# Gaia relativistic astrometry models in comparison


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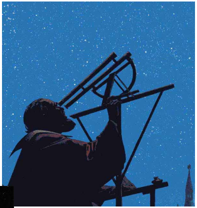
# Relativity in modern astronomy

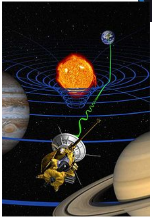
**Relativity** is already widely used (mostly as corrections) in **astronomy**

- VLBI, HIPPARCOS
- Planetary radar ranging and Lunar Laser ranging
- Satellite orbit determination



New generation of very accurate missions and observations

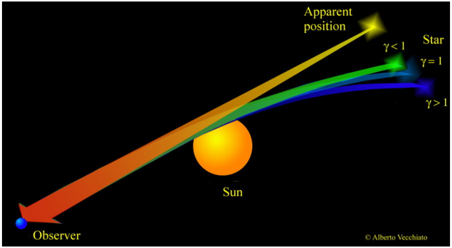
- Gaia → from $mas$ to $\mu as$
- ACES
- LLR with APOLLO → from $cm$ to $mm$
- Cassini, BepiColombo → from $m$ to $cm$



**More accurate** → **more sensitive** to Solar System gravitational field

A fully relativistic treatment of these observations is required

# Impact on astrometric measurements




| body | light bending ($\mu as$) | $\psi_{max}(1\ \mu as)$ (deg) |
|---|---|---|
| Sun | $1.75 \times 10^6$ | $180^o$ |
| Mercury | 83 | $9'$ |
| Venus | 493 | $4.5^o$ |
| Earth | 574 | $123^o$ |
| Mars | 116 | $25'$ |
| Jupiter | 16270 | $90^o$ |
| Saturn | 5780 | $17^o$ |
| Uranus | 2080 | $71'$ |
| Neptune | 2533 | $51'$ |

[Klioner, 2003]

- **Astrometry**: angular measurement of light direction
- **Aberration** (see AV): special relativity
- **Light deflection**: relativistic deflection by Solar System gravitational field

Relativistic models for space era

$\mu as$ accuracy

- **Gaia**: global map of the Milky way
- **AGP**: tests of General Relativity
- **Theia**: sub-$\mu$as astrometry

# Relativistic light propagation

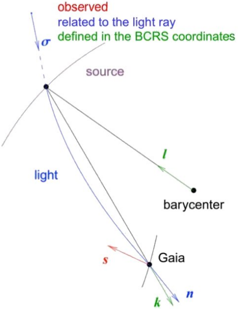
| **Post-Minkowskian** | **Post-Newtonian** |
| --- | --- |
| PM | PN |
| $h_{\mu\nu}(x) = \sum_n G^n \, h^{(n)}_{\mu\nu}(x)$ | $h_{\mu\nu}(x) = \sum_n \frac{1}{c^n} \, \tilde{h}^{(n)}_{\mu\nu}(x)$ |

Light propagation solutions use different approximations

- 1PN: Kopeikin and Schafer (1999), Blanchet et al (2001), Klioner (2003), Chauvineau et al (2005), ...
- 1.5PN / 2PN: de Felice et al (2006), Ashby and Bertotti (2010), Klioner and Zschocke (2010), Minazzoli and Chauvineau (2009), Bertone et al (2013)
- 1PM: Kopeikin and Schafer (1999), de Felice et al (2004), Crosta et al (2014)
- General PM development: Teyssandier and Le Poncin-Lafitte (2008), Hees et al (2014)

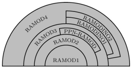
HUGE dataset ($\sim 10^{12}$ observations) $=$ analytical solutions

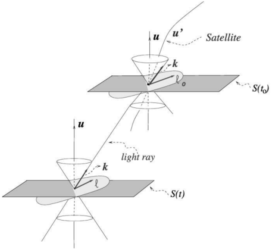
# Relativistic astrometric models for Gaia




GREM

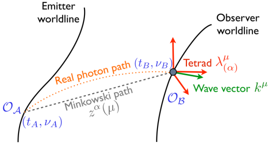
*[Klioner]*




RAMOD

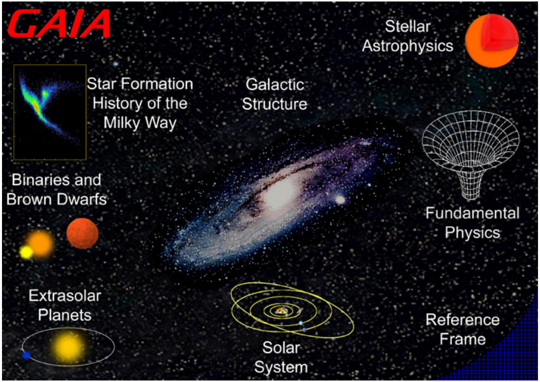
*[de Felice, Crosta, Vecchiato, . . . ]*




TTF

*[Teyssandier, Linet, . . . ]*

# Gaia's processing: multiple pipelines and approaches

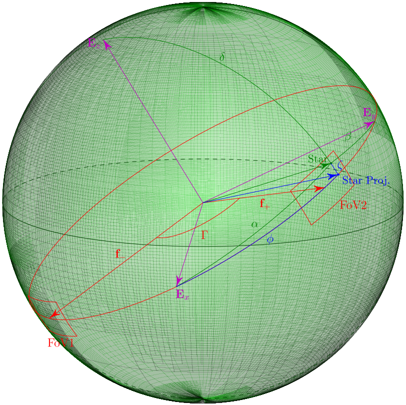



Why multiple pipelines and models?

- First fully-relativistic astrometric mission;
- No external verification possible at Gaia accuracy;
- Errors in the derivation or in the implementation of the relativistic formulae hard to detect;
- Systematic errors would affect multiple fields.

- Independent pipelines and groups within Gaia DPAC;
- Data analysis by independent models and pipelines and cross-check

# Short summary of the problem




- What we solve for:
  - star parameters $(\alpha, \delta, \omega, \mu_\alpha, \mu_\delta)$;
  - additional parameters $p$ - $e.g.$, corrections).
- What we have (see AV talk):
  - observables ($\Phi$, along-scan, and $\zeta$, across-scan);
  - $\cos\phi = \mathcal{F}(x_S, x_A, x_C, x_G)$
- Main ingredients (see AV talk):
  - description of the observer;
  - model for light propagation.

Every model should describe same set of observations as function of solve-for parameters

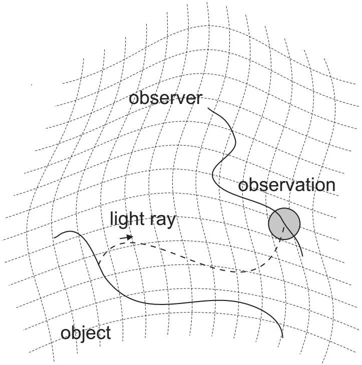
# Modeling of astrometric observations

**Light propagation** Null-geodesics equations

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} {\color{red}k^\alpha} k^\beta = 0 \quad , \quad k^\alpha k_\beta = 0$$

**Observer definition** Gaia position, velocity, attitude at observation

$$\vec{x}(t), \vec{v}(t), q_\alpha(t)$$




**Spacetime metric and frames**:
*IAU 2000 Resolutions on Relativity*
*[Soffel et al., 2003]* (see BB and CLPL talks)

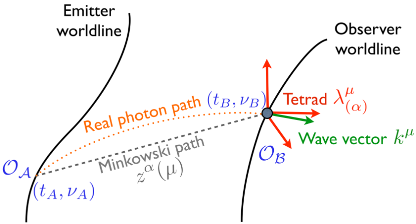
IAU framework introduced at different levels in the processing.

# Time Transfer Functions (TTF)

- Light propagation between 2 points as "simple" integral along straight line
- Avoids explicit computation of null-geodesic path
- Use TTF properties to define light direction at observation

**Light direction triple**

$$(\hat{k}_i)_{A/B} \equiv \left(\frac{k_i}{k_0}\right)_{A/B} = -c\frac{\partial \mathcal{T}_{e/r}}{\partial x^i_B}\left[1 - \frac{\partial \mathcal{T}_{e/r}}{\partial t_B}\right]^{-1}$$




**Projection on obs. ref. frame:**

$$n^{(i)} = -\frac{\lambda^0_{(i)} + \lambda^j_{(i)}\hat{k}_j}{\lambda^0_{(0)} + \lambda^j_{(0)}\hat{k}_j}$$

- General **closed form** equation exact at 2PM and well adapted for
  - numerical resolution for any weak field metric
  - systematic modeling of relativistic observables

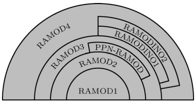
# Relativistic Astrometric MODel (RAMOD)

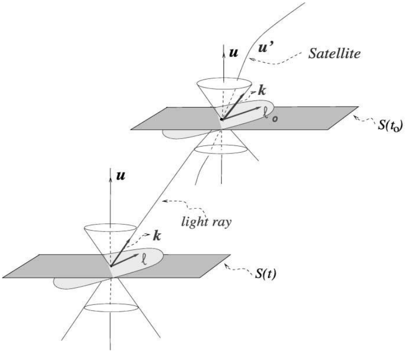
- Based on general weak-field assumptions for the metric ($g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ ).
- Strictly follows measurement protocol of relativity.
- Uses a projected tangent four-vector ($\bar{\ell}^{\alpha}$) and geodesic equations:

$$k^{\alpha} \rightarrow \bar{\ell}^{\alpha}$$

$$\frac{d\bar{\ell}^{\alpha}}{d\sigma} = F^{\alpha}(\bar{\ell}^{\sigma}\partial_p h_{\mu\nu})$$

- Projection of $\bar{\ell}^{\alpha}$ on observer tetrad $\lambda^{\alpha}_{\beta}$ at observation.
- Analytical solutions up to the $(v/c)^3$ order (currently).

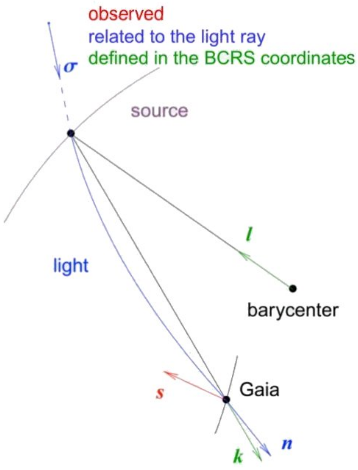



RAMOD

*[de Felice et al.]*

# Gaia RElativistic Model (GREM)




GREM

*[Klioner 2003]*

*[Klioner & Kopeikin 1992]*

- Completely compatible and strictly related to IAU 2000.
- "Relativistic theory of reference systems" to define an algorithm "quite similar to traditional methods" (KK92).
- As a consequence, the algorithm provides the following steps:
  1. take out the aberration ($s \to n$)
  2. take out the light deflection ($n \to \sigma \to k$);
  3. take out the parallax ($k \to l$);
  4. model the proper motion ($l \to l(t)$).

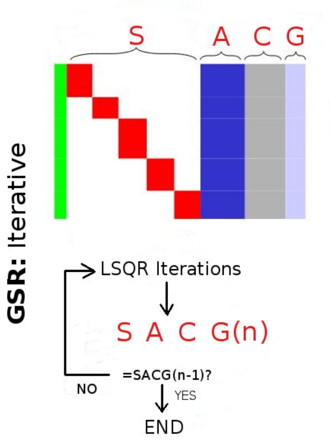
# Models summary

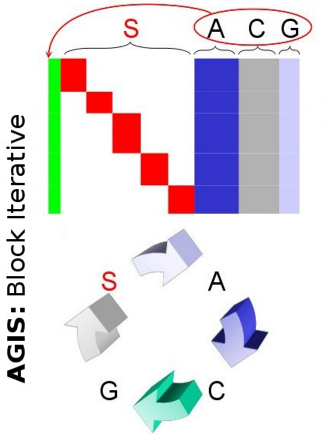
| Model | Observer | Light deflection | Ref. Frames | Pipeline |
|---|---|---|---|---|
| GREM | aberration parallax | differential $k^\alpha$ | at each step | AGIS |
| RAMOD | comoving tetrad | differential $\bar{\ell}^\alpha$ | BCRS SRS | GSR |
| TTF | comoving tetrad | integral $\hat{k}_i$ | BCRS SRS | GSR |

**Observable $\Phi$ and $\zeta$ are the same for all models. Gaia crosscheck requires to understand relations among models.**

- Linear System of Equation: $\mathbf{b} = A\mathbf{x}$, sparse, overdetermined

$$\mathbf{x} = \left(A^T A\right)^{-1} A^T \mathbf{b}$$




GSR approach: iterative

AGIS approach: block iterative

# Comparison of Gaia relativistic models

- Analytical model

- *are equations solving for the same theoretical values?*

- Implementations

- *are the implemented modelings equivalent?*

- *impact of "technical factors" (numerical noise, implementation errors, ...)?*
    - Computed observables and residuals
    - Determination of star coordinates and sphere reconstruction

Allows to correctly interpret discrepancies between results

# Analytical/formal equivalence

- **Observer modeling:**
  - GREM (IAU-like) description of the observer's reference system equivalent to the tetrad formulation (RAMOD, TTF) *[Klioner, 2004]*;
  - GREM aberration correction equivalent to RAMOD tetrad approach *[Crosta and Vecchiato, 2010]*.

- **Light propagation:**
  - equivalence of formulations for light deflection (TTF, GREM and RAMOD) at Gaia accuracy $v/c^2$ *[Bertone et al., 2014]*.

☑ **Relations proven for IAU metric chosen for Gaia ($\mu as$ accuracy)**

# Compare processing steps and results

- director cosines

$$n^{(i)} = -\frac{\lambda^0_{(i)} + \lambda^j_{(i)}\hat{k}_j}{\lambda^0_{(0)} + \lambda^j_{(0)}\hat{k}_j}$$

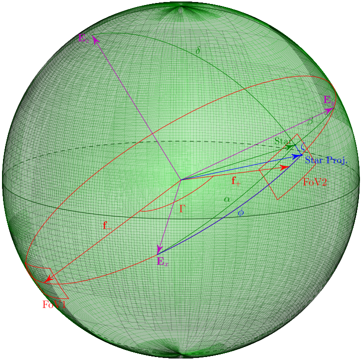
- abscissae (GSR/AGIS)

$$\cos\phi = \frac{n_{(1)}}{\sqrt{1 - n_{(3)}}}$$

$$\eta = \phi - \Gamma/2$$

- linearize and solve

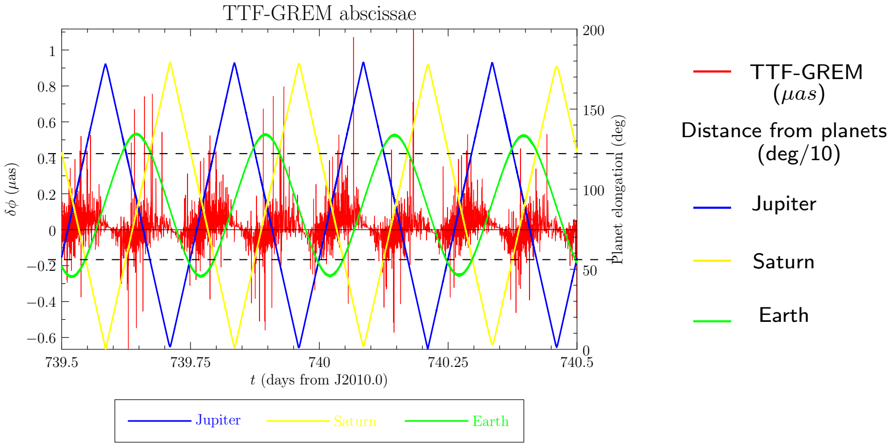
$$\cos\phi = \mathcal{F}(x_S, x_A, x_C, x_G)$$




Based on simulated data (CU2), we compare:

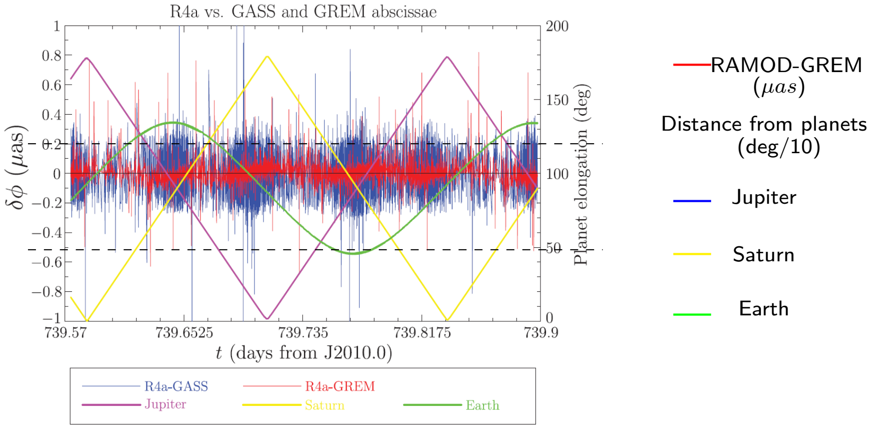
- Observable $\phi$ residuals
- Least square solution: source coordinates $x_S$ and Gaia's attitude $x_A$

# Computed $\Phi$: angle btw observation and Gaia axes




- Based on same catalog and synthetic observations
- AGIS vs GSR processing
- Differences below $\mu as$ accuracy

# Computed $\Phi$: angle btw observation and Gaia axes

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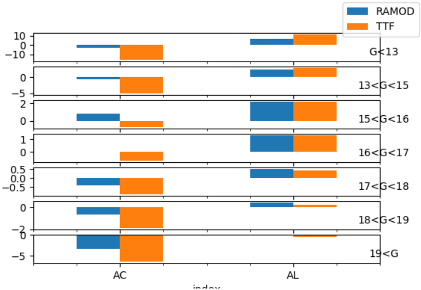
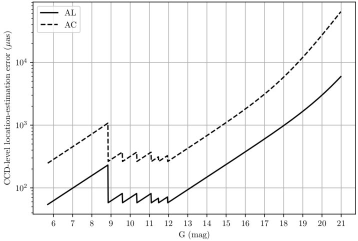


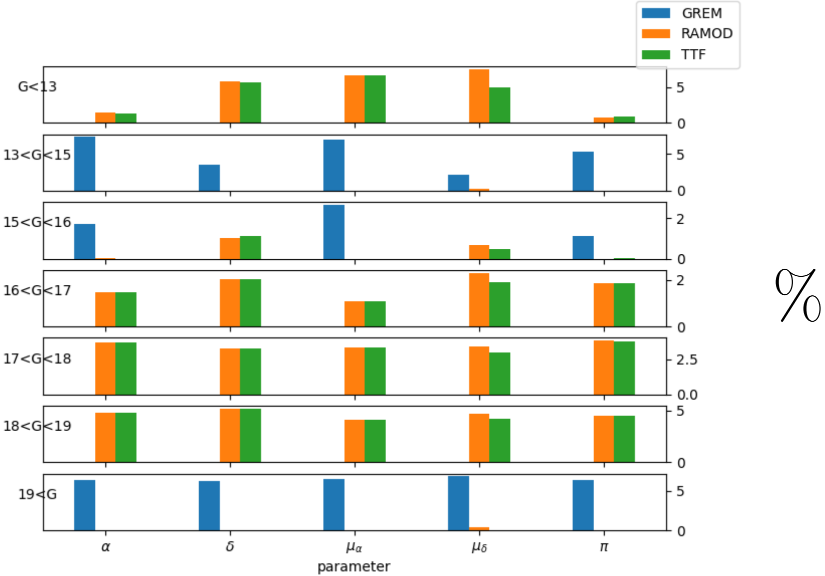

**Summary for stars of different magnitudes G**




**Noise budget vs magnitude G**

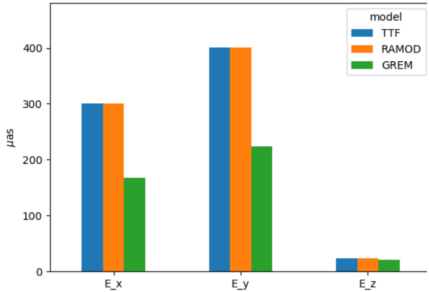
- Residuals differences (relative %) of RAMOD and TTF implementations in GSR (simulated $\phi$/AL and $\zeta$/AC)

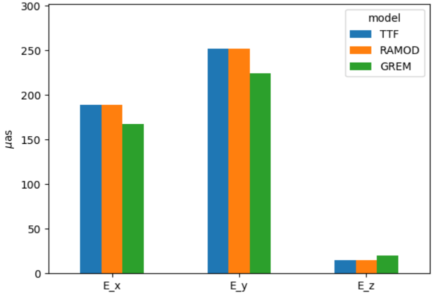
# Reconstruction of stellar parameters (S+A+I)




- Relative differences (%) w.r.t. best solution for each case

# Reconstruction of Gaia attitude




RESCALED

- AGIS Run: $2.3 \times 10^6$ stars, GSR Run: $0.9 \times 10^6$ stars;
- Combined solution with star parameters;
- Heavily depends on time coverage (rescaling only partially possible).

## Conclusions

- GREM: relativistic version of classical astrometric approach (ref. systems)
- RAMOD/TTF: fully relativistic definition of astrometric observable
- **Equivalence check** at analytical level (both observer and light deflection) and analysis of simulated data (input known).
- **(At Gaia level)** no good or bad model/approach: more intuitive for astrometrists "obliged" to take into account GR (GREM) or for relativists analysing astrometric data (RAMOD/TTF).
- **(Beyond Gaia)** RAMOD/TTF: general approach applicable to any metric; GREM: depends on new definition of reference systems and frames.

## Exercise tomorrow (with A. Vecchiato, B. Bucciarelli, and S. Bertone)

- Astrometric reduction in a simplified python3 implementation;
- Calculation of the abscissae $\phi$ based on synthetic catalog and observation epochs, residual analysis, stars-only solution;
- Check-out code and exercise https://github.com/steo85it/pygsr

GREM

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RAMOD

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TTF

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