Cosmology from galaxy clustering: the road to Euclid

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Giuseppe “Bepi” Tormen, 1962-2018

LECTURES OUTLINE

1. <u>Galaxy clustering in the context of the standard model of cosmology</u>

   - The accelerating Universe: why we say so?
   - Dark energy or dark gravity?
   - Statistical description of density fluctuations

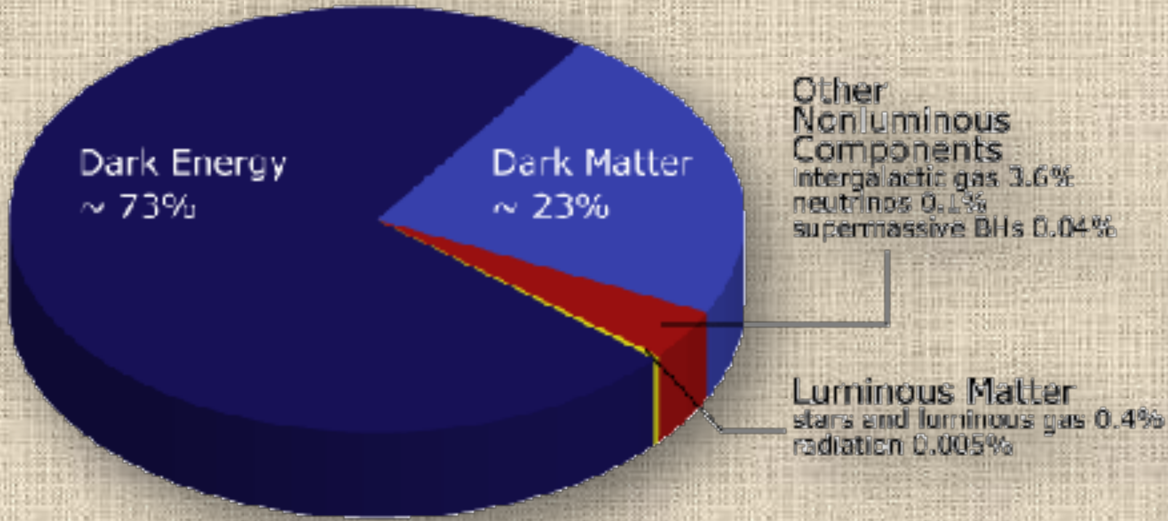
LECTURES OUTLINE

2. <u>Measuring galaxy clustering from redshift surveys</u>

- From observed galaxy counts to overdensities: masks, weights, windows, mock samples and all that
- From (properly measured) clustering to cosmological parameters: non-linearity, galaxy bias and redshift-space distortions
- Baryonic Acoustic Oscillations as a standard ruler
- Redshift-Space Distortions as a probe of the growth of structure
- Application to current and future surveys: the road to Euclid

The accelerating Universe: why we say so?

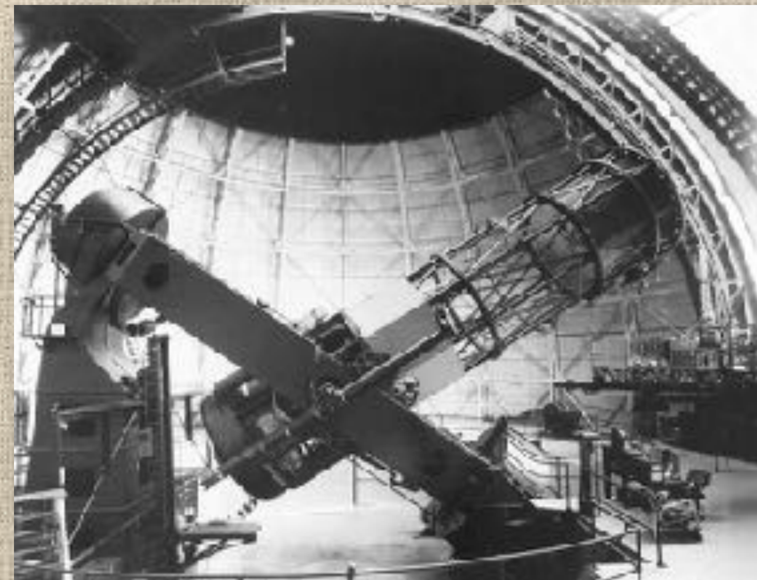
The "cosmic soup" of the 21st century: but who ordered it?

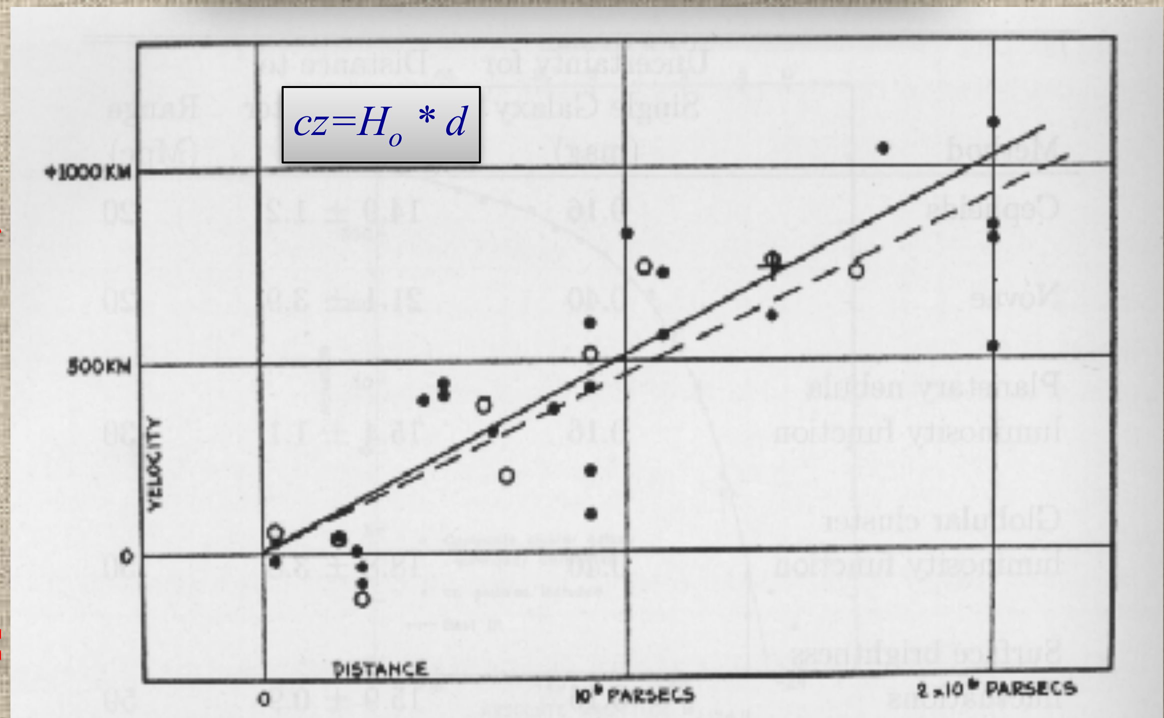


2011 Nobel Prize

This diagram reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart at a faster rate. Astronomers theorize that the faster expansion rate is due to a mysterious, dark force that is pushing galaxies apart.

E. Hubble 1929

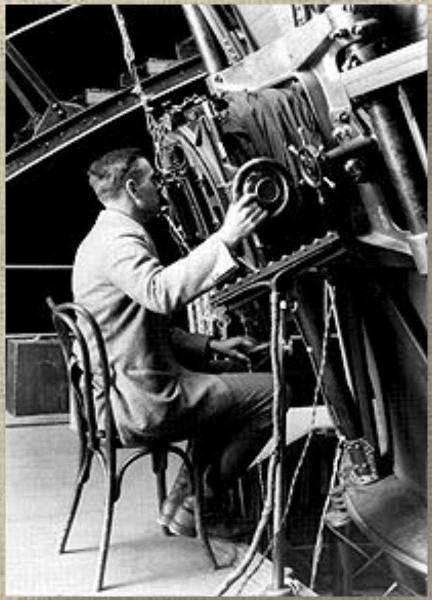


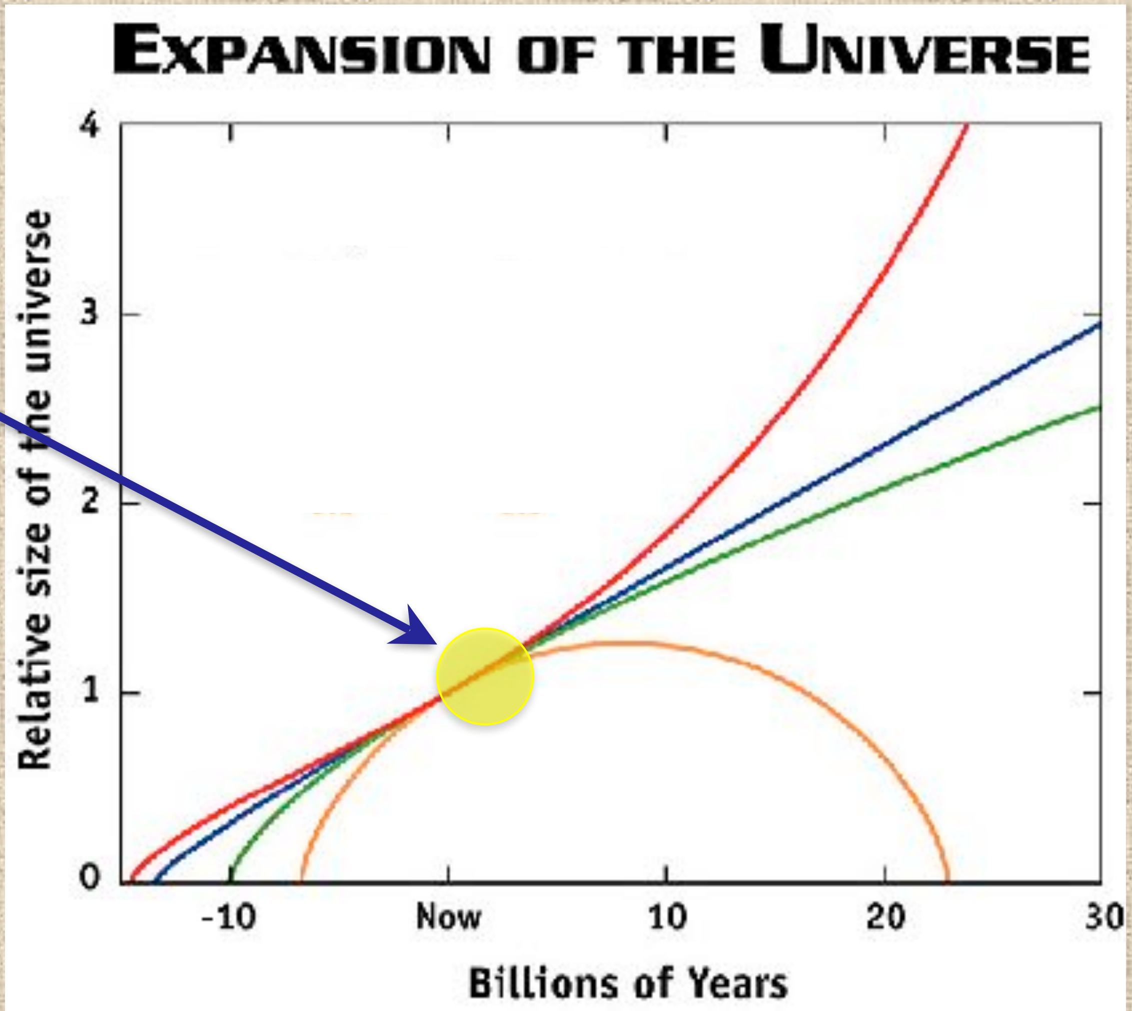


Apparent recession velocity v=cz

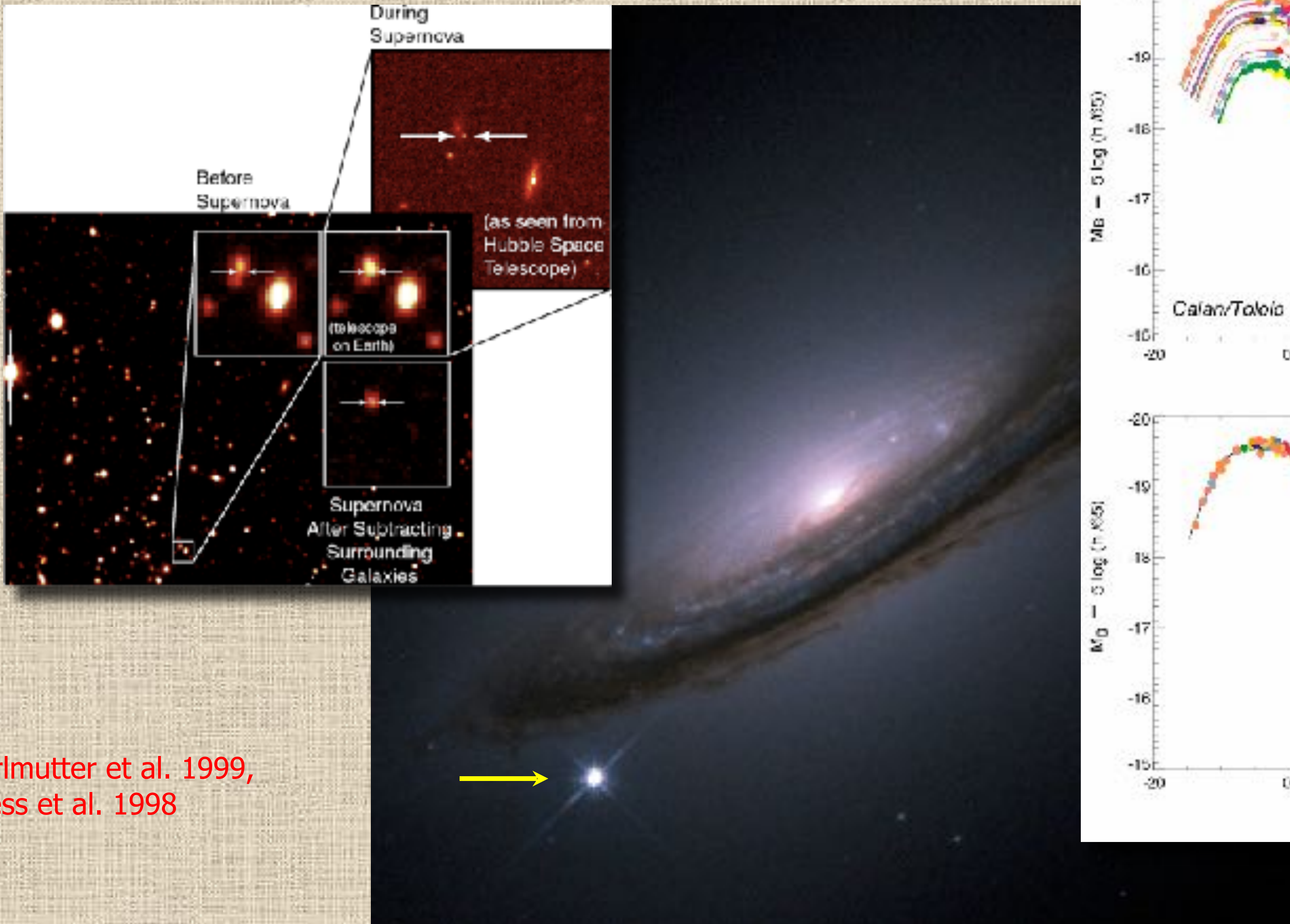
Distance

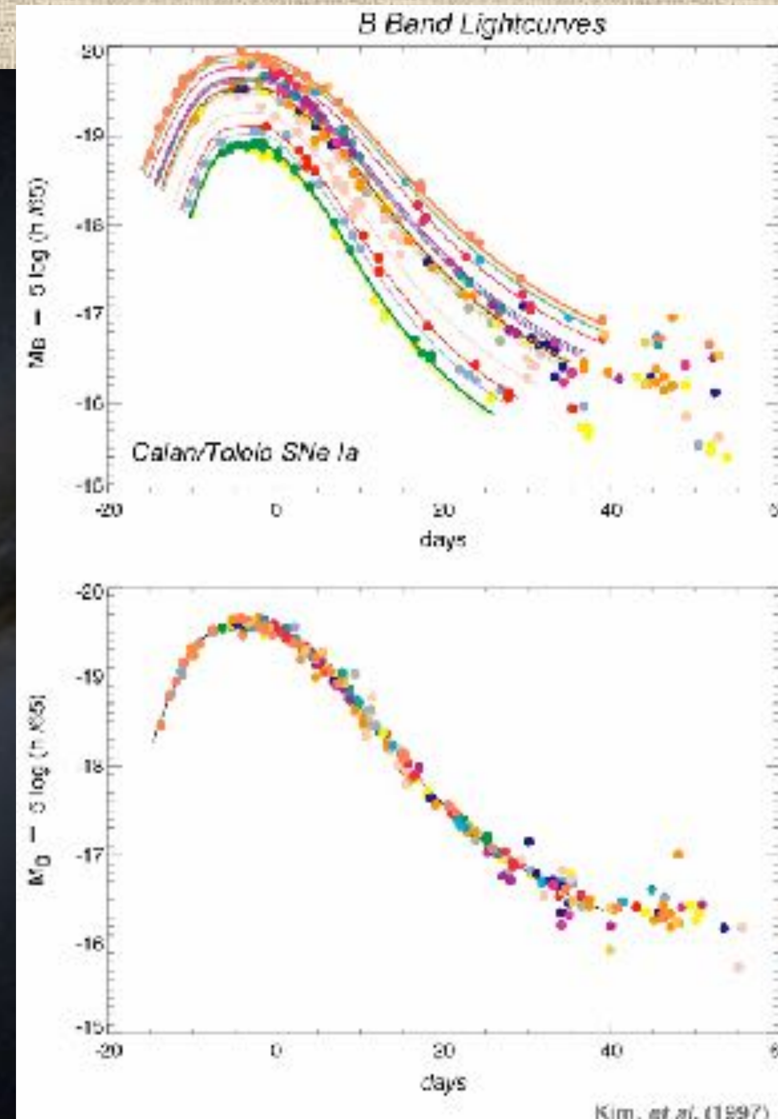
1990s: going further, what is the past history of the expansion rate?





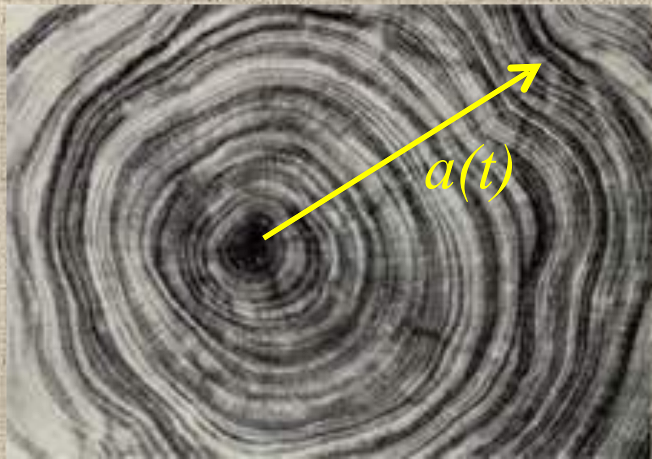
Beyond the Hubble law: Type-Ia supernovae as standard candles





Perlmutter et al. 1999,
Riess et al. 1998

All starts (again) with a "Hubble diagram", which, using Type Ia supernovae (1998), gives a surprising indication…



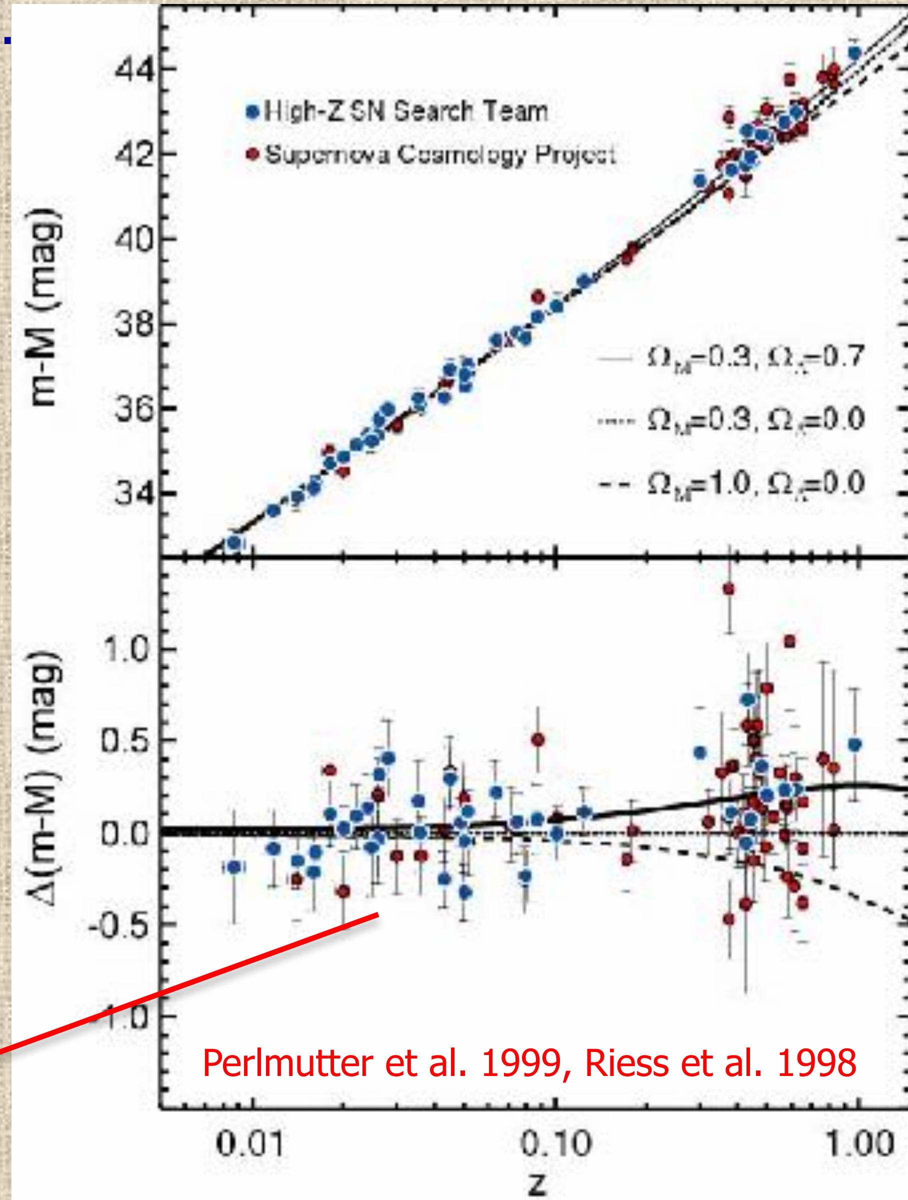
$$H \equiv \frac{\dot{a}}{a}$$

$$\boxed{d_L = (1+z)\int_0^z \frac{dz'}{H(z',\Omega_m,\Omega_\Lambda)}}$$

$$\boxed{H^2(z) = H_0^2\left\{\Omega_m(1+z)^3 + \Omega_\Lambda\right\}}$$

~0.3 ~0.7

log(Distance d_L)



Redshift of spectral lines

... i.e. that the expansion history *H(z)* given by the Friedmann equation:

$$\boxed{H^2(z) = H_0^2\{\Omega_m(1+z)^3 + \cancel{\Omega_k(1+z)^2} + \cancel{\Omega_\gamma(1+z)^4} + \Omega_x(1+z)^{3(1+w_x)}\}}$$

Matter | ~~**Curvature**~~ | ~~**Radiation**~~ | **Generic component**

$$\left(H \equiv \frac{\dot{a}}{a}; \quad w_x \equiv \frac{p_x}{\rho_x c^2}; \quad \Omega_i \equiv \frac{\rho_i}{\rho_c}\right)$$

matches the observations only if we add an extra component with equation of state $w_x = p/c^2\rho = -1$ corresponding to a cosmological constant Λ with energy density $\Omega_\Lambda \sim 3\Omega_\mathrm{m}$

$$\boxed{H^2(z) = H_0^2\{\Omega_m(1+z)^3 + \Omega_\Lambda\}}$$

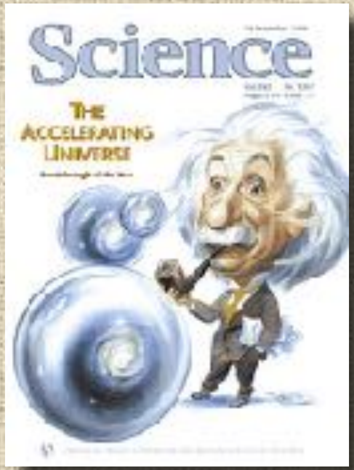
2011 Nobel prize in Physics

Adam Riess

Brian Schmidt

Saul Perlmutter

"… for the discovery of Cosmic Acceleration"



Why are we talking of acceleration?

• w describes the equation of state of the additional fluid contr
to $H(z)$: w = *pressure / density* = $p/c^2\rho$

• In GR both density and pressure contribute to gravity through the stress-energy tensor, as described by the dynamical equation

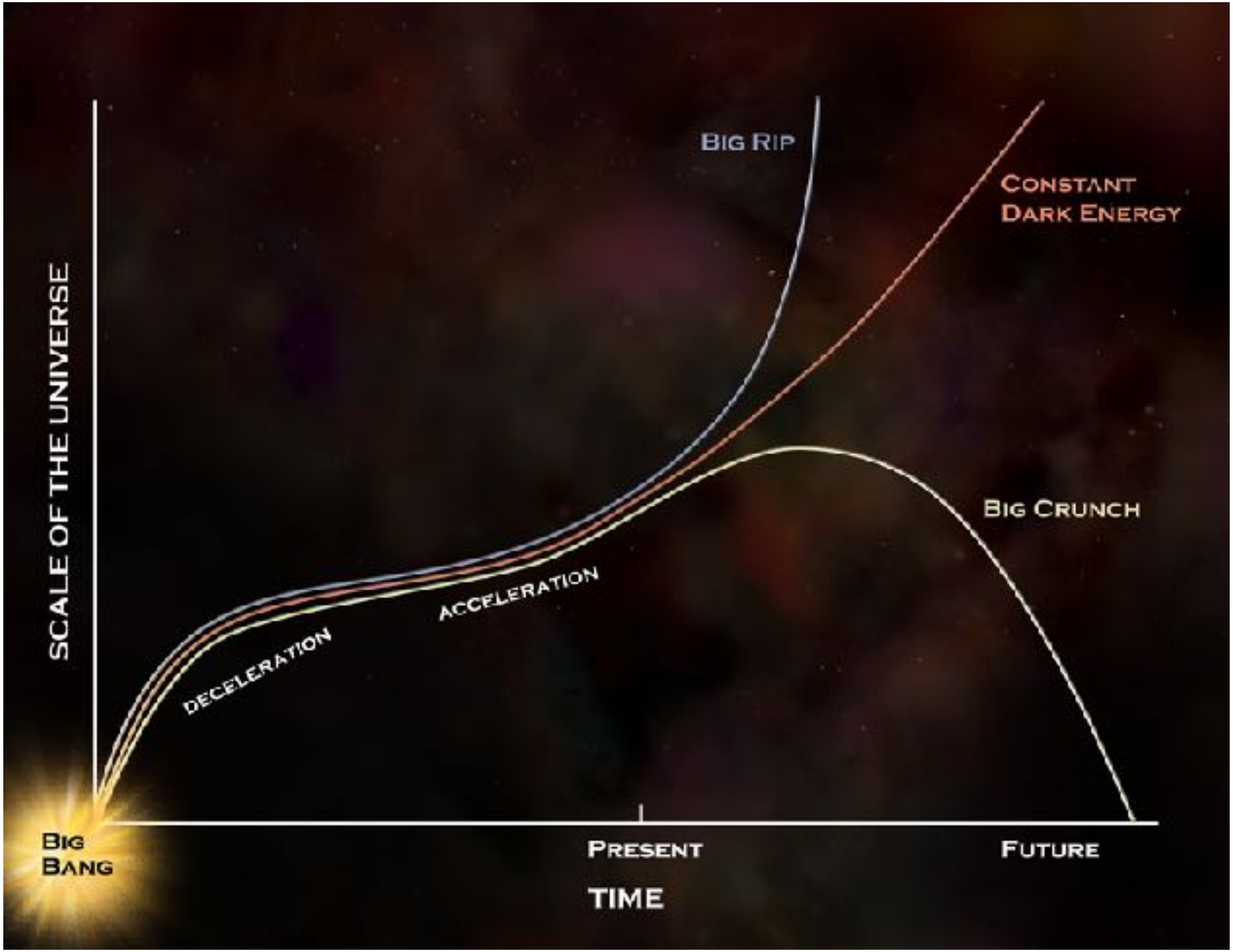
$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(c^2\rho_{TOT} + 3p_{TOT}\right)}$$

• But $w = -1$ implies $p = -c^2\rho_\Lambda$ and therefore, at z=0

$$\boxed{\frac{\ddot{a}}{a} = -\frac{H_0^2}{2}\left(\Omega_m - 2\Omega_\Lambda\right)}$$

$$\boxed{\Omega_i \equiv \frac{8\pi G}{3H_0^2}\rho}$$

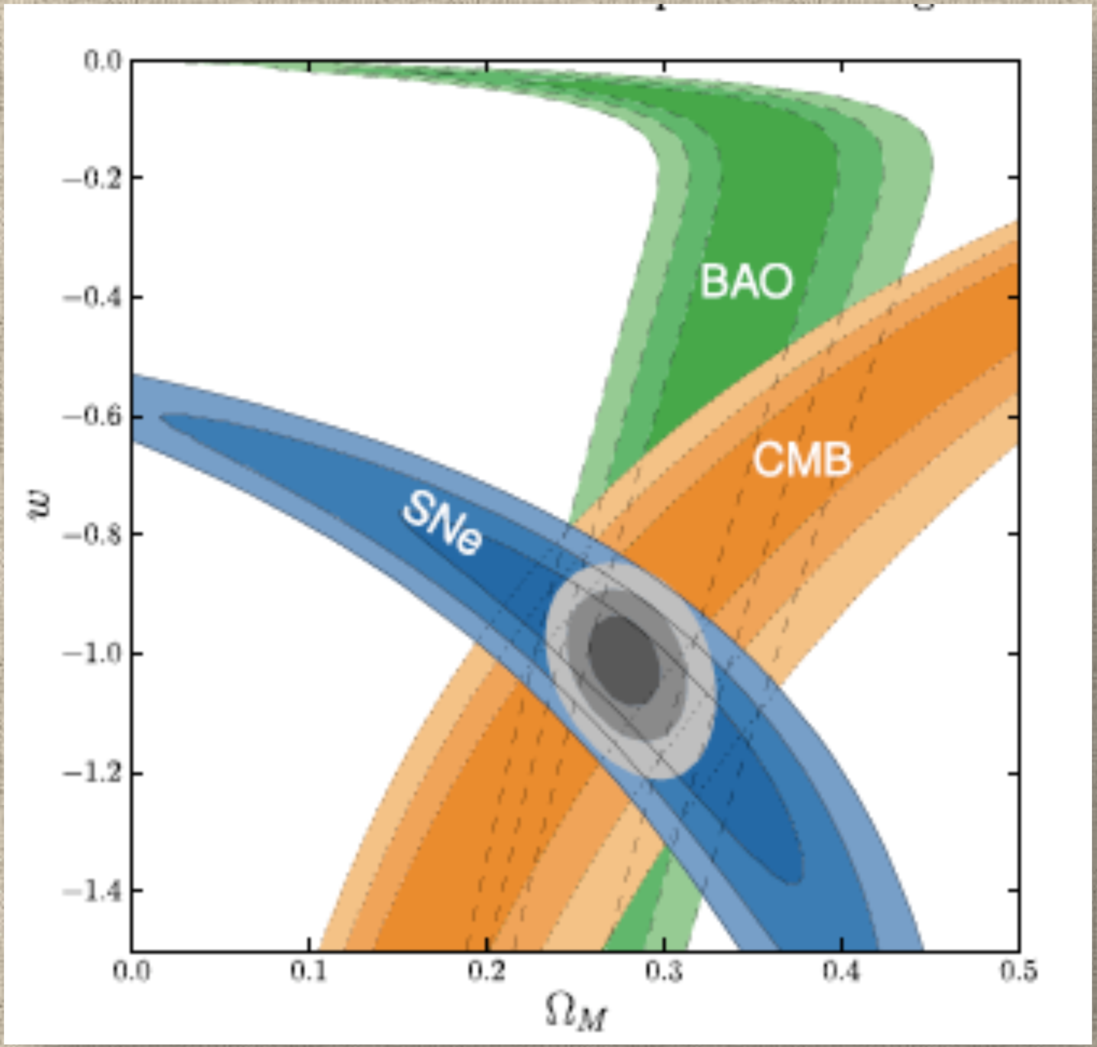
which for $\Omega_\Lambda > \Omega_m/2$ becomes positive --> **acceleration**



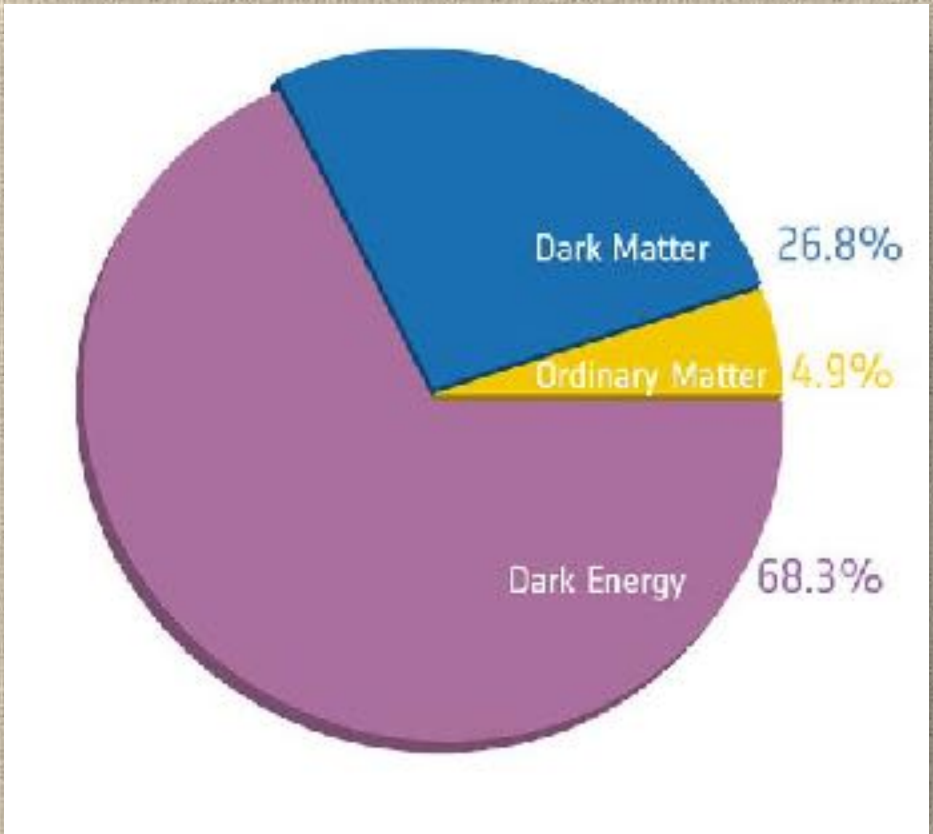
Big Rip
Constant Dark Energy
Big Crunch
Scale of the Universe
Deceleration
Acceleration
Big Bang
Present
Future
Time

A “Concordance Model”

Cosmic concordance: a w=-1 Universe?

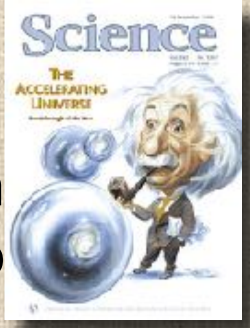


(Amanullah et al. 2010 - before Planck)

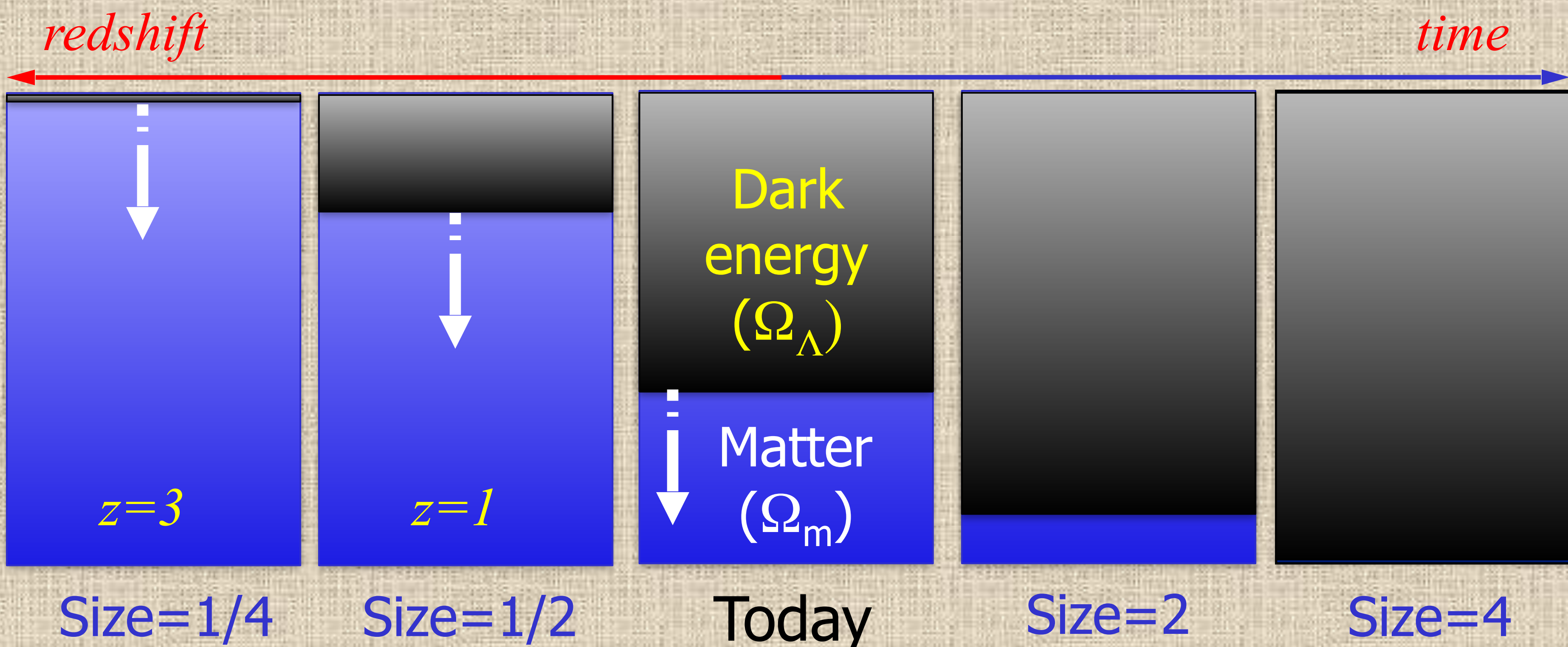


After Planck (The Planck Collaboration)

Cosmic coincidence



If *w=-1* and the cosmological constant corresponds to some sort of "quantum zero-point" (energy of vacuum), then its value today is a factor ~10^{120} too small, plus it is suspiciously fine-tuned: anthropic argument?



Thus could we have $w = w(z)$? --> e.g. quintessence, a cosmic scalar field slowly rolling to the minimum of its potential (e.g. Wetterich 1988), inducing an evolving $-1 < w(z) < -1/3$. Or more complex interactions between DM and DE (e.g. Amendola 2000; Liddle et al. 2008) ?

Remember the Friedmann equation for a flat Universe containing matter and a generic component with constant equation of state w (w=-1 for the Cosmological Constant case)

$$\boxed{H^2(z) = H_0^2\,\{\Omega_m(1+z)^3 + \Omega_x(1+z)^{3(1+w_x)}\}}$$

$$\left(H \equiv \frac{\dot{a}}{a};\quad w_x \equiv \frac{p_x}{\rho_x c^2};\quad \Omega_i \equiv \frac{\rho_i}{\rho_c}\right)$$

For a generic time-dependent equation of state $w(z)$, this modifies to:

$$\boxed{H^2(z) = H_0^2\,\{\Omega_m(1+z)^3 + \Omega_x \exp\left\{3\int_0^z [1+w(z')]\,d\ln(1+z')\right\}}$$

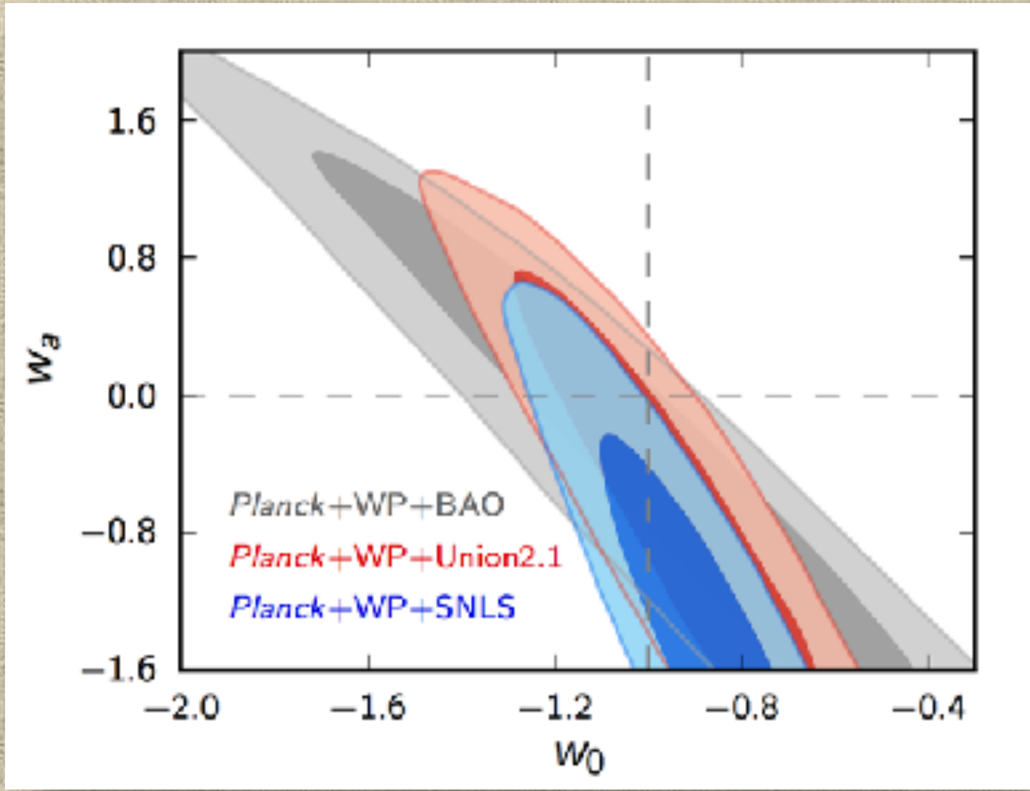
But how should we expect $w(z)$ to vary?

Λ is too small and fine-tuned: an evolving equation of state $w(a)$?

Parameterizing our ignorance:

$$w(a) = w_0 + w_a(1 - a)$$

[a = scale factor of the Universe = $(1+z)^{-1}$]



Planck Collaboration 2013, XVI

THE FIRST GOAL: MEASURE WHETHER $w=w(z)$

...IS THIS ALL?

...Lambda [or w(z)] is not the end of the story...

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

?

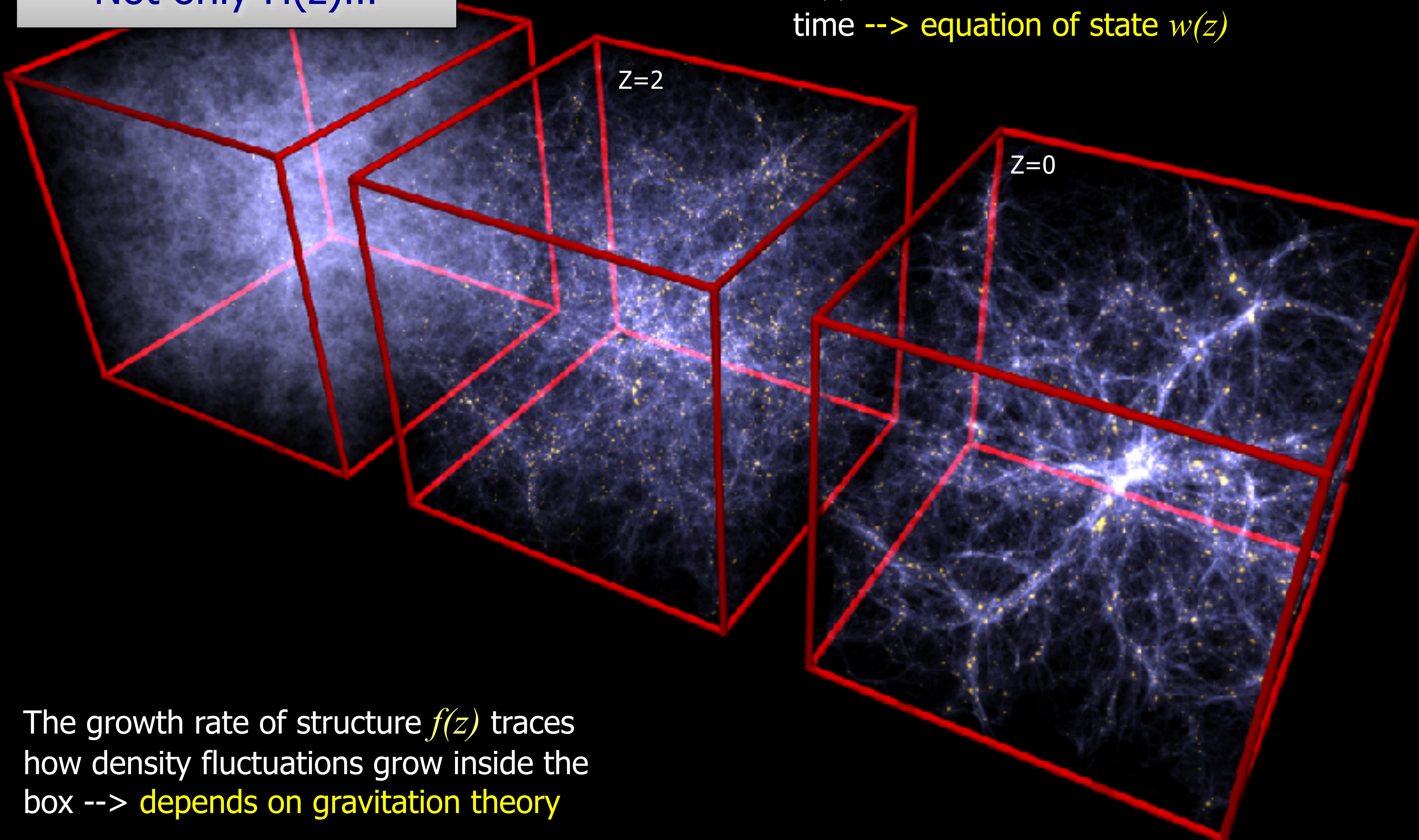
Modify gravity theory [e.g. ***R* → *f(R)***]

Add dark energy

"...the Force be with you"

Not only H(z)…

H(z) measures how the box expands with time --> equation of state *w(z)*



The growth rate of structure *f(z)* traces how density fluctuations grow inside the box --> depends on gravitation theory

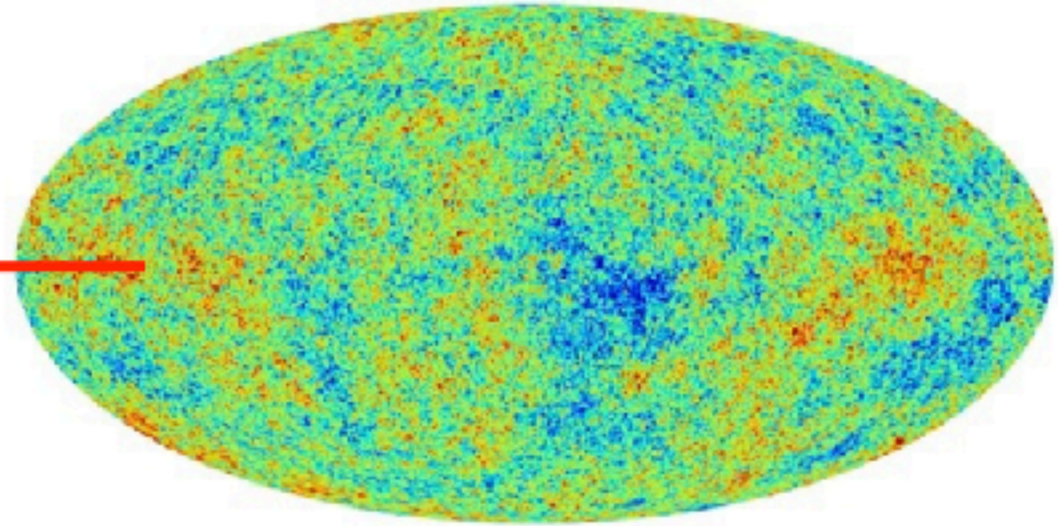
Springel et al.

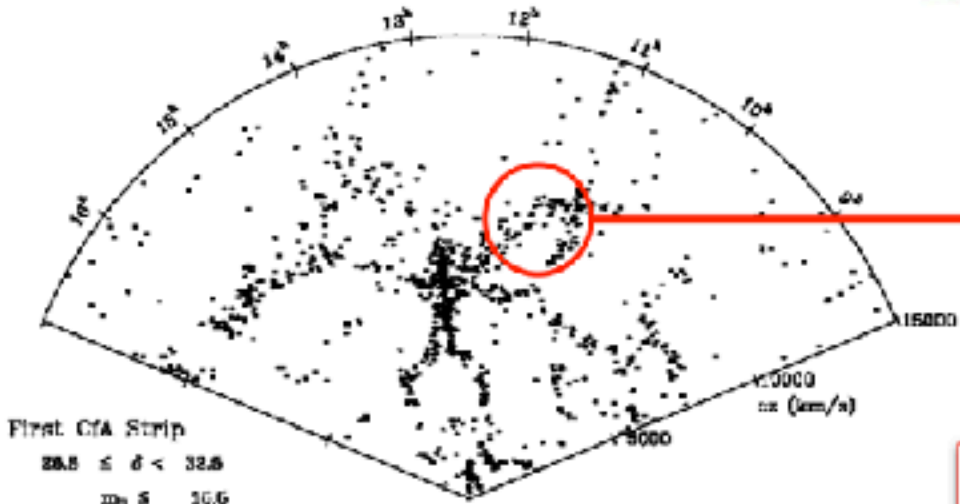
Cosmological perturbations

(Special thanks to Emiliano Sefusatti)

CMB temperature fluctuations

$T(\hat{n})$





$n_g(\vec{x})$

number density of galaxies

We can only study the **statistical properties** of cosmological perturbations

Mathematically, these are **random fields**

Random fields

If ϕ is a random **variable** with Probability Distribution Function (PDF) $\mathcal{P}(\phi)$ we can compute:

$$\langle \phi \rangle = \int d\phi \, \mathcal{P}(\phi) \, \phi$$ mean

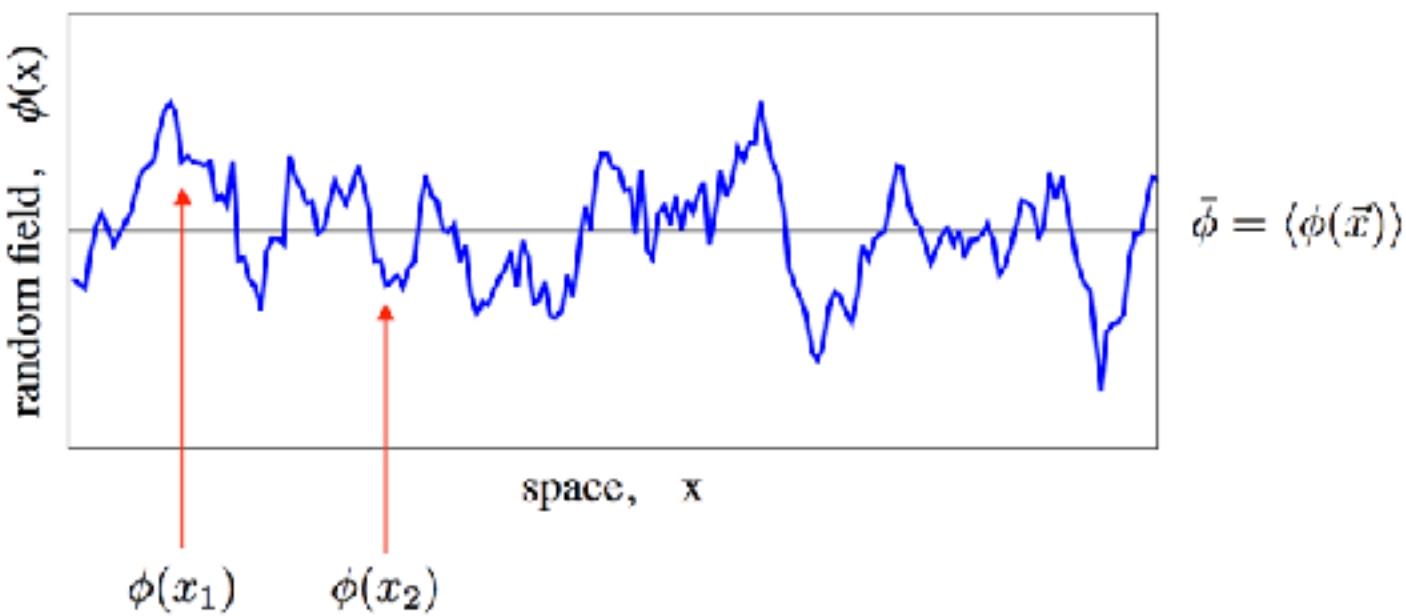
$$\langle \phi^2 \rangle = \int d\phi \, \mathcal{P}(\phi) \, \phi^2$$ 2-nd-order moment

$$\langle \phi^n \rangle = \int d\phi \, \mathcal{P}(\phi) \, \phi^n$$ n-th-order moment

$$\sigma_\phi^2 = \langle \phi^2 \rangle - \langle \phi \rangle^2$$ variance

Random fields

If $\phi(\vec{x})$ is a random *field* we can also compute **correlation functions**



two-point function $\quad \langle\phi(x_1)\phi(x_2)\rangle = \langle\phi(x_1)\rangle\langle\phi(x_2)\rangle + \langle\phi(x_1)\phi(x_2)\rangle_c$

three-point function $\quad \langle\phi(x_1)\phi(x_2)\phi(x_3)\rangle = \langle\phi(x_1)\rangle\langle\phi(x_2)\rangle\langle\phi(x_3)\rangle +$

$$+\langle\phi(x_1)\phi(x_2)\rangle_c\,\langle\phi(x_3)\rangle + \mathrm{perm.} +$$

$$+\langle\phi(x_1)\phi(x_2)\phi(x_3)\rangle_c$$

...

n-point function $\quad \langle\phi(x_1)\phi(x_2)\ldots\phi(x_n)\rangle$

The distribution of galaxies in the Universe

The galaxy number density and its perturbations:

$$n_g(\vec{x}) \equiv \bar{n}_g\,[1+\delta_g(\vec{x})]$$

$$\delta_g(\vec{x}) \equiv \frac{n_g(\vec{x})-\bar{n}_g}{\bar{n}_g}$$

$$\bar{n}_g \equiv \frac{N_g^{\rm tot}}{V}$$

mean galaxy number density over the volume V

galaxy **overdensity** or density contrast (random field!)

N.B. $\langle\delta_g(\vec{x})\rangle \equiv 0$

$\delta_g(\vec{x}) \geq -1$

galaxy number density (random field!)

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galaxy number density
(random field!)

Similarly, for the matter density we have

$$\rho(\vec{x}, t) = \bar{\rho}(t) \left[1 + \delta(\vec{x}, t)\right]$$

$$\delta(\vec{x}, t) \equiv \frac{\rho(\vec{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

mean matter density

matter **overdensity**

The galaxy two-point correlation function

What is the probability of finding two galaxies in the volume elements dV_1 and dV_2 ?

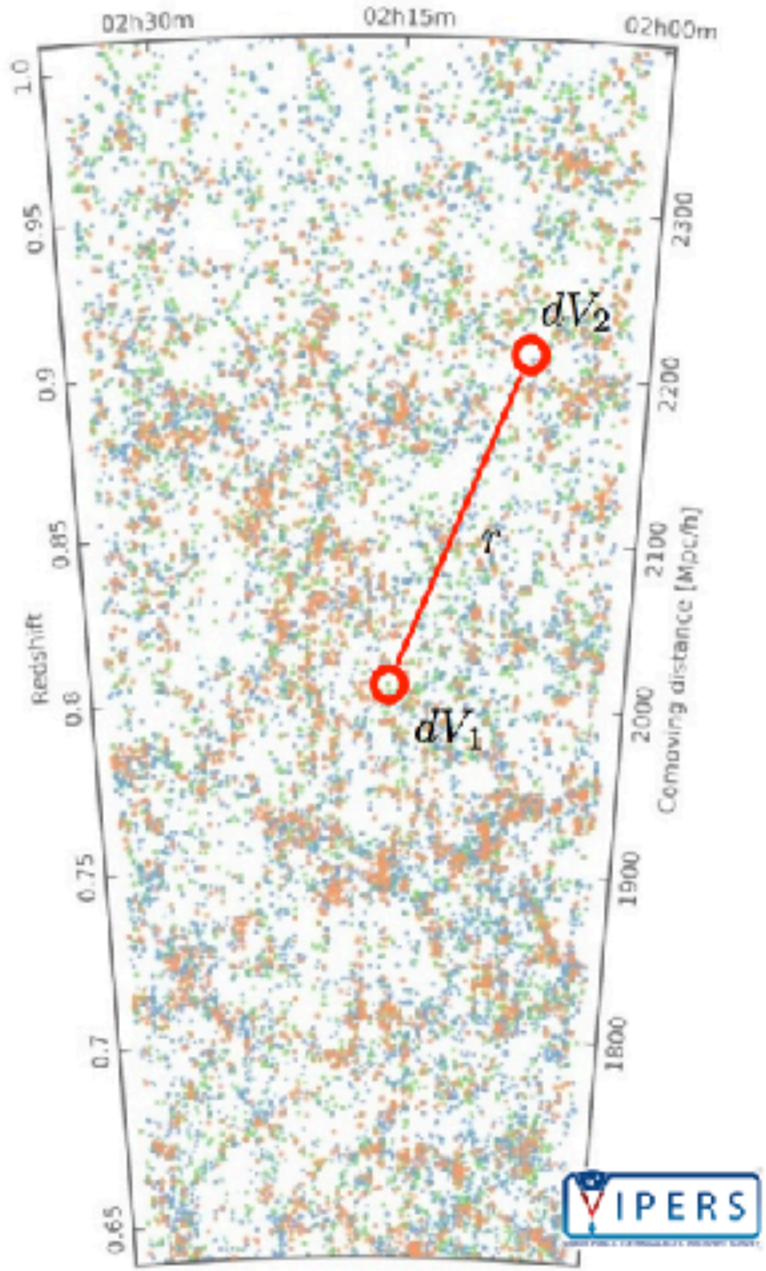
$$dP = dV_1\, dV_2\, \langle\, n_g(\vec{x}_1)\, n_g(\vec{x}_2)\, \rangle$$

$$= dV_1\, dV_2\, \bar{n}_g^2\, [1 + \underbrace{\langle\, \delta_g(\vec{x}_1)\, \delta_g(\vec{x}_2)\, \rangle}_{\text{excess probability}}]$$

We now make the assumption of ***statistical* homogeneity and isotropy**

$$\xi(|\vec{x}_1 - \vec{x}_2|) \equiv \langle\, \delta_g(\vec{x}_1)\, \delta_g(\vec{x}_2)\, \rangle$$

the two-point correlation function $\xi(r)$
only depends on the distance $r = |\vec{x}_1 - \vec{x}_2|$
between the two points



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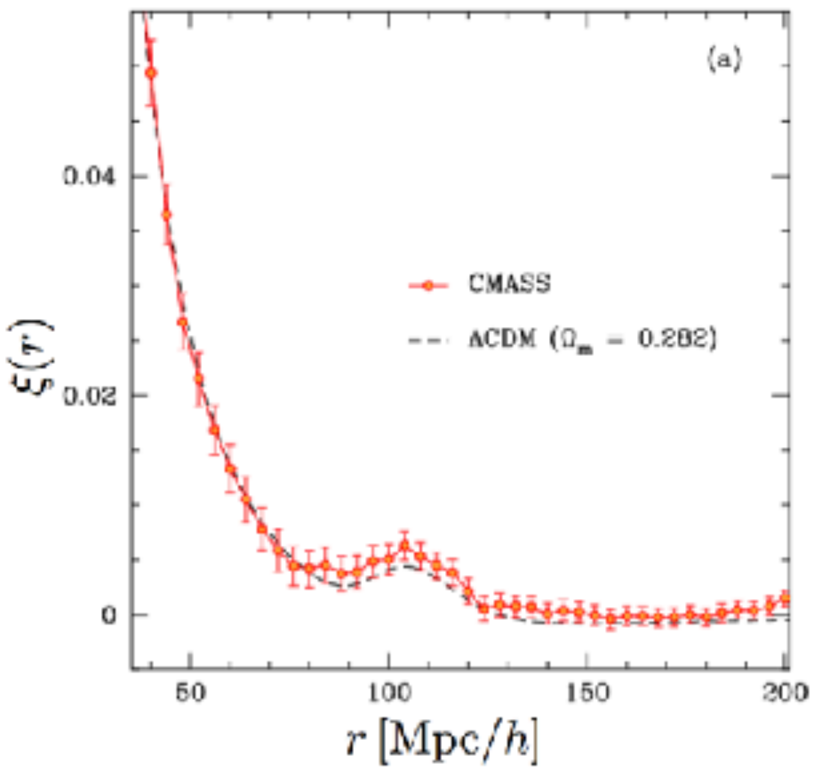
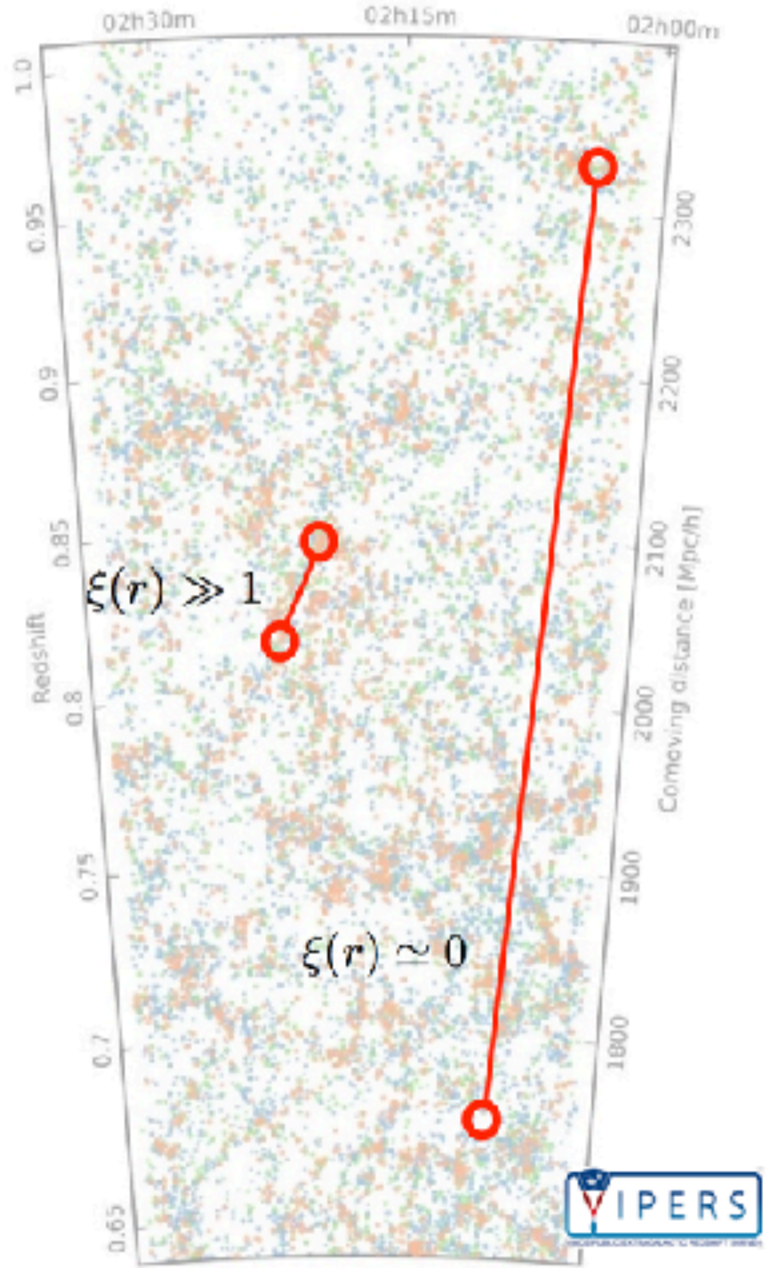
$$= dV_1\, dV_2\, \bar{n}_g^2\, [1 + \xi(r)\,]$$

excess probability





The galaxy three-point correlation function

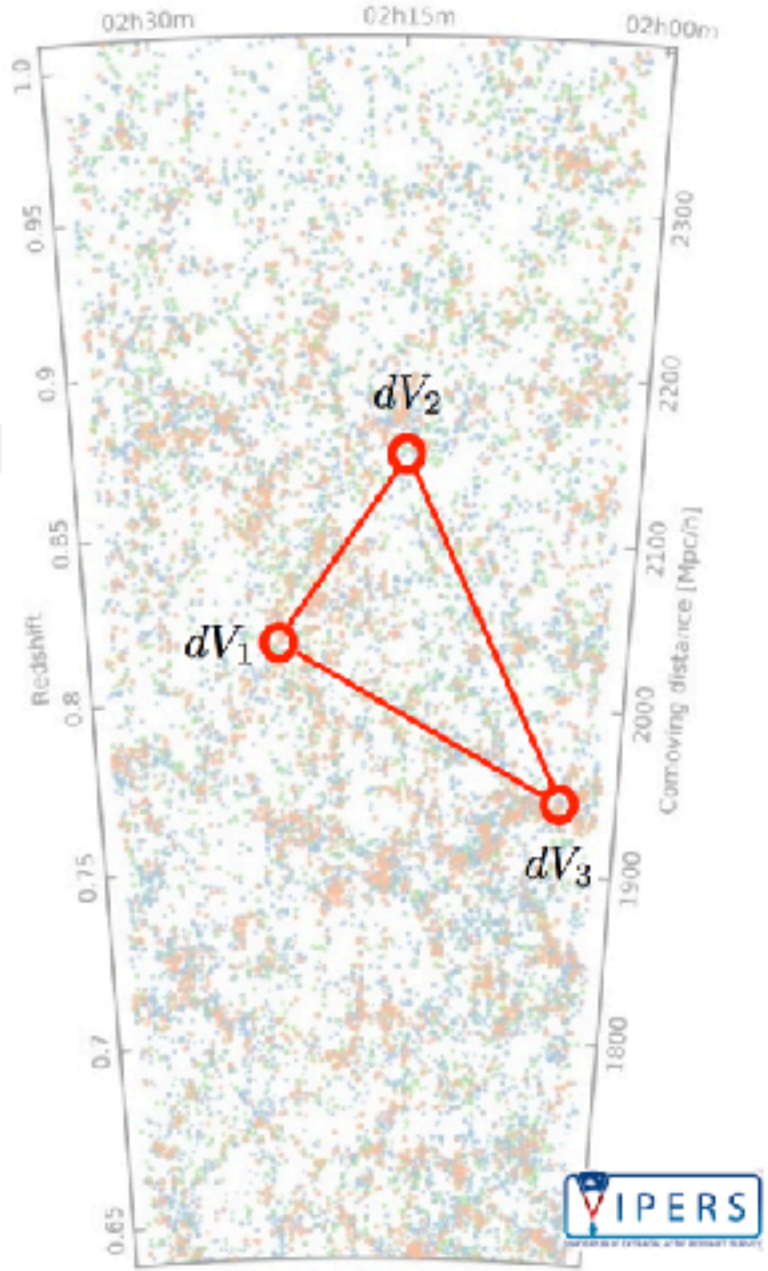
Similarly I can ask the probability of finding three galaxies in the volume elements dV_1 , dV_2 and dV_3

$$dP = dV_1 dV_2 dV_3 \langle n_g(\vec{x}_1) n_g(\vec{x}_2) n_g(\vec{x}_3) \rangle$$
$$= dV_1\, dV_2\, dV_3\, \bar{n}_g^3\, [1 +$$
$$+ \xi(r_{12}) + \xi(r_{13}) + \xi(r_{23}) + \zeta(r_{12}, r_{13}, r_{23})]$$

excess probability

$$\zeta(r_{12}, r_{13}, r_{23}) \equiv \langle \delta_g(\vec{x}_1) \delta_g(\vec{x}_2) \delta_g(\vec{x}_3) \rangle$$

the 3-point correlation function represents the (excess) probability to find 3 galaxies forming a triangle of a given shape and size

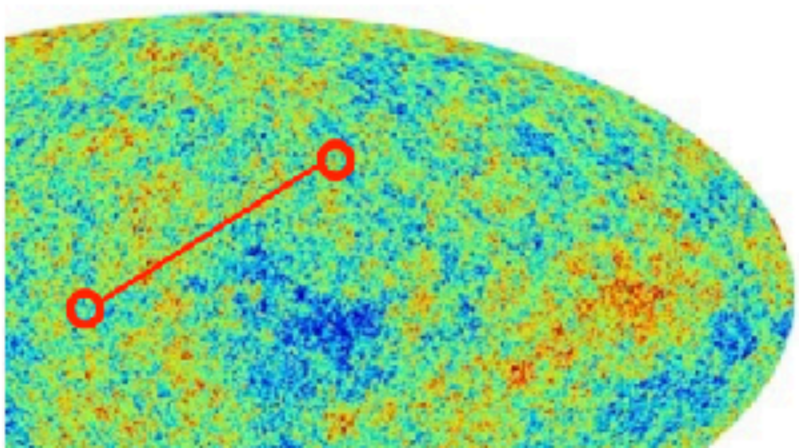


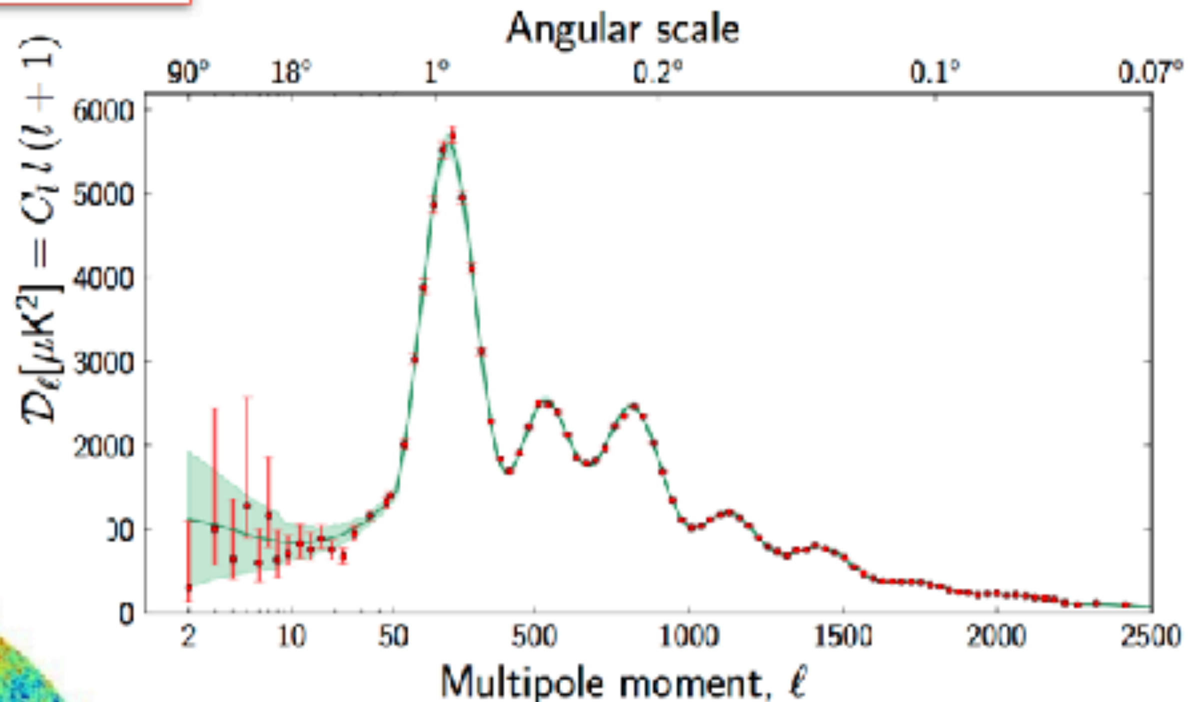
Gaussian and non-Gaussian random fields

The statistical properties of a Gaussian random field are completely characterised by its 2-point correlation function. All higher-order, *connected* correlation functions are vanishing

$$\delta_T(\hat{n}) = \frac{T(\hat{n}) - \bar{T}}{\bar{T}}$$

$$\mathcal{P}[\delta_T(\hat{n})] = \frac{1}{\sqrt{2\pi\sigma_T^2}} e^{-\frac{1}{2}\frac{\delta_T^2}{\sigma_T^2}}$$



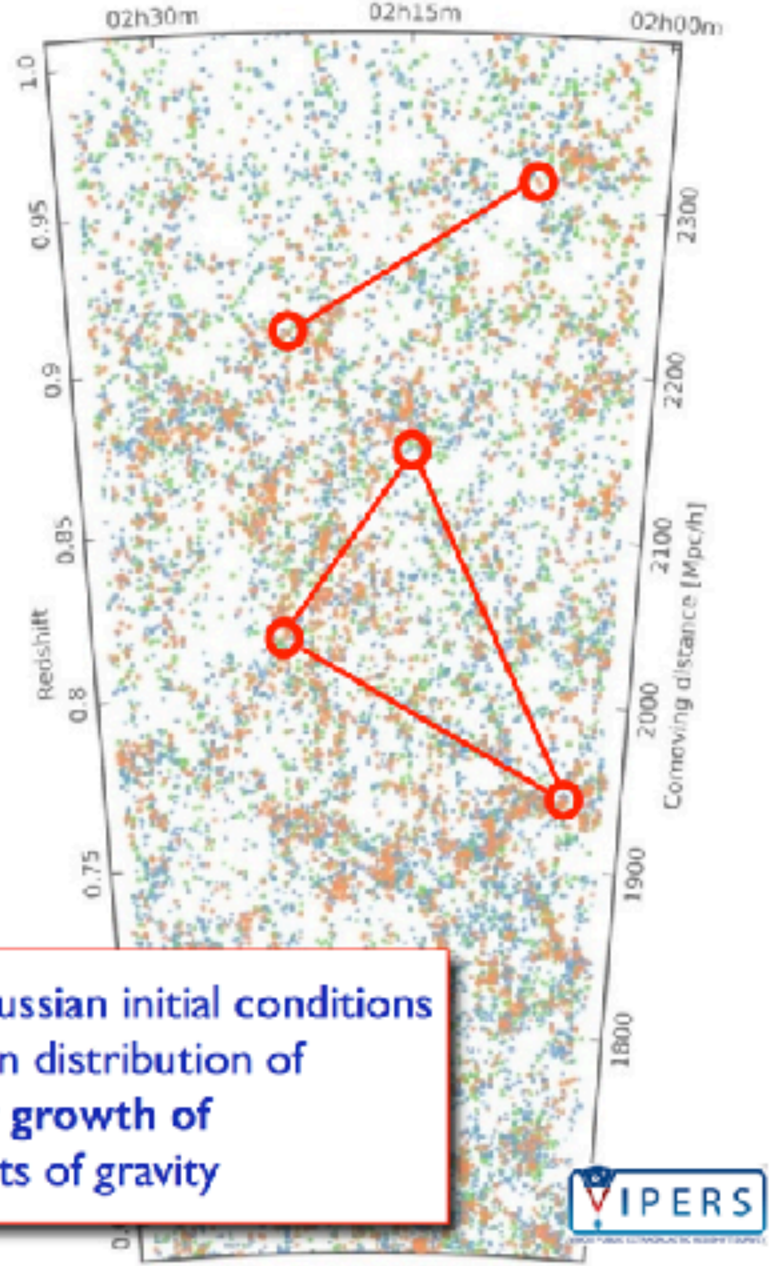


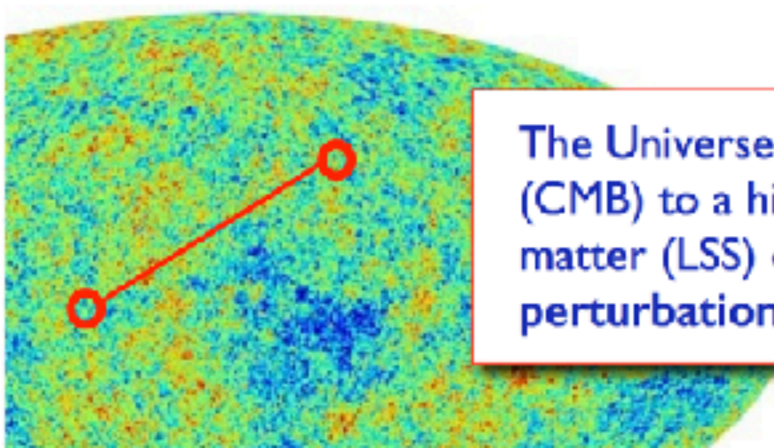
Perturbations in the CMB are one of the best examples of Gaussian random field!

Gaussian and non-Gaussian random fields

The statistical properties of a Gaussian random field are completely characterised by its 2-point correlation function. All higher-order, *connected* correlation functions are vanishing

all other random fields are non-Gaussian!





The Universe evolves from Gaussian initial conditions (CMB) to a highly non-Gaussian distribution of matter (LSS) due to **nonlinear growth of perturbations** under the effects of gravity

Ergodic hypothesis

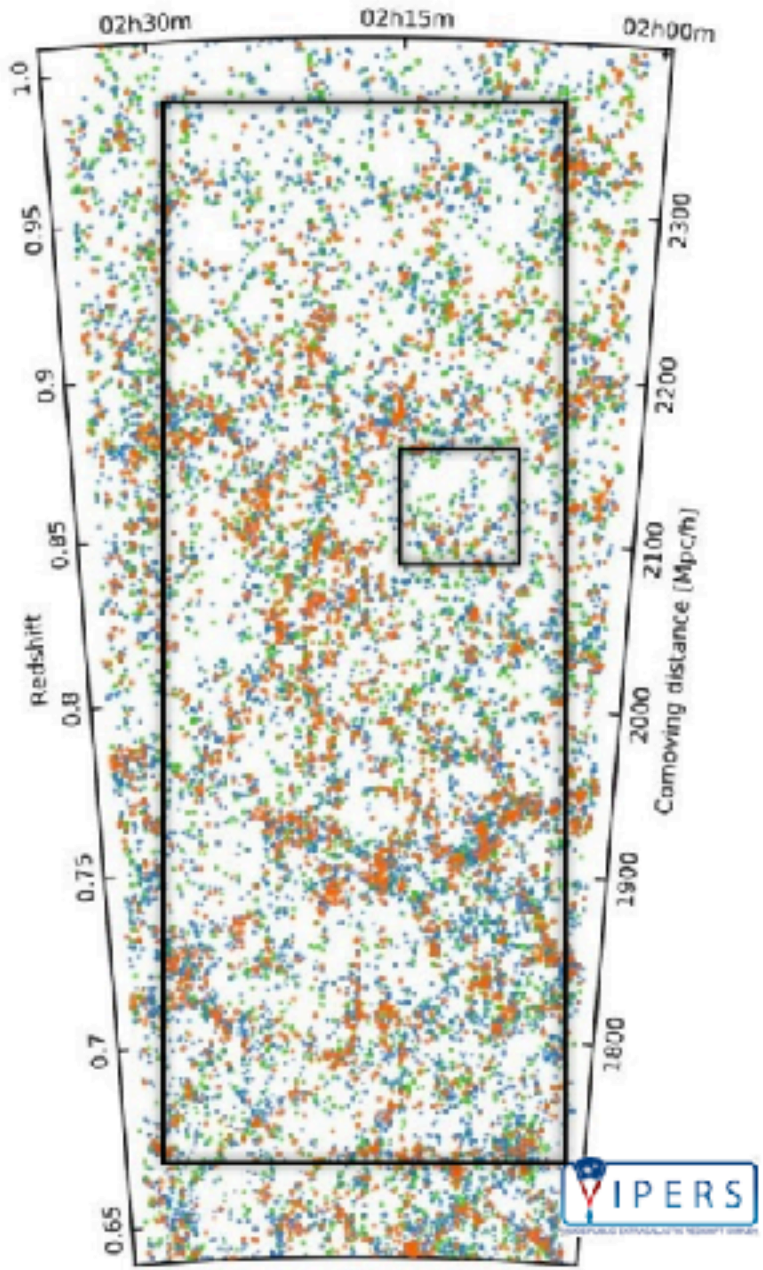
Expectation values, in principle, are to be intended as *ensemble averages*, i.e. averages over many "realisations of the Universe" …

… but we only have one Universe!

We have to assume the **ergodic hypothesis**: **ensemble averages are equal to spatial averages**

$$\langle \phi(\vec{x}) \rangle = \int d\phi \, \phi \, \mathcal{P}(\phi) = \frac{1}{V} \int_V d^3x \, \phi(\vec{x})$$

We should make sure, however, that the observed volume correspond to a "fair sample" of the Universe



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The galaxy number density and its perturbations:

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$$\delta_g(\vec{x}) \equiv \frac{n_g(\vec{x}) - \bar{n}_g}{\bar{n}_g}$$

$$\bar{n}_g = \frac{N_g^{\mathrm{tot}}}{V}$$

mean galaxy number density over the volume V

galaxy **overdensity** or density contrast (random field!)

N.B. $\langle \delta_g(\vec{x}) \rangle \equiv 0$

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$$\delta(\vec{x}, t) \equiv \frac{\rho(\vec{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

mean matter density

matter **overdensity**

Linear growth of density fluctuations

Jeans theory in co-moving expanding coordinates (non relativistic, Newtonian)

$$\text{Equation of Continuity} \quad : \quad \frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \boldsymbol{v}) = 0 \; ;$$

$$\text{Equation of Motion} \quad : \quad \frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\varrho}\nabla p - \nabla \phi \; ;$$

$$\text{Gravitational Potential} \quad : \quad \nabla^2 \phi = 4\pi G \varrho \; .$$

Linearize equations: $\rho = \rho_0[1+\delta]$

Linear growth equation
(neglecting pressure gradients):

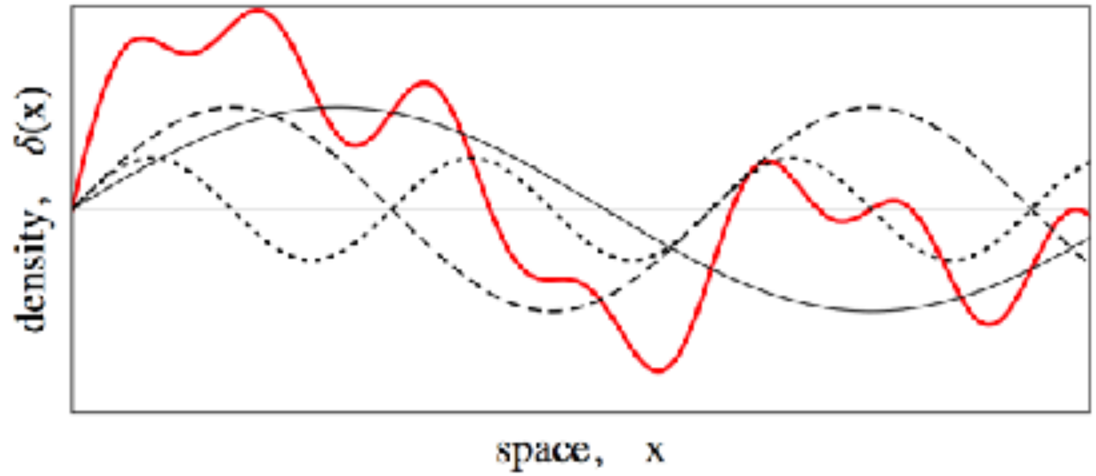
$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G \rho_b \delta.$$

Note: it is a linear differential equation –> superposition principle holds

Fourier space

Theoretical predictions for the matter correlation functions are performed in **Fourier space**

Fourier analysis naturally separates perturbations at different scales:



$$\delta_{\vec{k}} = \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \delta(\vec{x})$$

$$\delta(\vec{x}) = \int d^3k e^{i\vec{k}\cdot\vec{x}} \delta_{\vec{k}}$$

- Since $\delta(\vec{x})$ is a random field $\delta_{\vec{k}}$ is also a random field
- Since $\delta(\vec{x})$ is real $\delta^*_{\vec{k}} = \delta_{-\vec{k}}$

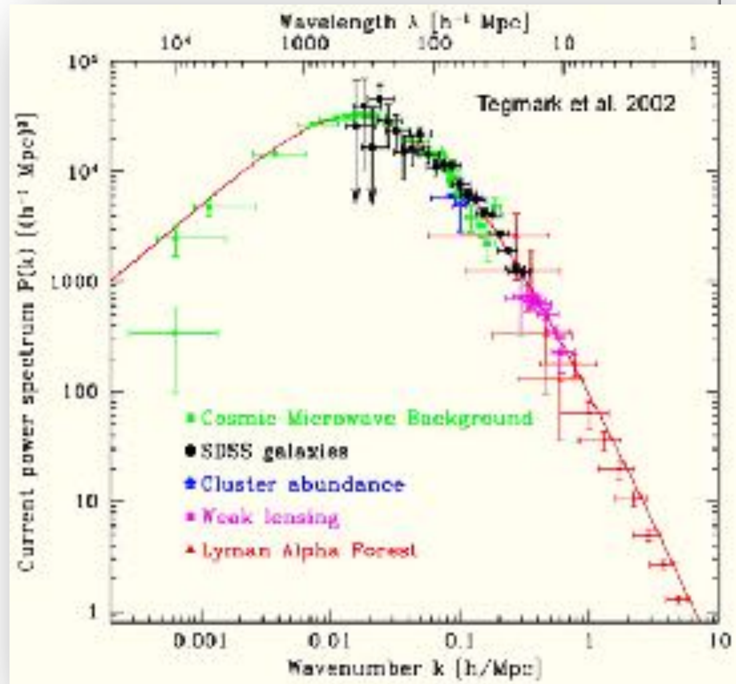
IMPORTANT: We can decompose our fluctuation field into "normal modes" and each of these will have to satisfy the growth equation

Fourier space: correlation functions

The 2-point function in Fourier space: the **power spectrum**

$$\langle \delta_{\vec{k}} \delta_{\vec{k}'} \rangle = \delta_D(\vec{k} - \vec{k}') P(\vec{k})$$

homogeneity & isotropy



$$P(k) = \int \frac{d^3x}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \xi(x)$$

The power spectrum is the Fourier transform of the 2-point correlation function

The power spectrum is a measure of the amplitude of perturbations as a function of scale

- Assuming isotropy, the Fourier transform simplifies to

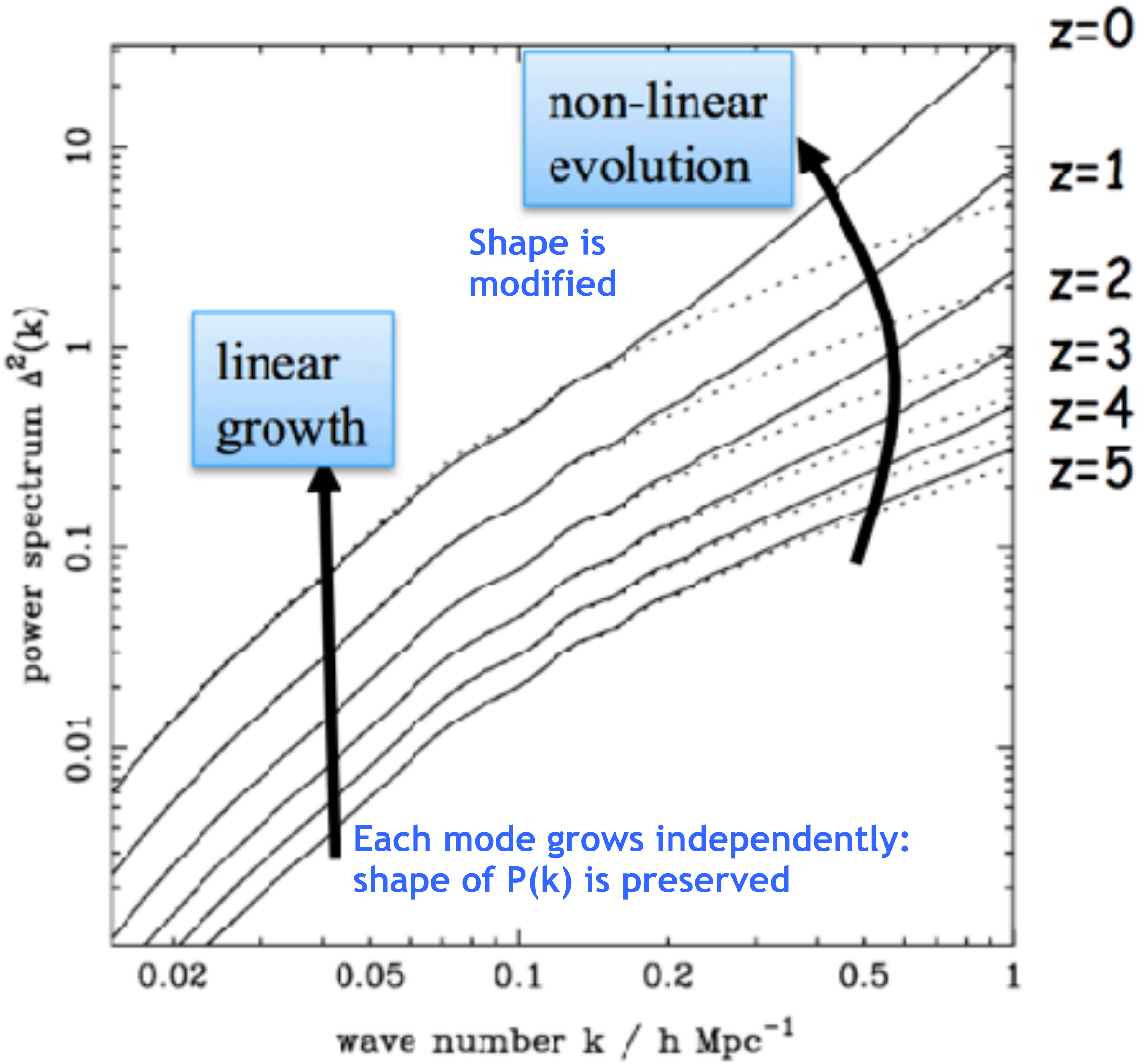
$$P(k) = 4\pi \int_0^\infty \xi(r) \frac{sin(kr)}{kr} r^2 dr$$

- You will sometimes find the power spectrum expressed as

$$\Delta^2(k) = \frac{1}{2\pi^2} P(k) k^3$$

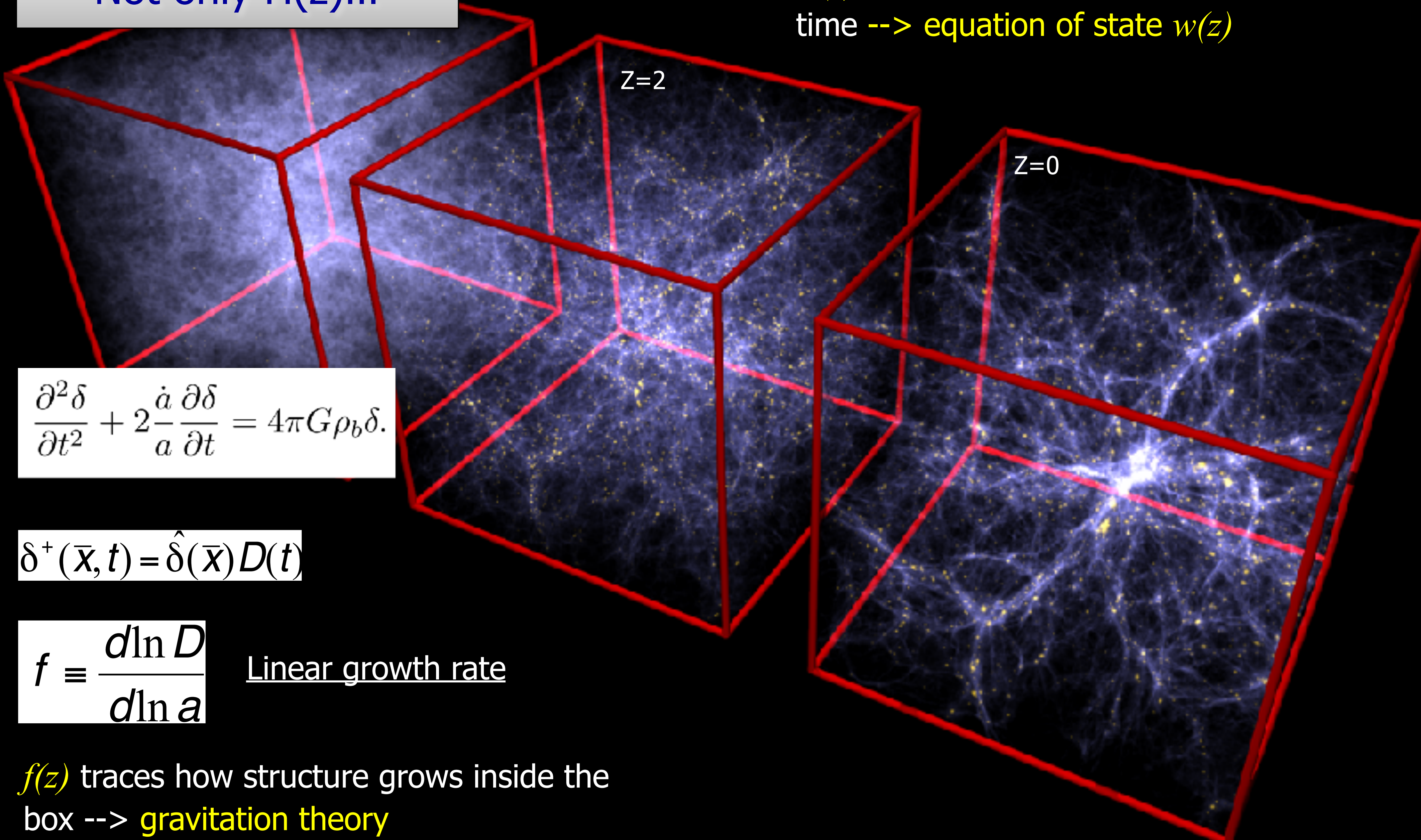
Which corresponds to the "power per octave" and shows the true contribution of each scale to the total variance of the field:

$$\sigma^2 = \int_0^\infty \Delta^2(k) d(\ln k)$$



Not only H(z)…

H(z) measures how the box expands with time --> equation of state *w(z)*



$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G \rho_b \delta.$$

$$\delta^+(\bar{x}, t) = \hat{\delta}(\bar{x}) D(t)$$

$$f \equiv \frac{d\ln D}{d\ln a}$$

Linear growth rate

f(z) traces how structure grows inside the box --> gravitation theory

Springel et al.

Understanding cosmic acceleration: the quest for two functions

$$w(z)$$

$$f(z)$$

End Lecture 1