

- Cumulants c_n: $c_1 = m_1, c_2 = m_2 - m_1^2, c_3 = m_3 - 3m_2 m_1 + 2m_1^3 \qquad c_{n>2} = 0, \text{if X is Gaussian}$

- Normalized Cumulants: $\lambda_n \equiv \dfrac{c_n}{\sigma^n}, \qquad \lambda_3, \lambda_4$ have special significance

PDF 2. THE CENTRAL LIMIT THEOREM (CLT)

- The PDF of the sum of N random variables, $X = X_1 + X_2 + X_3 + ... + X_N$ is **a convolution** of the individual PDFs of the N variables

It can be shown that: *"The sum, normalized by* $1/\sqrt{N}$ *of N random independent and identically distributed variables of zero mean and finite variance* σ^2 *is a random variable with a PDF converging to the Gaussian distribution with variance* σ^2 *The convergence is said to hold in the measure sense". (Sornette, 2000)*

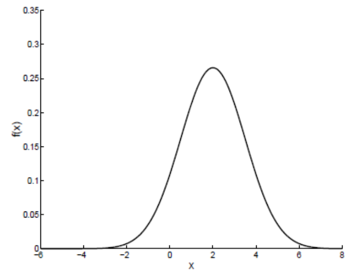
CLT holds when *(Sornette, 2000)*:

- Xi are independent or weakly correlated
- For different individual PDFs with finite variance **of the same order of magnitude**
- Does not say anything about the behaviour of the tails for finite N
- Strictly speaking holds for $N \to \infty$

CLT is an expression of collective behavior. It can be violated in many ways by linear and nonlinear correlations, intrinsic nonlinearities, external driving. However it also states a universal behavior for the collective statistics of independent, uncorrelated random variables.

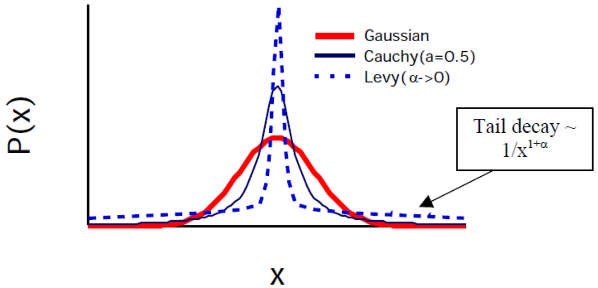
PDF 3. POWER LAW DISTRIBUTIONS, STABLE PDFS

- **Stable PDfs** have the property that $P_N(x)$, the PDF of the sum of N independent variables has the same functional form as the individiual PDF $P_1(x)$.

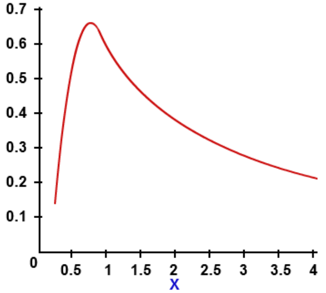


- **Gaussian law:** $$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

- **Levy law:** $$P(x) = \frac{C}{|x|^{1+\mu}}, \quad 0 < \mu < 2$$



- **Log-Normal** (stable under multiplications) $$P(x) = \frac{1}{\sqrt{2\pi}\sigma}\frac{1}{x}\exp\left(-\frac{\ln^2\left(x/x_0\right)}{2\sigma^2}\right)$$

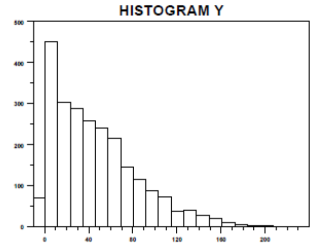


- **Truncated Ley laws:** cross-over property, convergence to Levy, to Gauss (as N increases)

PDF 4. PDFs – evaluation methods from time series analysis

- **Histograms and naive estimator:**

histogram: one defines an origin x_0 and a bin size h to construct a binning of data $[x_0+mh, x_0+(m+1)h]$. An estimation of the PDF is given by the normalized histogram:



$$\hat{f}(x) = \frac{1}{nh}(\text{no. of } X_i \text{ in the same bin as } x) \approx \frac{1}{n}\frac{(\text{no. of } X_i \text{ in the same bin as } x)}{(\text{width of bin containing } x)}$$

naive estimator: a box of width *2h* and height *(2nh)-1* is placed in each data point and then summed up:

$$\hat{f}(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h}w\left(\frac{x-X_i}{h}\right) \qquad w(x) = \begin{cases} \frac{1}{2}, & \text{if } |x|<1 \\ 0, & \text{otherwise} \end{cases}$$

(Silverman, 1998)

- **The kernel estimator:** the window is a kernel function, K, and not a box-car as for the naive kernel:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n}K\left(\frac{x-X_i}{h}\right)$$

$$\int_{-\infty}^{+\infty}K(x)dx = 1$$

- **Other methods** *(Silverman, 1998):* **The nearest neighbor method, Orthogonal series estimator, Maximum penalized likelihood estimator, General weight function estimators**

PDF 5. FLATNESS

- The flatness F is related to higher order moments of the fluctuations :

$$F = \frac{\left\langle \Delta P(t,\tau)^4 \right\rangle}{\left(\left\langle \Delta P(t,\tau)^2 \right\rangle\right)^2}$$

< > = mean on all data considered

➢ A fluctuating parameter is intermittent if the flatness F increases when considering smaller and smaller scales

➢ If F remains more or less constant whatever the scale, the fluctuations are self-similar

➢ F = 3 for Gaussian fluctuations

PDF 6. Multiscale analysis of space plasma data from PDFs estimated with a histogram based method

- Define the fluctuating field P(t)

$$\Delta P(t, \tau) = P(t+\tau) - P(t)$$

for a given value δ of the temporal scale. ($P=B_x, B_y, B_z, |B|$ or B^2)

- Probabilistic approach is justified under the hypothesis of ergodicity
- δ is the time that separates two observations of a fluctuating component :

$$\tau = \Delta t \,.\, 2^n$$

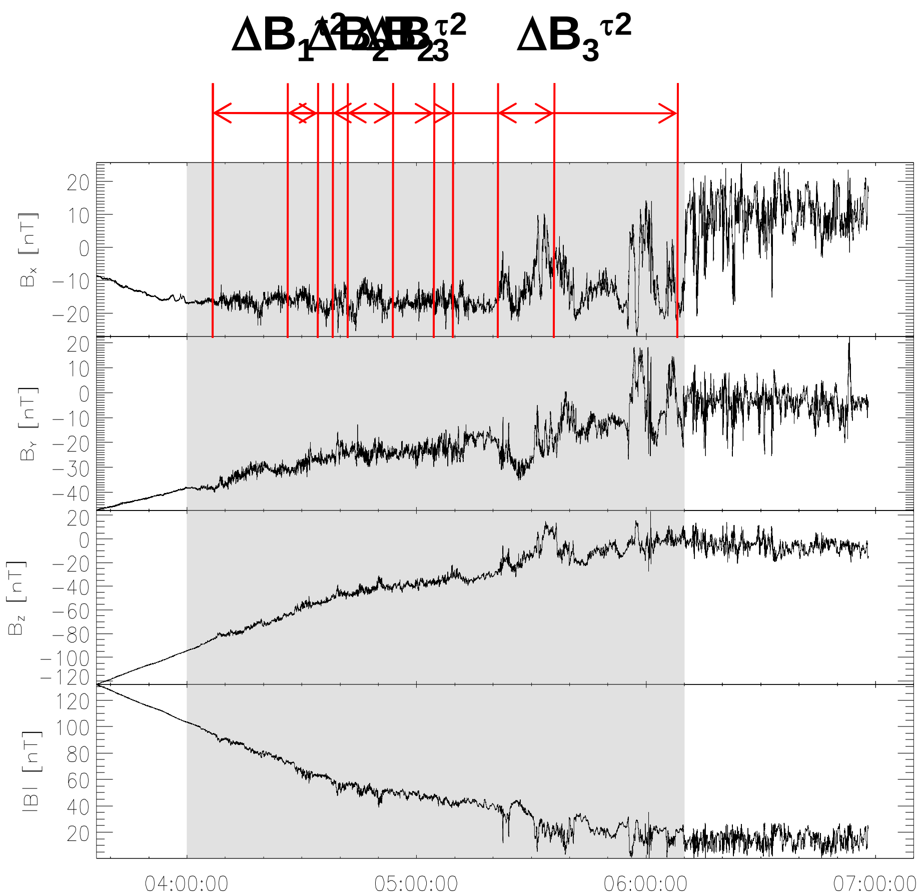
where Δt is the time resolution of the data.

- (Normalization of the PDFs with $\sigma(\delta)$ = the standard deviation of $\Delta P(t, \tau)$)

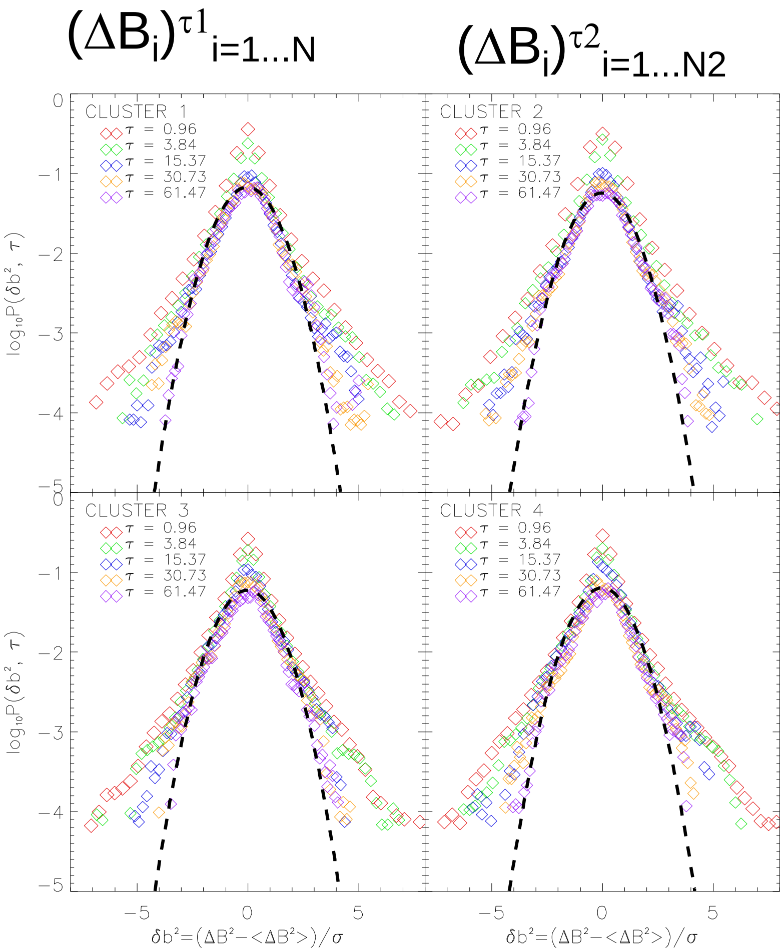
- Intermittency is associated with increasing departure of PDFs from gaussianity when the scale δ decreases.

- The Gaussianity threshold gives a hint on the topology of the energy transfer

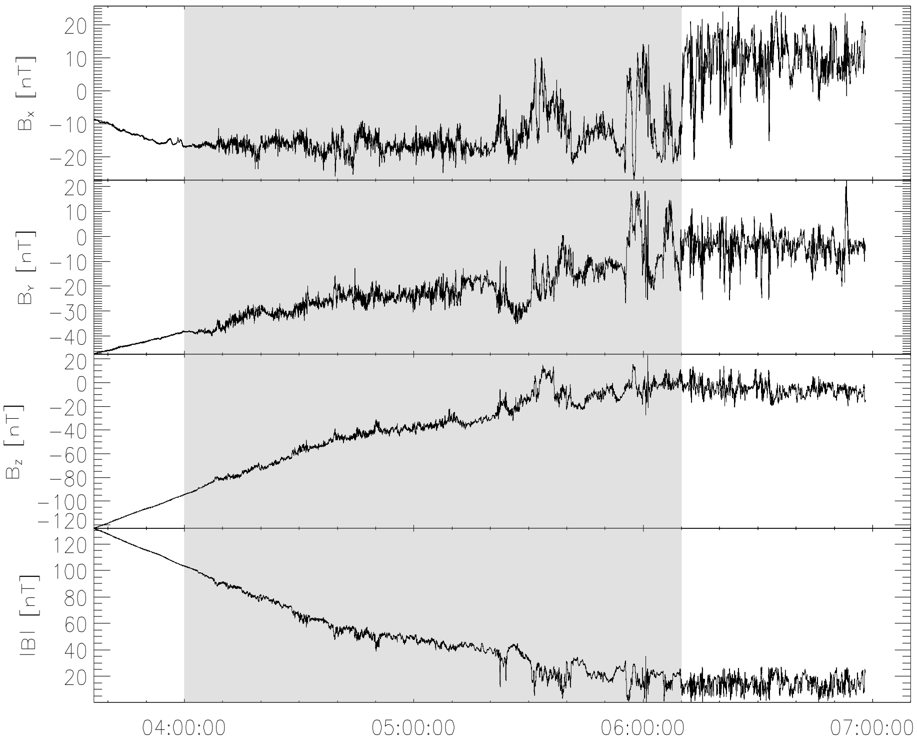
PDFs - Example, how to compute

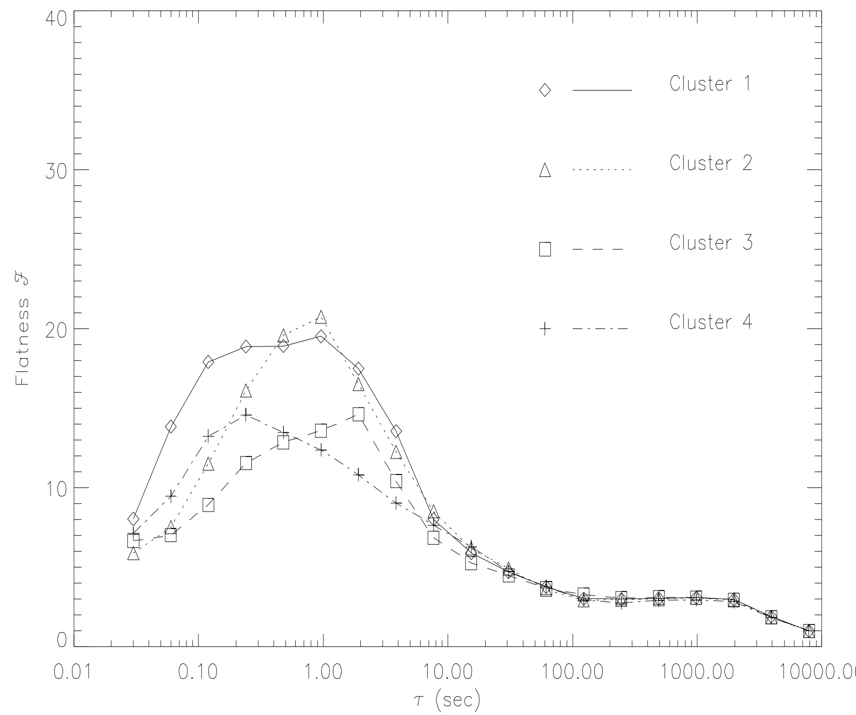


Cluster 1, 26/02/2001

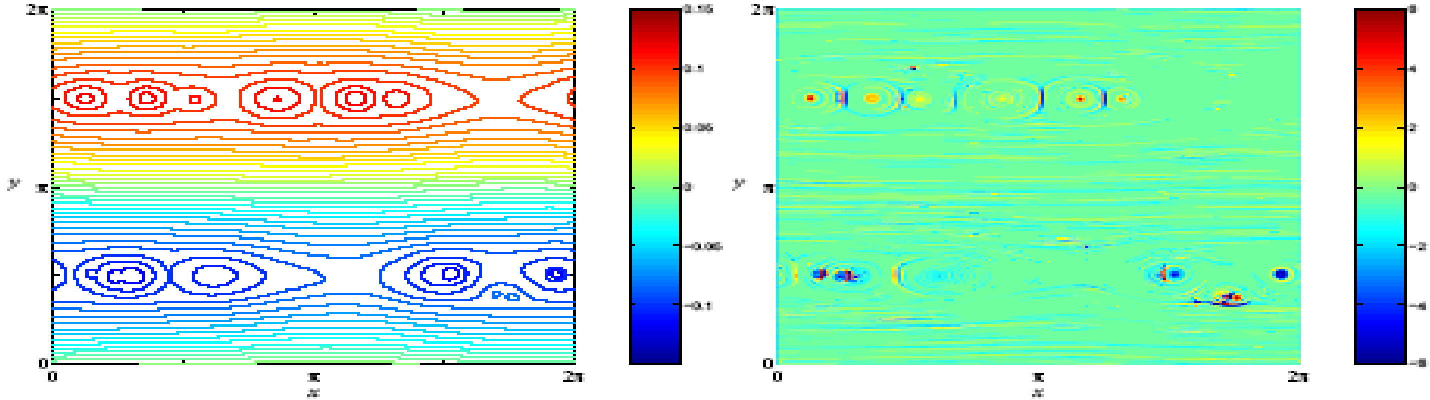


Flatness - Example

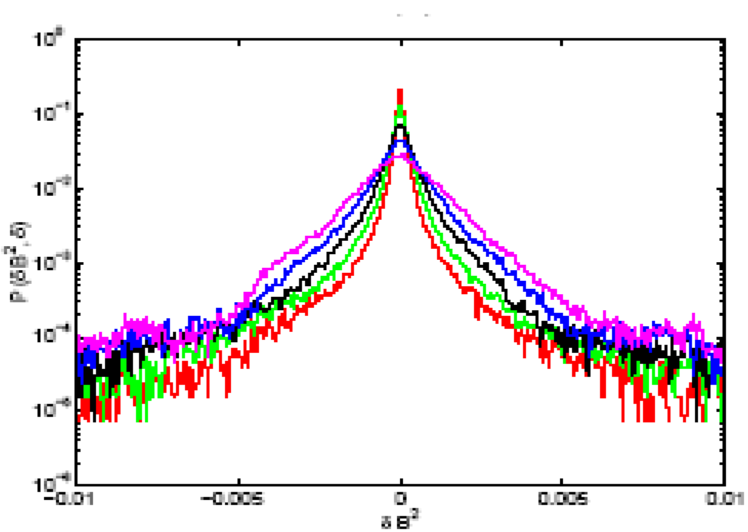




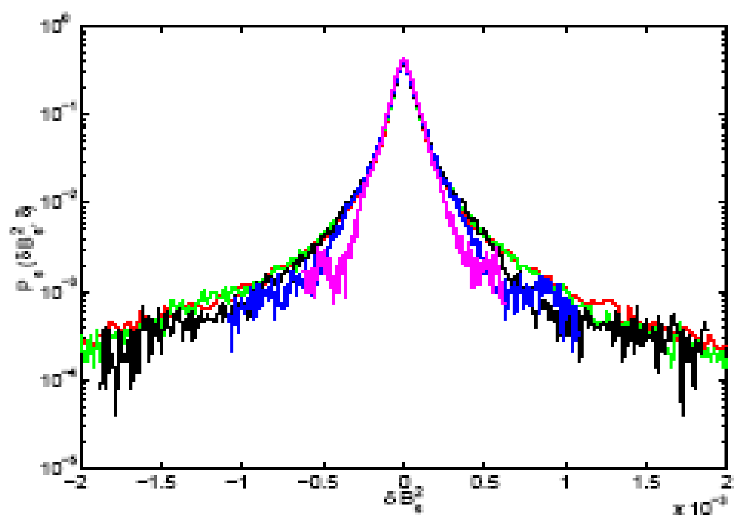
PDFs – Example from complexity MHD simulations



MHD simulation of coherent structures and the coresponding PDFs (Chang et al., 2004)



Non-scaled PDFs

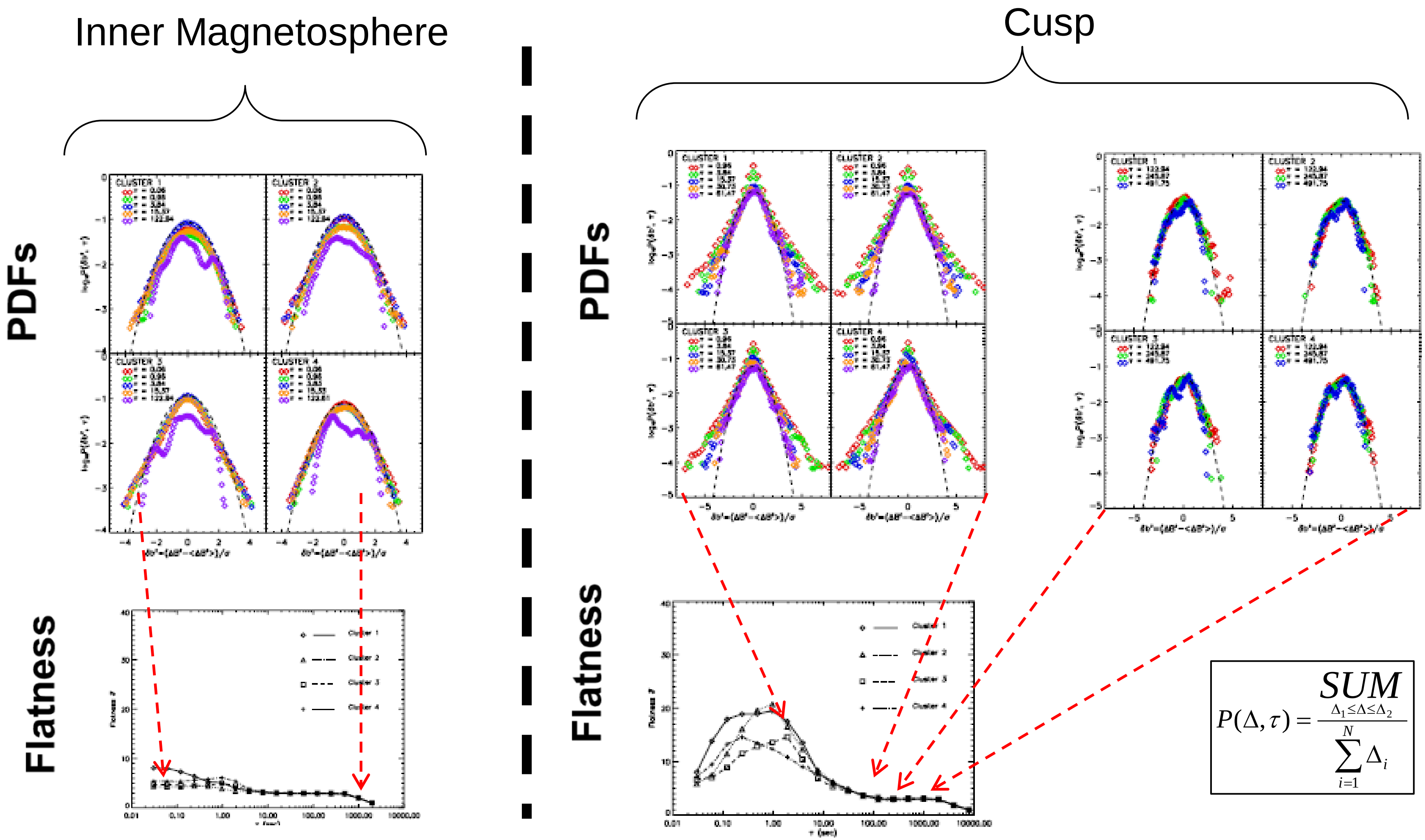


Scaled PDFs

PDFs in space data analysis

- Burlaga and Ogilvie (1970) analysed for the first time the PDFs of velocity fluctuations in the solar wind
- **Solar wind intermittency**: Marsch and Tu (1994), Ruzmaikin et al. (1995), Pagel and Balogh (2001), Hnat et al. (2002) Bruno et al. (2001, 2003), Sorriso-Valvo et al. (2001), Leubner and Voros (2005).
- **Intermittency** as departure of PDFs from Gaussianity has been also detected in the **terrestrial magnetosphere**, particularly in the **plasma sheet and magnetotail** (Lui, 2001; Voros et al., 2003; Weygand et al., 2005; Consolini et al., 2005).
- **Intermittency** of the magnetic field fluctuations is also detected in the terrestrial turbulent magnetic cusps (Yordanova et al., 2005; Nykyri et al., 2006; Echim et al., 2007)
- Intermittency of alfvenic turbulence is observed in planetary plasmas: e.g. Venus (Voros et al., 2008, Echim et al., 2011)

Gaussianity versus non-Gaussianity; statistics of fluctuations and “extreme” events



$$P(\Delta,\tau)=\frac{\underset{\Delta_1\le\Delta\le\Delta_2}{SUM}}{\sum_{i=1}^{N}\Delta_i}$$

Probability Distribution Functions (PDFs) of the magnetic field fluctuations measured by Cluster spacecraft in the inner magnetosphere (left) and the cusp (right). A threshold scale marks the transition from Gaussianity to non-Gaussianity. (data from Echim et al. 2007).

PDFs rescaling

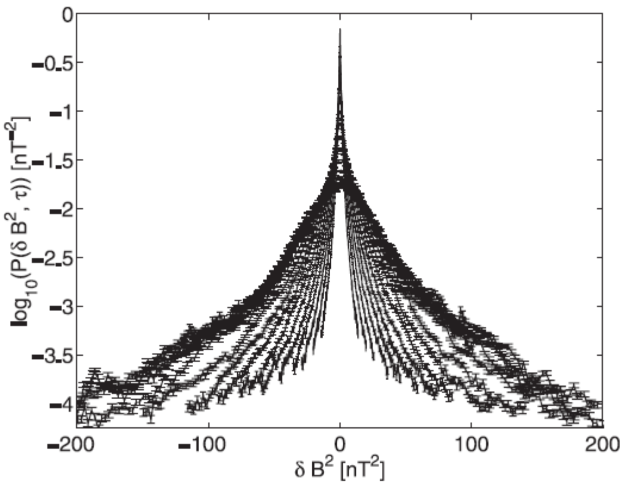
- Rescaling ⇔ self-similarity : finding a scale dependent relationship that collapses on a single master curve/PDF the PDFs corresponding to a range of scales -> it provides insight about the scaling of the higher order moments and the structure function

- Is part of the Dynamical Renormalization Group procedure; when converges leads to identification of a fixed point of the DRG (Gaussian PDF for CLT).

- The collapse of the rescaled PDF curves (e.g. Hnat et al., GRL, 2002 for solar wind)
  - Reveals the time scales over which similar physical processes occur, posible independent test of universality
  - Confirms any correlations suggested by the inverse power-law form of the power spectra (e.g., $k^{-5/3}$)
  - It quantifies the assymptotic behavior of the distribution of the fluctuations, may be essential for constraining turbulence models [Bohr et al.,1998]

➢ Rescaling methods:
  - Fit with an analytical function (Castaign et al., 1990, for laboratory fluid turbulence, Sorriso-Valvo at al. 1999, 2001 for solar wind turbulence)
  - Scaling exponents derived from data (Hnat et al., 2002, 2003, for solar wind)
  - Multifractal scaling

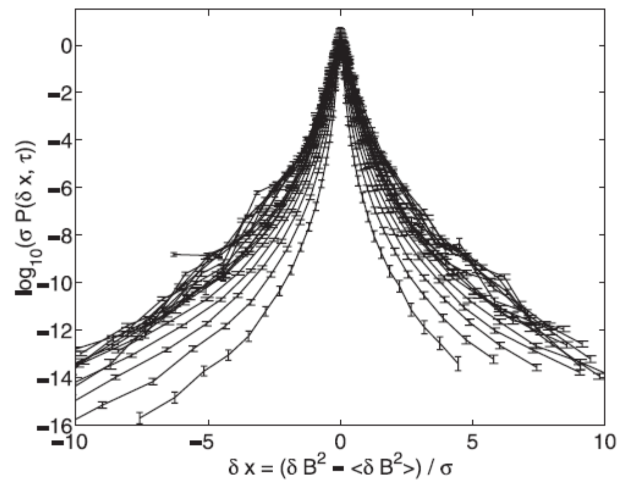
PDFs rescaling - examples



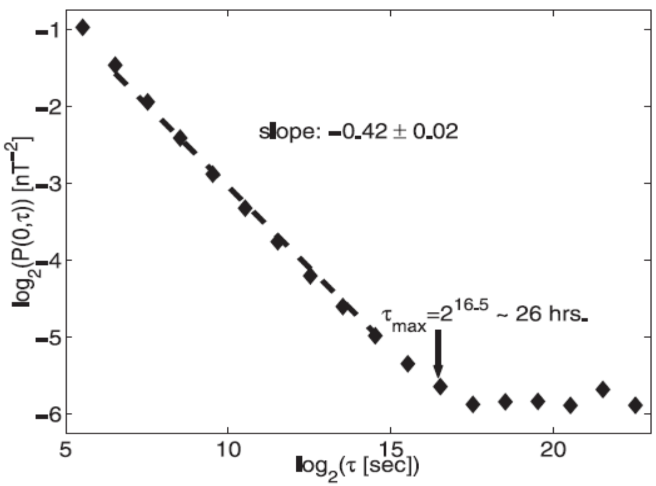
Unscaled PDFs of B^2 in the solar wind for scales between 46 seconds and 70 days

$$\delta b^2 = \frac{\Delta B^2(t,\tau) - <\Delta B^2(t,\tau)>}{\sigma(\tau)}$$

(Castaign, 1990)



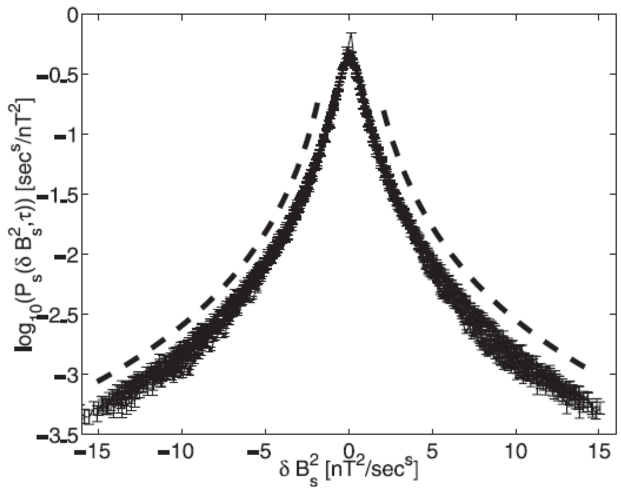
One parameter rescaled PDFs of B^2 in the solar wind for scales between 46 seconds and 70 days



Scaling of the maximum PDF, (P(0,τ)) for scales between 46 seconds and 70 days (Hnat et al., 2002)

$$P(\delta B^2, \tau) = \tau^{-s} P_s(\delta B^2 \tau^{-s}, \tau)$$

(Hnat et al., 2002)



Rescaled PDFs of B^2 in the solar wind for scales between 46 seconds and 70 days (Hnat et al., 2002)

Summary of lecture 1

[the full list of references is inserted at the end of Lecture #2]

- Dynamical complexity manifest itself as the simultaneous manifestation of mutually interacting nonlinear dynamical phenomena, possibly described as power laws of the distribution of dynamic observables
- Analysis methods devoted to complexity are mainly based on statistical analysis and aim at describing the scaling properties of fluctuating variables
- Power law behavior, typical for complex systems at or in the vicinity of criticality, can be probed with spectral and probabilitic approach
- The power spectrum can be recovered from one point satellite measurements only when (strong) assumptions on the fluctuations field/turbulence are applied
- The Welch algorithm reduces the spectral noise but reduces the range of probed frequencies
- Non-Gaussian PDFs are a signature of corrrelated random variables, v.CLT
- Incremental measures based on differences provides a scale description of the fluctuating field and are appropiate to probe intermittency
- A quantitative measure of intermittency is given by the moments of the PDFs, more specifically by the Flatness parameter.
- Rescaling of PDFs is a test of universality and can provide hints about the structure of the configuration space of the dynamical system (wheter or not in the vicinity of a fixed point).