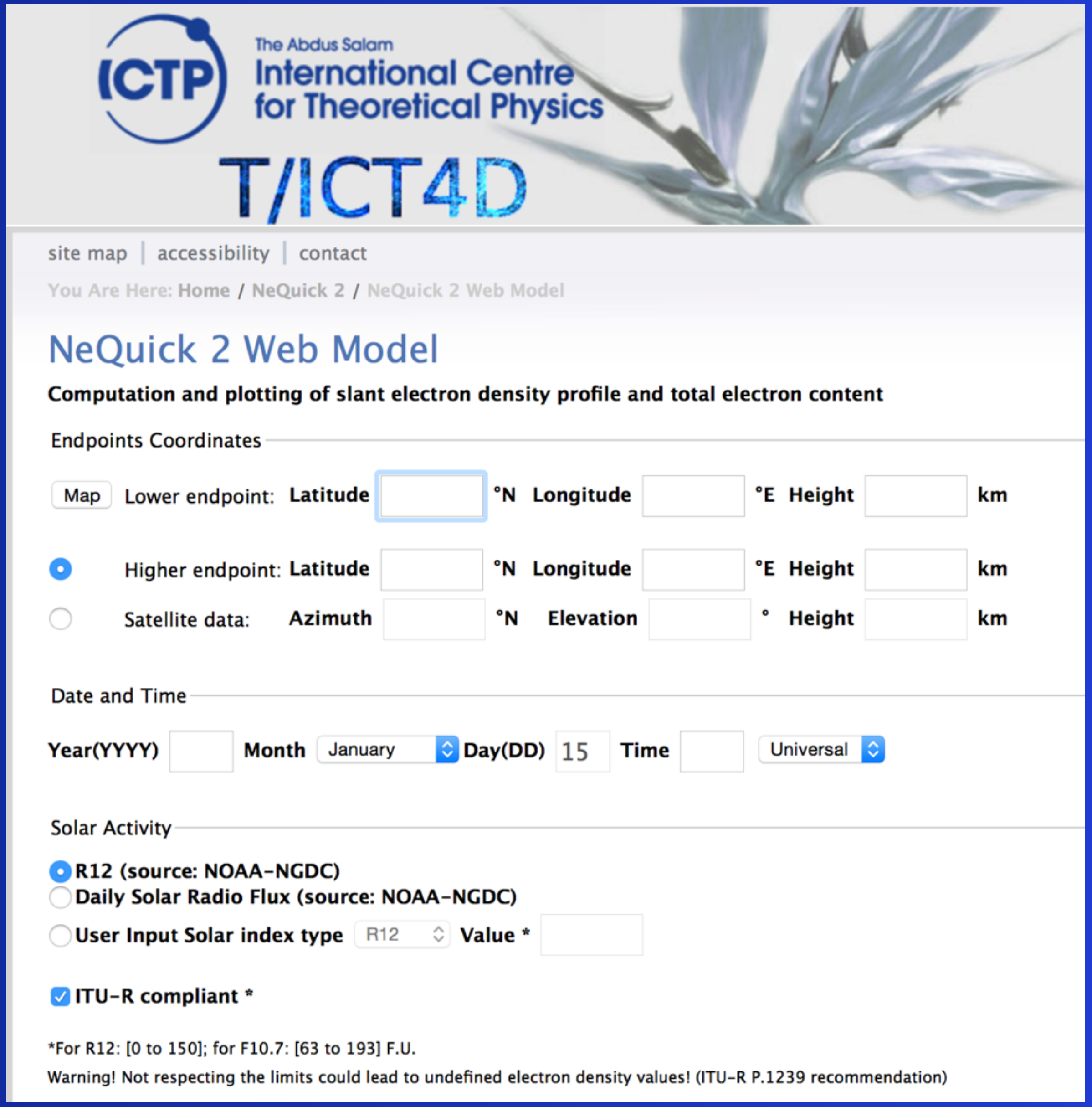
NeQuick model and data assimilation

Bruno Nava
ICTP, Trieste, Italy

International School of Space Science
THE POLAR UPPER ATMOSPHERE: FROM SCIENCE TO OPERATIONAL ISSUES
L'Aquila, 16-21 Sep 2018

NeQuick

- The NeQuick is an ionospheric electron density model developed at the T/ICT4D Laboratory of The Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy, and at the Institute for Geophysics, Astrophysics and Meteorology (IGAM) of the University of Graz, Austria.



- It is a quick-run empirical model particularly designed for trans-ionospheric propagation applications, conceived to reproduce the median behavior of the ionosphere ("climate").

- http://t-ict4d.ictp.it/nequick2

NeQuick 2

- The model profile formulation includes 6 semi-Epstein layers with modeled thickness parameters and is based on anchor points defined by foE, foF1, foF2 and M(3000)F2 values.

- These values can be modeled (e.g. ITU-R coefficients for foF2, M(3000)F2) or experimentally derived.

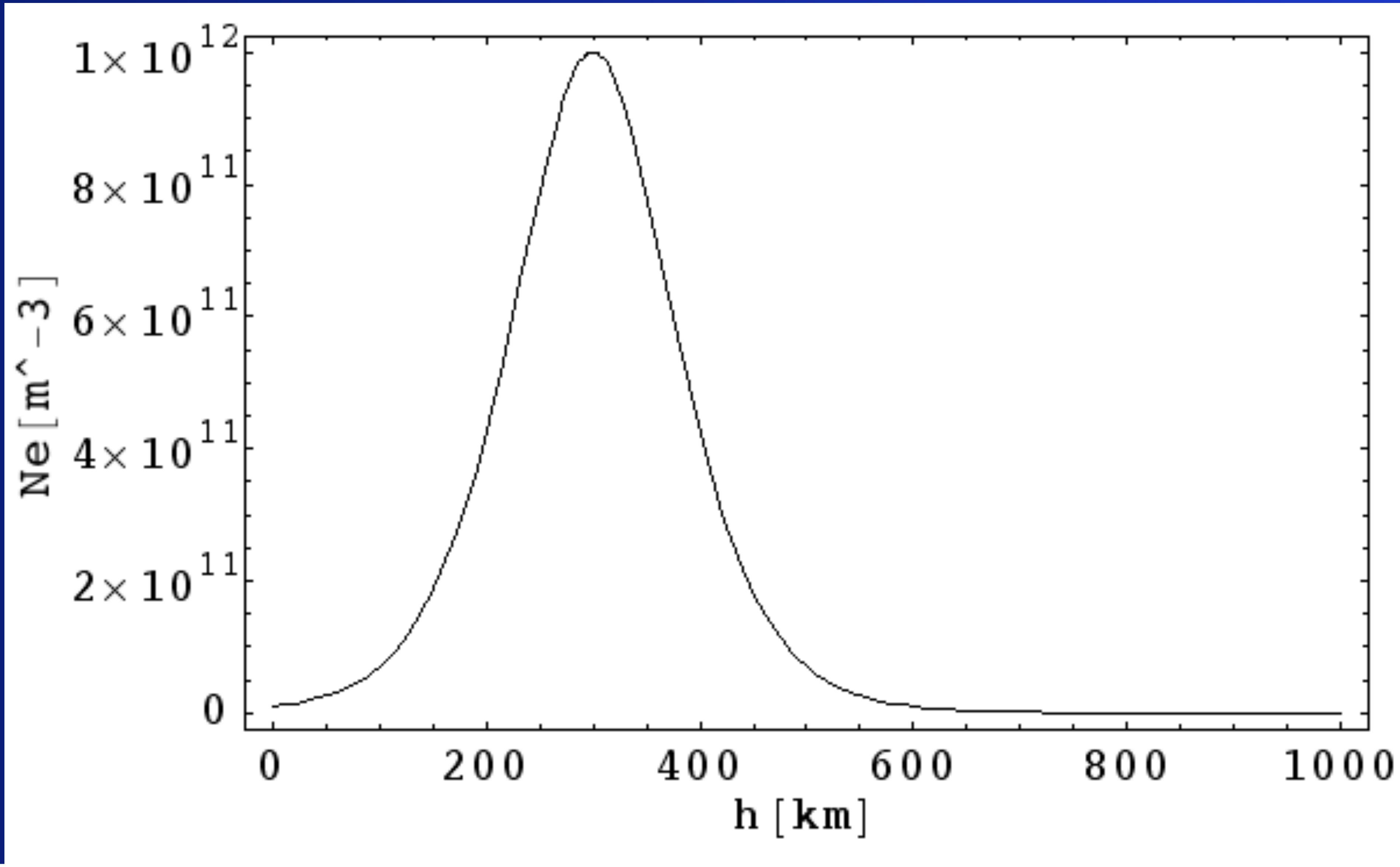
- NeQuick inputs are: position, time and solar flux; the output is the electron concentration at the given location and time.

- NeQuick package includes routines to evaluate the electron density along any “ground-to-satellite” ray-path and the corresponding Total Electron Content (TEC) by numerical integration.

B. Nava, P. Coïsson, S. M. Radicella, "A new version of the NeQuick ionosphere electron density model", Journal of Atmospheric and Solar-Terrestrial Physics (2008), doi:10.1016/j.jastp.2008.01.015

Epstein function

$$N(h; hmax, Nmax, B) = \frac{4Nmax}{(1+\exp(\frac{h-hmax}{B}))^2}\exp(\frac{h-hmax}{B})$$



NeQuick 2 formulation

The model is represented by a sum of Epstein functions for the E, F1 and F2 layers:

$$N_{bot}(h) = N_E(h) + N_{F1}(h) + N_{F2}(h)$$

where

$$N_E(h) = \frac{4Nm^*E}{\left(1+\exp\left(\frac{h-hmE}{BE}\xi(h)\right)\right)^2}\exp\left(\frac{h-hmE}{BE}\xi(h)\right)$$

$$N_{F1}(h) = \frac{4Nm^*F1}{\left(1+\exp\left(\frac{h-hmF1}{B1}\xi(h)\right)\right)^2}\exp\left(\frac{h-hmF1}{B1}\xi(h)\right)$$

$$N_{F2}(h) = \frac{4NmF2}{\left(1+\exp\left(\frac{h-hmF2}{B2}\right)\right)^2}\exp\left(\frac{h-hmF2}{B2}\right)$$

NeQuick 2 formulation

with

$$Nm^{*}E = NmE - N_{F1}\left(hmE\right) - N_{F2}\left(hmE\right)$$

$$Nm^{*}F1 = NmF1 - N_{E}\left(hmF1\right) - N_{F2}\left(hmF1\right)$$

and

$$\xi\left(h\right) = \exp\left(\frac{10}{1+1|h-hmF2|}\right)$$

is a function that ensures a "fade out" of the E and F1 layers in the vicinity of the F2 layer peak in order to avoid secondary maxima around hmF2.

NeQuick 2 formulation

The model topside is represented by a semi-Epstein layer with a height-dependent thickness parameter H:

$$N(h) = \frac{4NmF2}{(1+\exp(z))^2}\exp(z)$$

with

$$z = \frac{h - hmF2}{H}$$

$$H = H_0\left[1 + \frac{rg(h - hmF2)}{rH_0 + g(h - hmF2)}\right]$$

NeQuick 2 formulation

Peak heights

$$hmE = 120$$

$$hmF1 = \frac{hmE + hmF2}{2}$$

$$hmF2 = \frac{1490MF}{M + \Delta M} - 176$$

with

$$\Delta M = \begin{cases} 0.253/(foF2/foE \\ \quad -1.215) - 0.012, & \\ -0.012 & \text{if } foE = 0, \end{cases}$$

$$MF = M\sqrt{\frac{0.0196M^2 + 1}{1.2967M^2 - 1}} \qquad M = M(3000)F2.$$

NeQuick 2 formulation

Thickness parameters

$$BE_{bot} = 5$$

$$BE_{top} = \max\left(0.5\left(hmF1 - hmE\right), 7\right)$$

$$B1_{bot} = 0.5\left(hmF1 - hmE\right)$$

$$B1_{top} = 0.3\left(hmF2 - hmF1\right)$$

$$B2_{bot} = \frac{0.385 NmF2}{\left(dN/dh\right)_{max}}$$

$$H = kB2_{bot}\left[1 + \frac{rg\left(h - hmF2\right)}{rkB2_{bot} + g\left(h - hmF2\right)}\right]$$

NeQuick 2 formulation

where

$$\ln\left(\left(\frac{dN}{dh}\right)_{max}\right) = -3.467 + 1.714\ln\left(foF2\right) + 2.02\ln\left(M(3000)F2\right)$$

and

$$k = 3.22 - 0.0538 foF2 - 0.00664 hmF2 + 0.113\frac{hmF2}{B2_{bot}} + 0.00257 R12$$

NeQuick 2 formulation

Critical frequencies and propagation factor

$$(foE)^2 = \left(a_e\sqrt{F107}\right)^2 (\cos\chi_{eff})^{0.6} + 0.49$$

$$foF1 = \begin{cases} 1.4\ foE & \text{if} \quad foE \geq 2 \\ 0 & \text{if} \quad foE < 2 \\ 0.85\ 1.4\ foE & \text{if } 1.4\ foE > 0.85\ foF2 \end{cases}$$

$foF2$ modeled in terms of ITU – R coefficients

$M = M(3000)F2.$ modeled in terms of ITU – R coefficients

Data ingestion/assimilation

- Empirical models like NeQuick have been developed as climatological models, able to reproduce the typical median condition of the ionosphere.

- For research purposes and practical applications, in order to pass from “climate” to “weather”, there is a need to have models able to reproduce the current conditions of the ionosphere.

- Considering that there is an increasing availability of experimental data even in real time (ground and space-based GNSS, ionosondes), several assimilation schemes have been developed. They are of different complexity and rely on different kinds of data.

Data Assimilation

- “Data assimilation is an analysis technique in which the observed information is accumulated into the model state by taking advantage of consistency constraints with laws of time evolution and physical properties” (Bouttier, and Courtier, 1999).

- "Data assimilation is fundamentally a model specification and prediction technique that uses data to improve the fidelity of the model" (Bust and Mitchell, 2008).

- The ionospheric data assimilation schemes are based on different mathematical techniques; the most widely used are:
    - variational techniques (3D-VAR and 4D-VAR)
    - Kalman filters (extended Kalman filter (EKF), Ensemble Kalman filter (EnKF))

Assimilation schemes (example)

- Utah State University (USU) Global Assimilation of Ionospheric Measurements (GAIM) [Schunk et al., 2004] or the Jet Propulsion Laboratory (JPL)/University of Southern California (USC) Global Assimilative Ionospheric Model (GAIM) [Wang et al., 2004], or [Schunk et al., 2014], for example, are based on assimilation of data originating from different sources and imply the use of first principle models.

GAIM-band limited (BL)	Midlatitude to Low-Latitude Ionosphere
GAIM-Gauss Markov (GM)	Midlatitude to Low-Latitude Ionosphere
GAIM-4DVAR	Midlatitude to Low-Latitude Ionosphere with Drivers
GAIM-full physics (FP)	Midlatitude to Low Latitude Ionosphere-Plasmasphere with Drivers
Middle-low electro-DA	Midlatitude to Low Latitude Ionosphere with Drivers
IDED-DA	High-Latitude Ionosphere with Drivers
GTM-DA	Global Thermosphere Model-Data Assimilation

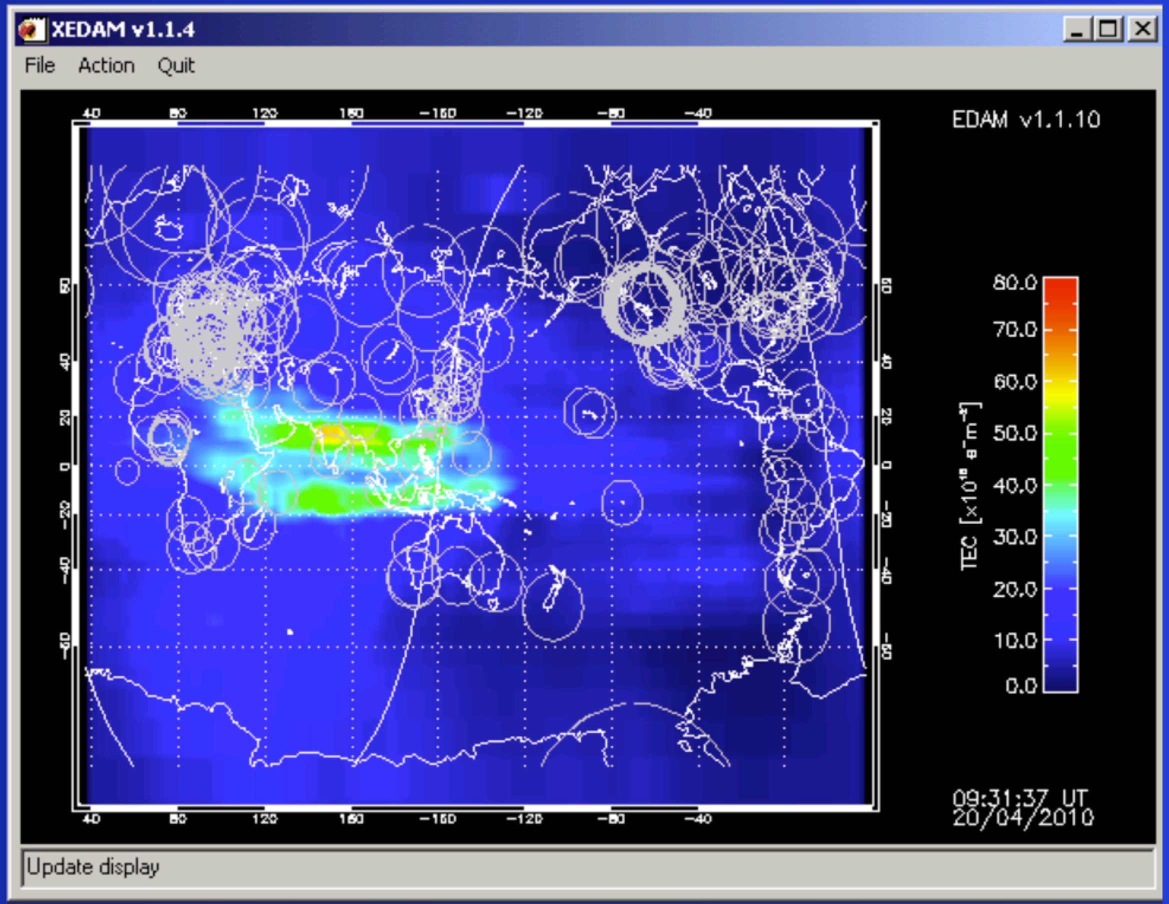
Ionosphere	Electrodynamics	Thermosphere
Ground-based GPS-TEC	Ground magnetometers	Satellite UV emissions
Satellite-based GPS occultation	DMSP cross-track velocities	In situ neutral densities and winds
Ionosonde and digisonde	SuperDARN line-of-sight velocities	Satellite accelerometer and drag
In situ N_e	Iridium magnetometers	FPI winds
911 Å, 1356 Å, limb, disk (UV)	ACE interplanetary magnetic field, *Dst*	ISR neutral parameters
Solar UV, EUV	Solar UV, EUV	Solar UV, EUV

Assimilation schemes (example)

- The Electron Density Assimilative Model (EDAM) [Angling and Khattatov, 2006; Angling, M. J., and N. K. Jackson-Booth, 2011] provides a mean to assimilate ionospheric measurements into a background ionospheric model.

- Assimilated data are: ground-based and space-based GPS-derived TEC, ionosondes-derived parameters
  - Currently IRI is used as a background model (electron density only)

- Extended, localised Gauss Markov Kalman Filter
  - BLUE + time evolution of the differences between the measurements and the background ionosphere
  - Model variances are propagated
  - Covariance are estimated as required



Assimilation schemes (example)

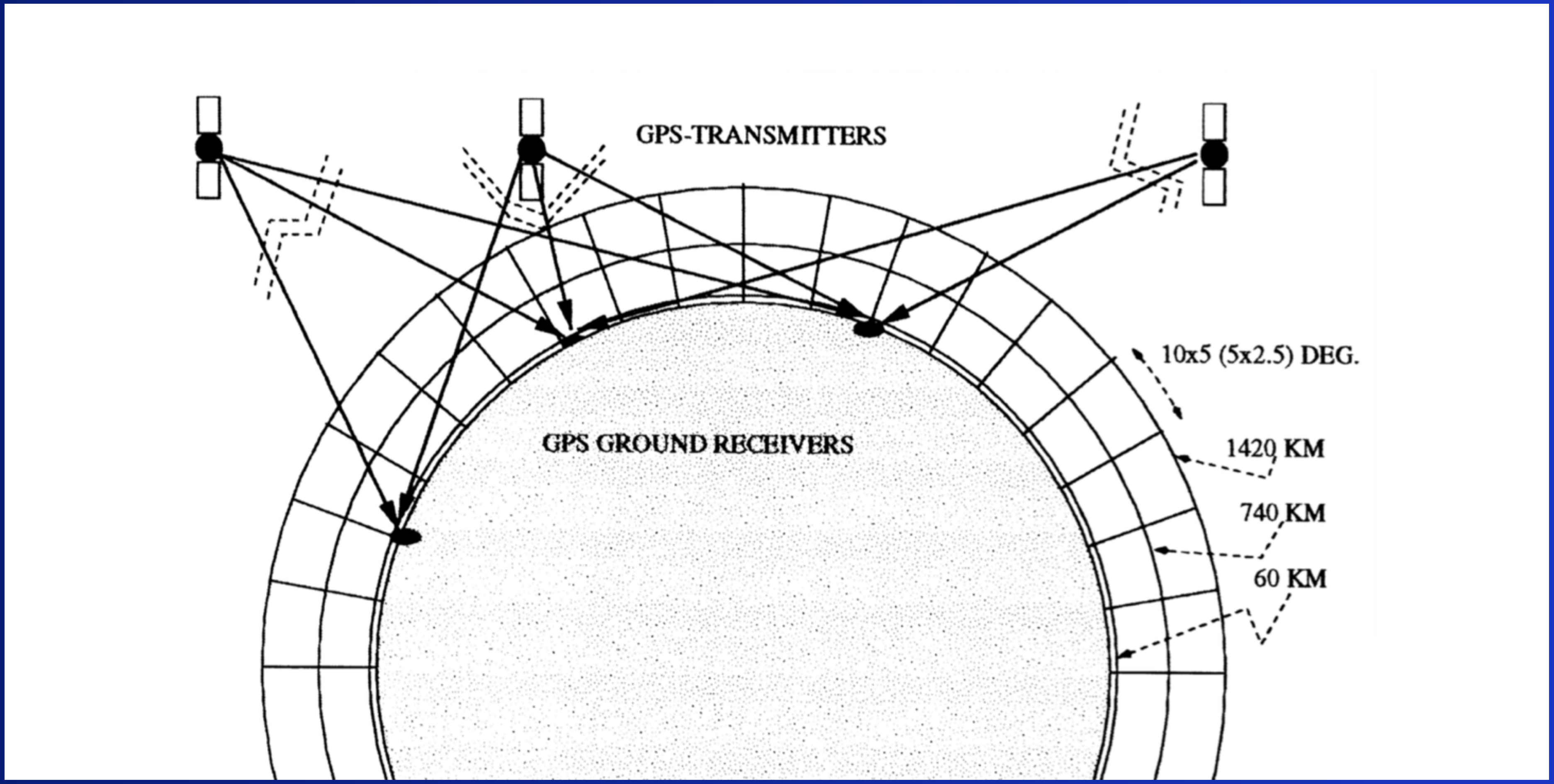
- The Multi Instrument Data Analysis System (MIDAS) [Mitchell C. N. and Spencer P. S. 2003] is a tomographic approach where TEC data are inverted to evaluate the distribution and time evolution of electron concentration.

  - Orthonormal basis functions and SVD are used to solve the inverse problem.

- Review paper: Bust, G. S., and C. N. Mitchell (2008), “History, current state, and future directions of ionospheric imaging, Rev. Geophys., 46,RG1003, doi:10.1029/2006RG000212.

Assimilation schemes (example)

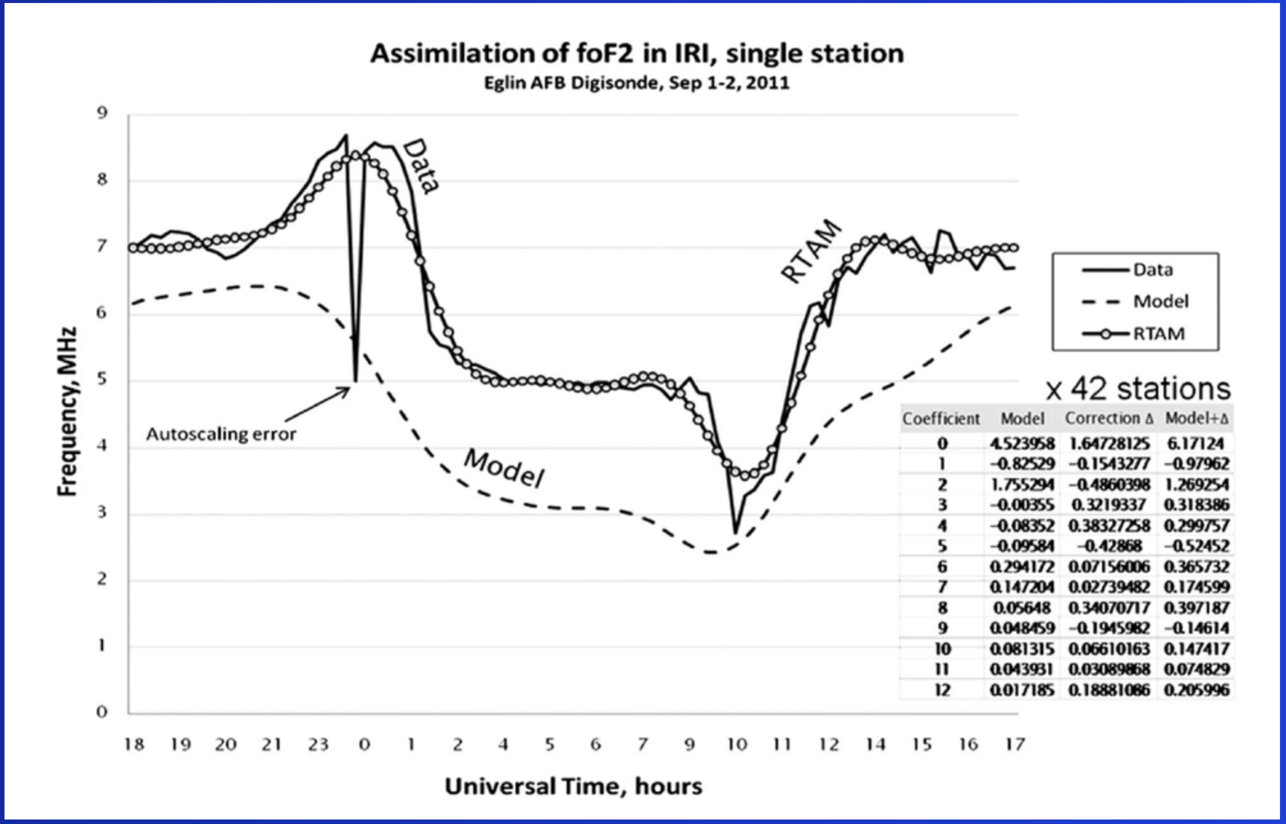
- TOMographic IONosphere model (TOMION), [Hernández-Pajares, M. et al., 1999] generates Global Ionospheric Maps (GIMs) of vertical TEC and includes an interpolation module using Kriging technique [Orús et al., 2005]. The ionosphere is represented by two or more layers of voxels and in each voxel the electron density is assumed to be constant. No background model is used.



Assimilation schemes (example)

- IRI Real Time Assimilative Model (IRTAM) [Galkin, I. A., et al. 2012], has been developed to assimilate Global Ionosphere Radio Observatory (GIRO) data (foF2, hmF2) in order to "update" the IRI electron density distribution, while preserving the IRI's typical ionospheric feature representations.

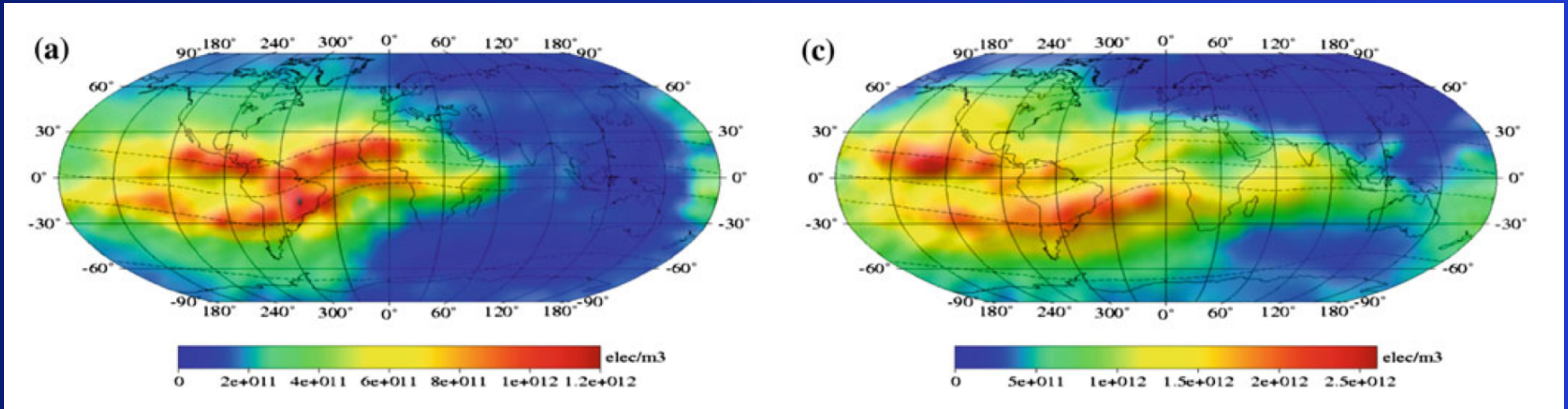
- The technique calculates the corrected coefficients for the spherical/diurnal expansion used by the CCIR-67/URSI-88 model to specify the global foF2 maps, and similarly the maps for all other IRI profile parameters.



Coefficient	Model	Correction Δ	Model+Δ
0	4.523958	1.64728125	6.17124
1	−0.82529	−0.1543277	−0.97962
2	1.755294	−0.4860398	1.269254
3	−0.00355	0.3219337	0.318386
4	−0.08352	0.38327258	0.299757
5	−0.09584	−0.42868	−0.52452
6	0.294172	0.07156006	0.365732
7	0.147204	0.02739482	0.174599
8	0.05648	0.34070717	0.397187
9	0.048459	−0.1945982	−0.14614
10	0.081315	0.06610163	0.147417
11	0.043931	0.03089868	0.074829
12	0.017185	0.18881086	0.205996

Assimilation schemes (example)

- A similar approach has been used by Brunini et al., [2013] in order to update the ITU-R database using radio occultation (COSMIC) electron density profiles.
- For this purpose the La Plata Ionospheric Model (LPIM) (after linearisation) is adjusted by Least Squares to every RO profile available for the time period of interest.



Global representation of the NmF2 estimated value within the 18–20 UT interval
a) NmF2 for the 2007 September equinox c) NmF2 for the 2011 December solstice

Assimilation schemes (example)

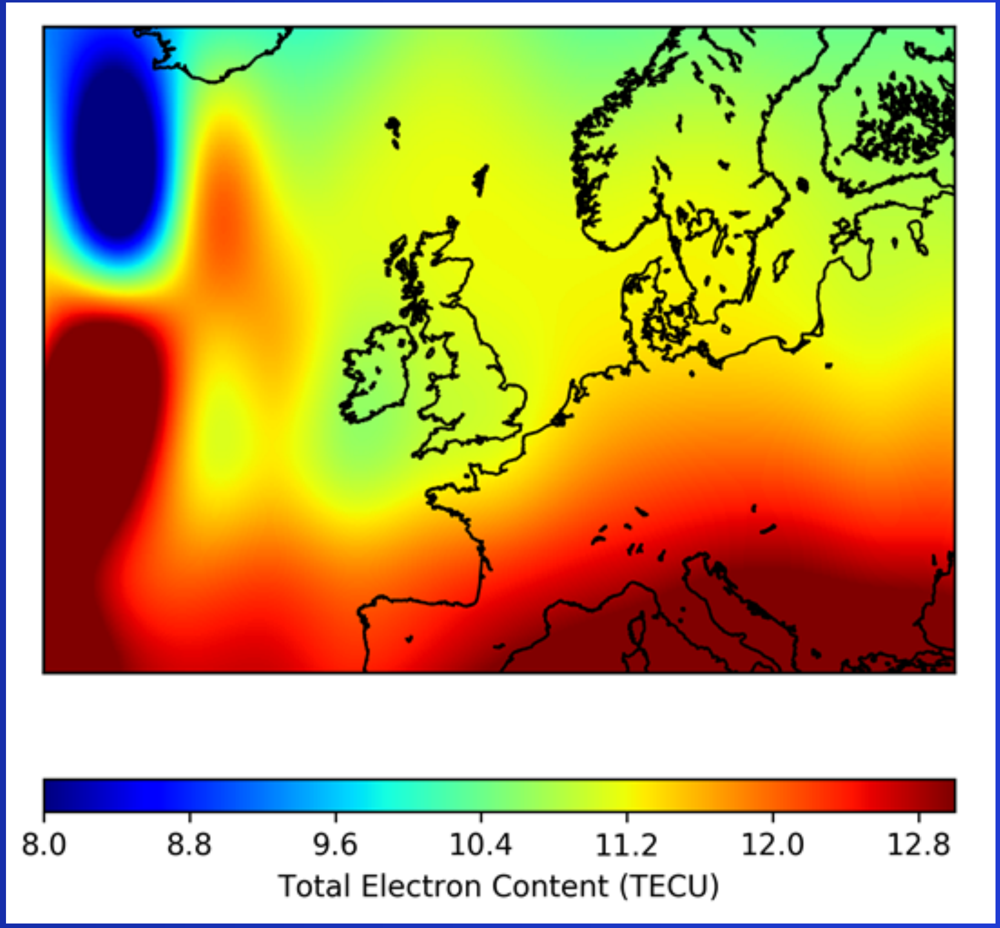
- The Ionospheric Data AssimilationThree-Dimensional (IDA3D), [Bust et al., 2004] uses a three-dimensional variational data assimilation technique (3DVAR).
- It is capable of incorporating ground based and space based GPS-TEC measurements and electron density measurements from radars and satellites.
- The background specification is based upon empirical ionospheric models, but IDA3D is capable of using any global ionospheric specification as a background. IDA3D produces a spatial analysis of the electron density distribution at a specified time. A time series of these specifications can be created using past specifications to determine the background for the current analysis.

Assimilation schemes (example)

TRAIN – A realtime ionospheric DA model

- TRAIN is based on the Malvern Mathematical Data Assimilation Model (MMaDAM)
  - Generalised data assimilation system
  - Hybrid approach of ensemble Kalman filter (EnKF) and particle filter (PFs)
- Background model
  - Ensemble of NeQuick models
- Observation operator
  - Direct integration through each ensemble member
- Forward operator
  - Collapse ensemble to the mean
  - ICTP method for NeQuick forecast

malvern mathematical



TRAIN electron density grid for 1200 UT, 4/6/2017 assimilating TEC data from ground based GPS receivers, a shipborne GPS receiver and a space based dual frequency altimeter.

Use of effective parameters

Pignalberi, A., Pezzopane, M., Rizzi, R. Galkin, I., "Effective Solar Indices for Ionospheric Modeling: A Review and a Proposal for a Real-Time Regional IRI"; Surv Geophys (2018) 39: 125. https://doi.org/10.1007/s10712-017-9438-y.

Effective parameters

- One of the first effective parameter that has been proposed is the “effective sunspot number” (SSNe). This parameter valid for a set of foF2 observations has been defined as the SSN value that, when used as input to the URSI foF2 model, gives a weighted zero-mean difference between the observed and the modeled foF2 values.

- T-index
  http://www.sws.bom.gov.au/Educational/5/2/1
  The T index can be best regarded as an "equivalent sunspot number" - the sunspot number which would best match the observations made by ionosondes.

Effective parameters

- IRI IG 12
  http://gauss2.gge.unb.ca/grads/attila/papers/52am/ion52am.pdf
  An effective sunspot number (inferred IG12 index) is used which is defined as the product of the IG12 index and the scaling factor

- Klobuchar-Style Ionospheric Coefficients
  http://aiuws.unibe.ch/ionosphere/#cgim
  Klobuchar-style alpha and beta coefficients best fitting VTEC data

Effective parameters can be defined using diverse kind of data/ models

Basic ideas

NeQuick features relevant to implement adaptation techniques based on the use of effective parameters.

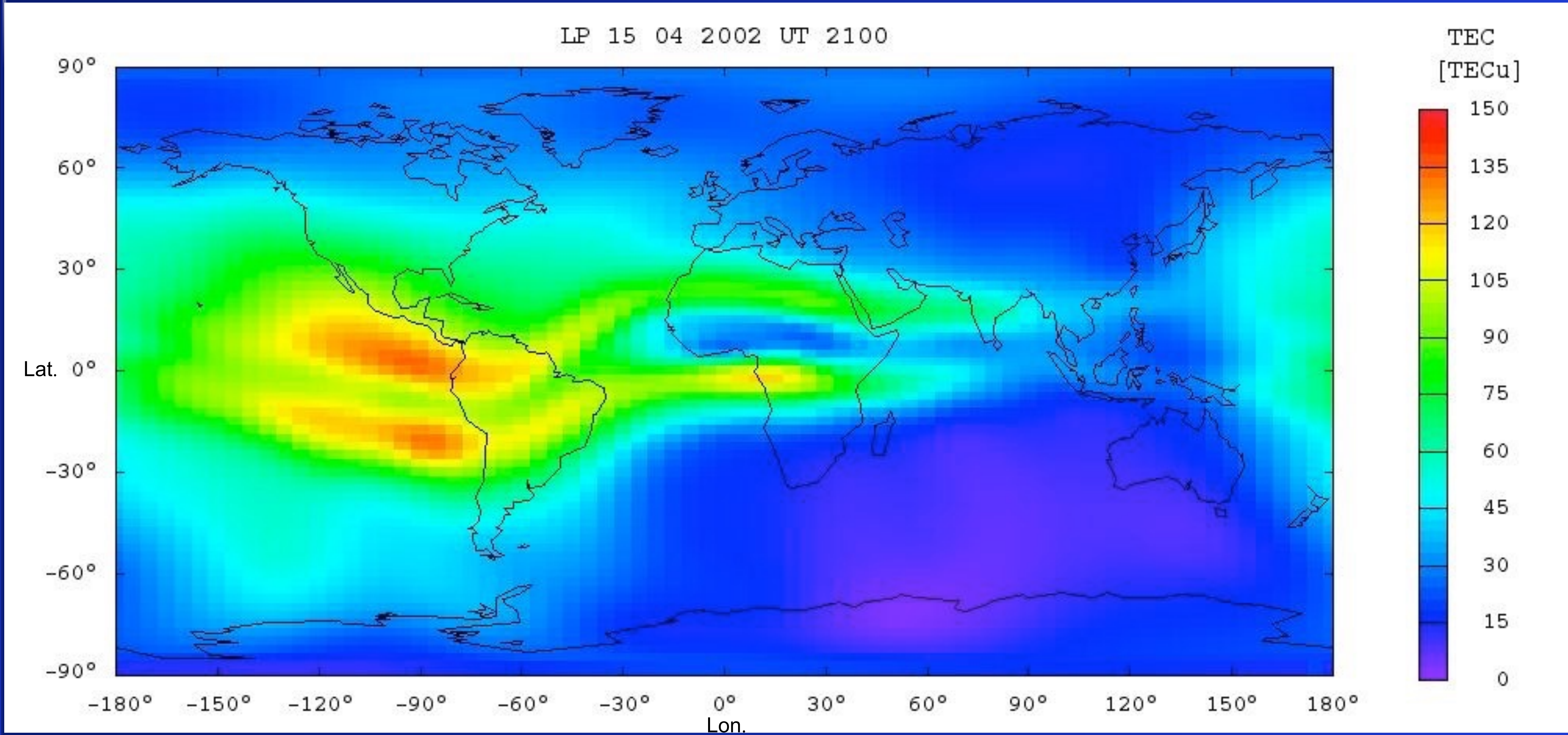
- For a given epoch & ray-path the model TEC is a monotonic function of the solar activity index, that can be regarded as an “effective ionization level” parameter.

∀ link, minimize TEC mismodeling → effective F10.7 (grid)

↓

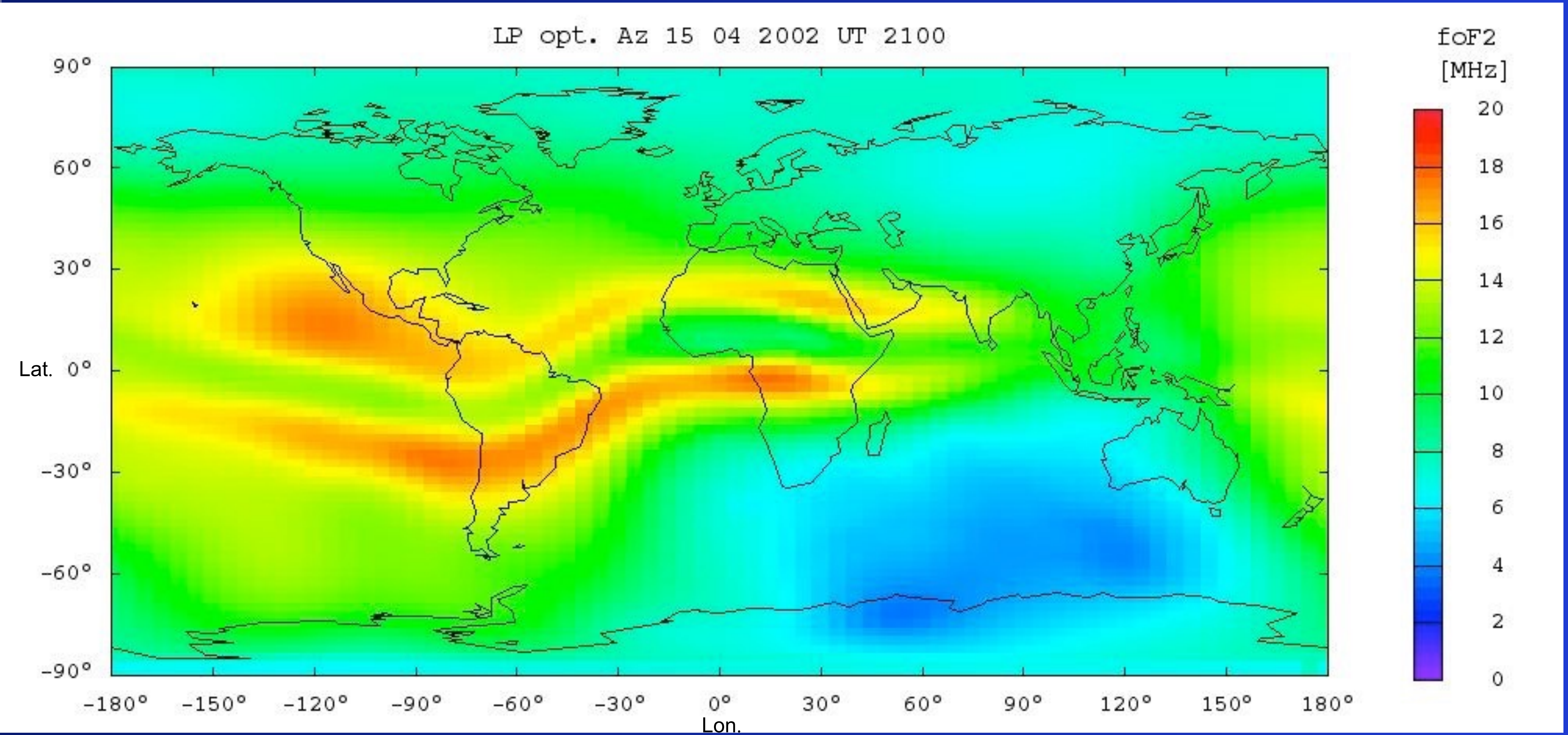
Use NeQuick to reconstruct the 3D electron density of the ionosphere and the other relevant parameters where needed

vTEC map



grid points:
lat.=-90°, 90° step 2.5°
lon.=-180°, 180° step 5°

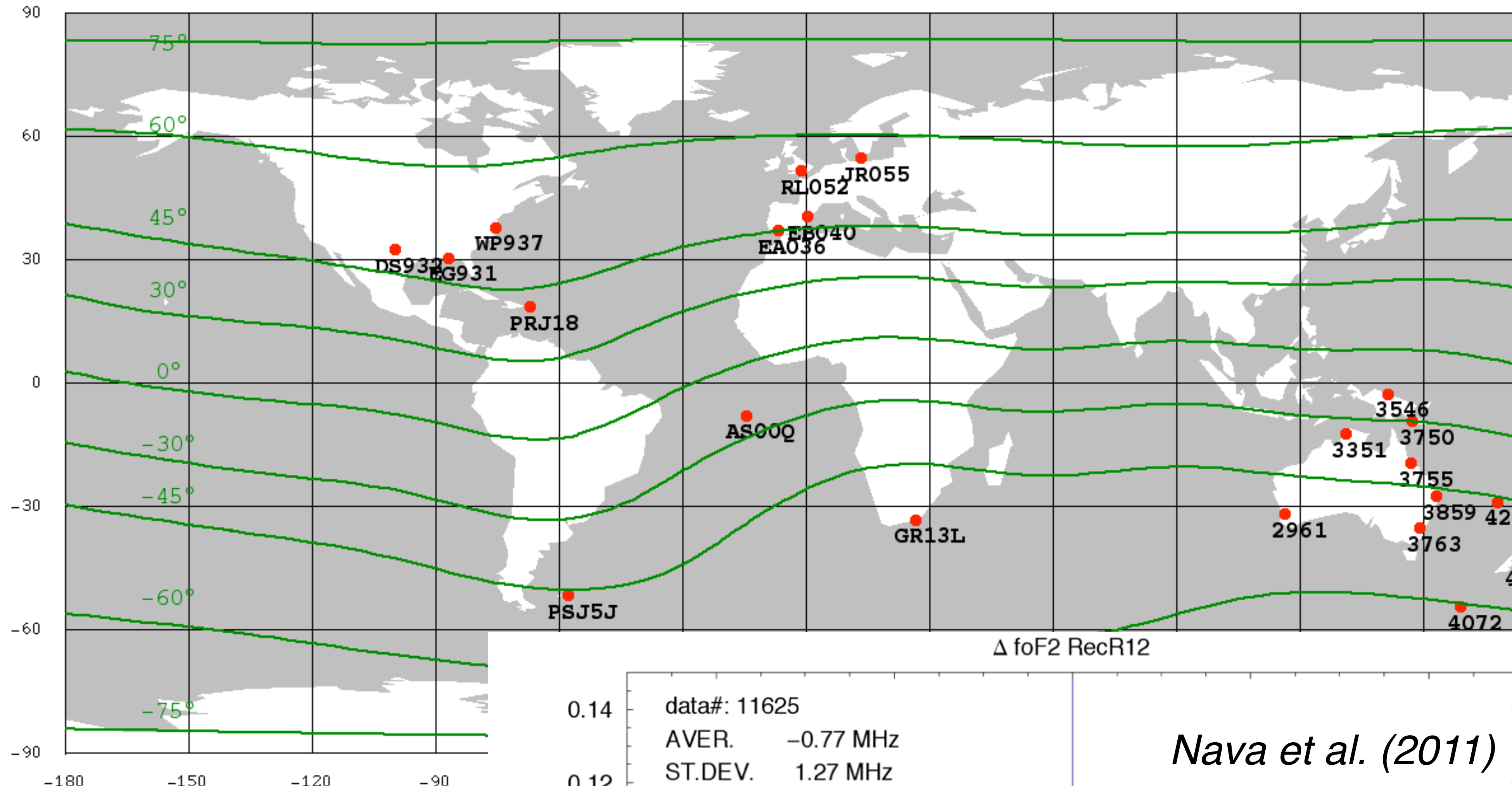
Reconstructed foF2 map

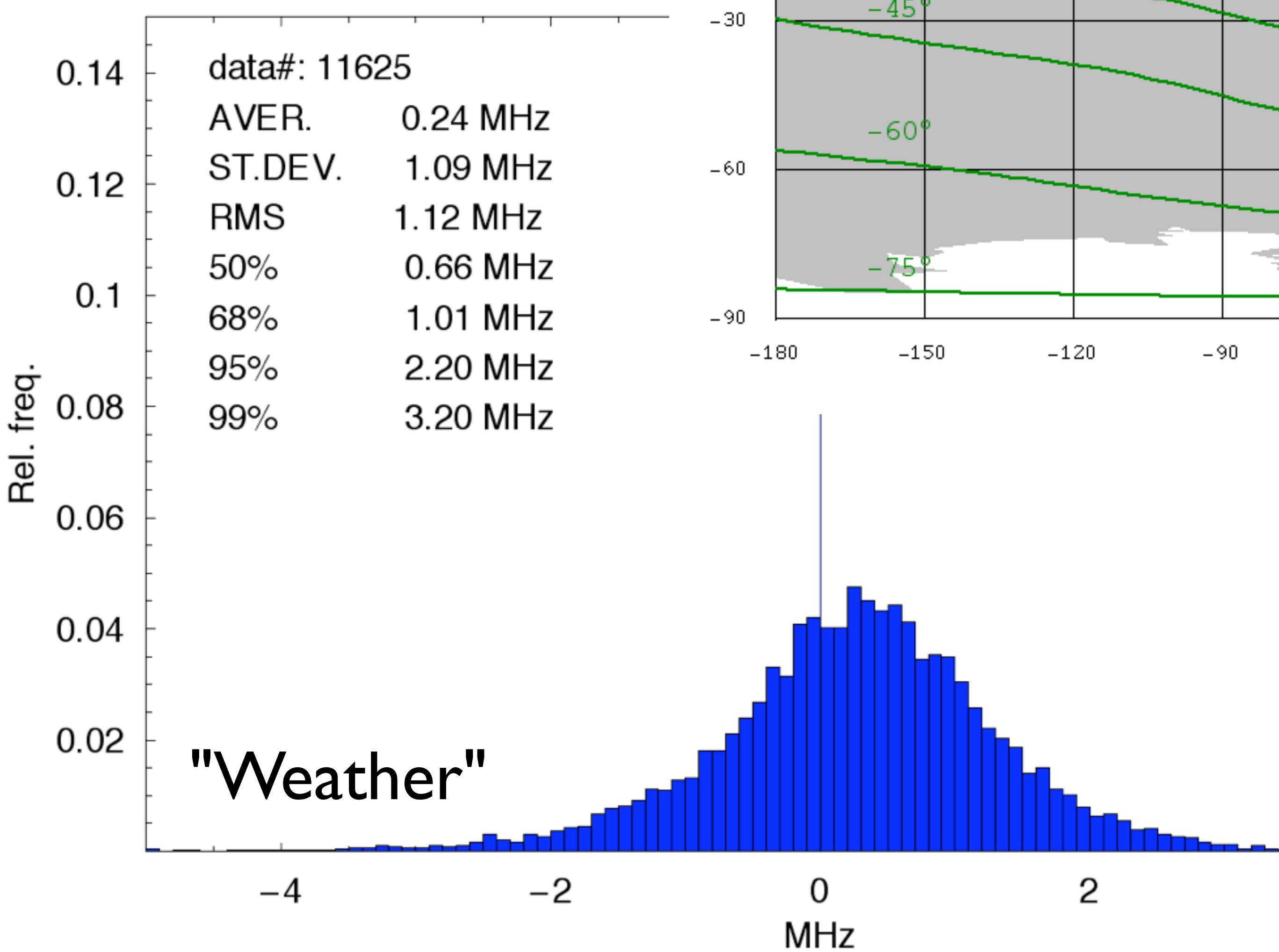


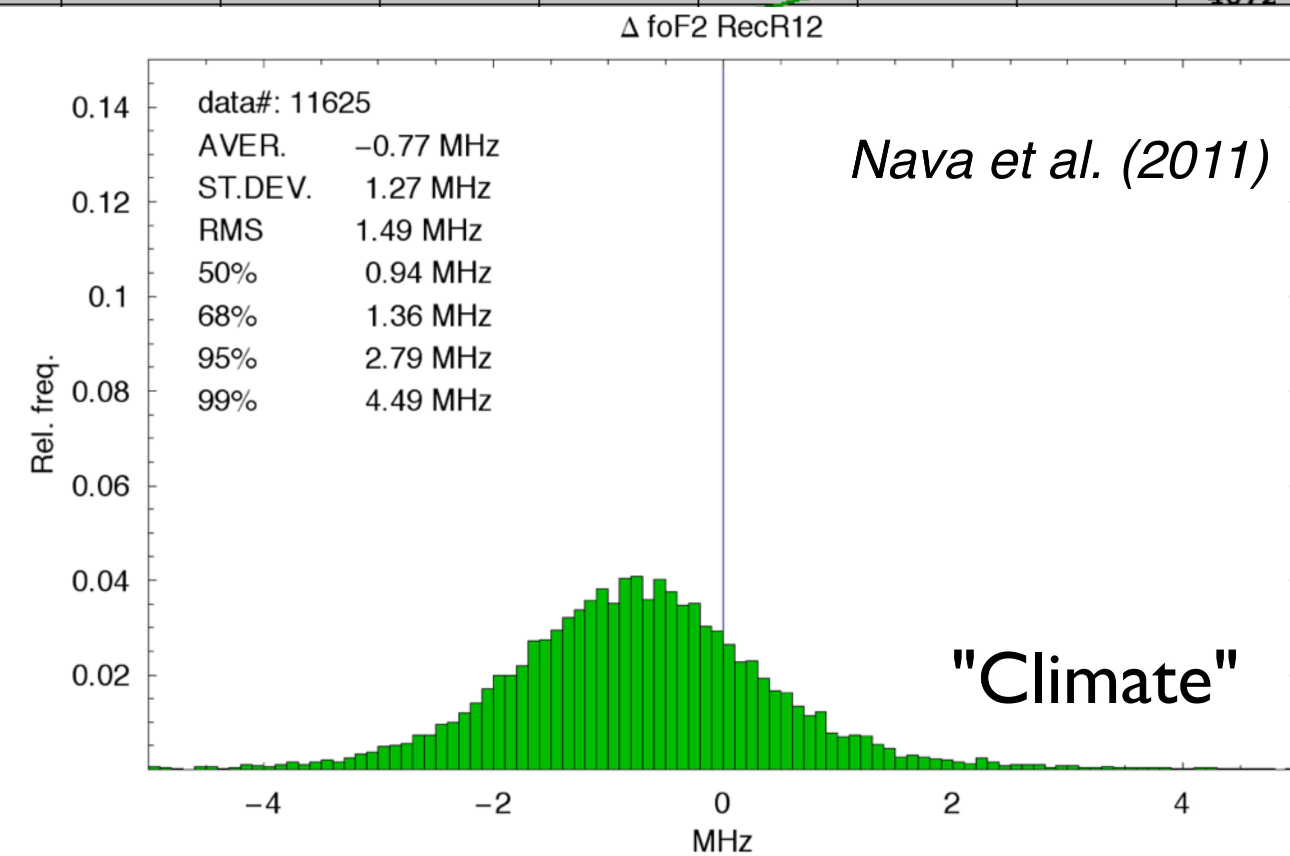
grid points:
lat.=-90°, 90° step 2.5°
lon.=-180°, 180° step 5°

NeQuick2: validation results (example: HSA)

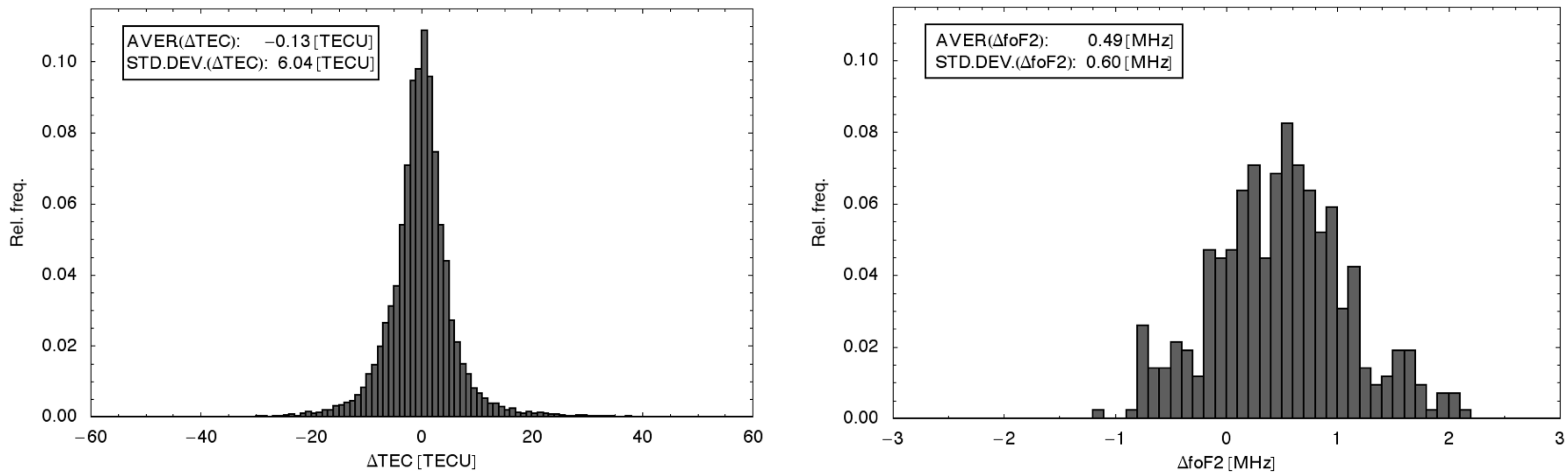
foF2 error statistics

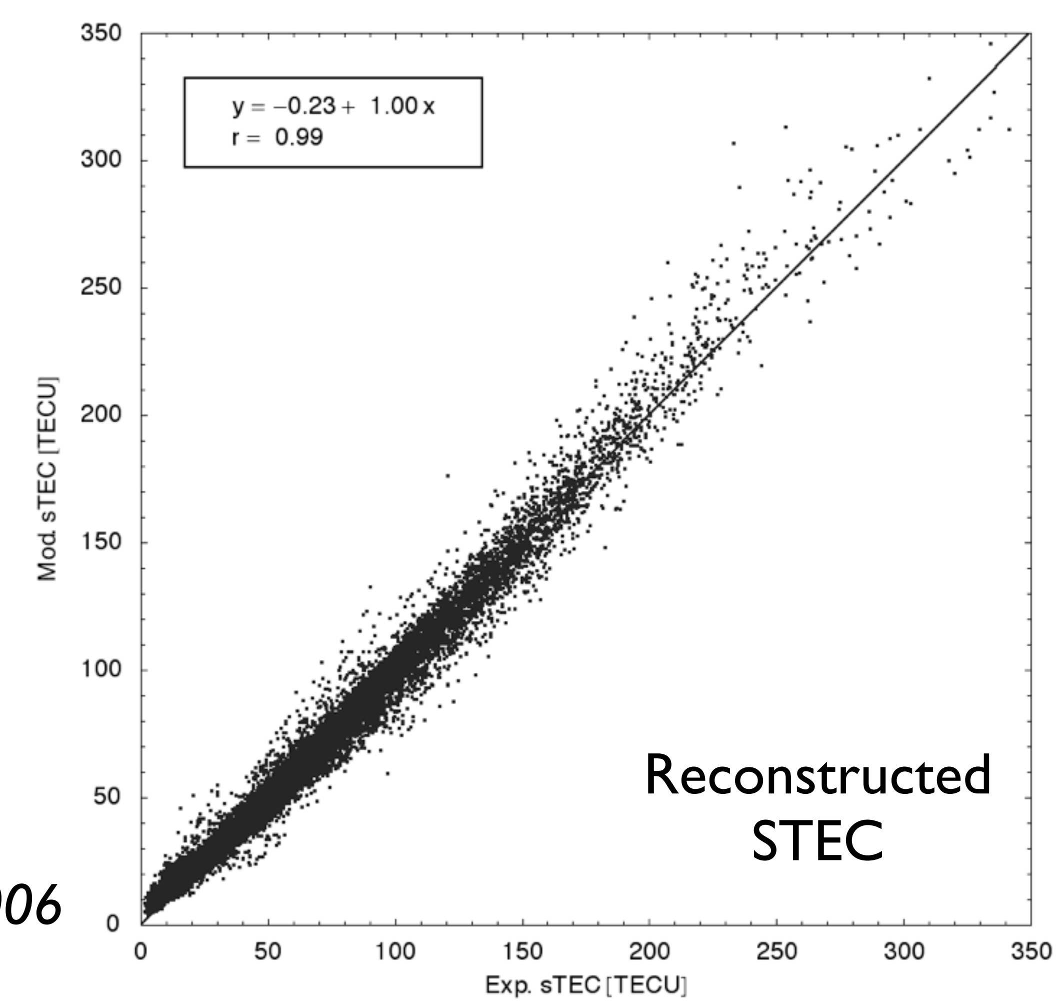


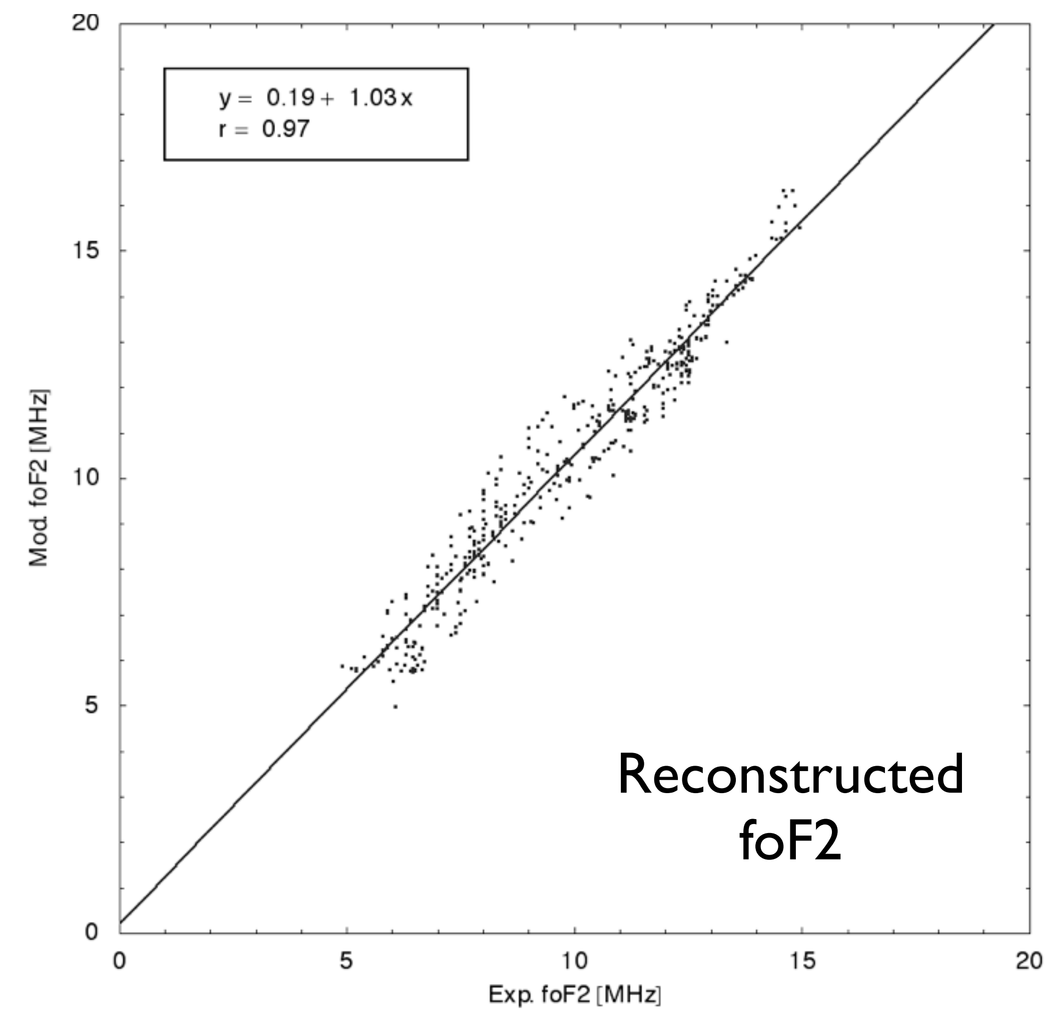




STEC data ingestion; error statistics (000405)

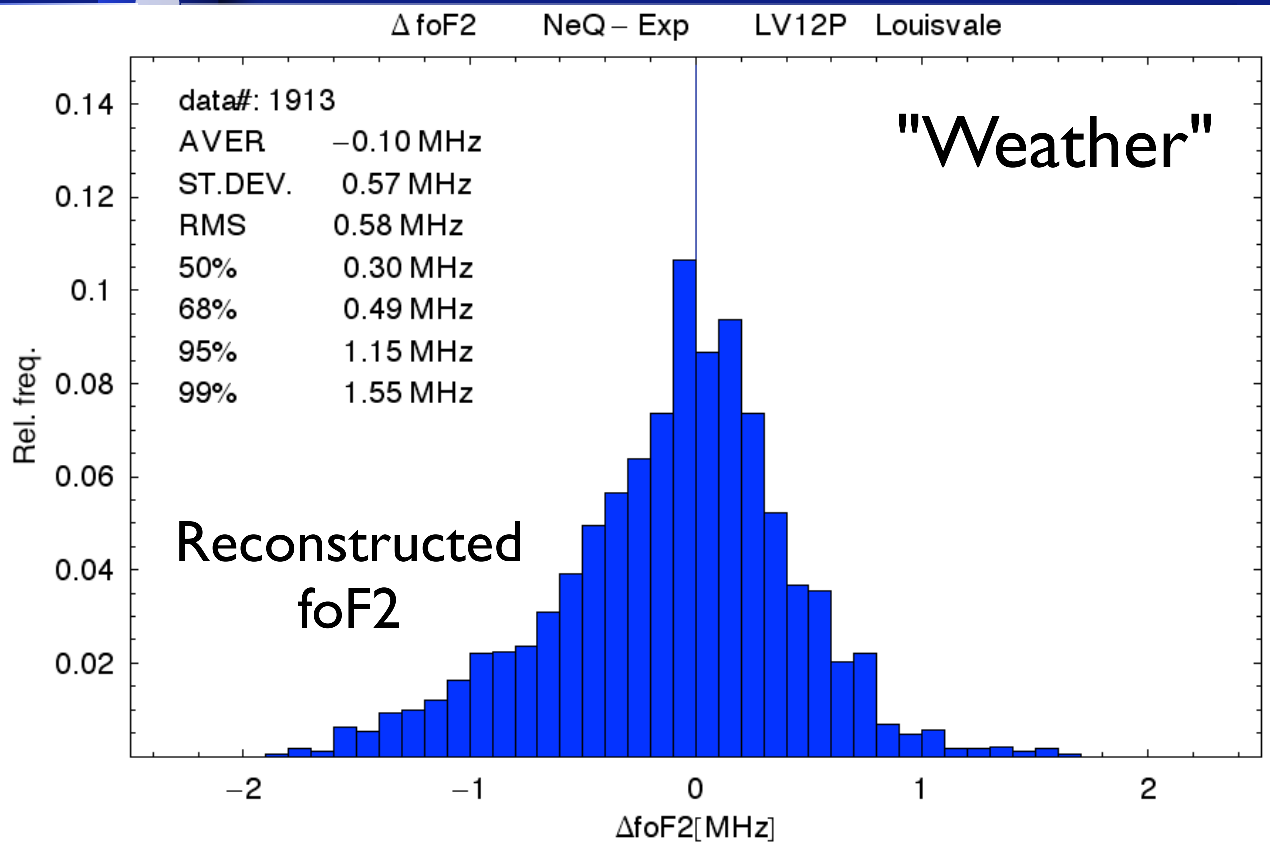


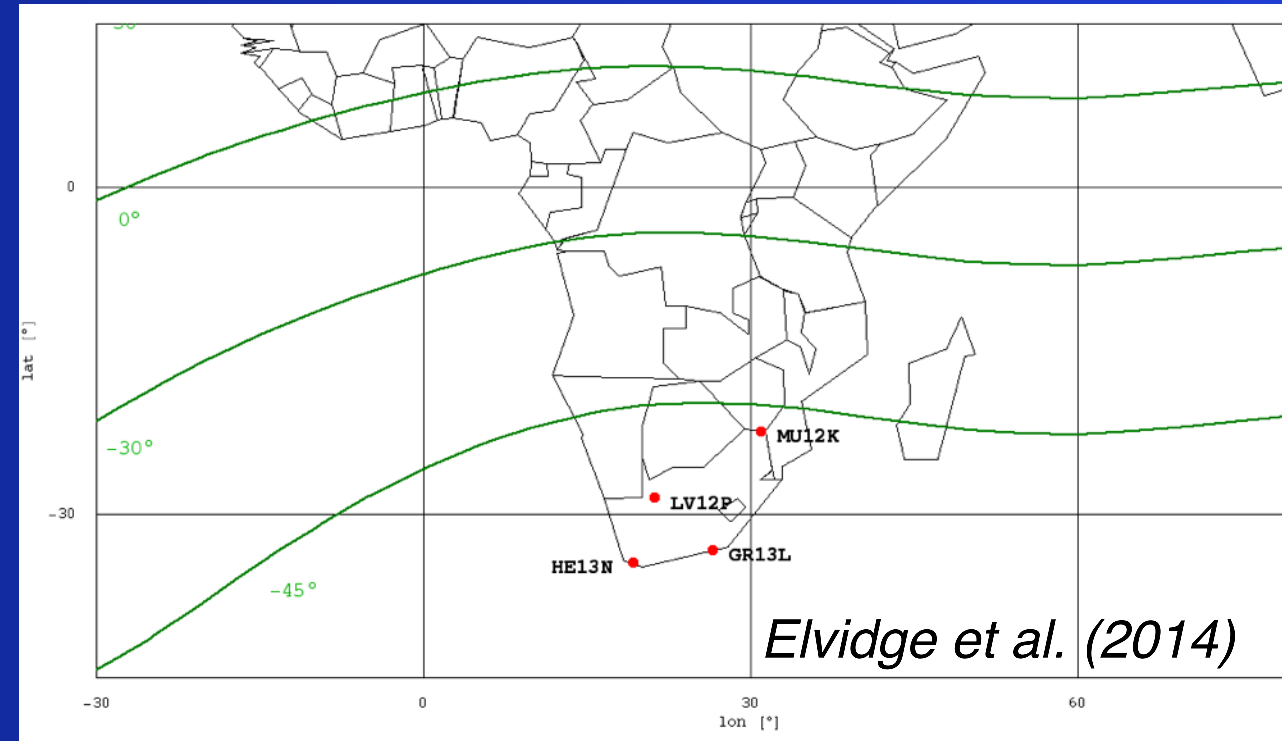


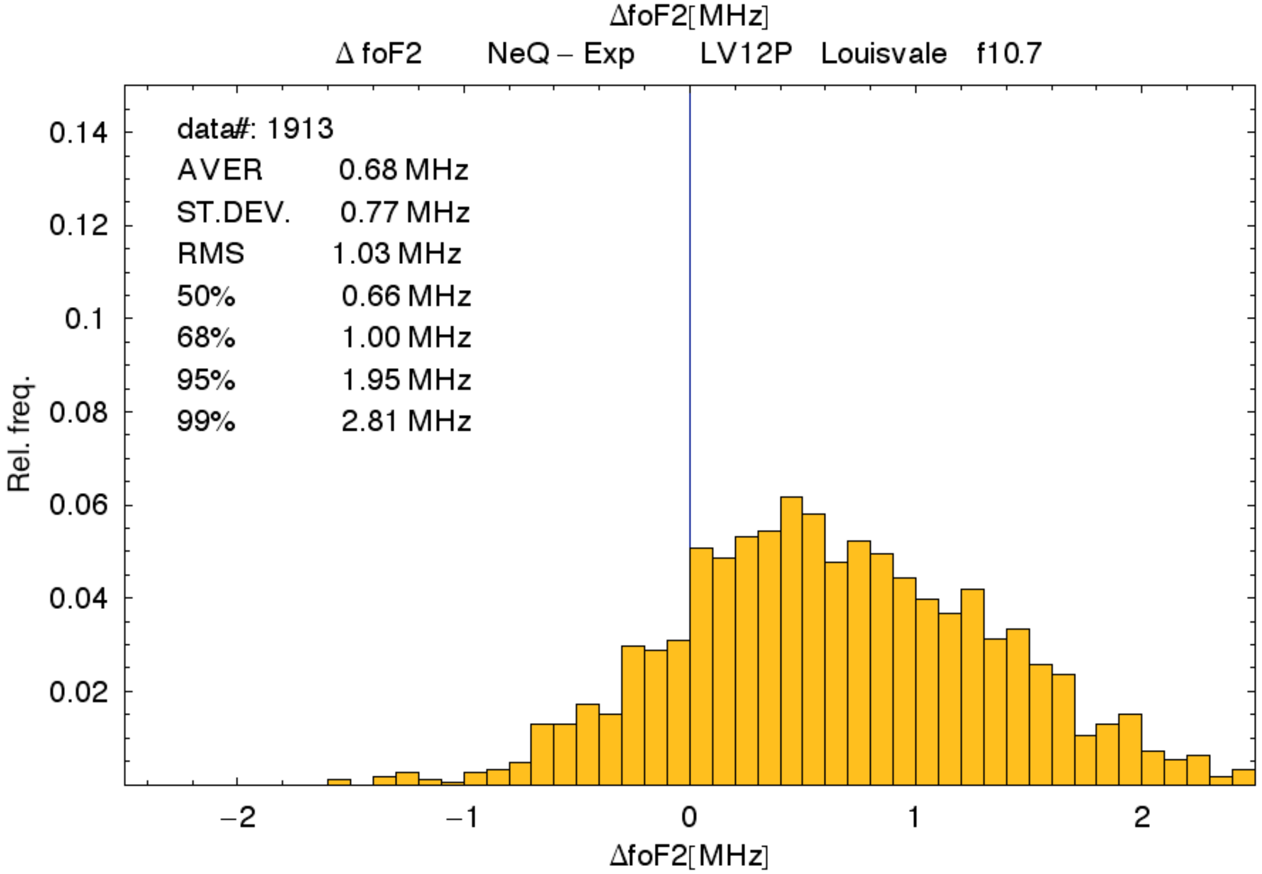


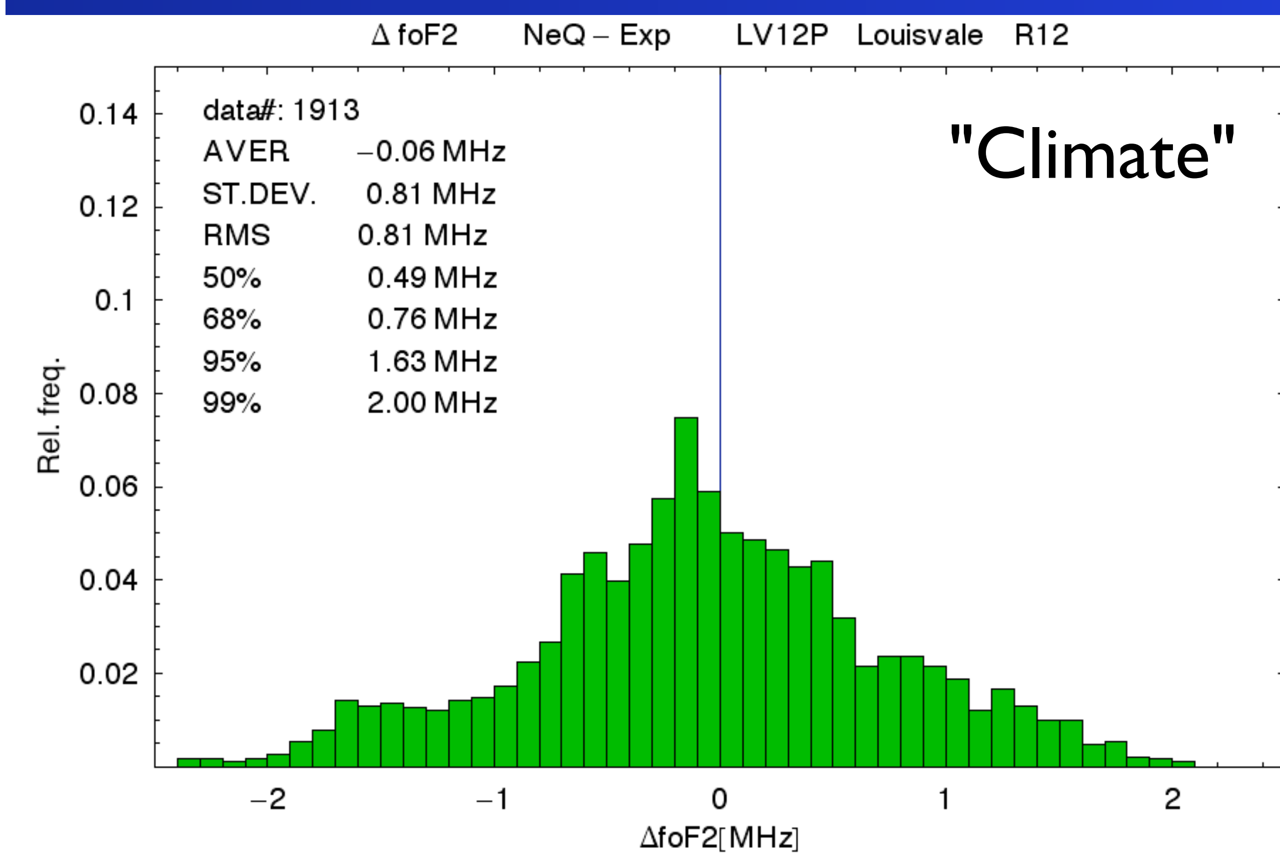
Nava et al., 2006

foF2 data ingestion; error statistics, Sep 2011





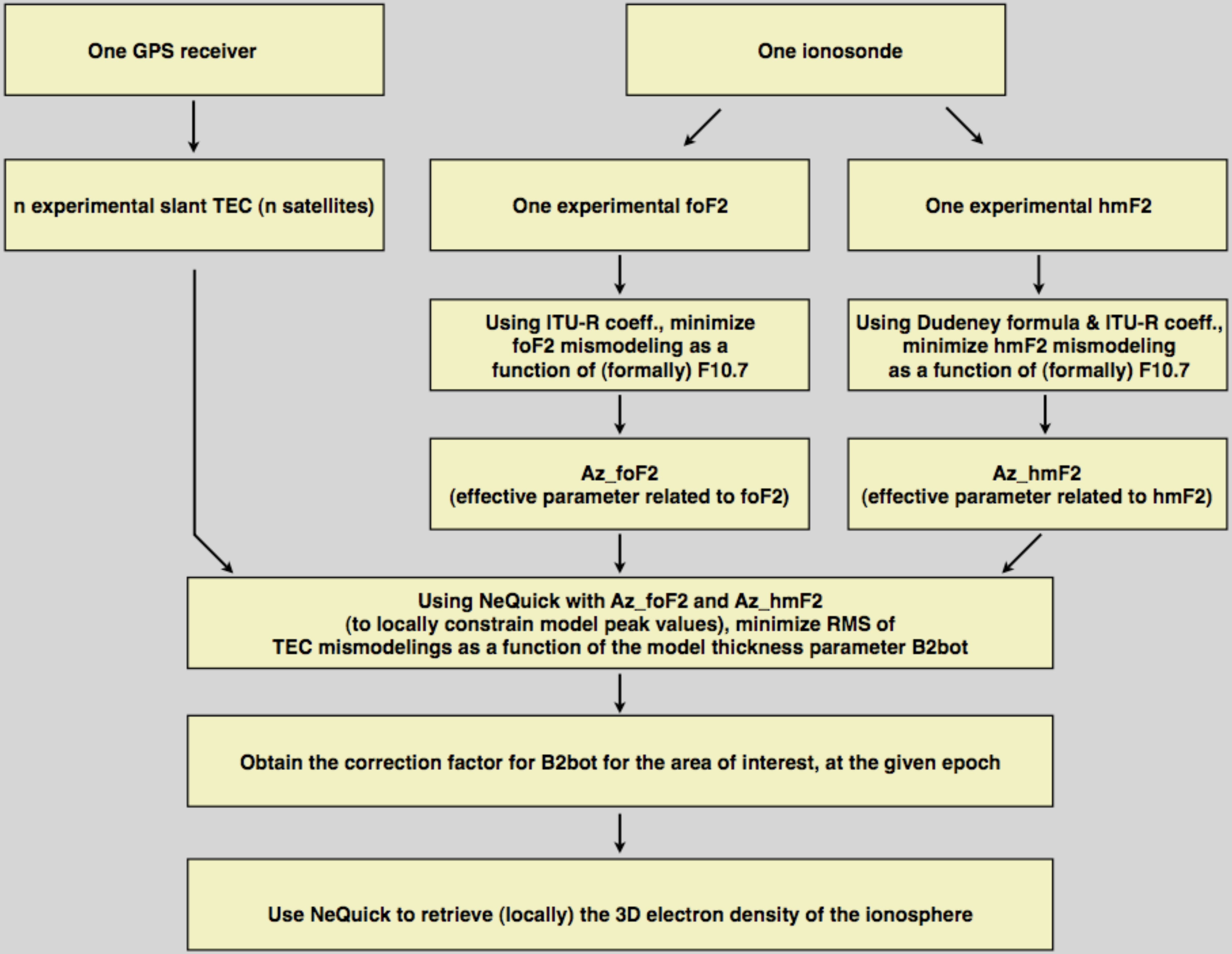




Adapting NeQuick model to experimental slant TEC and foF2 data at a given location

(Use of slab thickness to constrain the NeQuick profile shape parameter)

At a given epoch:



Remarks

- The use of two effective parameters has been considered in order to use the ITUR coefficients to estimate foF2 and hmF2 in a region surrounding the ground station.

- In this way the peak parameter values can be estimated for a slant TEC computation.

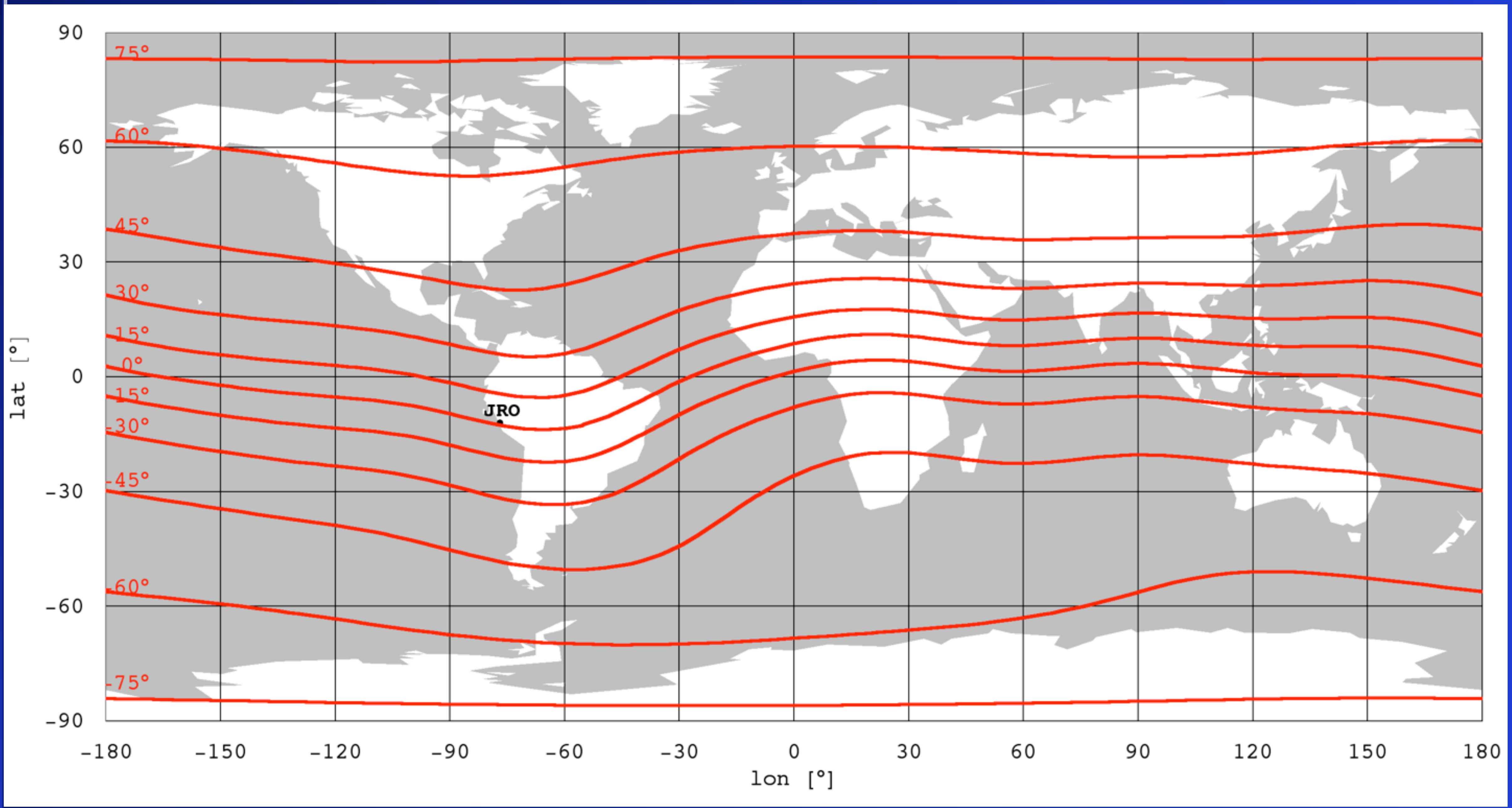
Use JRO profiles to simulate the process of adapting NeQuick to GPS derived TEC and ionosonde peak parameters data.

TEC and peak parameters are known from the profile.

↓

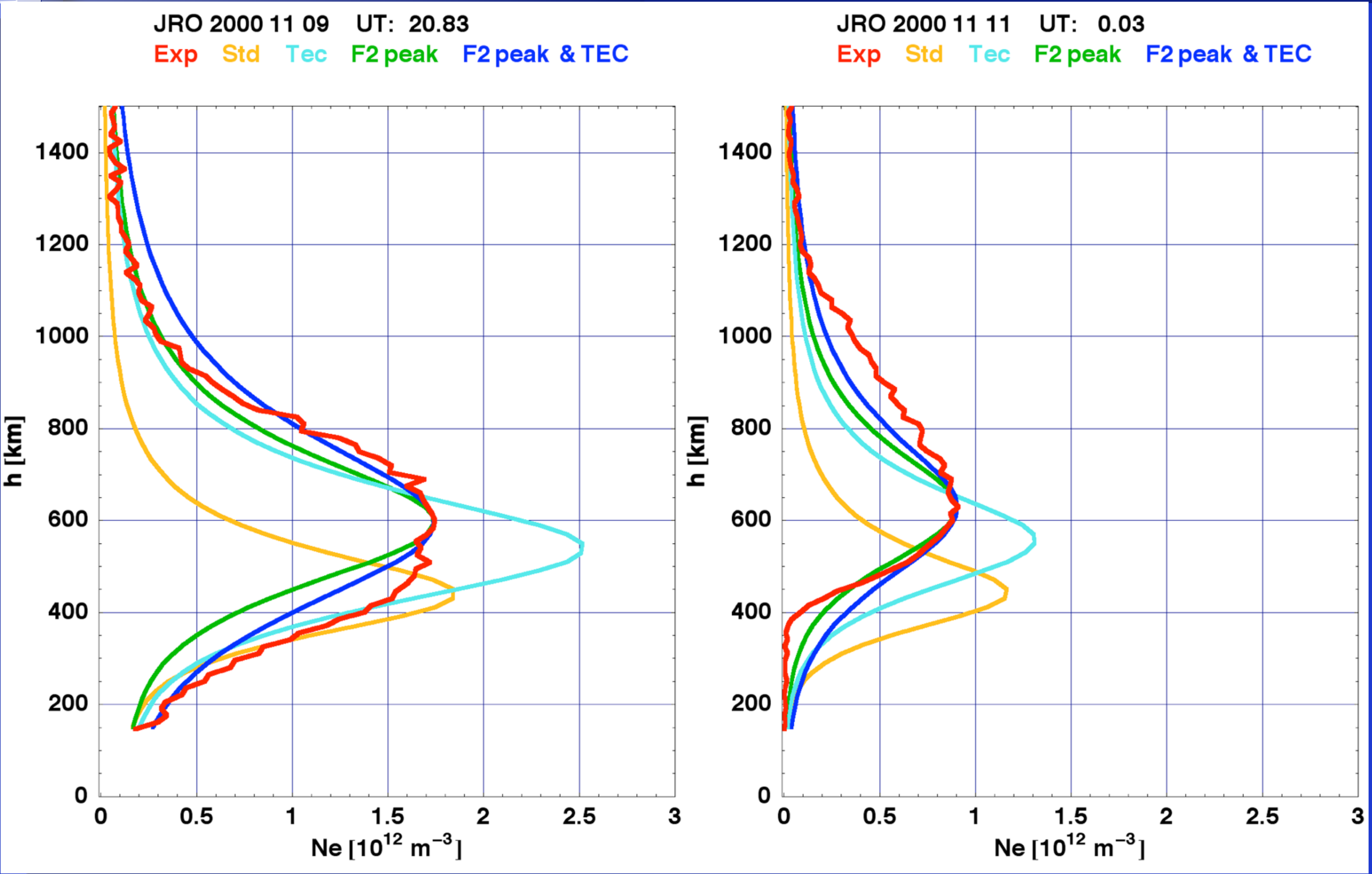
After model adaptation it is possible to compare profiles in order to evaluate the adaptation technique effectiveness.

Adaptation method validation



Jicamarca Radio Observatory (JRO) location

Adaptation method validation

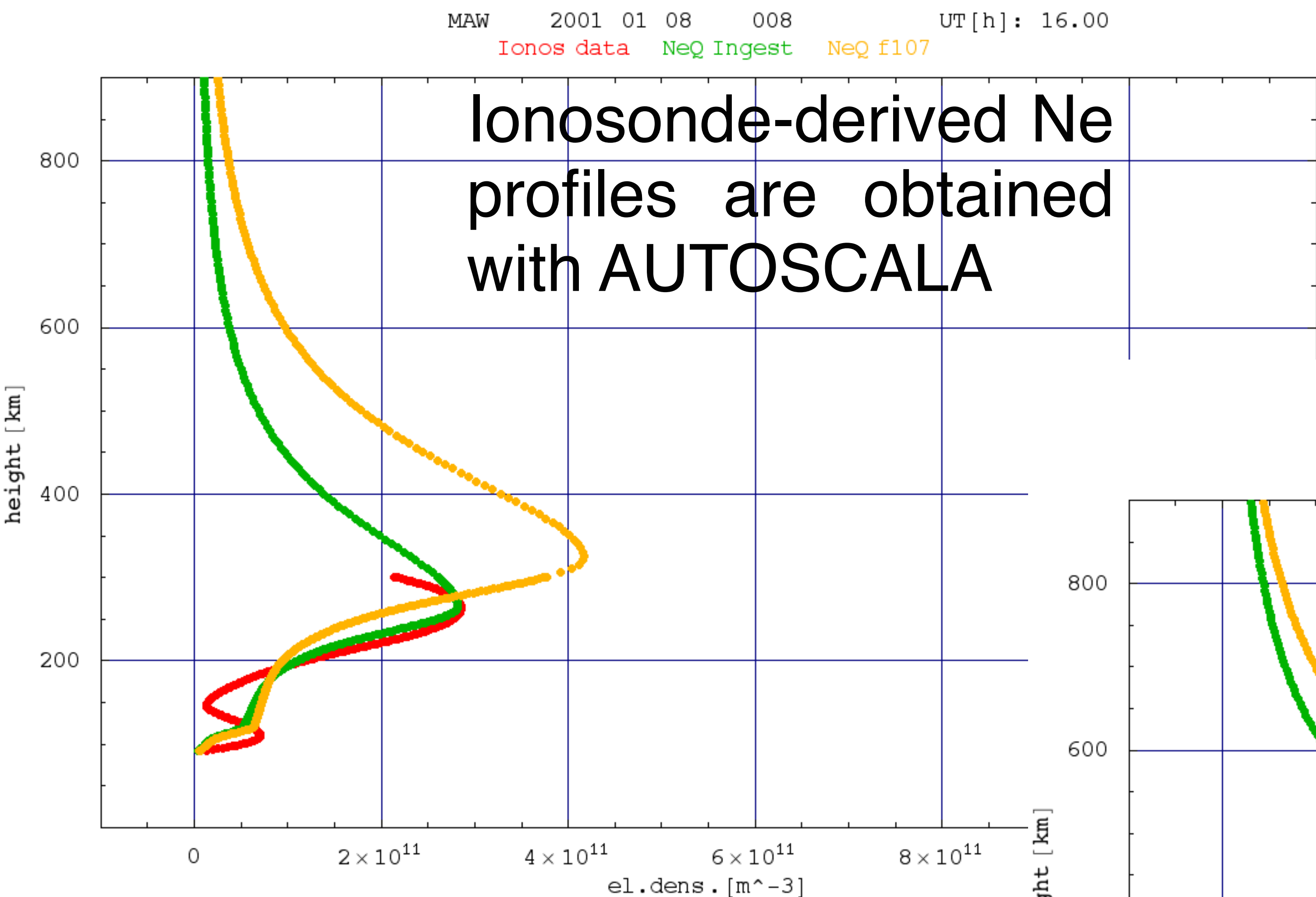


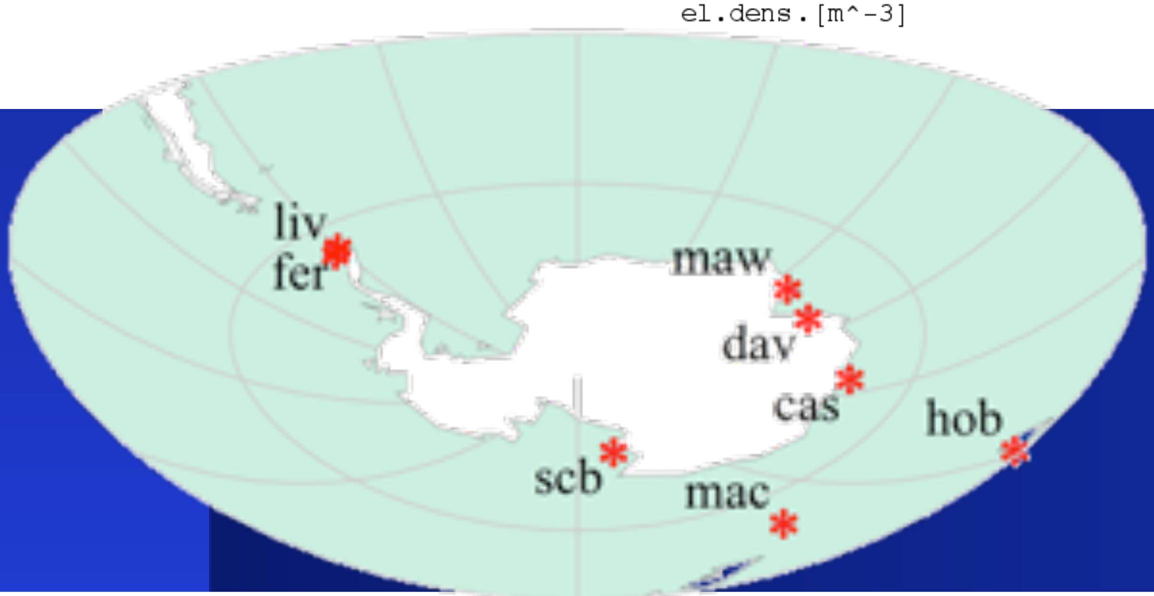
Model: NeQuick

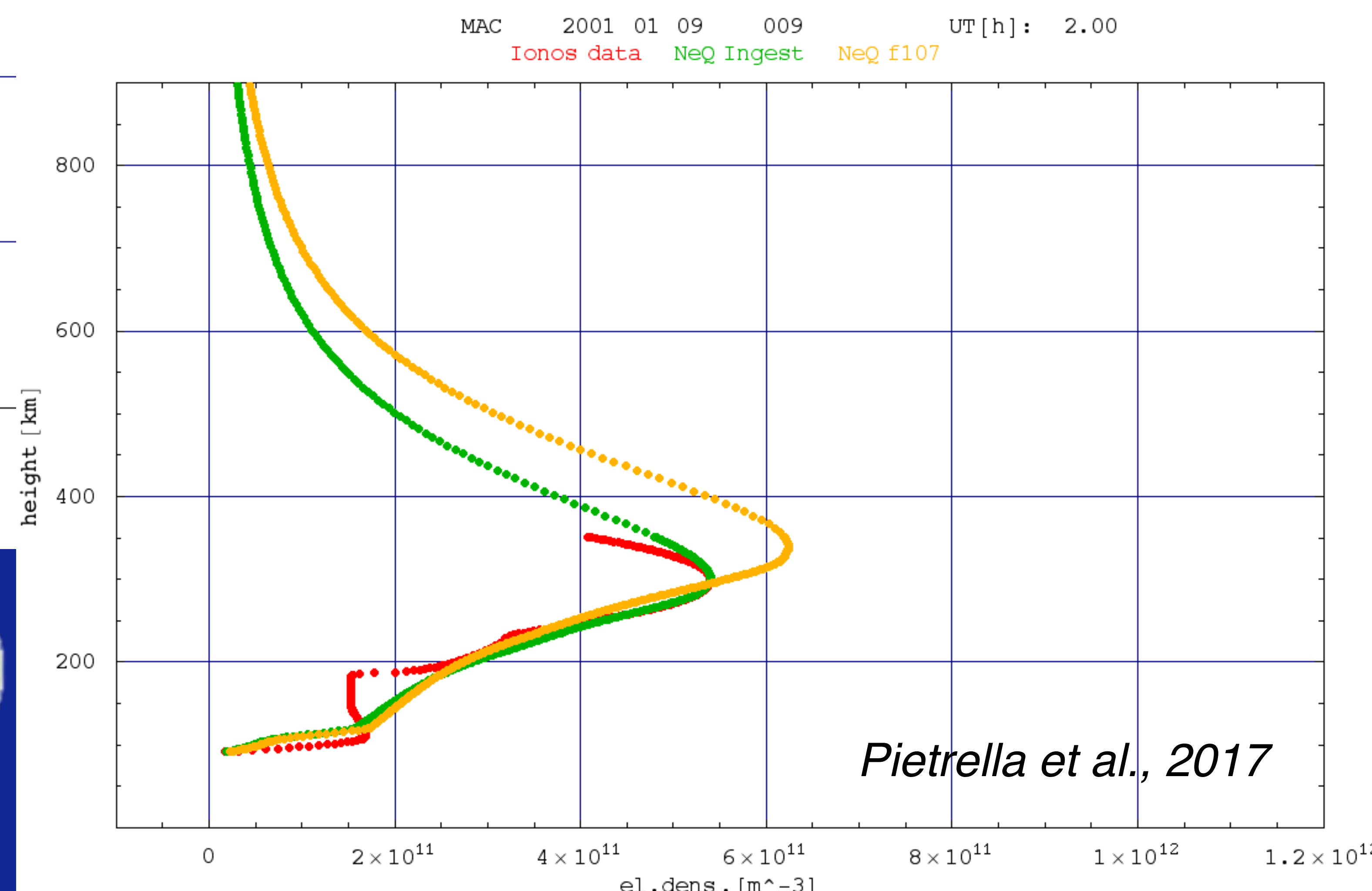
NeQuick adaptation to foF2 & hmF2

AUtomatic Scaling of Polar Ionograms and Cooperative Ionospheric Observations (AUSPICIO) Project

Ionosonde-derived Ne profiles are obtained with AUTOSCALA

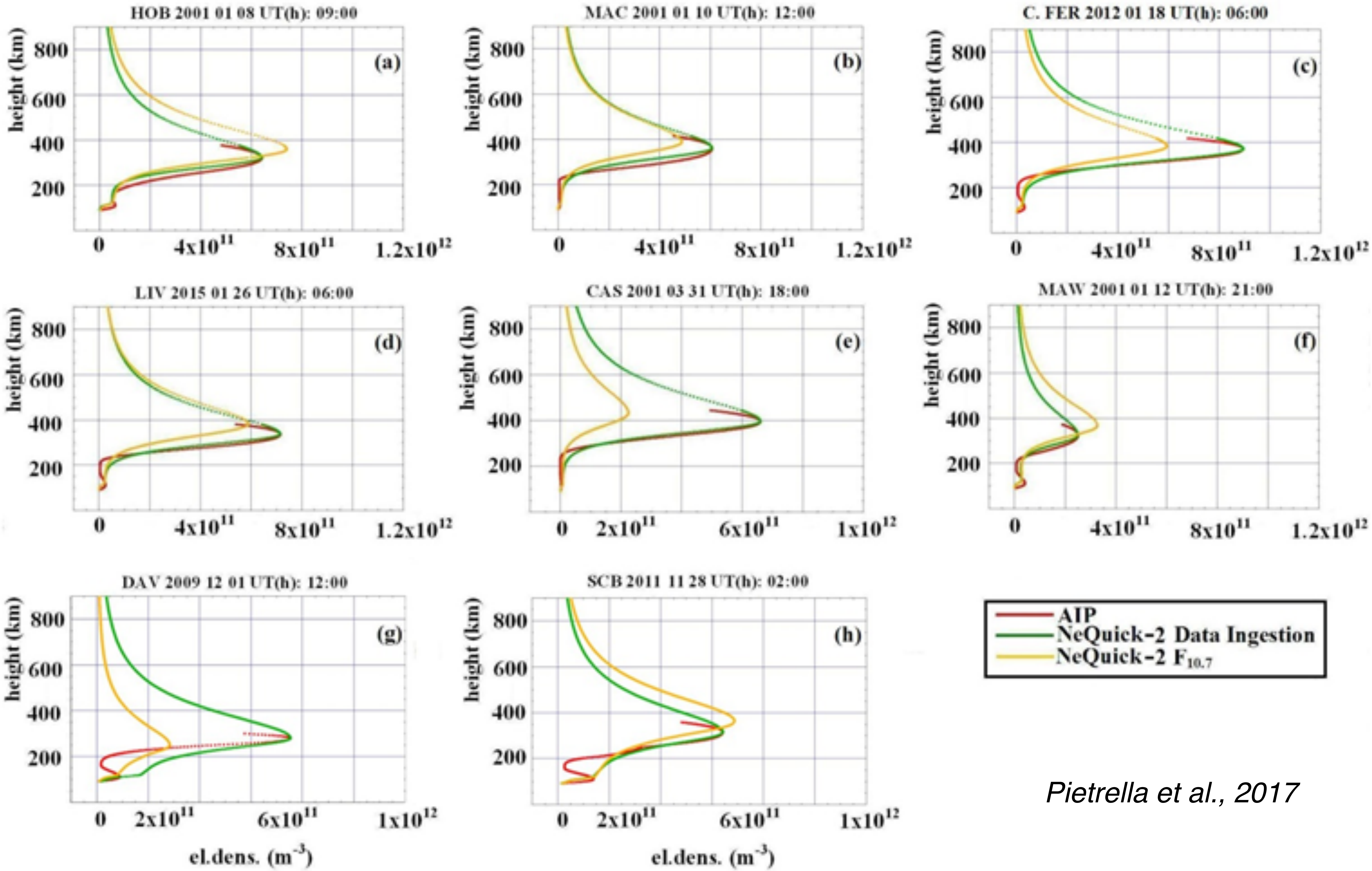






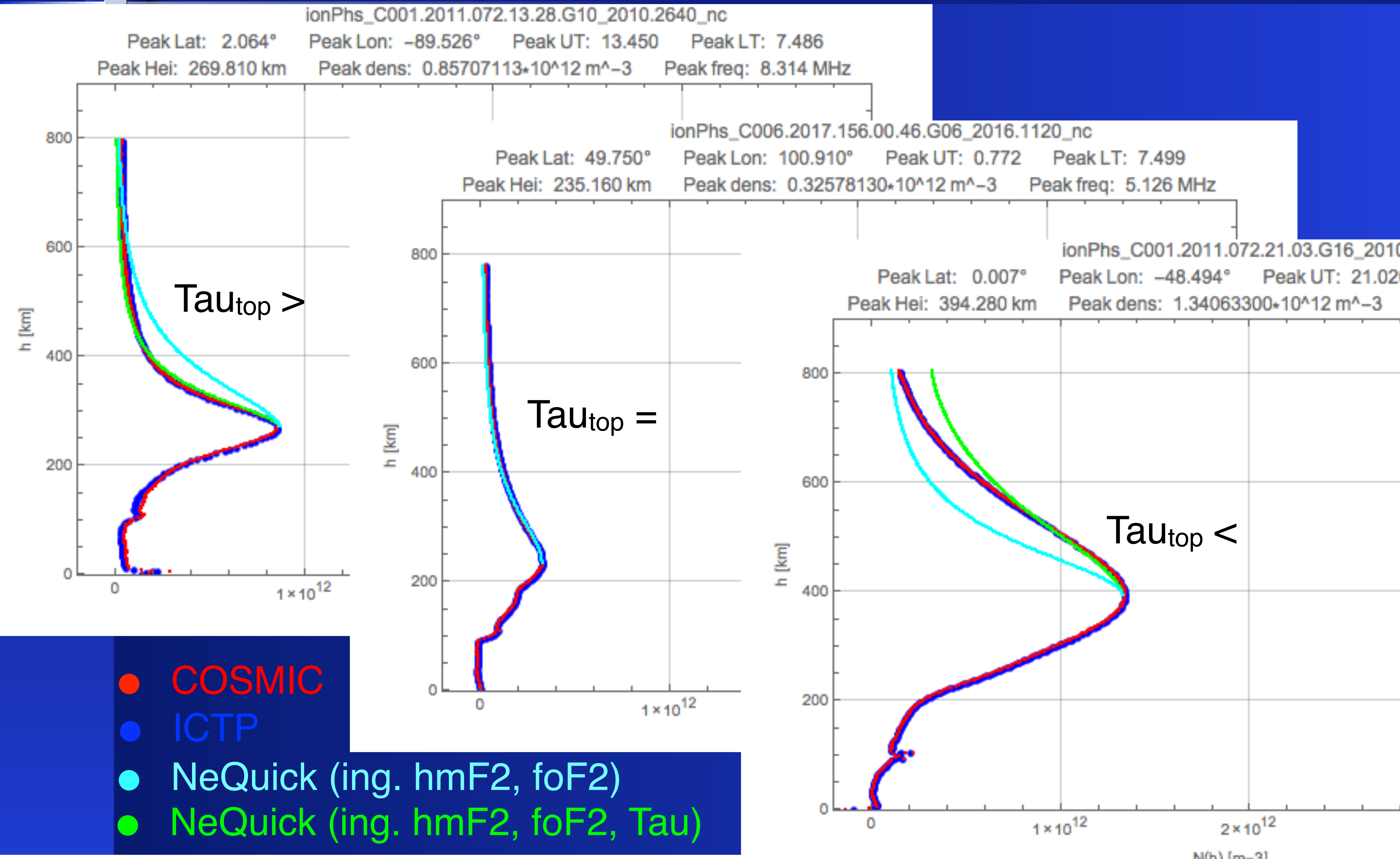
Pietrella et al., 2017

NeQuick adaptation to foF2 & hmF2



Pietrella et al., 2017

NeQuick adaptation to foF2, hmF2 (and Tau_{top})



Least Square Estimation

Least Square Estimation

Best Linear Unbiased Estimator (BLUE)*

$\mathbf{x}_t$ true model state (dimension n)

$\mathbf{x}_b$ background model state (dimension n)

$\mathbf{x}_a$ analysis model state (dimension n)

$\mathbf{y}$ vector of observations (dimension p)

$\boldsymbol{H}$ observation operator (dimension p x n)

B covariance matrix of background errors $\boldsymbol{\varepsilon}_b=(\mathbf{x}_b-\mathbf{x}_t)$ (dimension n x n)

R covariance matrix of observation errors $\boldsymbol{\varepsilon}_o=(\mathbf{y}-\boldsymbol{H}[\boldsymbol{x}_t])$ (dimension p x p)

A covariance matrix of analysis errors $\boldsymbol{\varepsilon}_a=(\mathbf{x}_a-\mathbf{x}_t)$ (dimension n x n)

*https://www.ecmwf.int/sites/default/files/elibrary/2002/16928-data-assimilation-concepts-and-methods.pdf

Least Square Estimation

- The following hypotheses are assumed:
  - *Linearized observation operator*: the variations of the observation operator in the vicinity of the background state are linear:
    - $\forall\, \mathbf{x}$ close enough to $\mathbf{x}_b$, $H(\mathbf{x}) - H(\mathbf{x}_b) = \mathbf{H}(\mathbf{x}-\mathbf{x}_b)$ where $\mathbf{H}$ is a linear operator
  - *Non-trivial errors*: **B** and **R** are positive definite matrices.
  - *Unbiased errors*: the expectation of the background and observation errors is zero:
    - $E[\mathbf{x}_b-\mathbf{x}_t] = E[\mathbf{y}-H[\mathbf{x}_t]] = 0$

Least Square Estimation

- The following hypotheses are assumed (cont.):
  - *Uncorrelated errors*: observation and background errors are mutually uncorrelated.
    - $E[(\mathbf{x}_b-\mathbf{x}_t)(\mathbf{y}-H[\mathbf{x}_t])^T] = 0$
  - *Linear analysis*: we look for an analysis defined by corrections to the background which depend linearly on background observation departures.
  - *Optimal analysis*: we look for an analysis state which is as close as possible to the true state in an r.m.s. sense (i.e. it is a minimum variance estimate).

Least Square Estimation

The optimal least-square estimator (BLUE analysis) is defined by

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}\,(\mathbf{y} - H[\mathbf{x}_b])$$

$$\mathbf{K} = \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}$$

K is called *gain* of the analysis

$$\mathbf{A} = (\mathbf{I}-\mathbf{K}\mathbf{H})\mathbf{B}$$

The BLUE analysis is equivalently obtained as a solution to the variational optimization problem:

$$\begin{aligned}
\mathbf{x}_a &= \text{Arg min} J \\
J(\mathbf{x}) &= (\mathbf{x}-\mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b) + (\mathbf{y}-H[\mathbf{x}])^T\mathbf{R}^{-1}(\mathbf{y}-H[\mathbf{x}]) \\
&= J_b(\mathbf{x}) + J_o(\mathbf{x})
\end{aligned}$$

where J is called the cost function of the analysis
J_b is the background term; J_o is the observation term

If the background and and observation errors pdfs are Gaussian, then $\mathbf{x}_a$ is also the maximum likelihood estimator of $\mathbf{x}_t$

Least Square Estimation

In our case:

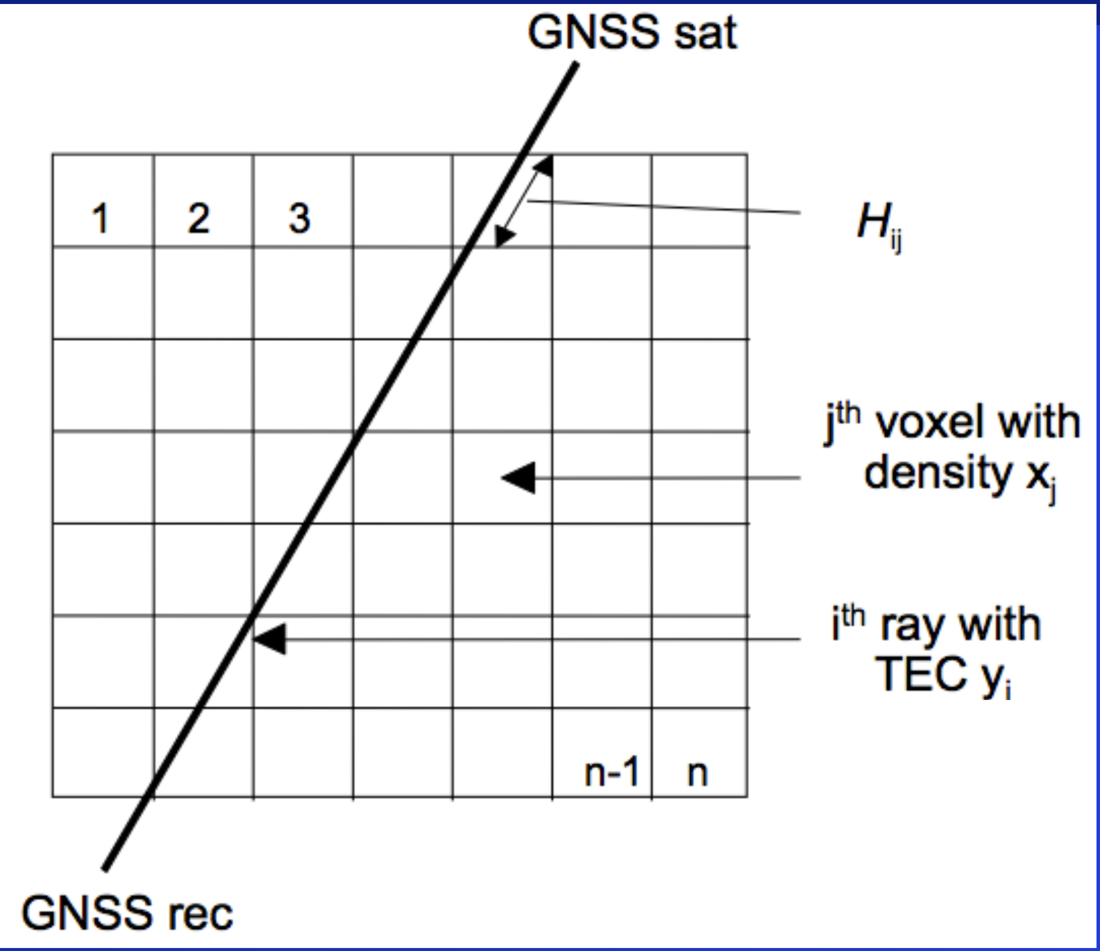
y ~ TEC

$\mathbf{x}_a$ ~ retrieved electron density

$\mathbf{x}_b$ ~ background electron density

H ~ “crossing lengths” in “voxels”

e.g. $\text{bckg\_TEC}_i = \mathbf{H}_i\mathbf{x}_b = \sum_j H_{ij}\, x_{bj}$



$R_{ij}=c_R\, \delta_{ij}\, y_i^2$ (measurements are independent)

$B_{ij}=c_B\, x_{bi}\, x_{bj}\, \mathrm{Exp}[-(z_{ij}/Lz)^2]\, \mathrm{Exp}[-(\alpha_{ij}/L\alpha)^2]$ (V & H correl. are separable)

z_{ij} ~ height difference between voxels i and j

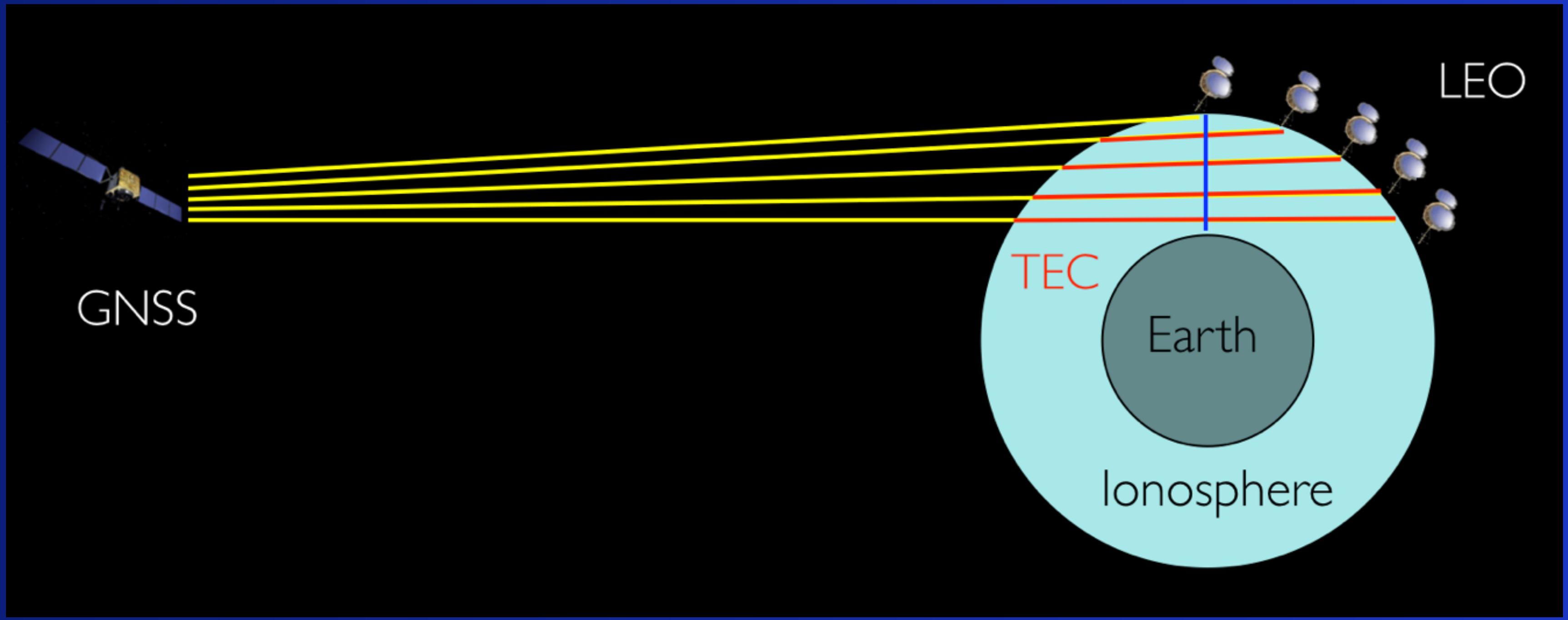
α_{ij} ~ angular (great circle) distance between voxels i and j

Lz ~ correl. distance in vert. direction (may depend on height)

Lα ~ correl. (angular) distance in hor. direction (may depend location,…)

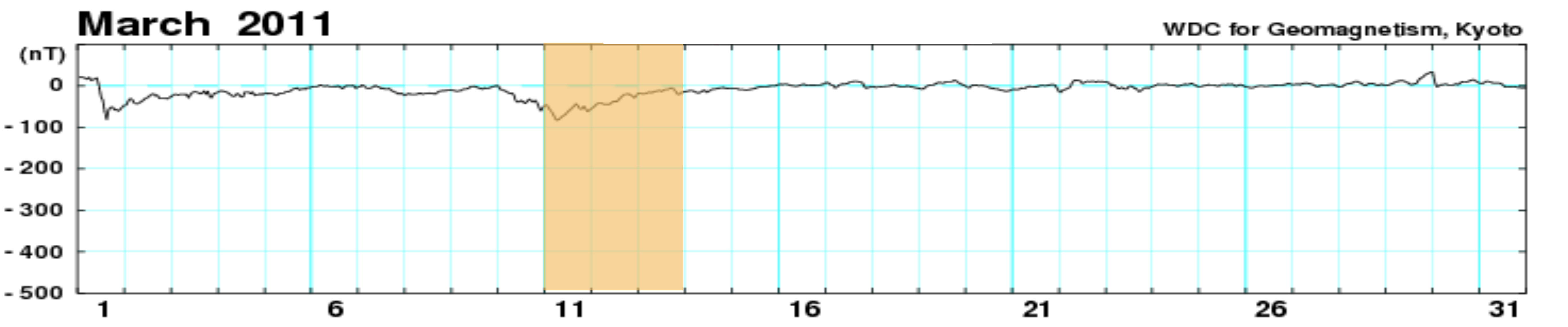
Data used

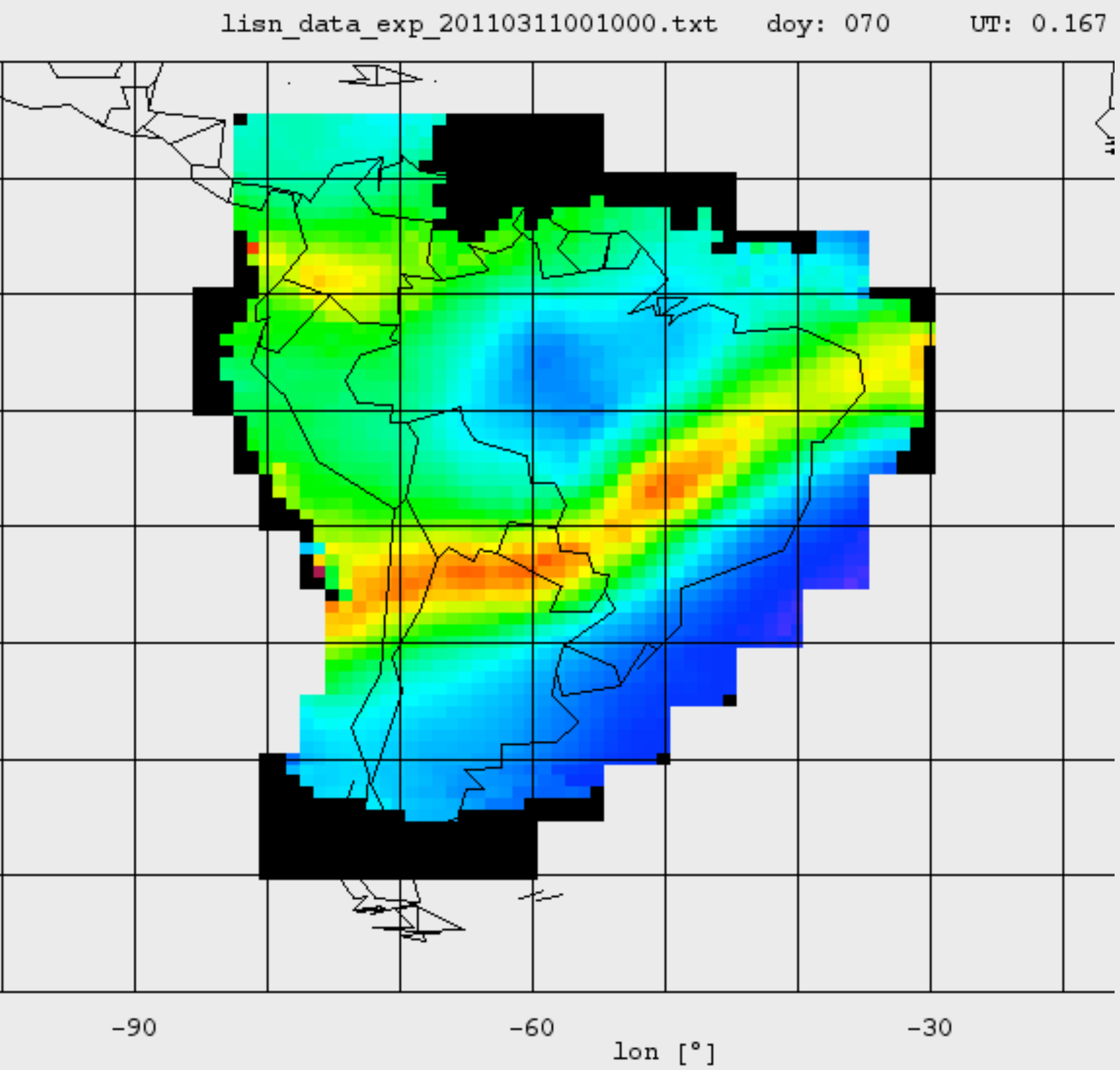
- For the assimilation
  - Ground-based GNSS-derived slant TEC data
  - Radio-Occultation-derived TEC data (calibrated TEC values along the LEO–to–GPS link below the LEO orbit)

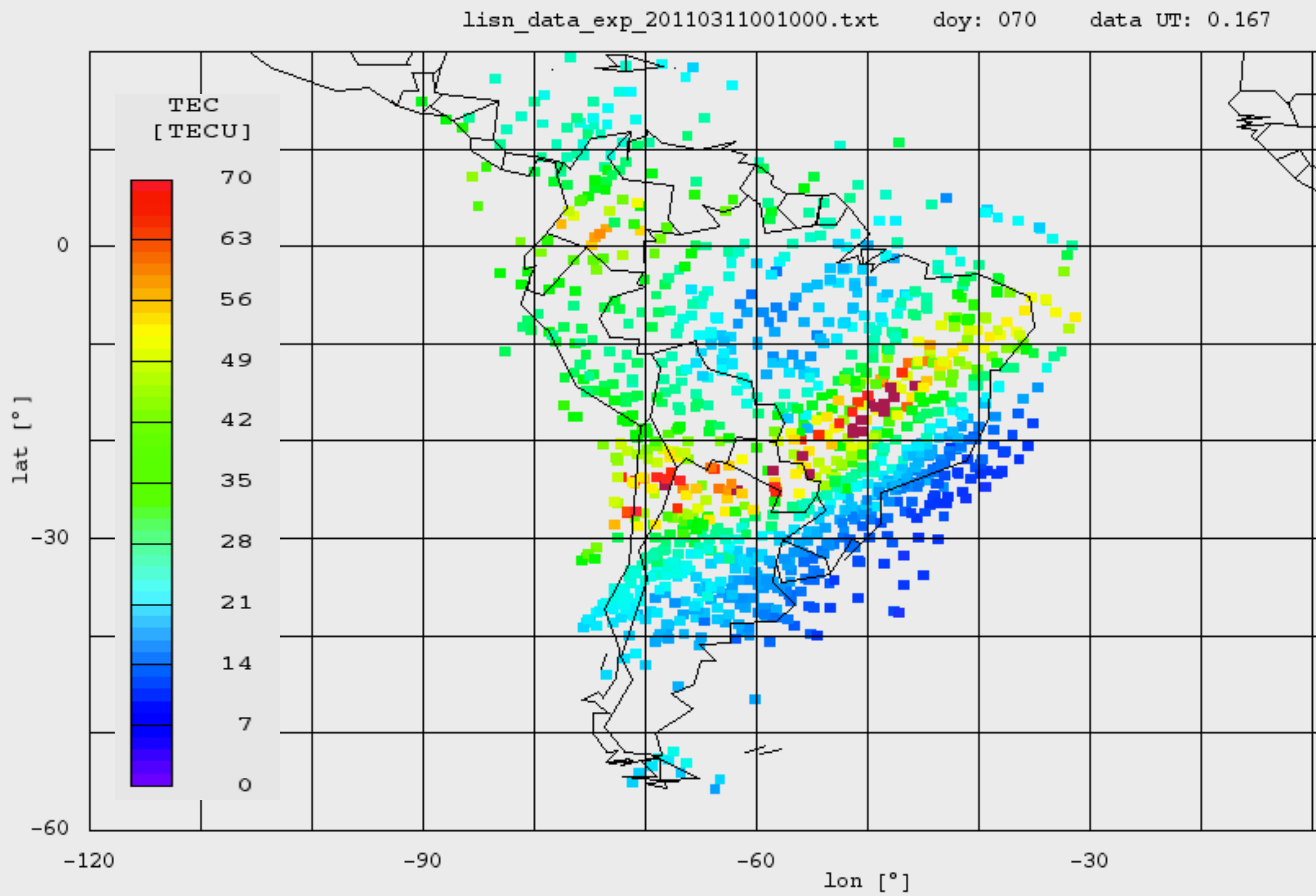


- For the validation
  - Manually scaled ionosonde derived foF2 and hmF2
  - Ground-based GNSS-derived slant TEC data (not used for assimilation)

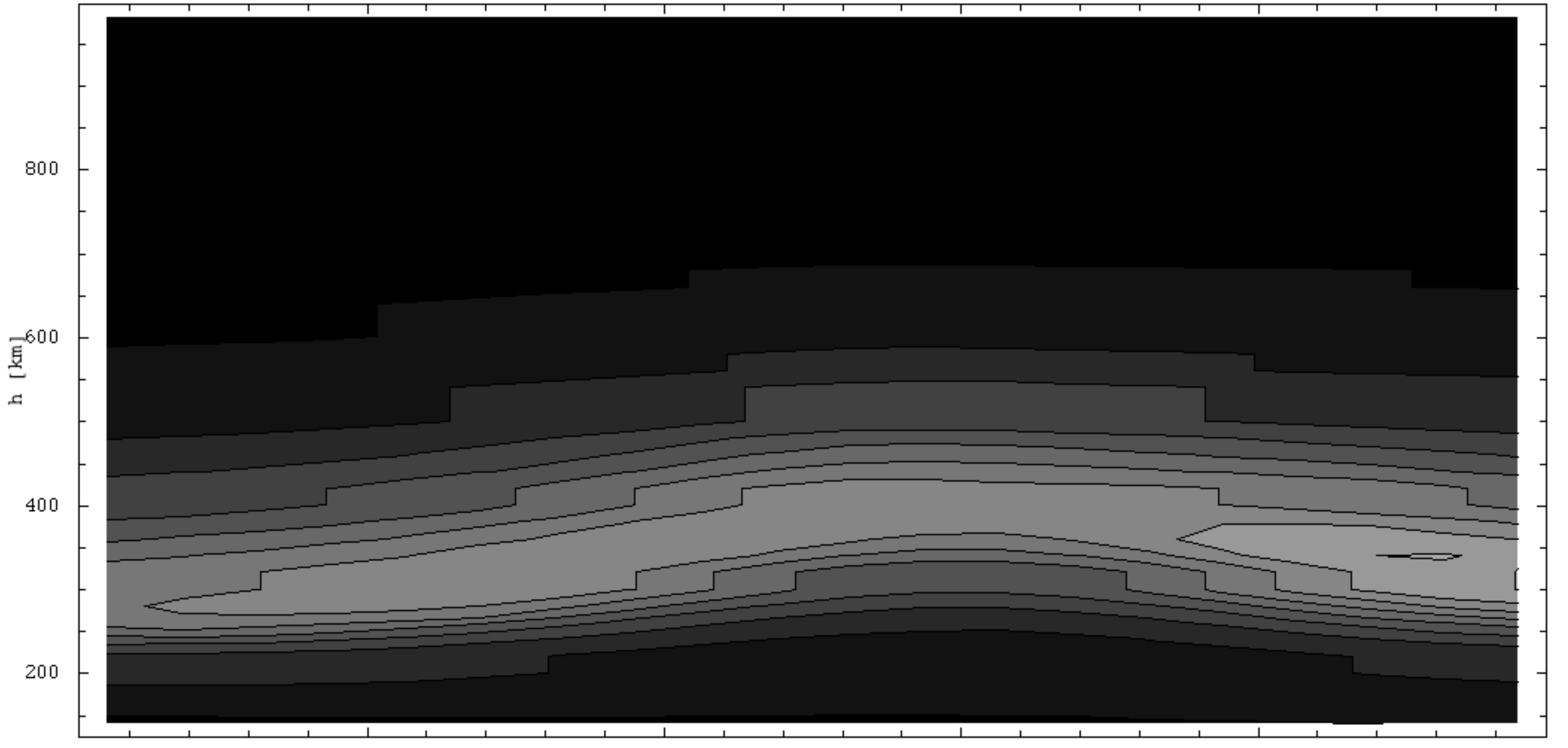
LISN: 3 days data (2011/03/11-12-13)



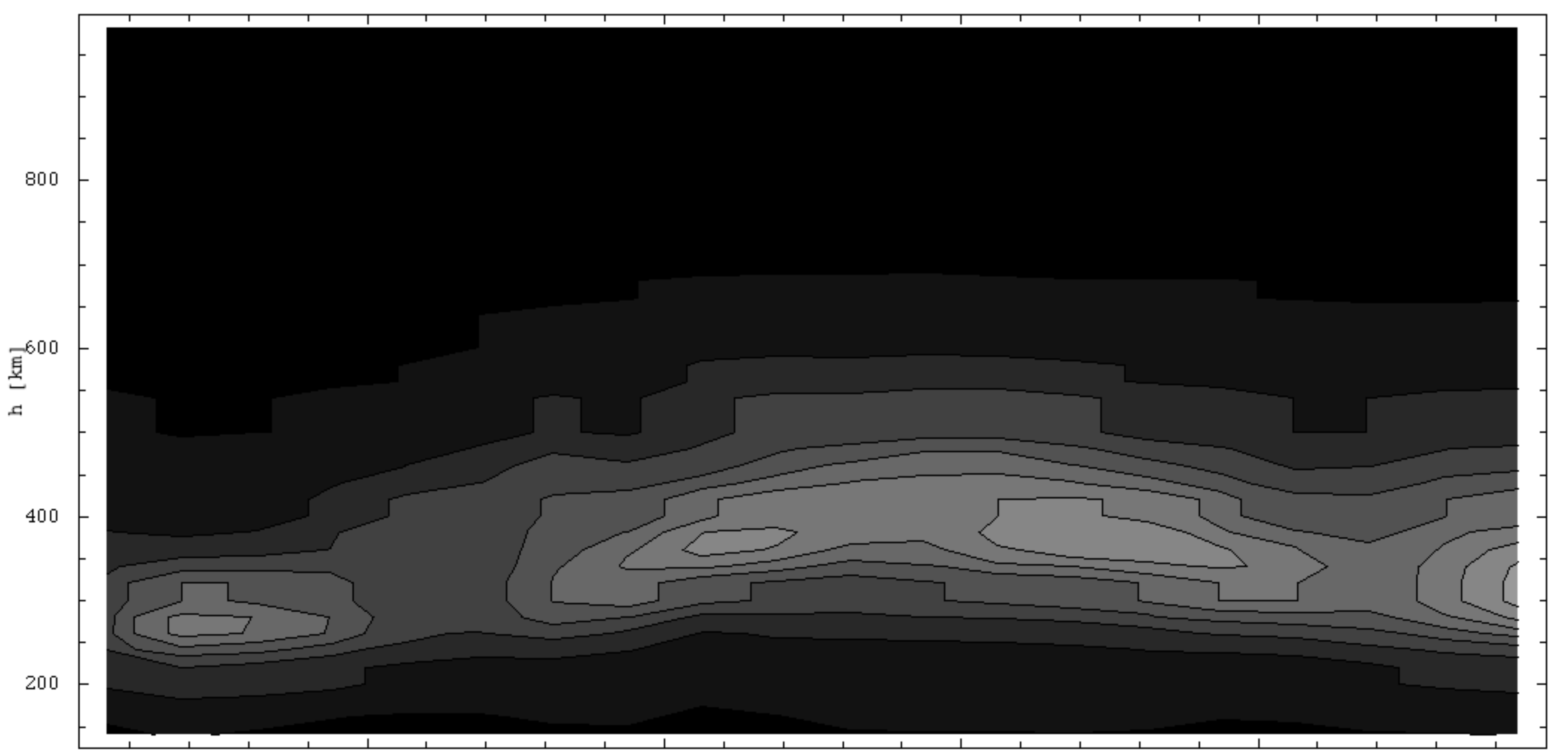




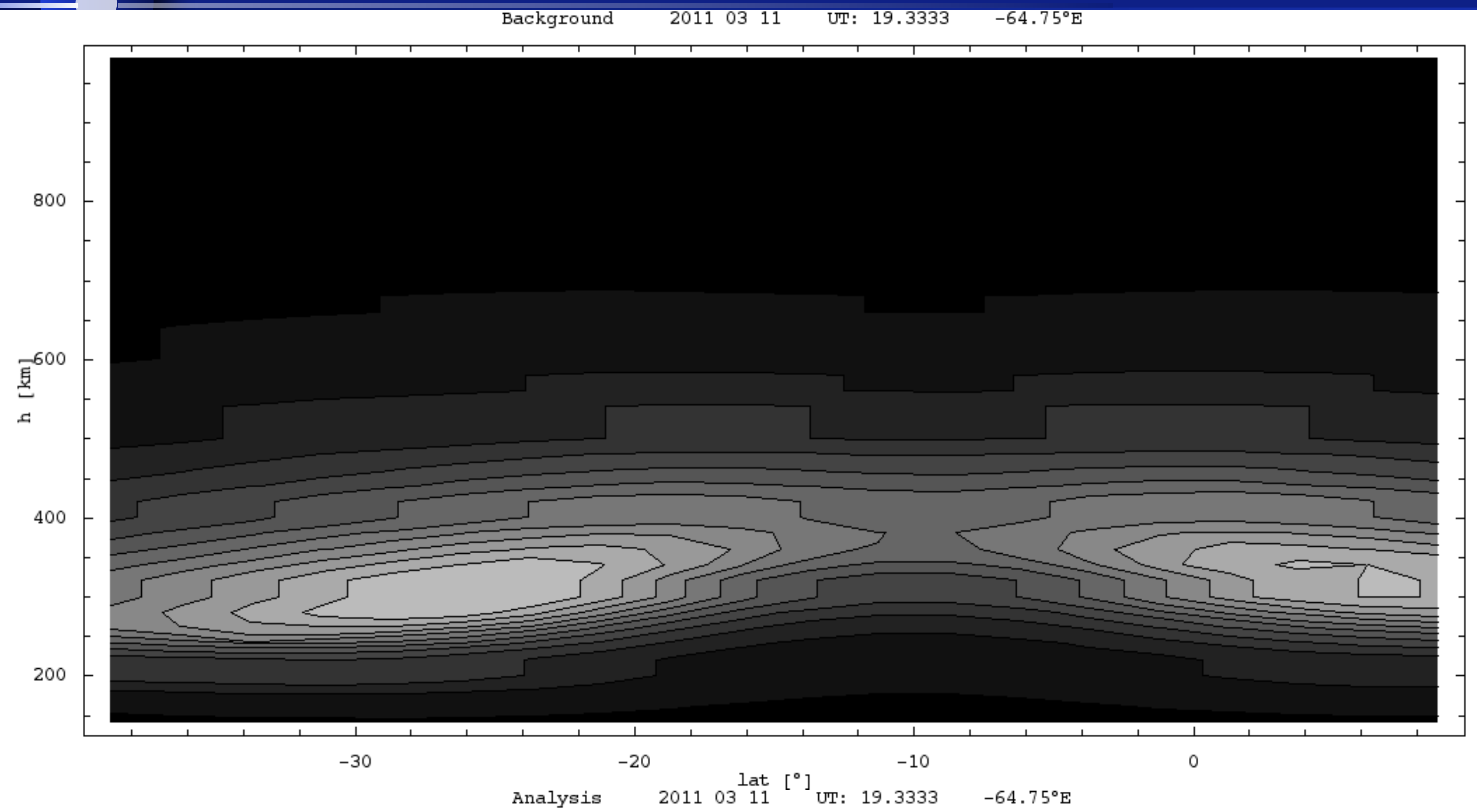
Background 2011 03 11 UT: 15. -64.75°E



Analysis 2011 03 11 UT: 15. -64.75°E

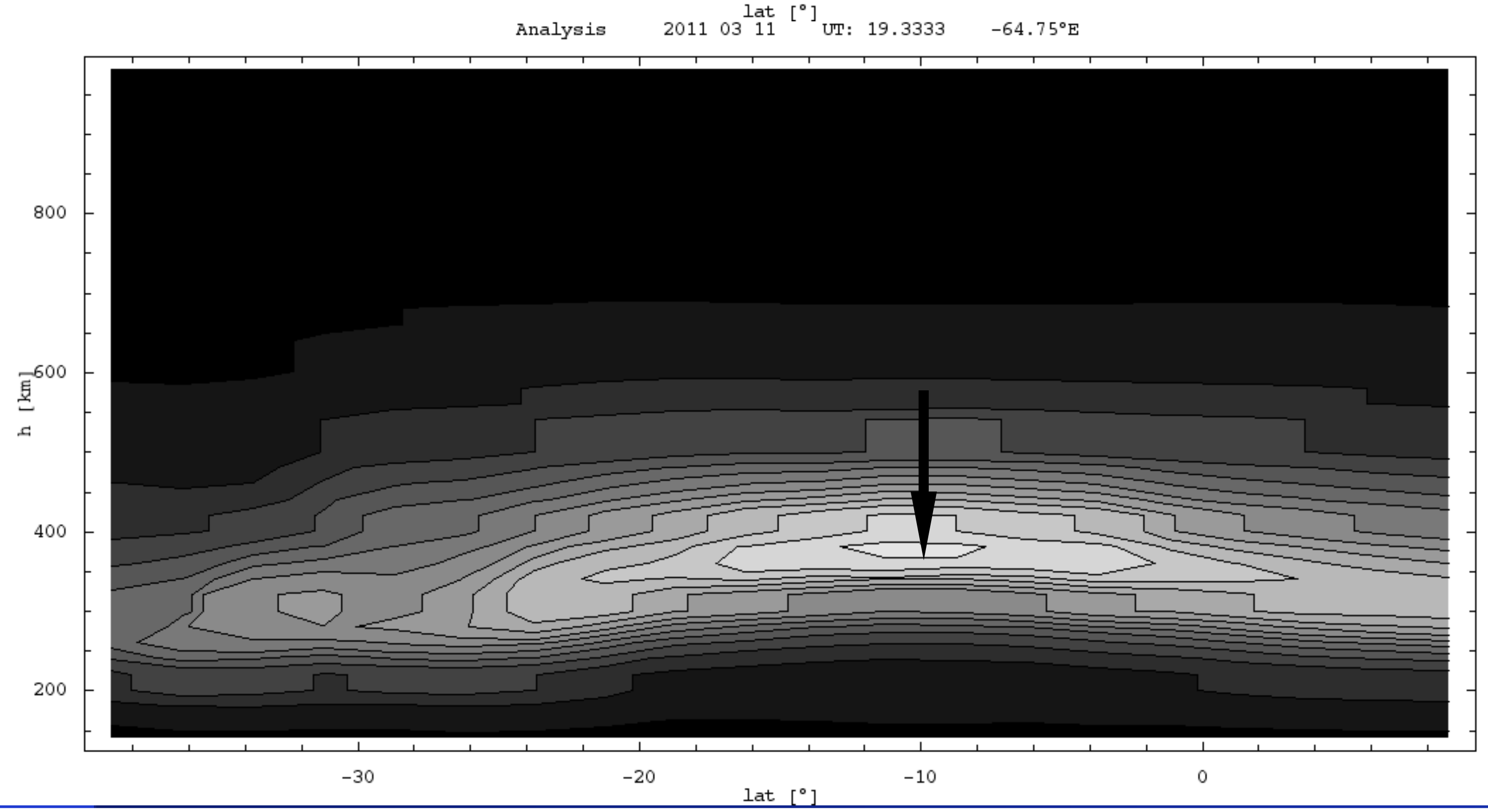


Assimilation effect

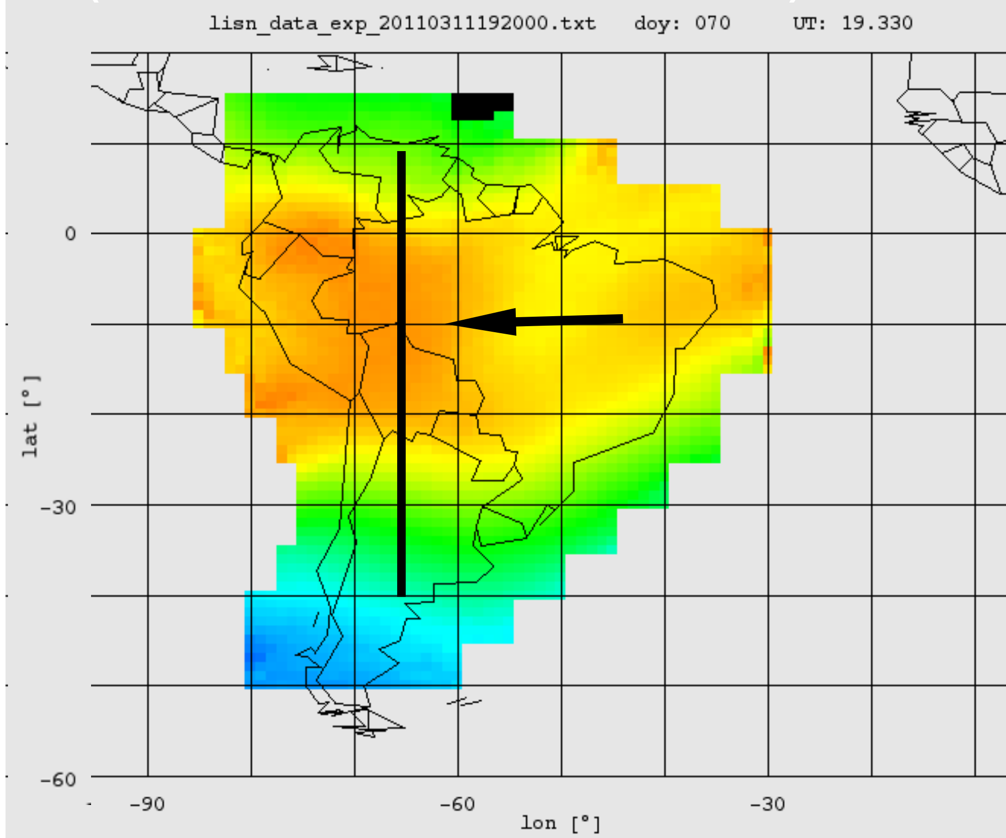


Background model
(before the assimilation)

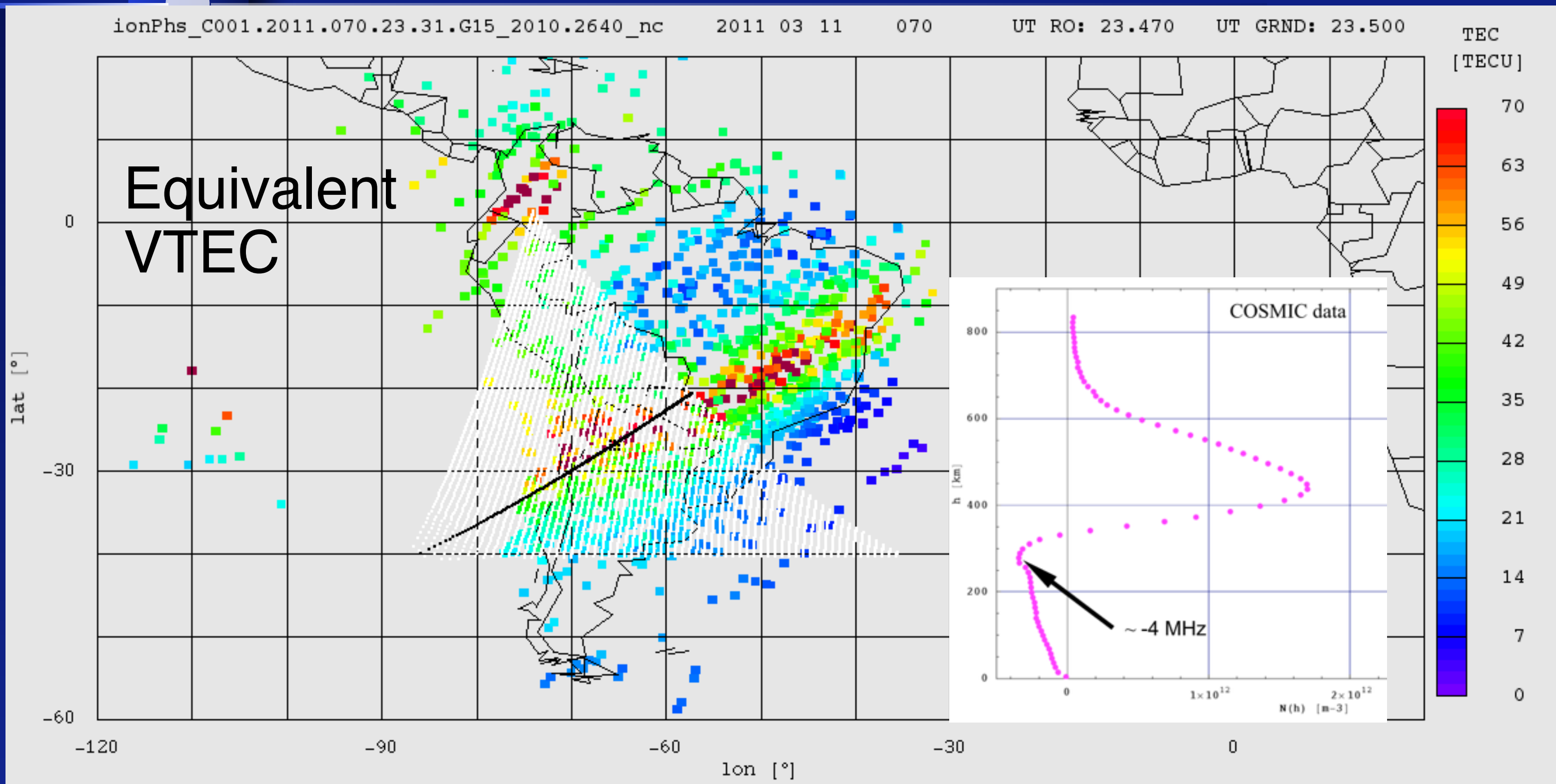
Cross section
19:33UT; -64.75°E
from -40°N to 10°N



Analysis
(after the assimilation)

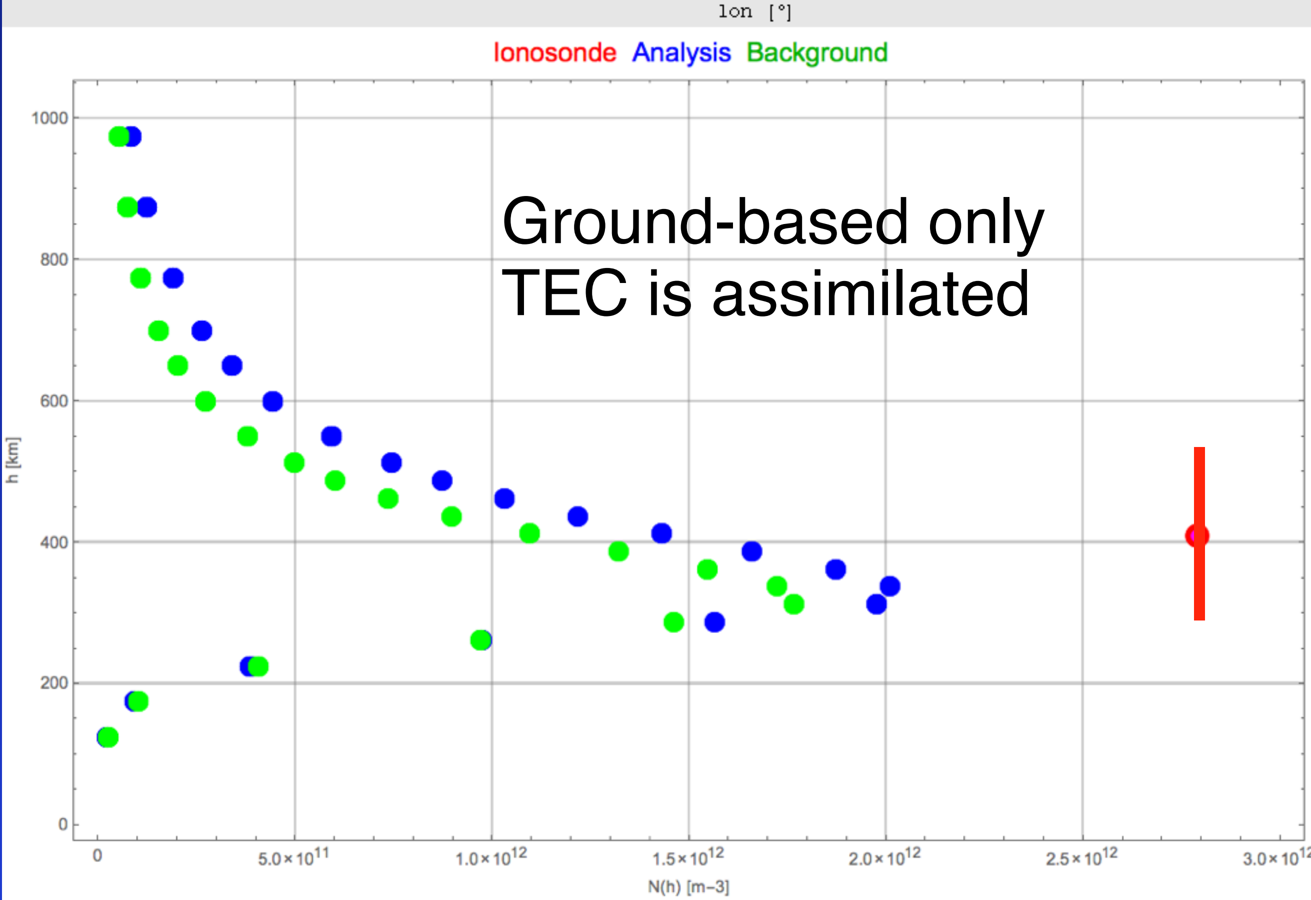


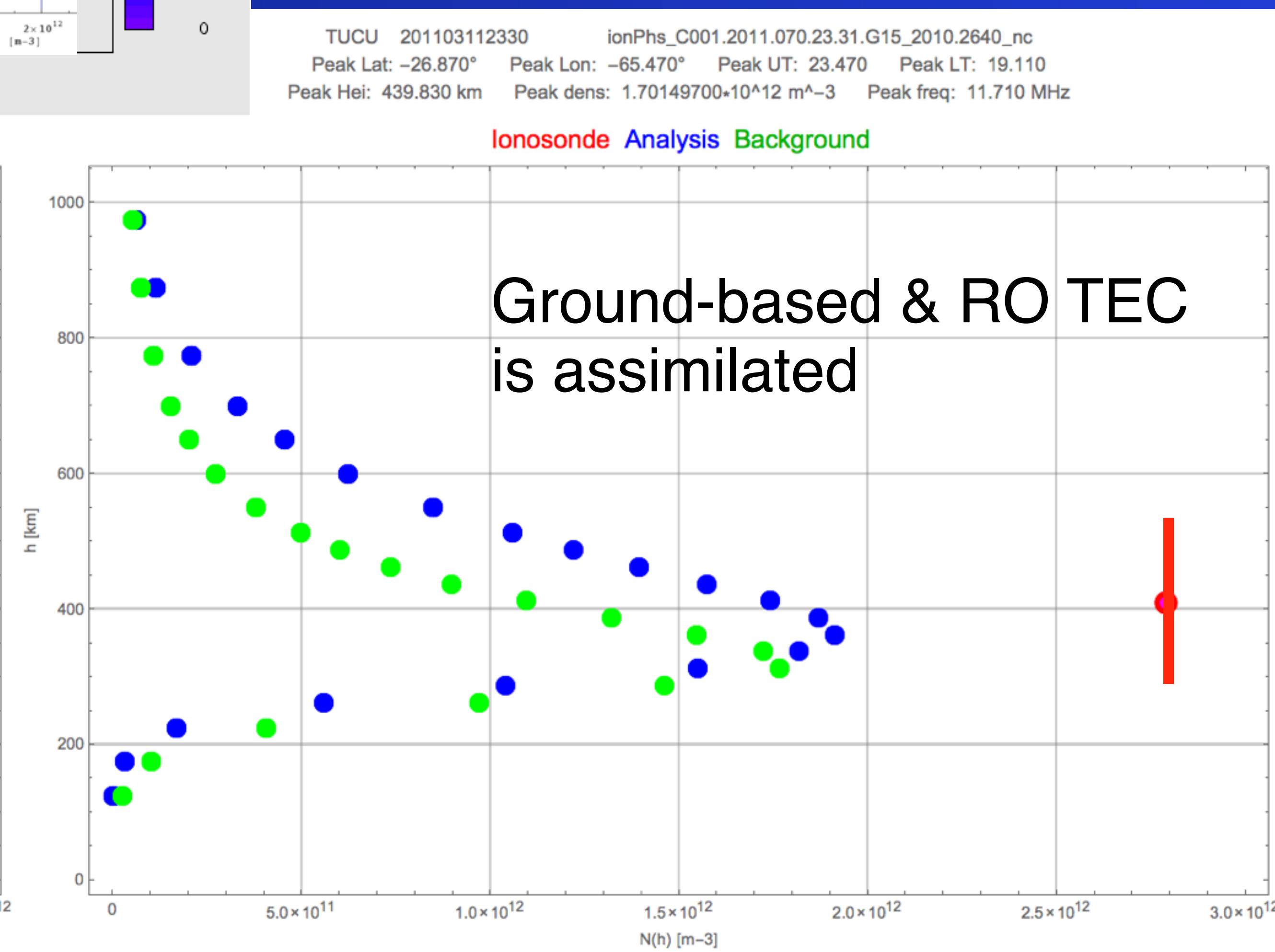
Ground-based & RO TEC DA



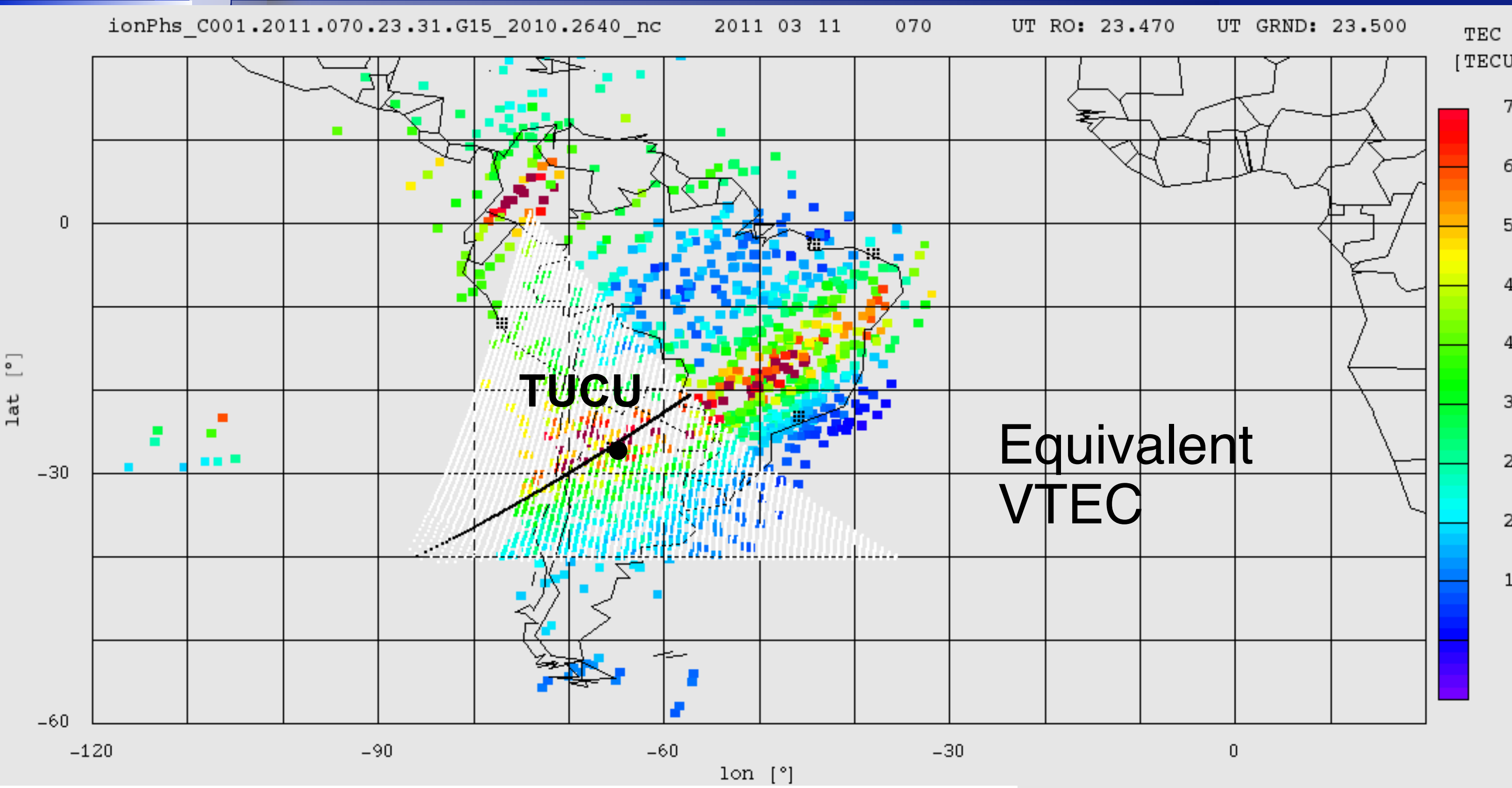
NeQuick used as a background with f10.7 input

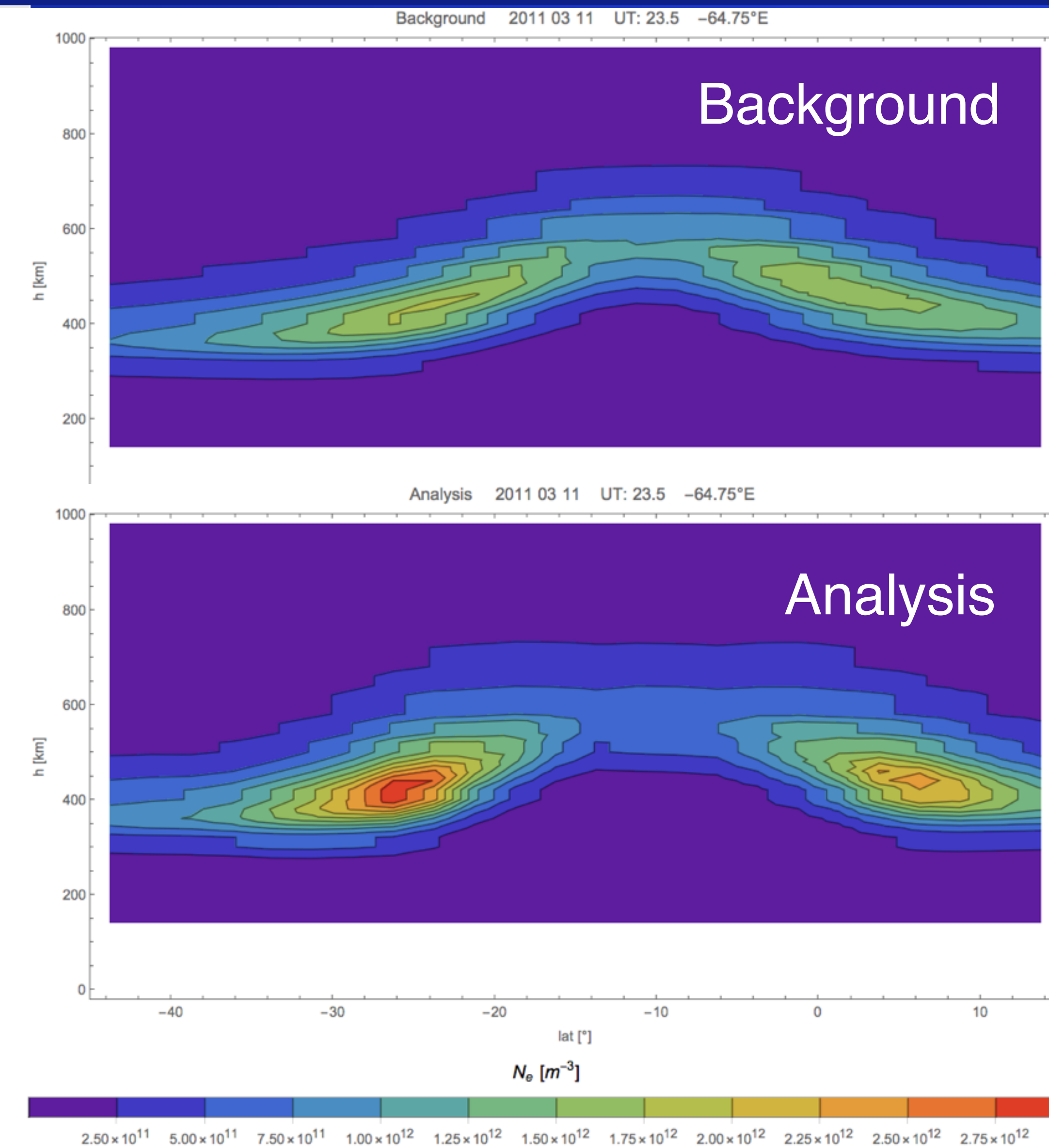
Ne profiles at Ionosonde location

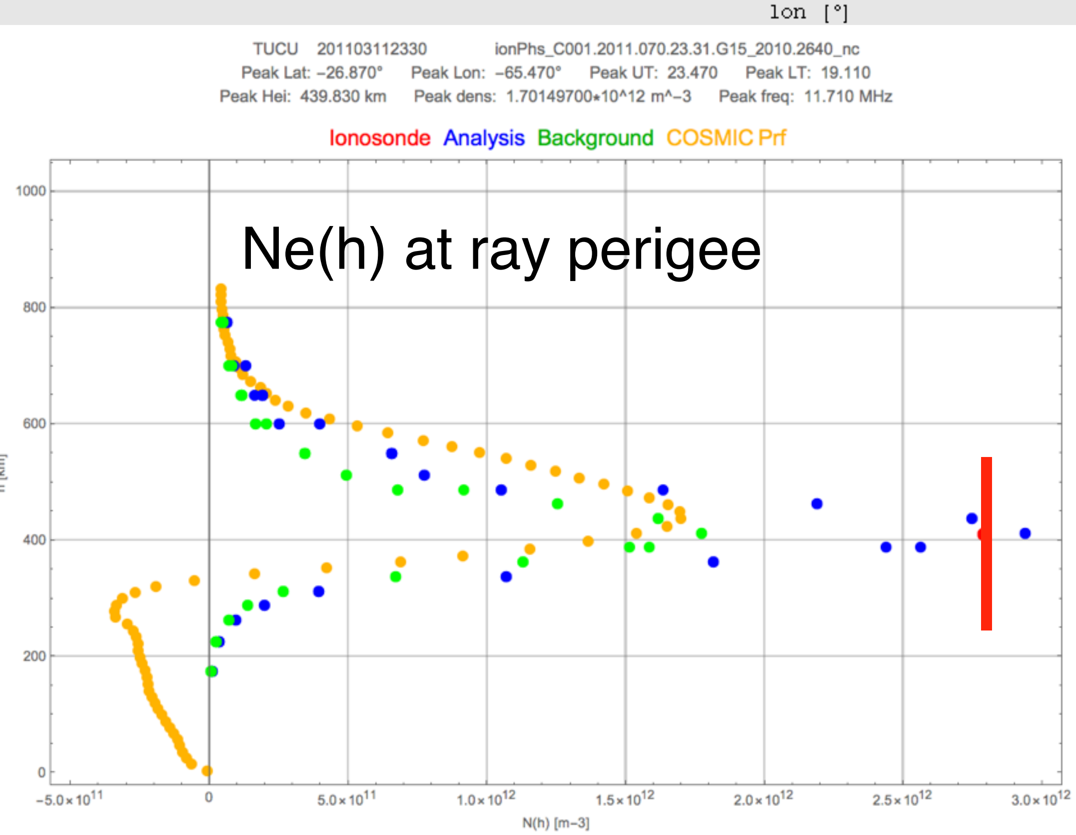


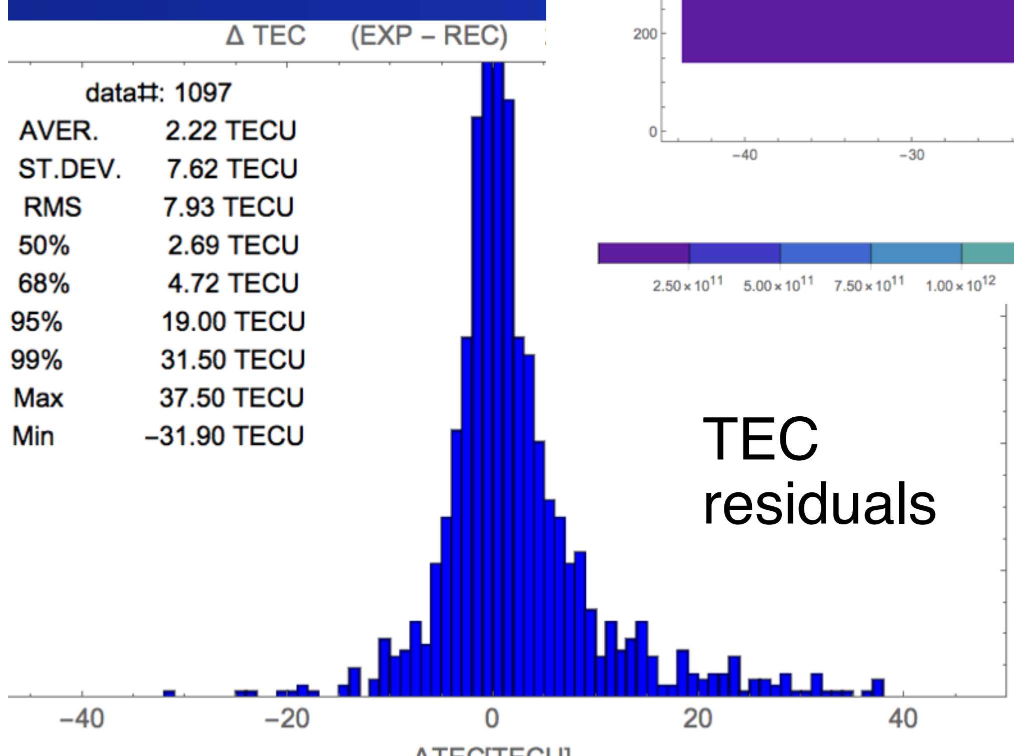


TEC DA with NeQuick pre-adaptation









Ne cross sections at ionosonde longitude

Thank you for your attention

References

- Aa, E., S. Liu, W. Huang, L. Shi, J. Gong, Y. Chen, H. Shen, and J. Li (2016), Regional 3−D ionospheric electron density specification on the basis of data assimilation of ground−based GNSS and radio occultation data, Space Weather, 14, 433–448, doi: 10.1002/2016SW001363.

- Angling, M. J., and N. K. Jackson-Booth (2011), A short note on the assimilation of collocated and concurrent GPS and ionosonde data into the Electron Density Assimilative Model, Radio Sci, 46 (RS0D13).

- Angling, M. J., and B. Khattatov (2006), Comparative study of two assimilative models of the ionosphere, Radio Sci., 41, doi:10.1029/2005RS003372

- D. Bilitza, B.W. Reinisch (2008), International Reference Ionosphere 2007: Improvements and new parameters, Advances in Space Research 42 (4), 599–609.

- Bouttier, F., and P. Courtier (1999), Data assimilation concepts and methods, meteorological training course lecture notes, Eur. Cent. for Med.Range Weather Forecasts, Reading, U. K.

References

- Brunini, C.; Azpilicueta , F.; Nava, B. A technique for routinely updating the ITU-R database using radio occultation electron density profiles, J Geod (2013) 87:813–823 DOI 10.1007/s00190-013-0648-x

- D. Buresova, B. Nava, I. Galkin, M. Angling, S. M. Stankov and P. Coisson, "Data ingestion and assimilation in ionospheric models", Annals of Geophysics, VOL. 52, N. 3/4, June/August 2009.

- Bust, G. S., T. W. Garner, and I. Gaussiran, T. L. (2004), Ionospheric Data Assimilation Three-Dimensional (IDA3D): A global, multisensor, electron density specification algorithm, J. Geophys. Res., 109, A11312, doi:10.1029/2003JA010234.

- Bust, G. S., and C. N. Mitchell (2008), “History, current state, and future directions of ionospheric imaging, Rev. Geophys., 46,RG1003, doi:10.1029/2006RG000212

- Chartier, A. T., T. Matsuo, J. L. Anderson, N. Collins, T. J. Hoar, G. Lu, C. N. Mitchell, A. J. Coster, L. J. Paxton, and G. S. Bust (2016), Ionospheric data assimilation and forecasting during storms, J. Geophys. Res. Space Physics, 121, 764–778, doi:10.1002/2014JA020799.

- Datta-Barua, S., G. S. Bust, G. Crowley, and N. Curtis (2009), Neutral wind estimation from 4-D ionospheric electron density images, J. Geophys. Res. Sp. Phys., 114(A6), n/a–n/a, doi:10.1029/2008JA014004.:10.1029/2008JA014004.

References

- Elvidge, S., M. J. Angling, and B. Nava (2014), On the use of modified Taylor diagrams to compare ionospheric assimilation models, Radio Sci., 49, 737–745, doi: 10.1002/2014RS005435.

- Galkin, I. A., B. W. Reinisch, X. Huang, and D. Bilitza (2012), Assimilation of giro data into a real-time IRI, Radio Science, 47 (4), doi: 10.1029/2011RS004952

- M. Hernández-Pajares, J.M. Juan, J. Sanz (1999), New approaches in global ionospheric determination using ground GPS data, Journal of Atmospheric and Solar-Terrestrial Physics,
- Volume 61, Issue 16, 1237-1247

- Komjathy, A., B. Wilson, X. Pi, V. Akopian, M. Dumett, B. Iijima, O. Verkhoglyadova, and A. J. Mannucci (2010), JPL/USC GAIM: On the impact of using COSMIC and ground-based GPS measurements to estimate ionospheric parameters, J. Geophys. Res., 115 (A2), A02307, doi: 10.1029/2009JA014420

- Migoya-Orue,Y.O.; Radicella, S.M.; Nava, B. Comparison of topside electron density computed by ionospheric models and plasma density observed by DMSP satellites, Adv. Space Res, 52 (2013) 1710–1716.

- Minkwitz, D., K. G. van den Boogaart, T. Gerzen, M. Hoque, y M. and Hernández-Pajares (2016), Ionospheric tomography by gradient-enhanced kriging with STEC measurements and ionosonde characteristics; Ann. Geophys. 34, 999-1010.

References

- Mitchell CN, Spencer PSJ (2003), A three-dimensional time-dependent algorithm for ionospheric imaging using GPS, ANN GEOPHYS-ITALY 46 (4): 687-696

- Nava, B., S. M. Radicella, and F. Azpilicueta (2011), Data ingestion into NeQuick 2, Radio Sci., 46, RS0D17, doi:10.1029/2010RS004635.

- Nava, B., S. M. Radicella, R. Leitinger, and P. Coïsson (2006), A near-real-time model-assisted ionosphere electron density retrieval method, Radio Sci., 41, RS6S16, doi: 10.1029/2005RS003386.

- Nava, B., P. Cosson, and S. Radicella (2008), A new version of the NeQuick ionosphere electron density model, Journal of Atmospheric and Solar-Terrestrial Physics, 70 (15), 1856-1862.

- Nigussie, M., S. M. Radicella, B. Damtie, B. Nava, E. Yizengaw, and L. Ciraolo (2012), TEC ingestion into NeQuick 2 to model the East African equatorial ionosphere, Radio Sci., 47, RS5002, doi:10.1029/2012RS004981.

- M. Nigussie, S.M. Radicella, B. Damtie, B. Nava, E. Yizengaw, K. Groves. Validation of the NeQuick 2 and IRI-2007 models in East-African equatorial region, Journal of Atmospheric and Solar-Terrestrial Physics 102 (2013) 26–33.

References

- O.J. Olwendo, C. Cesaroni (2016), Validation of NeQuick 2 model over the Kenyan region through data ingestion and the model application in ionospheric studies, Journal of Atmospheric and Solar-Terrestrial Physics, 145, 143-153.

- Orús, R., M. Hernández-Pajares, J. Juan, and J. Sanz (2005), Improvement of global ionospheric VTEC maps by using kriging interpolation technique, Journal of Atmospheric and Solar-Terrestrial Physics, 67(16), 1598–1609.

- Pietrella, M., Nava, B., Pezzopane, M., Migoya Orue, Y., Ippolito, A. and Scotto, C. (2017), "NeQuick2 and IRI2012 models applied to mid and high latitudes, and the Antarctic ionosphere". Antarctic Science, pp. 1–12. doi: 10.1017/S0954102016000602.

- Pignalberi, A., Pezzopane, M., Rizzi, R. Galkin, I., "Effective Solar Indices for Ionospheric Modeling: A Review and a Proposal for a Real-Time Regional IRI"; Surv Geophys (2018) 39: 125. https://doi.org/10.1007/s10712-017-9438-y.

- Scherliess, L., R. W. Schunk, J. J. Sojka, D. C. Thompson, and L. Zhu (2006), Utah state university global assimilation of ionospheric measurements gauss-markov kalman filter model of the ionosphere: Model description and validation, J. Geophys. Res., 111, doi: 10.1029/2006JA011712.

References

- Schunk, R. W., L. Scherliess, J. J. Sojka, D. C. Thompson, D. N. Anderson, M. Codrescu, C. Minter, T. J. Fuller-Rowell, R. A. Heelis, M. Hairston, and B. M. Howe (2004), Global Assimilation of Ionospheric Measurements (GAIM), Radio Sci., 39, doi: 10.1029/2002RS002794.

- Schunk, R. W., et al. (2014), Ensemble Modeling with Data Assimilation Models: A New Strategy for Space Weather Specifications, Forecasts, and Science, Space Weather, 12, doi: 10.1002/2014SW001050.

- Shaikh, M.M., Nava, B., Kashcheyev, A., (2017). A model-assisted radio occultation data inversion method based on data ingestion into NeQuick. Adv. Space Res. 59 (1), 326-336.
- http://dx.doi.org/10.1016/j.asr.2016.09.006.

- Wang, C., G. Hajj, X. Pi, I. G. Rosen, and B. Wilson (2004), Development of the global assimilative ionospheric model, Radio Sci., 39, doi:10.1029/2002RS002854.

- Yue, X., W. S. Schreiner, Y. Lin, C. Rocken, Y. Kuo, and B. Zhao (2011), Data assimilation retrieval of electron density profiles from radio occultation measurements, 262 J. Geophys. Res., 116.