Space plasma complexity: approaches and methods – II

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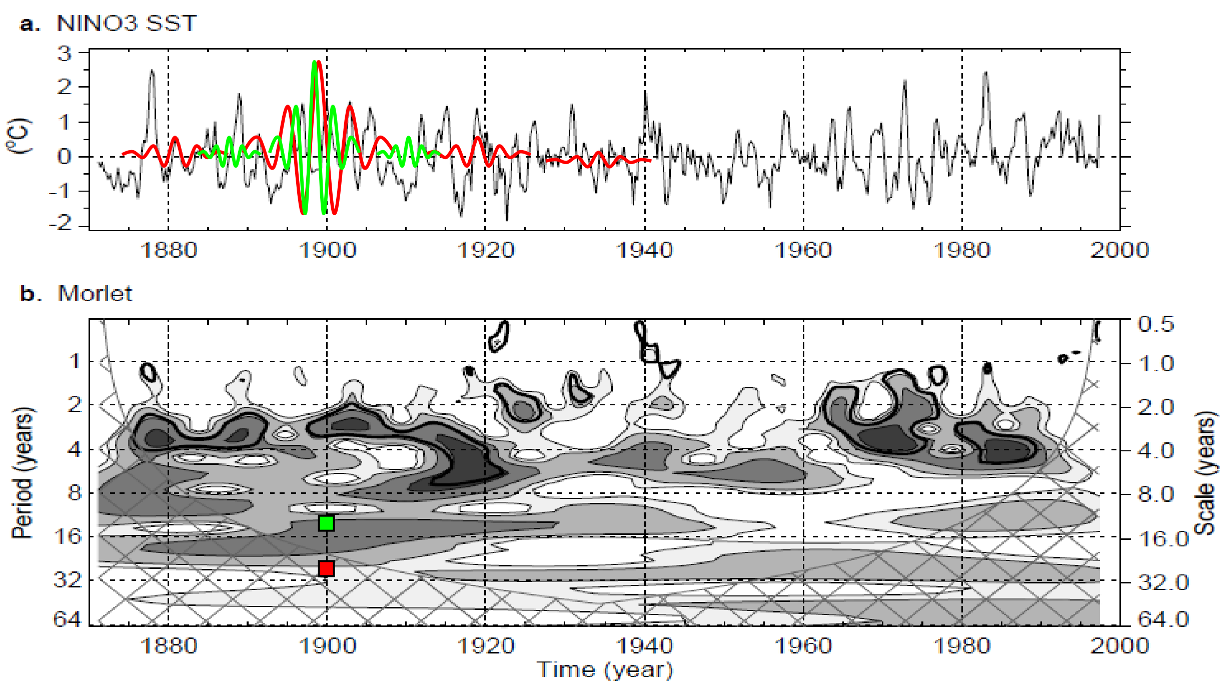
1. **Approaches and Methods for Dynamical analysis with Wavelets applied on time series analysis**
2. **Approaches and Methods adapted for the Multifractal Analysis of time series**
3. **Brief Introduction to the Interactive Nonlinear Analysis (INA) library**
4. **Summary of the Lectures; homework**

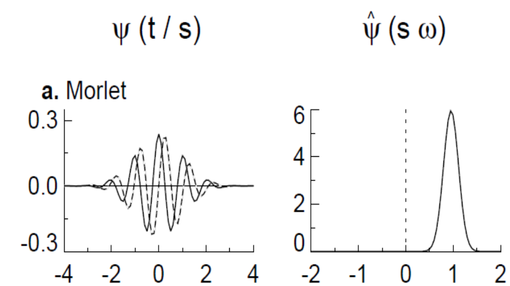
WA 1. Wavelet analysis, introduction

- Computation of wavelet spectrum of the signal (time series) Q in the Ψ (“mother”) wavelet basis for each moment t and scale τ: the family of analyzing wavelets $\Psi(t,\tau)$ may be compared to ***a mathematical microscope***, for which Ψ characterizes the optics, τ- is the resolution, t the position (in addition to the dilation, τ, and translation t, the wavelet transform may also imply a rotation)
- The wavelet coefficients, $C(\tau,t)$ from the continuous transform:

C(15yr, 1900)
C(30yr, 1900)

$$C(\tau,t)=\frac{1}{\tau}\int_{t_1}^{t_2}\Psi\left(\frac{t'-t}{\tau}\right)Q(t')dt'.$$





$$\psi_0(\eta)=\pi^{-1/4}e^{i\omega_0\eta}e^{-\eta^2/2},$$

(Torrence and Compo, 1997)

WA 2. Wavelets, general mathematical properties/conditions

- **Admissibility**: $$C_\psi = \int_0^\infty \left|\widehat{\psi}(k)\right|^2 \frac{\mathrm{d}k}{|k|} < \infty \,, \quad \widehat{\psi}(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi(x)\mathrm{e}^{-i2\pi kx}\mathrm{d}x \,, \quad \int_{-\infty}^{\infty} \psi(x)\mathrm{d}x = 0$$

- **Similarity**: the scale decomposition is obtained by the translation and dilation of only one mother wavelet
  -> very good spatial resolution at small scales
  -> very good scale resolution at large scales *(Farge, 1992)*

- **Invertibility**: there should be at least one reconstruction possibility to recover the signal from its wavelet representation (*Farge, 1992*)

- **Regularity**: the wavelet should be concentrated in a finite spatial domain and be sufficiently regular (*Farge, 1992*).

- **Wavelets can be real of complex**

- **Cancelations**: in addition to admissibility condition the , the wavelet should have vanishing moments up to order M (particularly for turbulence studies)

$$\int_{-\infty}^{\infty} x^m \psi(x)\mathrm{d}x = 0 \qquad \text{for} \quad m = 0, M$$

WA 3. Wavelets, (physics) motivation

- Unfolding the space-time structure of a signal/measurement in complex/turbulent systems; provides meaningful analysis of spatial properties at each scale *(Meneveau, 1991)*

- Direct estimation of the (spectral/scale) energy density by the wavelet coefficients, easy to intuitively visualise and interpret

- Quantitative estimation of the spatial spotiness and intermittency of the energy flux and transfer and identification of “active” scales

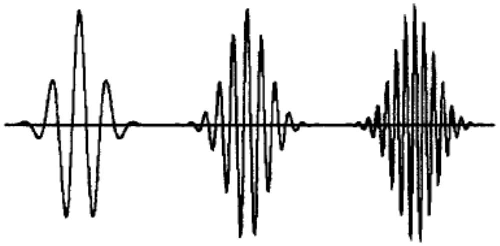
- [Hydrodynamic turbulence] Quantitative estimation of the energy flux and energy transfer rate between scales and/or between spatial regions of the investigated dynamical system

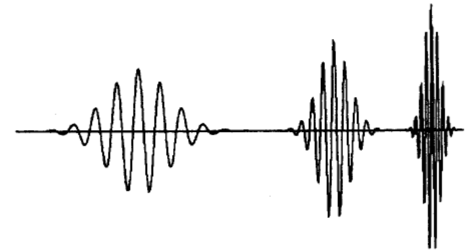
- Relevant tool for complexity and turbulence analysis as they are able to reveal the spatio-temporal structure of the dynamical coherent structures and their multi-scale interaction/structure

WA 4. Continuous and Discrete wavelet transforms *(Meneveau, 1991)*

	Continuous Wavelet transform (WT)	Discrete Wavelet transform (DWT)
Transform	$Wf(s,r) = \frac{1}{\sqrt{rC_\psi}} \int_{-\infty}^{+\infty} f(x) \psi^*(\frac{x-r}{s}) dx$	*Discrete wavelet basis* $g^{(m)}{}_{[i]} = \psi^{(m)}{}_{[i]}(x) = \frac{1}{\sqrt{{a_0}^m}} \psi\left(\frac{x - ib_0{a_0}^m}{{a_0}^m}\right)$ $\psi^{(m)}(x) = \frac{1}{\sqrt{2}} \psi\left(\frac{x}{2^m}\right)$ *with the property:* $\sum_{k=-\infty}^{\infty} g^{(m)}(k - 2^m i) g^{(n)}(k - 2^n j) = \delta_{ij}\delta_{mn}$ *wavelet coefficients* $w^{(m)}[i] = \sum_{j=-\infty}^{\infty} g^{(m)}[i - 2^m j] f[j]$
Invertibility	$f(x) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_0^{\infty} s^{-2} Wf(s,u) \psi_{s,u}(x) ds du$	$f[j] = \sum_{m=1}^{\infty} \sum_{i=-\infty}^{\infty} w^{(m)}[i] g^{(m)}[i - 2^m j]$
Energy preserving	$\int_{-\infty}^{\infty} \|f(r)\|^2 dr = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_0^{\infty} \|Wf(s,x)\|^2 s^{-2} ds dx$	$\sum_{-\infty}^{\infty} f[j]^2 = \sum_{m=1}^{\infty} \sum_{i=-\infty}^{\infty} \left(w^{(m)}[i]\right)^2$
Parseval theorem	$Wf(\xi,r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\omega) \hat{\psi}^*{}_{s,r}(\omega) d\omega$	

WA 5. Wavelet versus (windowed/short time) Fourier spectrum





ST Fourier transform

$$\Phi_\gamma(u) = g_{\xi,t}(u) = e^{i\xi u} g(u-t)$$

$$Sf(\xi,t) = \int_{-\infty}^{+\infty} f(u) e^{-i\xi u} g(u-t)\, du$$

Properties of g:

- Usually real, and even function
- Compact support
- its energy is concentrated in the low-frequency components,
- it has unit energy, i.e. $\int |g(u)|^2 du = 1$

Wavelet transform

$$\Phi_\gamma(u) = \psi_{s,t}(u) = \frac{1}{\sqrt{s}} \psi\left(\frac{u-t}{s}\right)$$

$$Wf(s,t) = \int_{-\infty}^{+\infty} f(u) \frac{1}{\sqrt{s}} \psi^*\left(\frac{u-t}{s}\right) du$$

Properties of ψ:

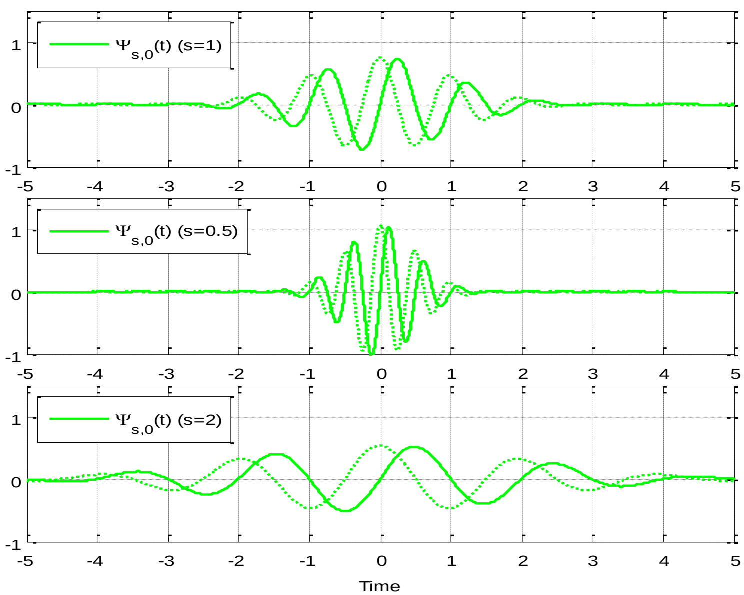
- Compact support or fast decay
- Zero mean ($\int \psi(u) dt = 0$)
- It has unit energy, i.e. $\int |\psi(u)|^2 dt = 1$

(P. Kovacs, STORM Workshop and School, 2015)

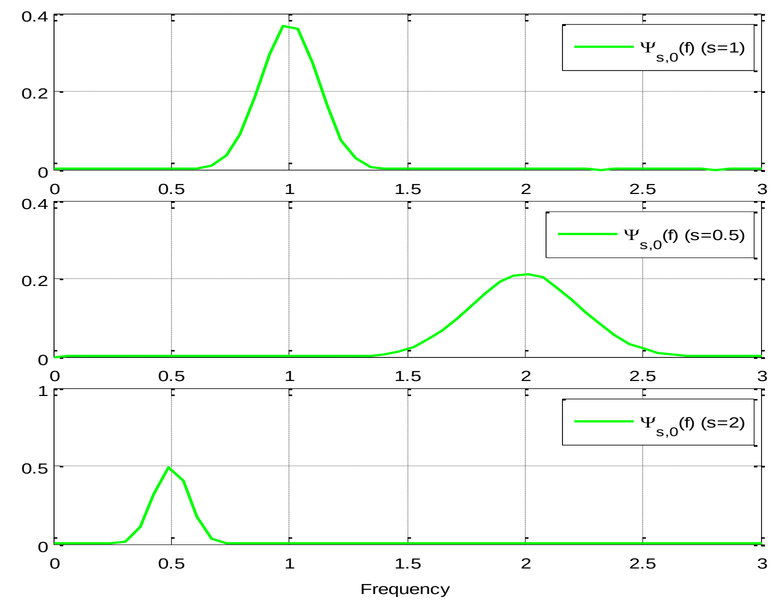
WA 7. Sample kernel function of WT in time and frequency space

Morlet mother wavelet: $\psi(t) = \pi^{-\frac{1}{4}} e^{-i\omega_0 t} e^{-\frac{t^2}{2}}$

Dilated kernel function in time-space

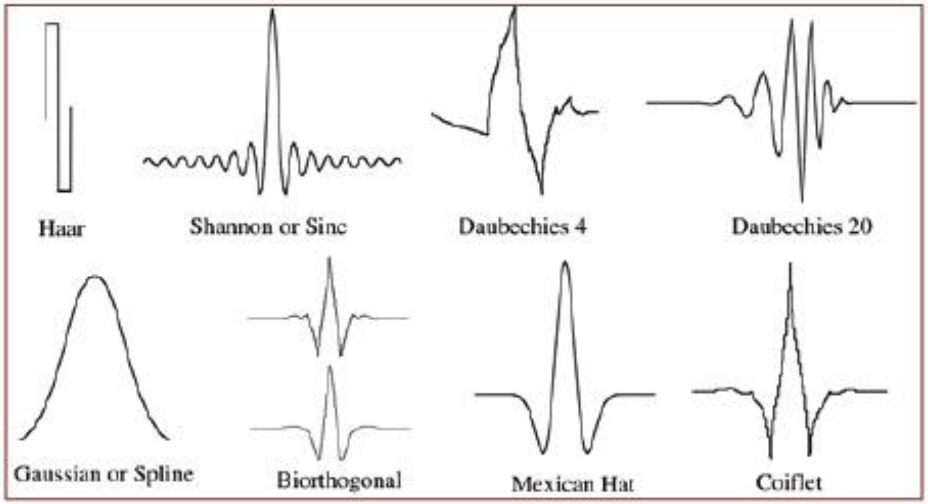


Dilated kernel function in freq. space

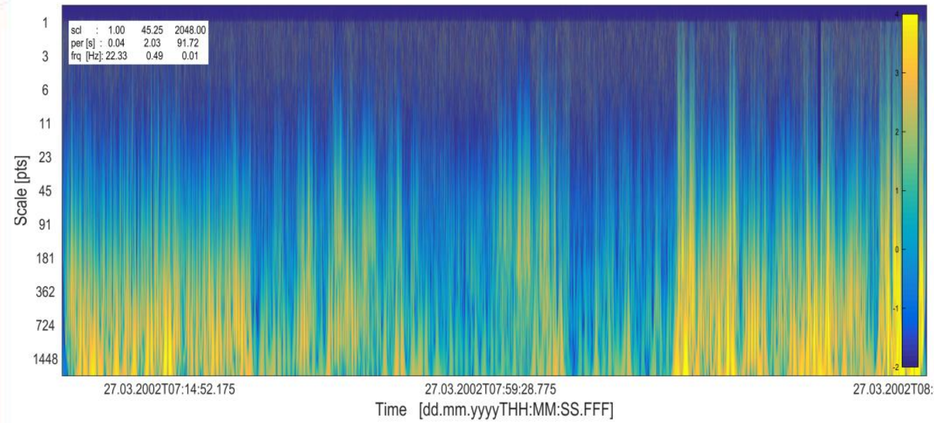


(P. Kovacs, STORM Workshop and School, 2015)

WA 6. Wavelets, some examples



Graphical illustration of a few examples of wavelets *(from Daubechies, 1992)*



Wavelet spectrum (in Haar wavelets) of B_z component of the magnetic field measured by Cluster-1.

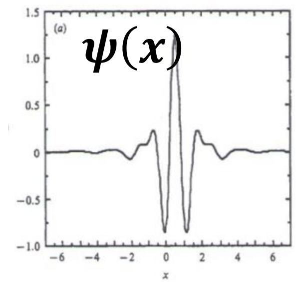
- **Complex versus real valued wavelets :**
  - <u>Complex wavelets</u> **t**ake complex values whose modulus gives the energy while the phase detects singularities and measures instantenous frequencioes> Ecamples : Morlet wavelet
  - <u>Real valued wavelets</u> examples: Haar, Maar wavelet (Mexican hat)

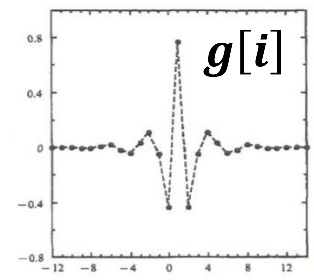
- **Reprezentation of wavelet coefficients:** generally one takes a linear scale for the spatial ordinate and logarithmic scale for the scale coordinate, full color range for each scale, normalization if wavelet coefficients/energy at different scales need to be intercompared

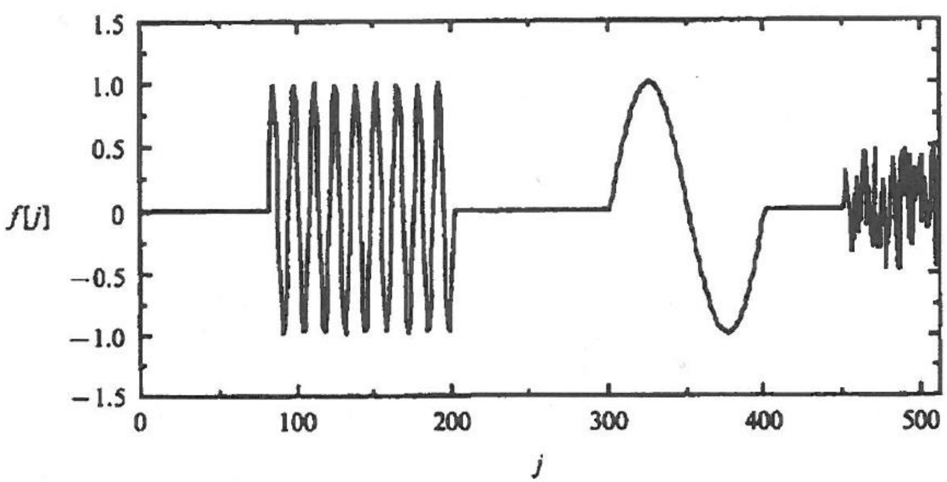
WA 8. Example of a discrete WT transform (LMR mother)

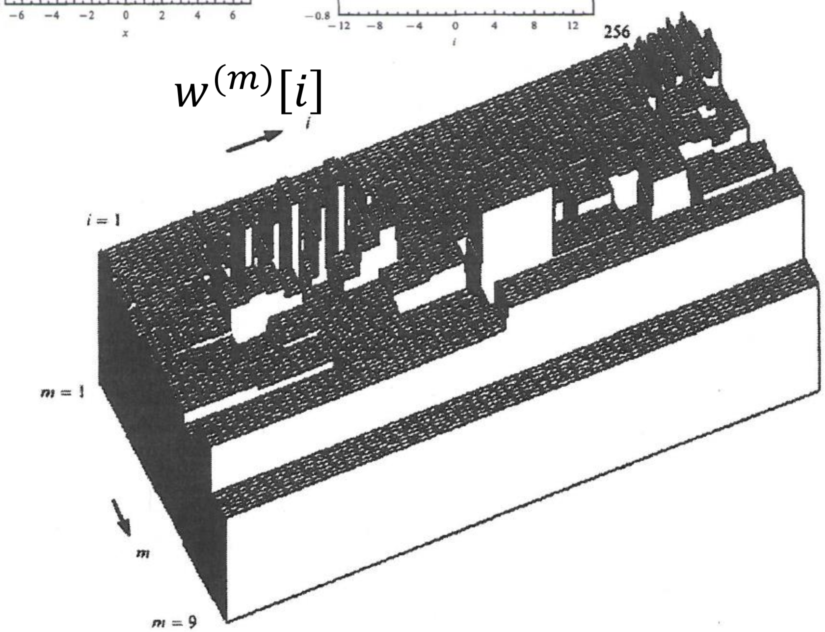
Lemarie, Meyer and Battle (LMR) wavelet:

$$\hat{\psi}(\omega) = \frac{e^{-i(\omega/2)}}{\omega^4}\frac{\sqrt{\Sigma_8(\frac{\omega}{2}+\pi)}}{\sqrt{\Sigma_8(\omega)\Sigma_8(\frac{\omega}{2})}},$$









Sample signal illustrating different oscillations at different scales and positions and white noise at the right (from Meneveau, 1991)

Three-dimensional map of the wavelet coefficients $w^{(m)}[i]$
computed with the LMR mother wavelet (from Meneveau, 1991).

WA 9. Wavelet analysis and the Local Intermittency Measure (LIM)

• $|C(t,\tau)|^2$ is a measure of the energy density of the field Q(t) at the given scale and position (when the Taylor hypothesis is satisfied in turbulence studies).

• The **Local Intermittency Measure** (LIM) or normalized power is derived from wavelet coefficients (Farge et al., 1990) :

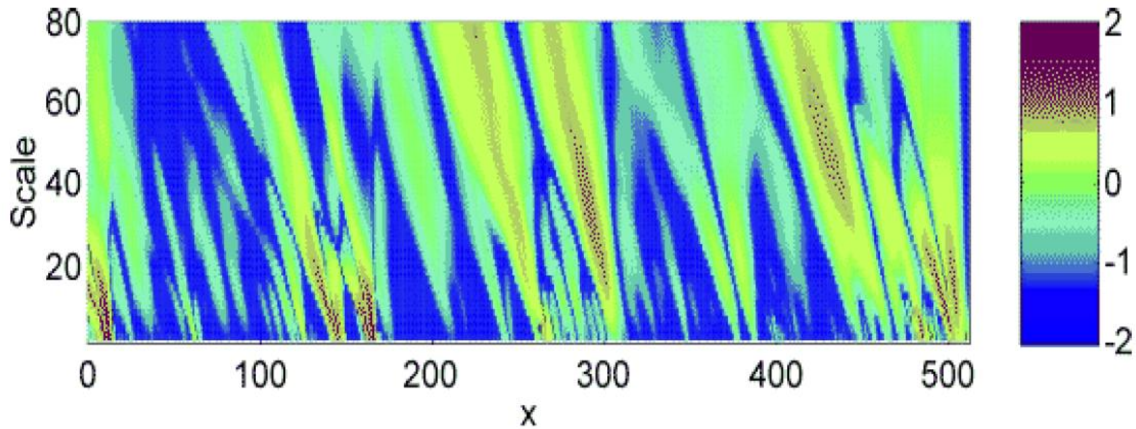
$$LIM(\tau,t) = \frac{|C(\tau,t)|^2}{< |C(\tau,t)|^2 >}$$

• LIM(τ,t) = 10 means that the point t contributes 10 times more than the <average over the entire time interval of the Fourier energy spectrum> at scale τ; the global wavelet energy spectrum corresponds to the Fourier energy spectrum smoothed by the wavelet spectrum at each scale (Farge et al., 1990)

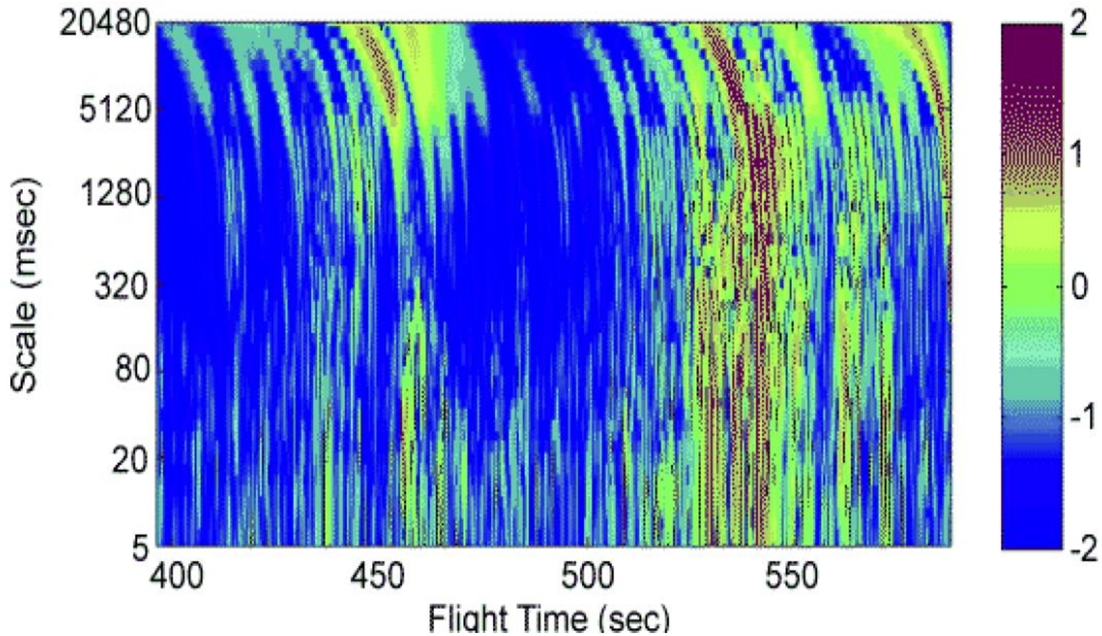
WA 10. Local Intermittency Measure (LIM) of plasma complexity – examples from numerical simulations and auroral activity

$$LIM(\tau, t) = \frac{|C(\tau, t)|^2}{< |C(\tau, t)|^2 >}$$

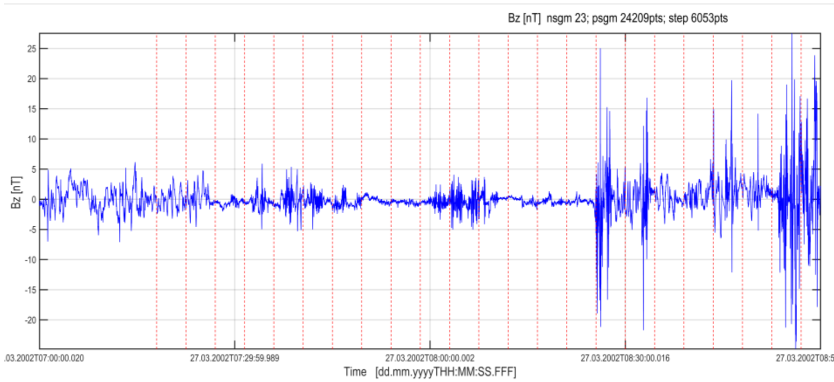
LIM of 2D MHD simulations, Haar base (Chang et al, 2006)



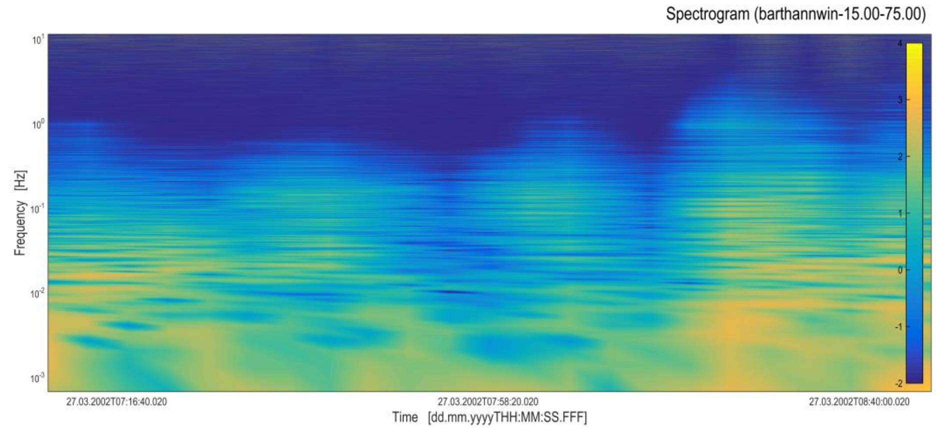
LIM of auroral electric field from Haar wavelets decomposition (Chang et al, 2006)



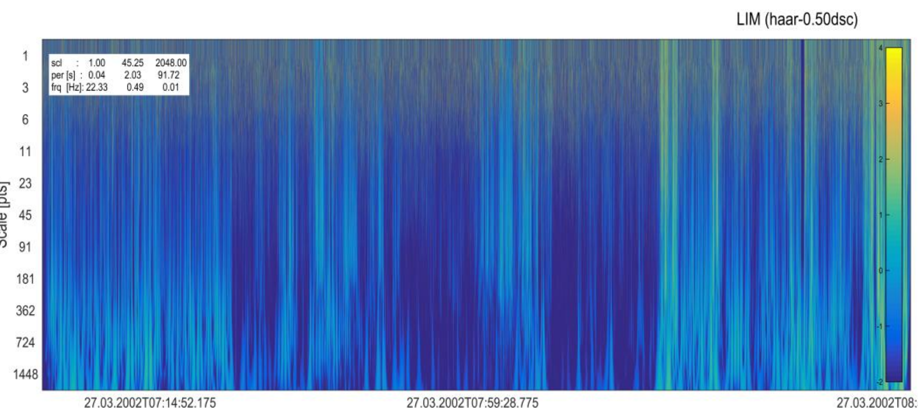
WA 11. Wavelet analysis of complexity from Cluster observations in the solar wind , 27/03/2002



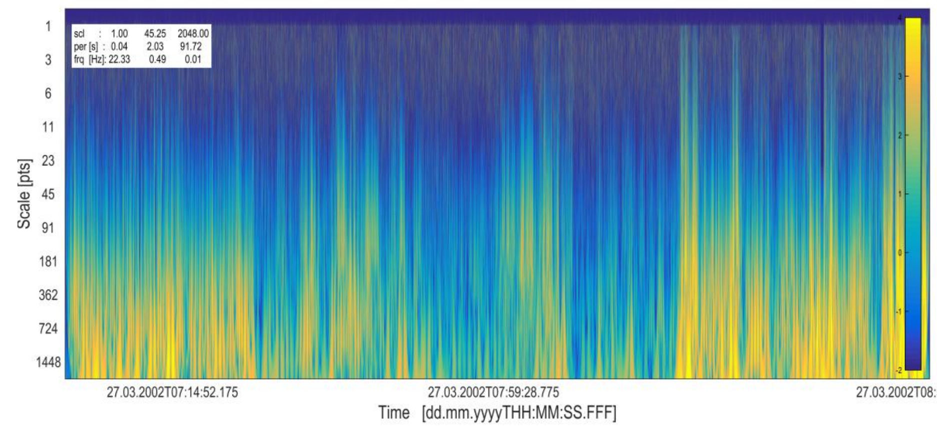
Bz component of the magnetic field measured by Cluster-1 FGM.



Windowed Fourier analysis (spectrogram) of Bz component of the magnetic field measured by Cluster-1.



The Local Intemittency Measure (LIM) estimated with Haar wavelet analysis of B_z fluctuations measured by Cluster-1.



Wavelet spectrum (in Haar wavelets) of Bz component of the magnetic field measured by Cluster-1.

FRACTALS and complexity: definitions

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line."

B. Mandelbrot (cited also by Sornette)

"Fractals are sets of points embedded in space and having a particular, non-Euclidean topology described by a fractional dimension "(J. Feder).

- ***Scaling properties are one of the key descriptors of dynamical complexity.***

- *Scaling law* - power law with a scaling exponent α describing the behavior of a quantity *F* as a function of a scale *s*, $F(s) \sim s^{\alpha}$ (for a large range of *s*)

- *Fractal system* - system characterized by a non-integer scaling exponent α

- *MultiFractal system* - system characterized by scaling laws with an infinite number of different fractal exponents; the scaling is valid for the same range of scale parameter

- *Crossover* - change point in scaling law (related to singularities in the dynamical description); one exponent applies for small scale parameters, another one applies for the large scales

- *Persistence – (in time series)* a large/mall value is usually followed by a large/small value

(Kantelhardt, in Springer Encyclopedia of Complexity and System Science, 2009)

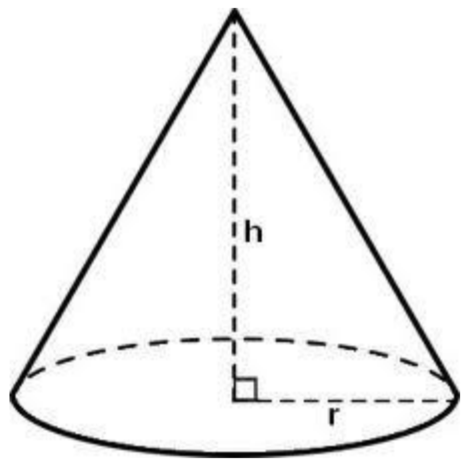
Fractals, modern motivation/applications

- Fractals can model complex disordered objects (clouds, stock market, …) and can reveal scale invariance and symmetry
- Fractals are used to explain anomalous phenomena in condensed matter and solid state physics
- Fractal objects are key element for models of non-equilibrium growth (Laplace equation and electrical discharge, patterns of bacterial growth, viscous fingering)
- Applications to scale-free networks of infinite dimensions, phase transitions in disordered media (e.g. magnetization of spins in Ising model)
- Fractals are relevant for the analysis of processes characterized by scale invariance and leading to power law distributions; applications to time series analysis in turbulence, complexity, finance, geology, analysis of DNA sequences, path integrals of quantum theory of space-time.
- Contribute to the study and understanding of nonlinear systems, strange attractors and chaos.
- Many practical applications in, e.g., image compression, medicine, colloids and polymers

FRACTAL GEOMETRY

$$V = \frac{4}{3}\pi R^3$$

$$D = 3$$



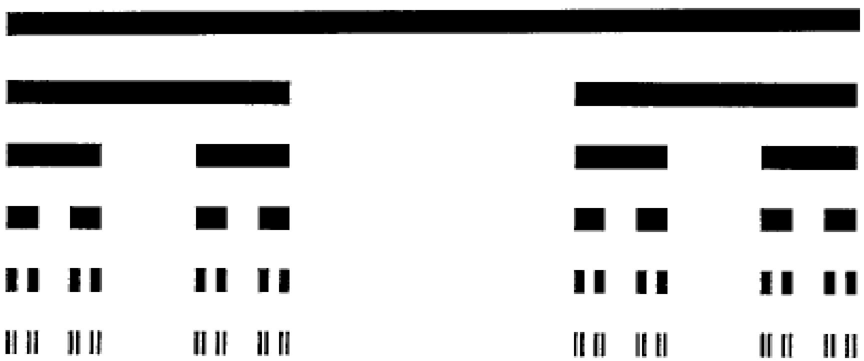
$$V = \frac{1}{3}\pi r^2 h$$

$$D = 3$$

?

D=2,37

ANOTHER FRACTAL EXAMPLE: THE CANTOR SET



INITIATOR : the unit length interval [0,1]
GENERATOR : divide the interval into three equal parts and delete the open middle interval leaving its endpoints in the set

THE LENGTH : $(2/3)^n = \delta^{(1-D)}$

$$D = \frac{\ln 2}{\ln 3} = 0.6309$$

THE MEASURE :

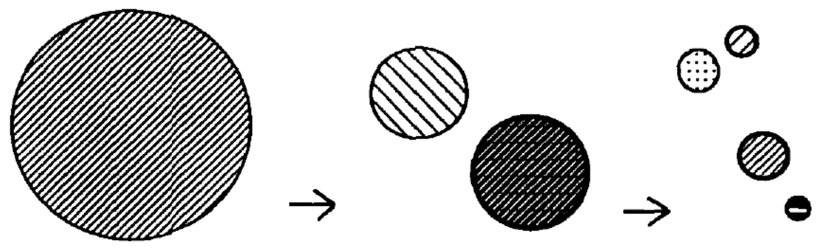
$$M_d = \sum_{i=1}^{N} \delta^d = 2^n \left[\left(\frac{1}{3} \right)^n \right]^d = \delta^{d-D}$$

THE FRACTAL DIMENSION :

$$D = \frac{\ln 2}{\ln 3} = 0.6309$$

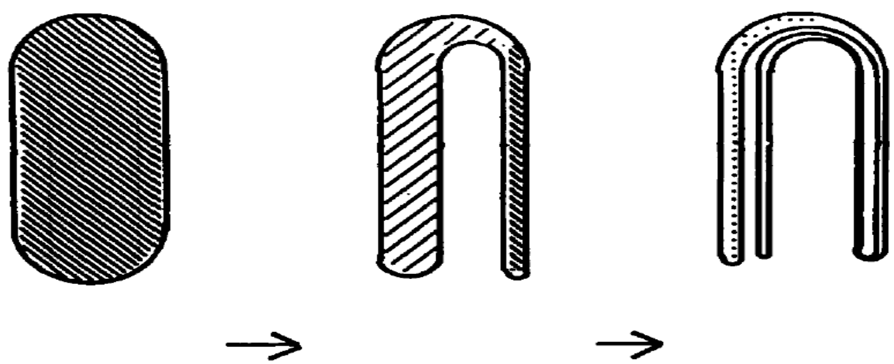
MULTIFRACTALS: multiplicative processes

A process that generates populations through multiplicative sequences (e.g. eddies in turbulent flows) may exhibit multifractal properties that describe the relative abundance of the population on (fractal) subsets of the set.



(a) (b) (c)

Isotropic multiplicative process (Richardson cascade): division continues to smaller and smaller scales (lognormal and β-models of turbulence)

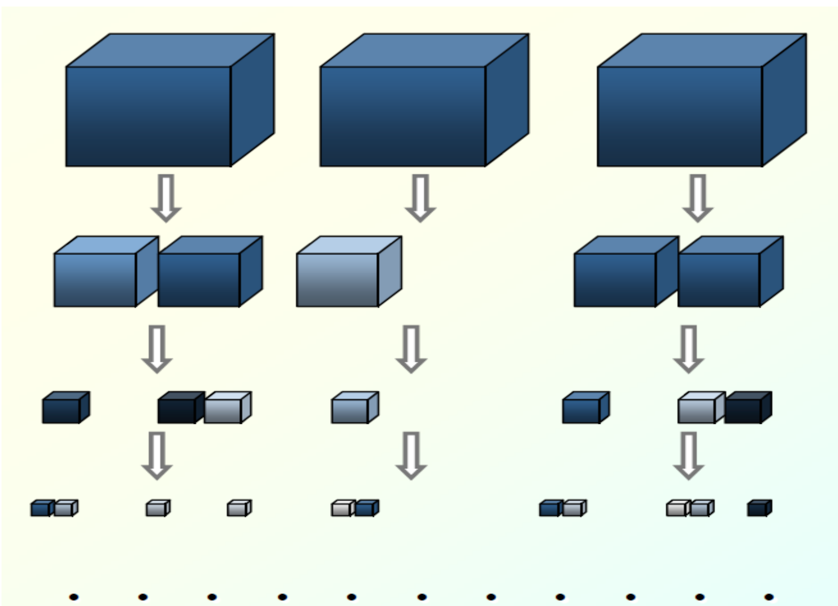


(a) (b) (c)

Break-down of isotropy: stretching and folding , the thickness and density of blobs results from "successive multipliers"(Meneveau and Sreenivasan, 1991)

MULTIFRACTALS: multiplicative processes

The multiplicative process proceeds as follows : at each step a new generation of the population is created from the previous generation by division of the length interval by 2 -> at step n there are 2^n cells of length 2^{-n}. The population is described by the measure of the set given by the "content" of each of the 2^n cells, μ_i.



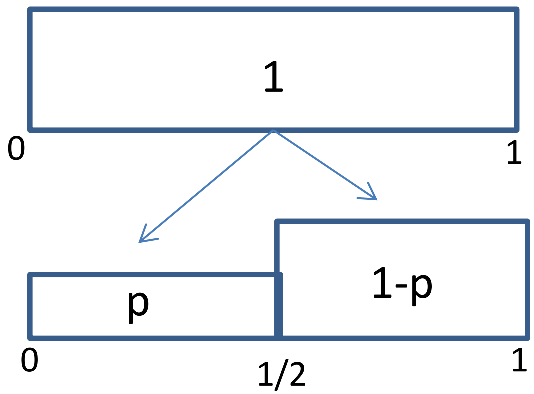
By writing $\mu_\zeta = \delta^\alpha$ one can compute the spectrum of mono-fractal dimension for each α, the $f(\alpha)$ multifractal spectrum as the fractal dimension of the set of measure μ_ζ; α is also known as the Lipschitz-holder exponent.

A subset is defined as the set of $N_n(\zeta)$ segments that have the same measure μ_ζ. The fractal dimension of the subset is determined from:

$$\mu_i = \frac{N_i}{N}$$

$$M_d(S_\zeta) \sim \delta^{-f(\zeta)} \delta^d$$

AN EXAMPLE : the binomial multiplicative (*p*) cascade



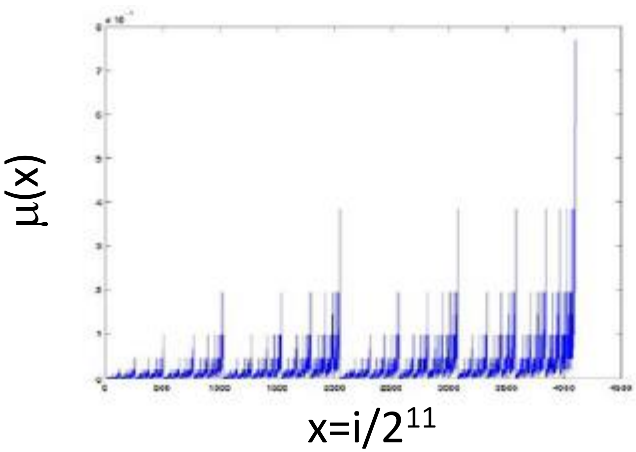
The unit mass is redistributed at each multiplication: a fraction p goes to the left and (1-p) to the right

("space filling and non-uniform energy transfer rate" – the p-model of turbulence)

After n iterations the number N_ε of segments that have the same measure μ_ε is equal to

$$N_n(\xi) = \frac{n!}{((\xi n)!)((1-\xi)n)!} \qquad \xi = \frac{k}{n}$$

$$\mu_\xi = \Delta^n(\xi), \Delta(\xi) = p^\xi (1-p)^{1-\xi}$$



The measure μ(x) after 11 iterations

$$\mu_i = \frac{N_i}{N}$$

Figure from Rudolf Reidi, Rice University

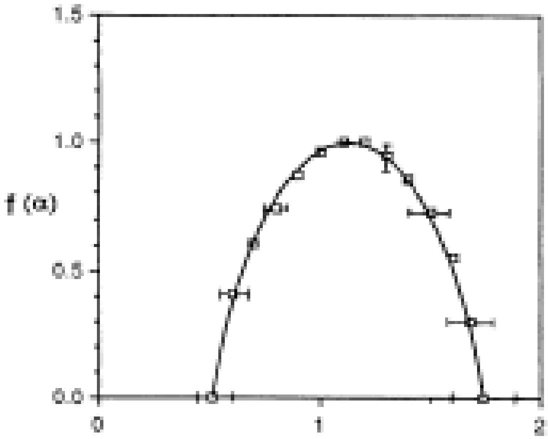
This measure has scaling properties

$$M_d(S_\zeta) \sim \delta^{-f(\zeta)} \delta^d$$

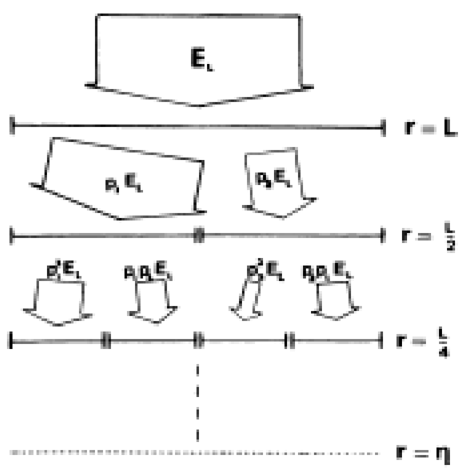
$$f(\zeta) = \frac{\xi \ln \xi + (1-\xi)\ln(1-\xi)}{\ln 2}$$

(Feder, 1988)

The f(α) multifractal spectrum



Multifractal spectrum of p-model of Hydrodynamic model of turbulence (Meneveau and Sreenivasan, 1987).



Space-filling, non-uniform energy transfer rate cascade of p-model (Meneveau and Sreenivasan, 1987)

ε defined for the binomial multiplicative process has not a clear useful significance .

$\mu_\xi = \delta^\alpha$ is expressed by definition as a function of α – the Lipschitz-Holder exponent

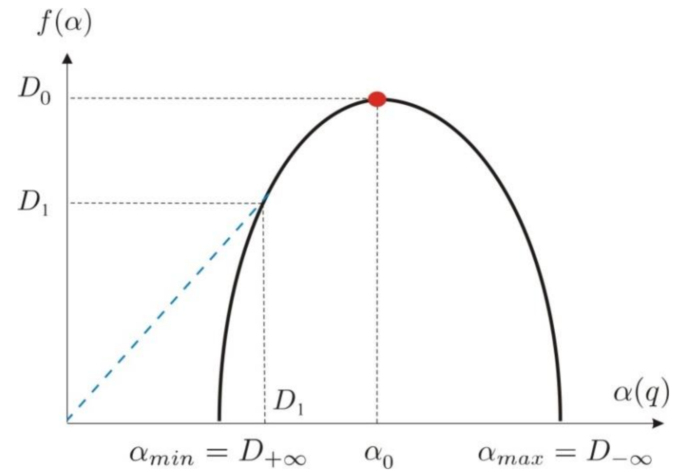
$$\alpha(\zeta) = \frac{\ln \mu_\zeta}{\ln \delta} = -\frac{\xi \ln p + (1-\xi)\ln(1-p)}{\ln 2}$$

α has a one-to-one relationship with ξ

-> the f(α) is derived directly from f(ξ)

f(a) is the fractal dimension of the set characterized by the singularity α, i.e. the power law behavior described by the Holder exponent, $\mu_\varepsilon = \delta^\alpha$

$\alpha_{max} - \alpha_{min}$ = (multifractal) measure of intermitency *(Macek & Wawrzaszek, 2009)*



Multiscale analysis with the Structure Function – Generalized Hurst exponent

- The intermittent behaviour is analyzed in terms of high order moments of the PDFs : **<u>the structure function (SF)</u>**

$$S_q\left(\delta B^2,\tau\right)=\int_0^{\delta B^2_{\max}}\left(\delta B^2\right)^q P\left(\delta B^2,\tau\right)d\delta B^2=\left\langle\left|B^2\left(x_i+\tau\right)-B^2\left(x_i\right)\right|^q\right\rangle$$

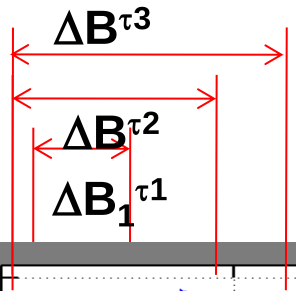
- For each SF S_q, we associate a fractal Hurst exponent ζ_q for a range of scales τ

$$\xi_q=d\left(\log S_q\left(\delta B^2,\tau\right)\right)/d\left(\log\tau\right)$$

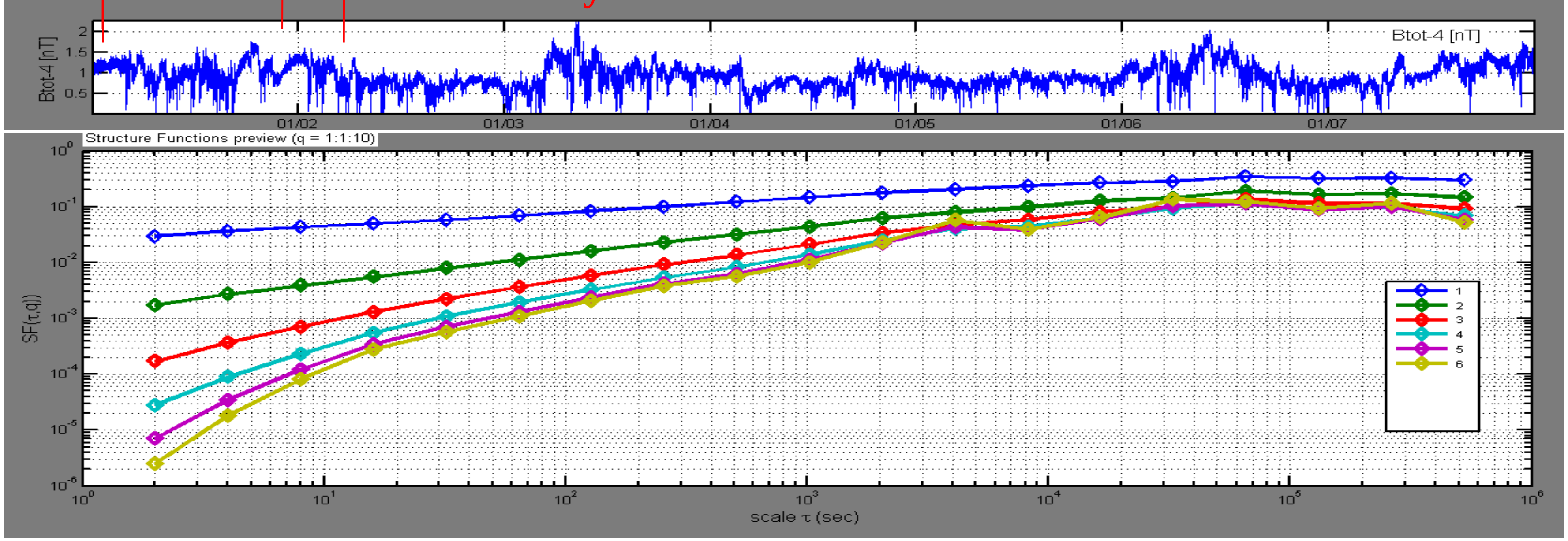
- If $\zeta_q = \zeta_1 q$, the fractal properties of the fluctuating series are fully described by the value of ζ_1 : mono-fractal/self-similar fluctuations. For intermittent turbulence → ζ_q is a non-linear function of q : multifractal case

- SFs can be evaluated for any positive values of q but will generally diverge for q < 0

(Standard) Structure Function analysis



Ulysses data 01/01 - 07/01/2007



Moments of various orders of the probability distribution functions:

$$S_q(\tau) = \langle|\delta B(\tau)|^q\rangle = \int_0^{\delta B_{max}} |\delta B(\tau)|^q \, P(\delta B, \tau) d\delta B$$

Power law scaling:

$$\boldsymbol{S_q(\tau) \sim \tau^{\zeta_q}}$$

- $\zeta_q = sq$, the process is self-similar/monofractal with fractal dimension (or Hurst exponent) s.
- The statistics and scaling of the structure function is dominated by the most numerous fluctuations..

Multifractal analysis with partition function

1. define an "incremental measure" $\delta\mu_j = |B(t_j + \delta) - B(t_j)|$

2. subdivide the total time interval T *into* M=T/τ *segments with* $\tau=\kappa\delta$ and calculate the normalized scale-dependent "segmental measure" (i.e., the scale dependent measure that we seek to define)

$$\mu_i(\tau) = \sum_{j=(i-1)k+1}^{ik} \delta\mu_j/\mu \quad \text{with} \quad \mu = \sum_{j=1}^{N} \delta\mu_j$$

3. Assume that each such measure varies with the scale τ in a singular manner as a power law, τ^{α}. We can now form the q^{th} moment order of the coarse grained probabilities, traditionally called the "partition function" (Macek & Szczepaniak 2008)

$$\Gamma(q,\tau) = \sum_{i=1}^{M} \mu_i^q(\tau)$$

4. search for the dominant singular behavior of $\Gamma(q,\tau)$ as characterized by a power law in τ with exponent $\gamma(q)$,$\tau^{\gamma(q)}$, for small τ similar to that for the structure function analysis. In general, for each moment order, $\gamma(q)$ is a different number characterizing the particular fractal behavior of the subset of fluctuations, which dominates the (singular) scaling behavior of that particular moment order.

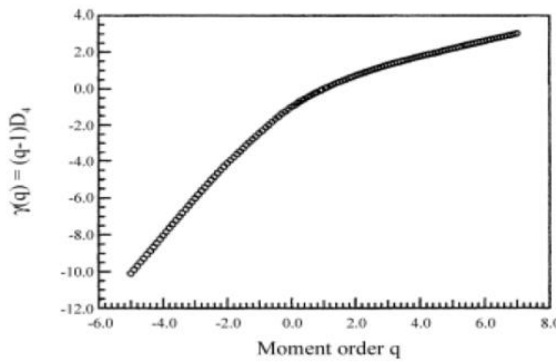
The multifractal spectrum – different representations

Link to standard mathematical representation of multifractals:

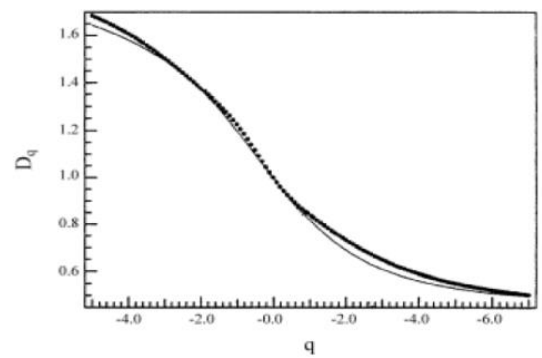
- generalized dimensions: $D_q(q) = \gamma(q)/(q-1) + 1$

- The multifractal spectrum $f(\alpha)$: $f(\alpha) = q\alpha\,(q)-(q-1)/\,D_q(q)$
  $\alpha = d/dq[(q-1)\,D_q(q)]$

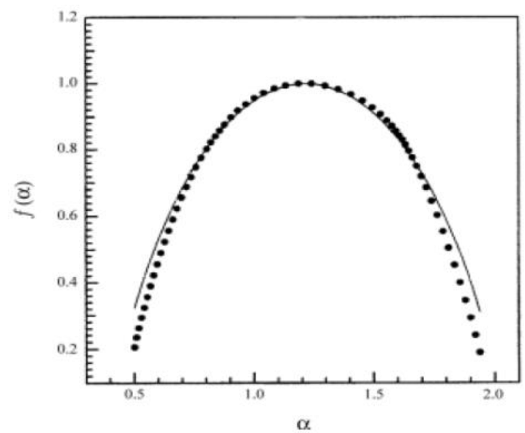
Top left Partition function exponent $\gamma(q)$ for a time series of the AE (auroral electrojet) index.



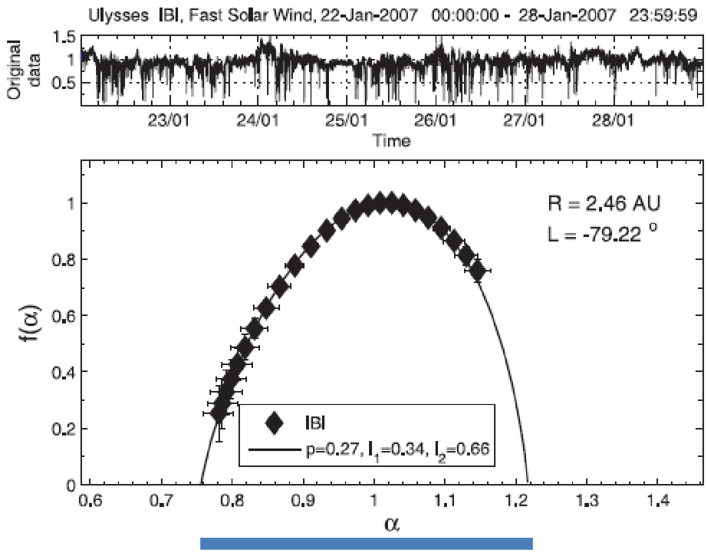
Top right: Generalized dimension *Dq for the same AE time series. Solid line is the best fit using the P-model.*



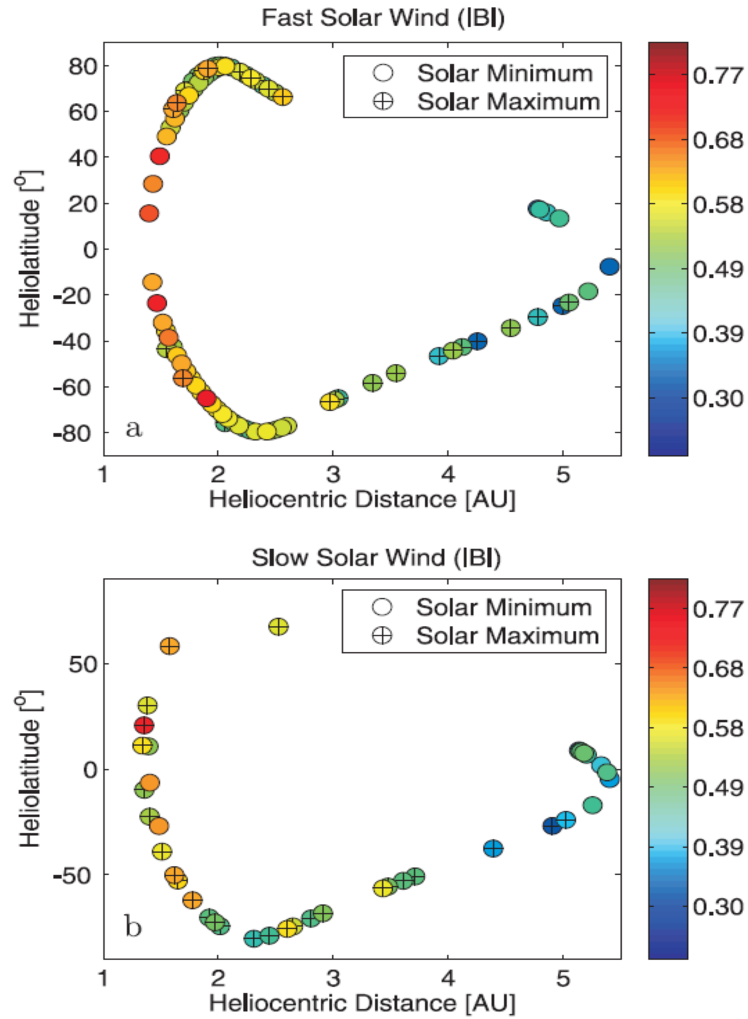
Bottom*: Singularity* spectrum $f(\alpha)$ for the same AE time series obtained from the Legendre transform.
(from Consolini et al 1996)



Multifractal analysis of the radial evolution of the solar wind intermittency



Multifractal spectrum f(α) as a function of singularity strength α (diamonds) determined for the magnetic field strength of the fast solar wind measured by Ulysses in 2007 at 2.46 AU, −79.22◦. Continuous line shows a theoretical two-scale model fitted with observations *(Wawrzsasek et al., 2015)*.



Map of the degree of multifractality as a measure of intermittency determined for fast (a) and slow (b) solar wind during two solar minima (1997-1998 and 2007-2008) and a solar maximum (1999-2001), respectively. Color denotes the values of the parameter determined for data at different heliocentric distances and heliographic latitudes. *(Wawrzsasek et al., 2015)*.

Single-Parameter Scaling

- Monofractal condition can be satisfied by a one-parameter scaling with the parameter s [Chang et al., 1973]:

$$P(|\delta E|, \tau) = (\tau/\tau_0)^{-s} P_s\left(|\delta E|(\tau/\tau_0)^{-s}\right)$$

One can show that $\zeta_q = q\zeta_1 \quad \Rightarrow \quad s = \zeta_1 = H$

- *For monofractal fluctuations, the single-parameter scaling is able to provide a clear description of how the strength of the fluctuations varies with the time scale.*

(S. Tam, Workshop on Multifractal Turbulence, Brussels, 2010)

Rank-Ordered Multifractal Analysis (ROMA)

- Technique introduced by Chang and Wu [2008]
- Technique retains the spirit of structure function analysis and single-parameter scaling
- Divide ***(Rank-Order)*** the domain of $Y=|\delta E|(\tau/\tau_0)^{-s(Y)}$

  (Note: $s=s(Y)$) into separate ranges and, for each range, look for one-parameter scaling

$$P(|\delta E|,\tau) = (\tau/\tau_0)^{-s(Y)} P_s\left(|\delta E|(\tau/\tau_0)^{-s(Y)}\right)$$

- Scaling function $(\tau/\tau_0)^{s(Y)} P(|\delta E|,\tau)$ and scale invariant Y

(S. Tam, Workshop on Multifractal Turbulence, Brussels, 2010)

To solve for $s(Y)$, the scaling parameter s for the range $Y = [Y_{low},\ Y_{high}]$:

- construct the range-limited structure functions with prescribed s

$$S'_q(\tau) = \int_{Y_{low}(\tau/\tau_0)^s}^{Y_{high}(\tau/\tau_0)^s} |\delta E(\tau)|^q P\left(|\delta E|, \tau\right) d|\delta E|$$

- Look for the scaling behavior

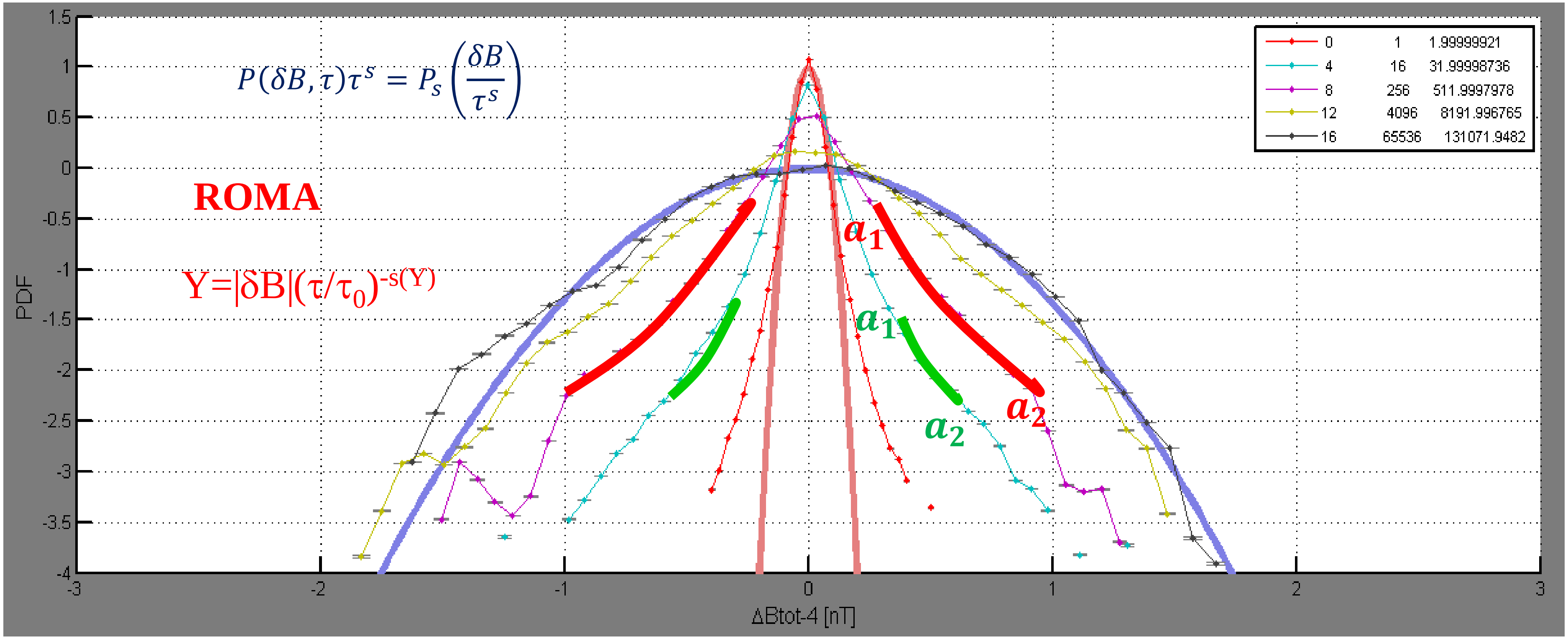
$$S'_q(\tau) \sim \tau^{\zeta'_q}$$

- The solution s will satisfy

$$\zeta'_q = qs$$

(S. Tam, Workshop on Multifractal Turbulence, Brussels, 2010)

Rank-Ordered Multifractal Analysis (ROMA)



Range limited structure function:

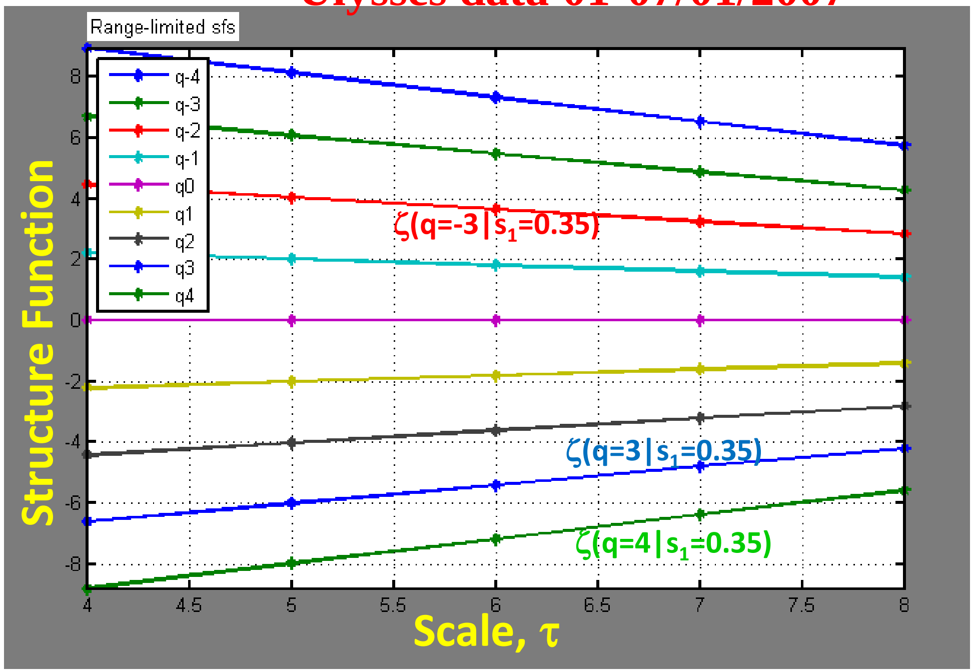
$$S_q(\delta B,\tau) = \langle|\delta B(\tau)|^q\rangle = \int_{a_1}^{a_2} |\delta B(\tau)|^q \, P(\delta B,\tau)d(\delta B) \quad a_1 = Y_1\tau^s\,,\ a_2 = Y_2\tau^s$$

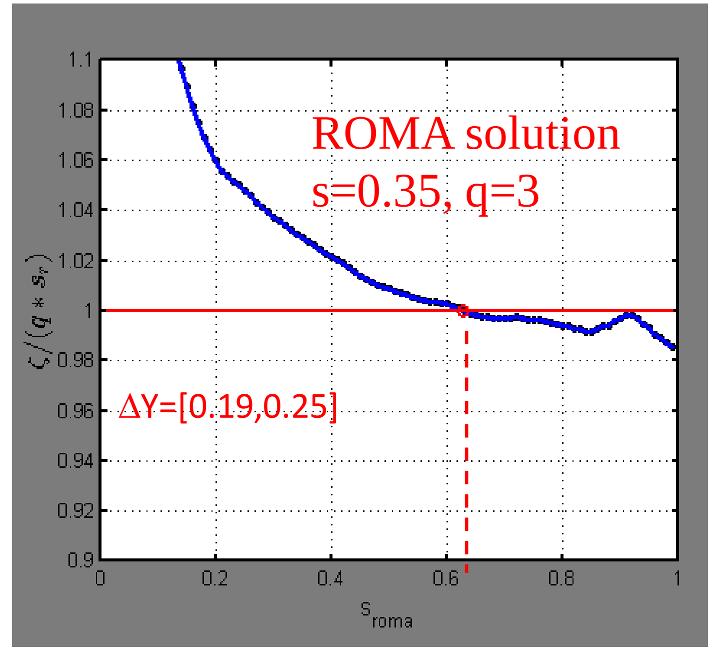
Divide ***(Rank-Ordering)*** at each scale the domain of fluctuations, ΔB, in subsets Y ordered such that each subset is mono-fractal and has the fractal dimension s(Y), Chang and Wu, 2008

$$Y=|\delta B|(\tau/\tau_0)^{-s(Y)}$$

Rank-Ordered Multifractal Analysis (ROMA)

Ulysses data 01-07/01/2007





$\Delta Y=[0.19, 0.25]$, scales: 16 : 256 [sec], surogate $s_1=0.35$

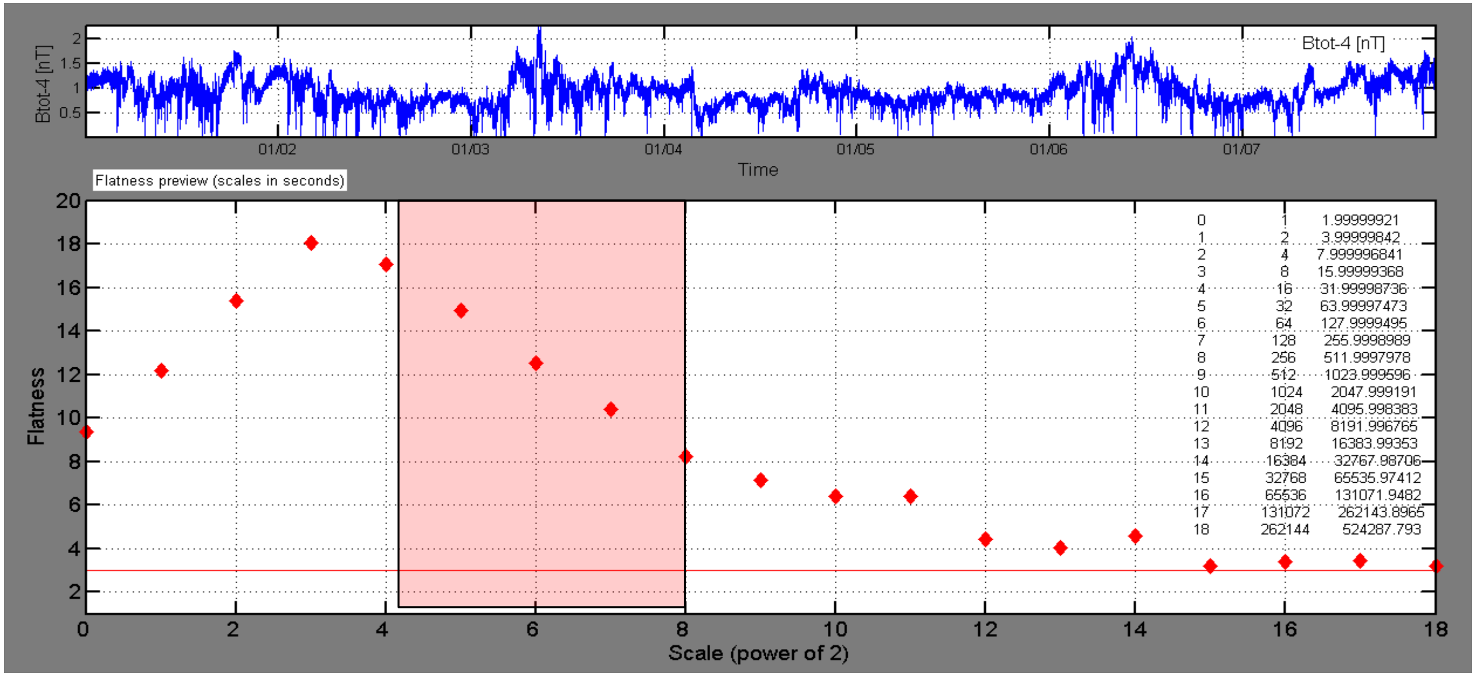
$\zeta(q=3, s)$ for DY = [0.19, 0.25],
fast solar wind from Ulysses, 01-07/01/2007

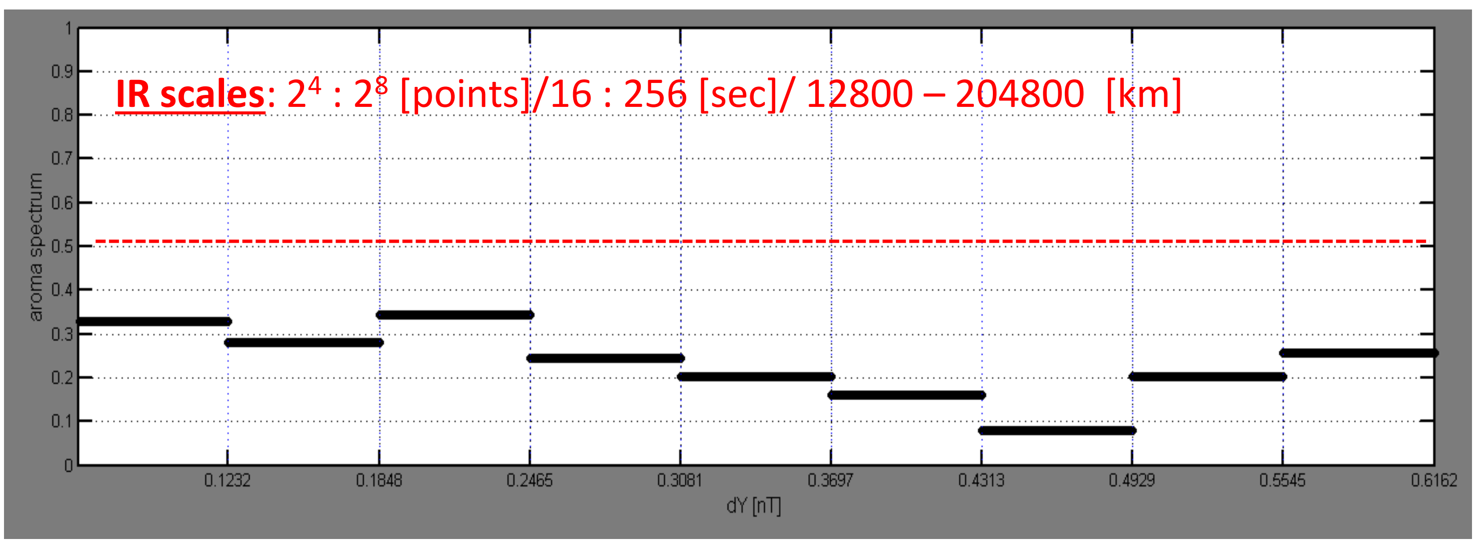
Range limited structure function:

$$S_q(\delta B, \tau) = \langle |\delta B(\tau)|^q \rangle = \int_{a_1}^{a_2} |\delta B(\tau)|^q \, P(\delta B, \tau) d(\delta B) \qquad a_1 = Y_1 \tau^s \, , \; a_2 = Y_2 \tau^s$$

Look for the scaling behavior: $\zeta_q = qs$

ROMA spectrum: Ulysses 01 – 07/01/2007

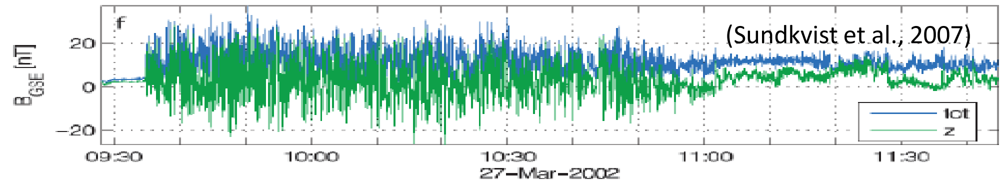




A Multi-Level Data Analysis Strategy

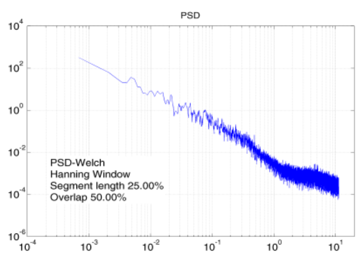
Observation

In-situ Data

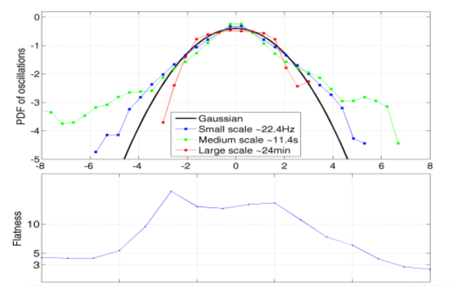


Analysis

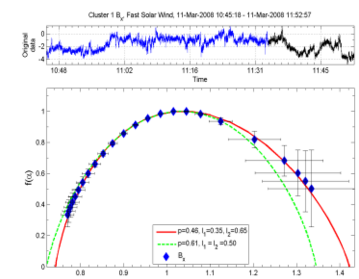
Power Spectral Density



Probability Distribution Functions, Wavelets



Structure Functions, Fractals/Multifractals



Classification

Periodicities

Scaling, Spatio-temporal structures

Scale invariance, topology

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