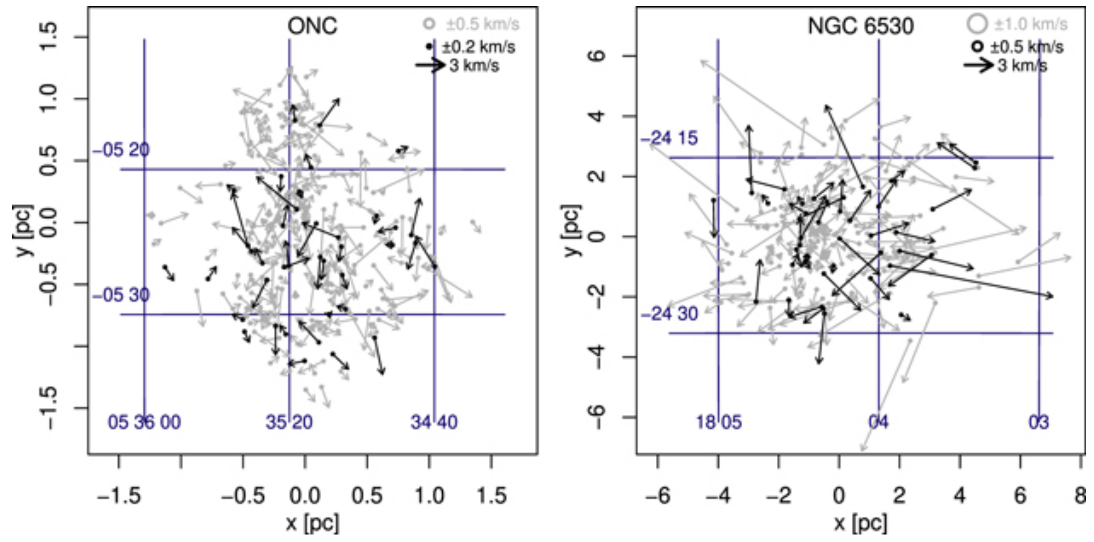
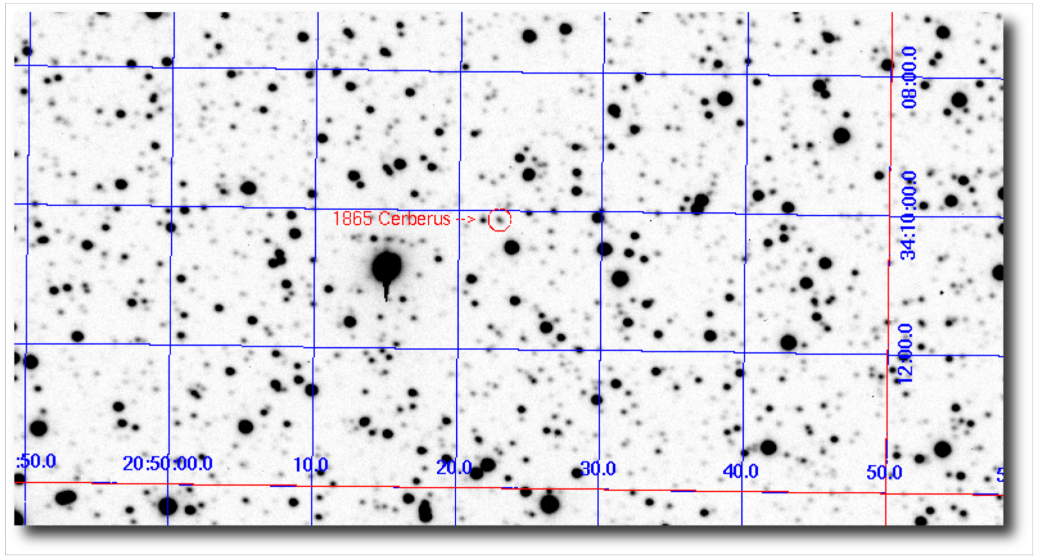
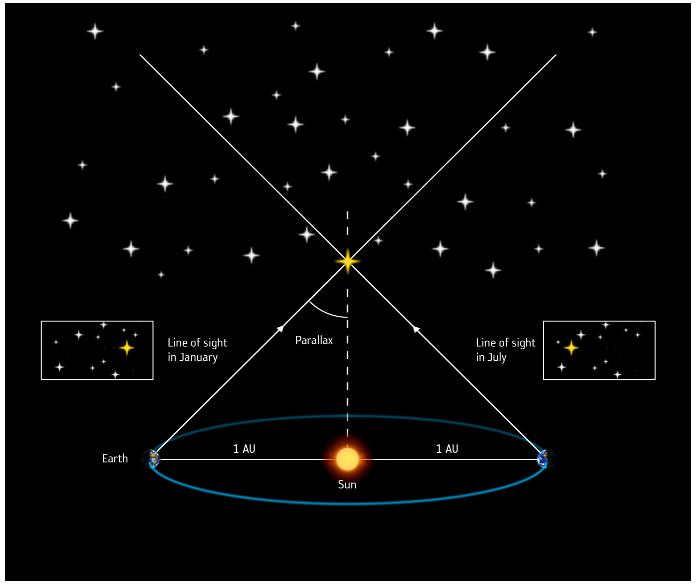




DIFFERENTIAL ASTROMETRY: PRINCIPLES AND TECHNIQUES



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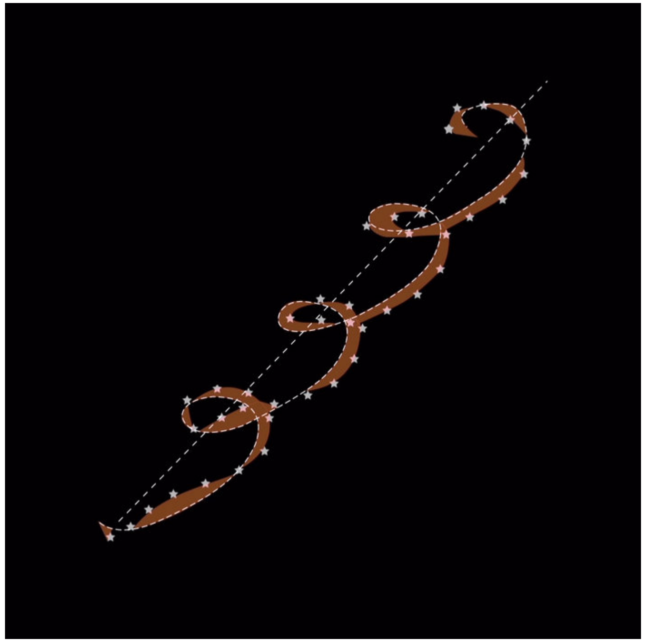


WHEN

- I AM DEALING WITH RELATIVELY SMALL FIELD OF VIEWS
- I WANT TO DETERMINE THE COORDINATES OF SOME OBJECTS WITH RESPECT TO OTHERS WHOSE POSITION IS KNOWN IN A GLOBAL REFERENCE SYSTEM
- I AM INTERESTED IN THE RELATIVE MOTION OF THE OBJECTS OF STUDY

WHY

- DENSIFICATION OF THE REFERENCE FRAME
- RELATIVE PARALLAXES/PROPER MOTIONS
- KINEMATICS OF STELLAR SYSTEMS
- EPHEMERIDES OF SOLAR SYSTEM BODIES
- BINARY ORBITS
- ASTROMETRIC MICROLENSING
- RELATIVISTIC LIGHT DEFLECTION

WHAT

- POINT SOURCES

WHERE

- OPTICAL/NEAR-INFRARED IMAGING

HOW

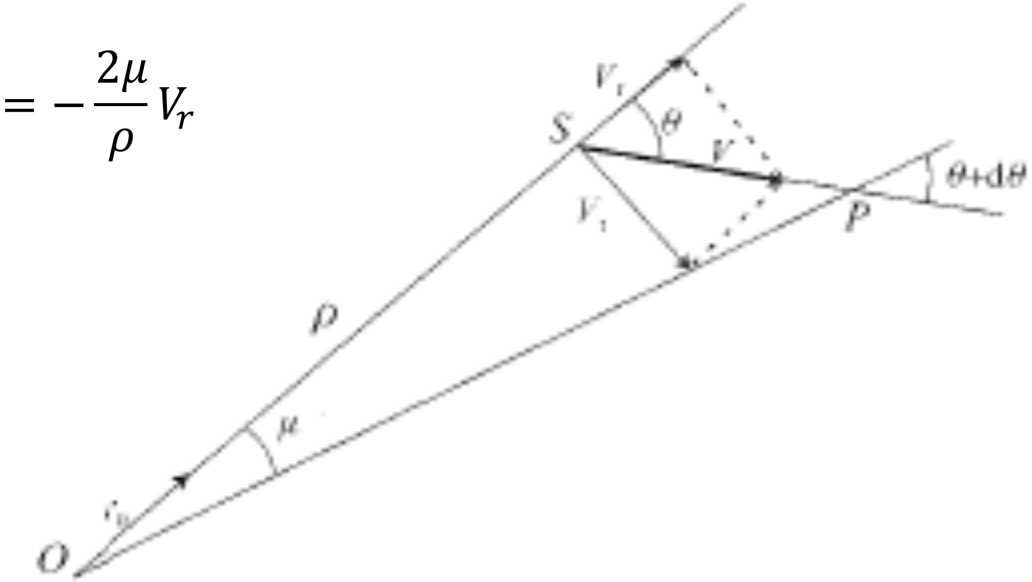
- PRINCIPLES OF DIFFERENTIAL ASTROMETRY
  - Stellar motions (‘true’ displacements)
  - Apparent displacements (observer-induced)
  - Instrumental/environmental effects
- PRINCIPLES OF DATA REDUCTION
  - Observation equation
  - Parameter estimation

Proper Motion

A star's *proper motion μ* is its **yearly angular displacement**.
Unless the center of mass of the system represented by the star does not concide with the star (e.g., in a multiple system), The star's **space velocity** is assumed to be **constant.**
A star with **tangential velocity** V_T (*Km/s)* at a distance of ρ *parsecs* has a proper motion $\mu=V_T/(4.74047\ \rho)$ *arcseconds/year*; however, the star's *radial motion* with respect to the observer induces a ***variation*** in the proper motion called ***secular (or perspective) acceleration***

Let V be the star space velocity and θ its angle with the direction of motion, then the modulus of the proper motion is $\mu=V\sin\theta/\rho \rightarrow d\mu/dt = -V/\rho^2\sin\theta d\rho/dt+V/\rho\cos\theta d\theta/dt$.
Noting that $d\theta/dt = -\mu$ and $V\cos\theta = V_r = d\rho/dt$, one gets

$$\rightarrow \quad \frac{d\mu}{dt} = -\frac{\mu}{\rho}\frac{d\rho}{dt} - \frac{\mu}{\rho}V_r = -\frac{2\mu}{\rho}V_r$$



If μ is given in arcsec/year, the radial velocity in Km/s and the distance in parsecs, the **perspective acceleration amounts to**

$$d\mu/dt= -0.205\ 10^{-5}\ \mu V_r/\rho \quad \text{arcsec/year}^2$$

significant only for **nearby stars** with **high radial velocities**.

Parallactic displacement

The **apparent displacement** produced by the motion of the observer with respect to a fixed reference system is called *parallactic displacement*.
Annual parallax: even for the nearest stars it can be **modelled** to better than a **microarcsecond** by first-order formulas in the small quantity R/ρ ($< 10^{-6}$ rad), where R is the **distance of the observer** from the solar system barycenter (*geocentric* or *horizontal parallax* is only relevant for bodies within the solar system).
For a star in the **p direction,** the vectorial displacement due to annual parallax is given by

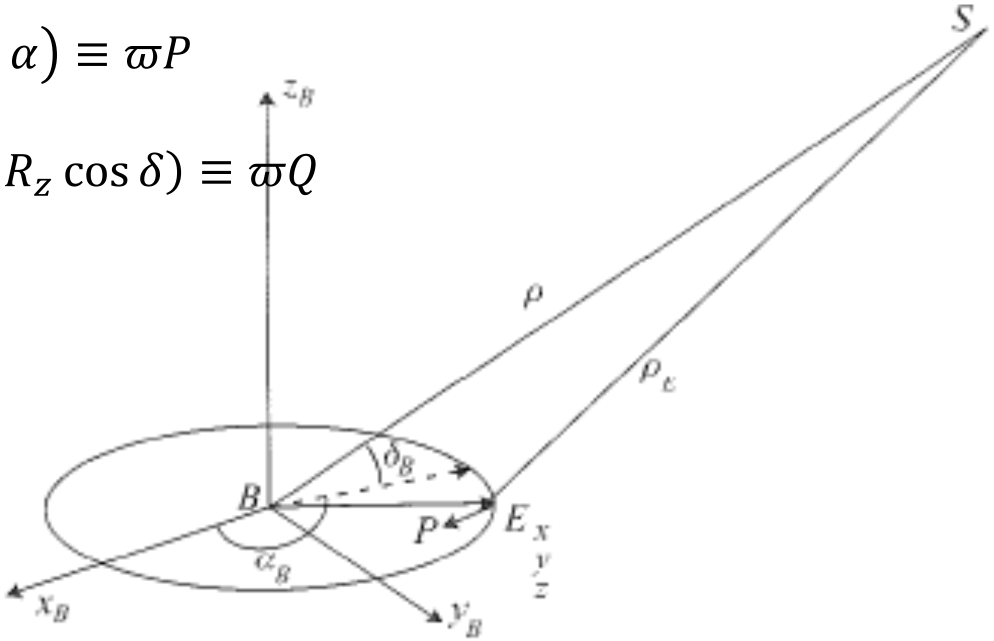
$$d\vec{p} = \vec{p}' - \vec{p} = \frac{1}{\rho}\vec{p}\times(\vec{p}\times\vec{R})$$

If the barycentric **spherical coordinates** of the star are (α,δ), the **corrections** to the barycentric coordinates derived from the above formula reads

$$\Delta\alpha\cos\alpha = \frac{1}{\rho}\left(R_x \sin\alpha - R_y \cos\alpha\right) \equiv \varpi P$$

$$\Delta\delta = \frac{1}{\rho}\left(\left(R_x \cos\alpha + R_y \sin\alpha\right)\sin\delta - R_z \cos\delta\right) \equiv \varpi Q$$

where P, Q are called *parallax factors* and ϖ = 1/ρ is the star's **parallax**.
[when ***R*** is given in *Astronomical Units* and ρ in *parsecs,* the parallax ϖ is in *arcseconds*]

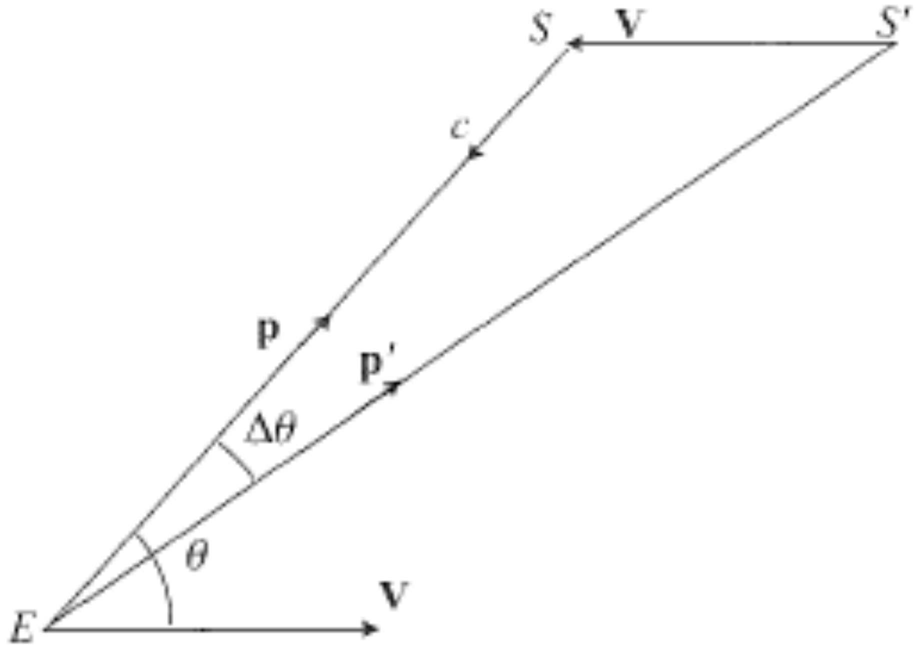


Aberration

Apparent effect induced by the **velocity of the observer** in the fixed frame. Depending on the relative motion of the observer and object of study we can distinguish different effects:

first-order newtonian formula → $$d\vec{p} = -\frac{1}{c}\vec{p}\times(\vec{p}\times\vec{V})$$

- Transforming the vector components into spherical coordaintes, keeping only **first-order** terms in V/c, one obtains the aberrational correction in equatorial coordinates, accurate at the **milliarcsecond level,**



$$\Delta\alpha\cos\alpha = -\frac{1}{c}\left(V_x\sin\alpha + V_y\cos\alpha\right)$$

$$\Delta\delta = -\frac{1}{c}\left(\left(V_x\cos\alpha + V_y\sin\alpha\right)\sin\delta - V_z\cos\delta\right)$$

- *Earth Diurnal aberration* $V_d \cong 10^{-6}\,c$ → *0.2''*
- *Earth Annual aberration* $V_a \cong 10^{-4}\,c$ → *20''*
- *Galactic Secular Aberration* $V_c \cong 10^{-3}\,c$ → *150''*

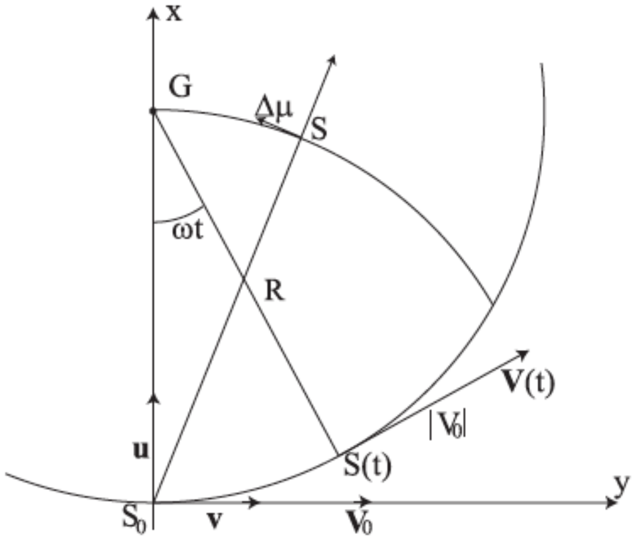
present in Gaia's catalog (BCRS origin)

- the **curvature of the galactocentric orbit** introduces a variability in the Sun's velocity of the order of $V_c\omega t$, where ω is the Sun's angular velocity in *radians* (≅ 2.4 10^{-8}) ; this gives rise to an ***apparent proper motion*** $\mu=\Delta V_c/c \cong 4$ muas/year, which would appear as a residual *proper motion* **vectorial field** in the direction of the galactic center.

Galactic secular aberration →

$$V_x(t) = V_c \sin\omega t = V_c\omega t + \ldots$$

$$V_y(t) = V_c \cos\omega t = V_c - \frac{1}{2}\omega^2 t^2 + \ldots$$



- In **Special Relativity** (SR), *Lorentz transformations* for velocities apply, while in the GR framework one must take into account the effect of gravitational potential in the expression of the observer's velocity.
  Let **p** be the geometric direction to the star , **V** the vectorial velocity of the observer, and **p'** the *aberrated direction*, the exact special relativistic aberration equation is given by

$$\vec{p}' = \frac{\gamma^{-1}\vec{p}}{1+\vec{p}\frac{\vec{V}}{c}} + (1+\gamma^{-1})\frac{\vec{V}}{c}$$

where $\gamma=(1-V^2/c^2)^{-1/2}$, and *c* the light velocity in vacuum, from which the **aberration angle** developed to **third order** in V/c reads

$$\sin \Delta\theta \sim \frac{V}{c}\sin\theta - \frac{1}{4}\frac{V^2}{c^2}\sin 2\theta + \frac{1}{4}\frac{V^3}{c^3}\sin 2\theta \cos\theta$$

- This formula agrees with the **newtonian aberration only to first order** in V/c, which is at the ***mas*** **precision**.
- The **GR contribution** is proportional to w/c^3, where *w* is the gravitational potential at the observer $\cong GM_{Sun}/|\mathbf{r}_0-\mathbf{r}_{Sun}|$, and is at the level of **1 microarcsecond**.

Rigorous calculation of the aberration displacement in (α,δ)

- The aberration displacement Δθ can be decomposed into corrections to the star's spherical coordinates (α,δ) referred to the inertial frame by applying a rotation of Δθ to the star direction vector **p** around the axis normal to the plane defined by **p** and the direction vector of the observer's velocity $\mathbf{p}_0$; such axis is given by **n** = **p** x $\mathbf{p}_0$, so the rotation reads

$$\vec{p}' = \vec{p}\cos\Delta\theta + (\vec{n}\times\vec{p})\sin\Delta\theta$$

Since $\vec{p} = \begin{bmatrix}\cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta\end{bmatrix}$ and $\vec{p}' = \begin{bmatrix}\cos\alpha'\cos\delta' \\ \sin\alpha'\cos\delta' \\ \sin\delta'\end{bmatrix}$, one can easily derive the corrections Δα = α'- α, Δδ = δ' – δ.

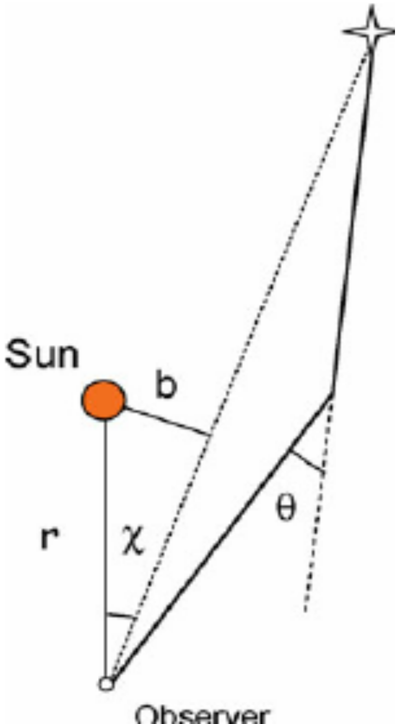
<u>However, keep in mind that</u>, *since we are dealing with* ***differential astrometry****, we only need to consider the* ***differential aberration*** *effect* ***within the field of view*** *of our observations. → quantifying the variation of each astrometric effect across the field of view can help setting up properly what is called the '****plate model****'*

Gravitational light deflection

- In high-accuracy differential astrometry, another effect which changes the apparent direction of the incoming photons is due to relativistic light deflection from the major solar system planets
- The main contribution is given by the **spherically symmetric part** of the gravitational field of each body, and corresponds to a ***deflection angle*** of

$$\Delta\theta = (1+\gamma)\frac{GM}{c^2 r}\tan^{-1}\chi$$

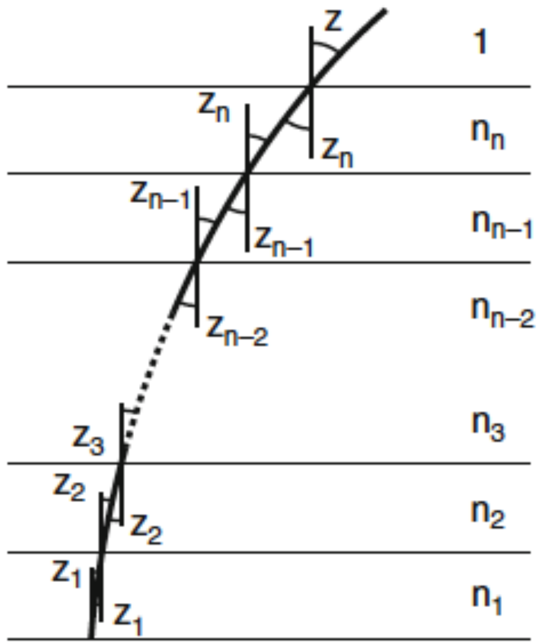
where G is the gravitational constant, M the mass of the perturbing body, r the distance observer-perturbing body, χ the separation angle between source and perturbing body, and γ the PPN parameter (=1 in GR)
- The *quadrupole* term is the first non-zero term of the multipole expansion that depends on the asphericity of the mass



- Due to its smallness, the **quadrupole component** of of light deflection has been so far impossible to detect by means of astrometric measurements

Body	Monopole		Quadrupole	
	grazing mas	χ $\delta\theta = 1\ \mu$as	grazing μas	χ $\delta\theta = 1\ \mu$as
Sun	17,000	180°		
Mercury	0.083	0.15°		
Venus	0.49	4.5°		
Mars	0.12	0.4°		
Jupiter	16.3	90°	240	8 R_J
Saturn	5.8	17°	95	4 R_S
Uranus	2.1	1.2°	8	2 R_U
Neptune	2.5	0.9°	10	2 R_N

Atmopheric Refraction (plays a role only in ground-based observations!)



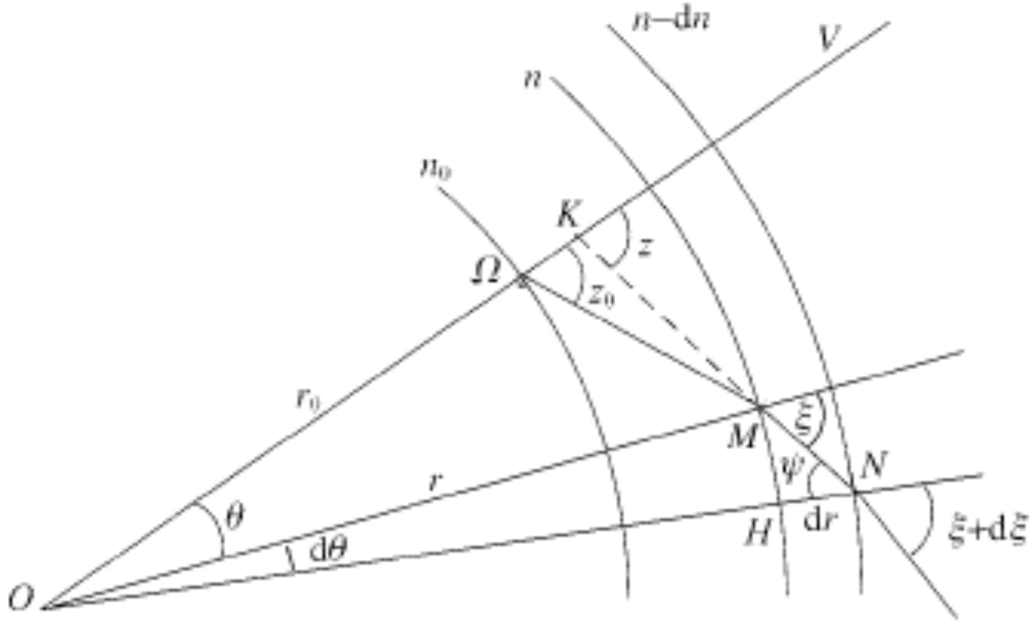
- **Snell's law** for different isotropic media: $(n+dn)\sin(\xi+d\xi)=n\sin\xi$
  In the case of *thin plane layers* where ξ is equal to the zenith distance z, one obtains by iterative application of Snell's law $n_0 \sin z_0=\sin z$ being $\mathbf{n_0}$ the **refractive index** at the observer, and $\mathbf{z_0}$ the **apparent zenith distance**; Naming $\mathbf{R=z-z_0}$ the **total refraction angle** and neglecting second order terms in R, we have the *first-order* refraction formula

$$R = (n_0 - 1)\tan z_0$$

- For **accurate** astrometric work one needs to take into account **curvature effects**. Using a radially symmetric atmosphere model around the local vertical, O being the Earth center, r_0 the radius at the observer Ω and z_0 the apparent zenith distance of the star, one can derive the following integral equation

$$R = R_0 n_0 \sin z_0 \int_1^{n_0} \frac{dn}{n(r^2n^2 - r_0^2 n_0^2 sin^2 z_0)^{1/2}}$$

  which is **exact** but assumes the knowledge of n(r). In *practice*, this formula is semplified by introducing *two small parameters:* the refractivity at the ground $\alpha=(n_0-1)$, and η defined by $(nr/n_0r_0)^2=1+2\eta$

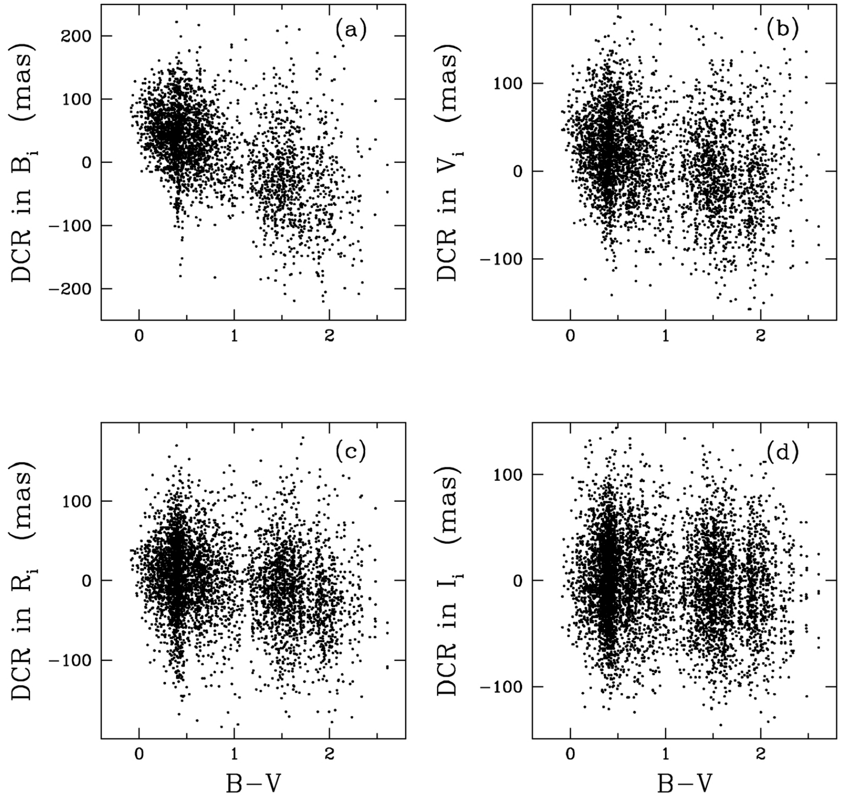


Non-standard conditions and Cromatic effects

- Developing *R* with respect to α and η and defining $\beta = L/r_0$ (where L is the scale height of an exponentially decreasing atmosphere density) after some calculations one obtains the famous ***Laplace formula***

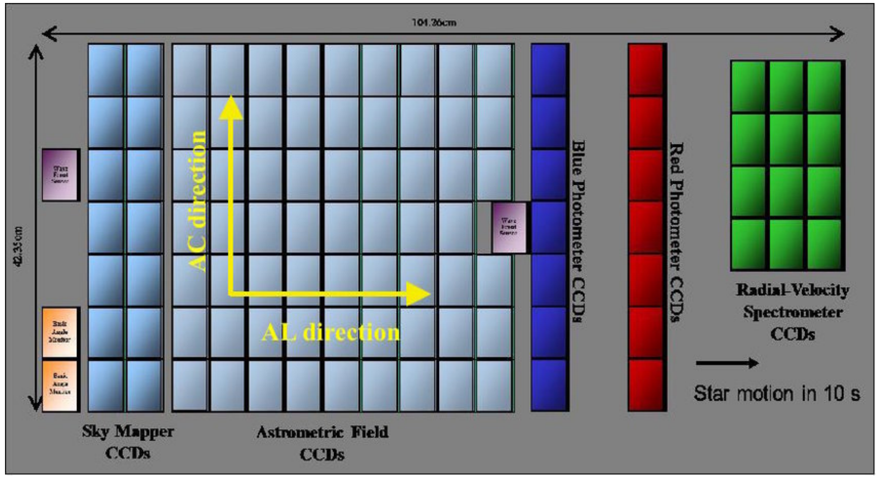
$$R = \alpha(1-\beta)\tan z_0 - \alpha\left(\beta - \frac{\alpha}{2}\right) tan^3 z_0$$

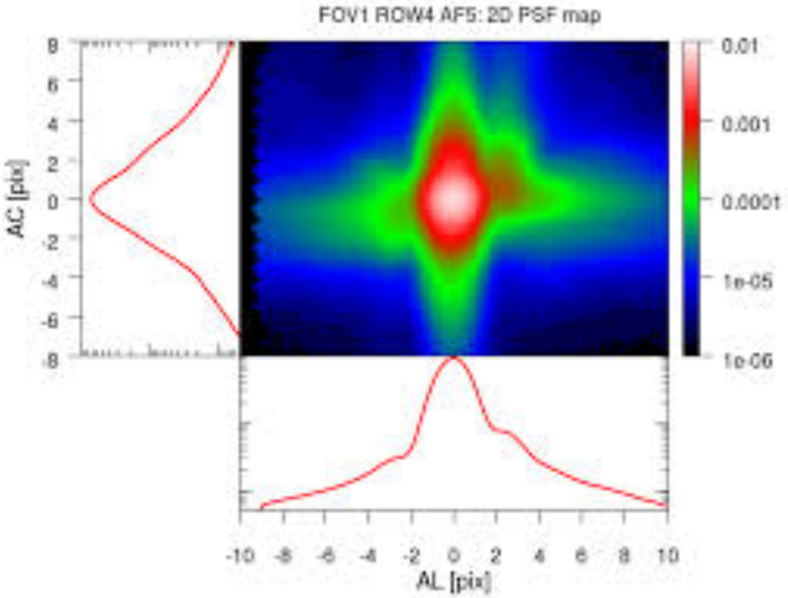
- The quantities $A=\alpha(1-\beta)$ and $B=\alpha(\beta-\alpha/2)$ in the previous formula are functions of the atmospheric conditions as well as of the wavelength of the incident light.
- For **standard atmospheric conditions** (T=15°C, p=1 atm, λ=590 nm) **A**=60''.236 and **B**=0''.0675
- **Pulkovo Observatory** in 1985 has produced **accurate refraction tables** for varying atmospheric conditions and as function of zenith distance
- The dependence of the refractive index on the wavelength is a complicated function of λ^{-2}, but also the finite passband of the filters and the stellar spectral distribution must be taken into account
    - the resulting astrometric effect goes under the name of ***Differential Color Refraction*** (DCR), which is best determined by observations
    - to **alleviate** the effect of **DCR** one should prefer **near-infrared** observations
- The above models are at best **good to a few mas** for small zenith distances, and reach the level of 50 mas for z=70°
- With **ad-hoc** techniques it is possible to reach **sub-ma**s accuracy, but one needs to go into **space** for **micro-arcsecond accuracy**



<u>Instrumental Effects</u>

- **Optical aberrations**, and in particular off-axis aberrations, of the telescope *optical train* can generate astrometric effects, as they impact the determination of the image *centroid*
    - *coma* ($y^2\theta$)*, astigmatism* ($y\theta^2$)*, distortion* (θ^3) [y=aperture radius, θ=field angle); *distortion* acts only on position, while the first two affects also image quality; all of them change the telescope Point Spread Function (PSF)
- Various kinds of **mechanical obscurations** of the FOV give rise to *vignetting*, which also affects the PSF
- CCD observations made in *Time Delay Integration* mode (TDI), as opposed to the most common *stare mode,* can also introduce **distortions** in the PSF of the image, and therefore on its centroid
    - in TDI mode, the CCD is read out at the same rate as the star motion across the detector
    - in this case, the distortion comes from a *smearing* of the PSF in both the read-out, and across-scan direction
- A first approach to correcting **instrumental effects** is to represent them by **power series of the image location** on the focal plane; **distortions of the PSF** can be corrected by using different **PSF templates as function of location** in the optical system → need optimal distribution of reference stars





Observation Equation

- **First step** is to correct the *reference stars*'s coordinates (the *astrometric parameters* α and δ) and their associated *uncertainties* from the reference catalog values to those at the *epoch* and *location* of the observation
  - this includes ***proper motion*** and ***parallax*** effects
  - the know ***refraction*** and ***aberration*** effects should also be evaluated and pre-corrected
  - the **observed positions of the i-th reference star** can be expressed as

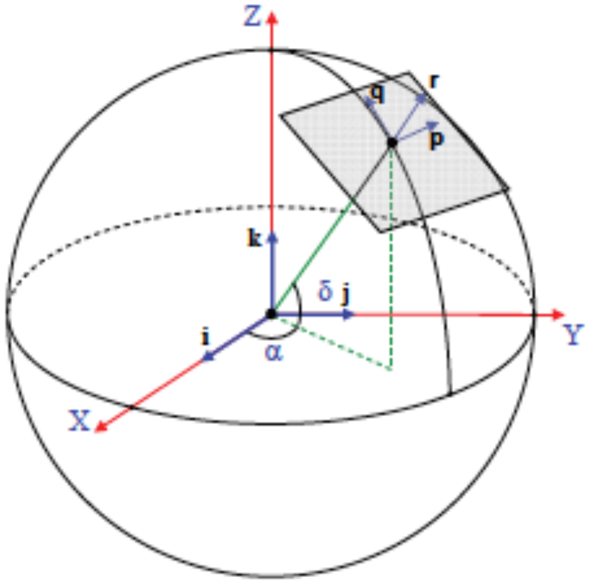
$$\alpha_i = \alpha_{i0} + \mu_{\alpha i} t_j + \varpi_i P_j + L_{ij}$$
$$\delta_i = \delta_{i0} + \mu_{\delta i} t_j + \varpi_i Q_j + M_{ij}$$

  where $t_j = T_j - T_0$, and L_{ij}, M_{ij} are non-linear displacements of the star position from the reference epoch T_0 to the observation epoch T_j

- **Second step** involves the ***geometric transformation*** which maps the sky coordinates to *tangent plane* coordinates, i.e., the plane tangent to the unit sphere in the direction of the optical axis.
  - *gnomonic projection* (astrograph-like telescope optics, most common)
  - *equidistant projection* (Schmidt telescopes)
- **Third step**, commonly called ***plate model***, consists in determining the functional relation that links the measured quantities (*x,y*) identifiyng the object's *location* onto the detector to some *field angles* directly related to the sky coordinates of that object at the epoch of observation.

First step: (1) Propagation of the astrometric parameters to the observation epoch

- We introduce the normal triad at the star direction relative to the equatorial system (**p**, **q**, **r**)



$$\vec{p} = \begin{pmatrix} -\sin\alpha \\ \cos\alpha \\ 0 \end{pmatrix} \quad \vec{q} = \begin{pmatrix} -\cos\alpha\cos\delta \\ -\sin\alpha\cos\delta \\ \cos\delta \end{pmatrix} \quad \vec{r} = \begin{pmatrix} \cos\alpha\cos\delta \\ \sin\alpha\cos\delta \\ \sin\delta \end{pmatrix}$$

- Let the astrometric parameters of the star at the reference epoch T_0 be (α_0, δ_0, $\mu_{\alpha 0}$, $\mu_{\delta 0}$, ϖ_0) and V_{r0} its radial velocity; we can calculate the star's vectorial proper motion as

$$\vec{\mu}_0 = \vec{p}_0\mu_{\alpha*0} + \vec{q}_0\mu_{\delta 0}$$

where $\mu_{\alpha*}=\mu_\alpha\cos\delta$. Writing the star's space velocity as

$$\vec{v} = \vec{\mu}_0\frac{A}{\varpi_0} + \vec{r}_0 V_{r0}$$

- With the previous notations, the direction to the star at a time t=(T_0-T_j) is given by

$$\vec{r}(t) = \langle \vec{r}_0 + (\vec{p}_0\mu_{\alpha*0} + \vec{q}_0\mu_{\delta 0} + \vec{r}_0\zeta_0)t \rangle = \langle \vec{r}_0(1+\zeta_0) + \vec{\mu}_0 t \rangle$$

where ζ_0=$V_{r0}\varpi_0$/A is called *radial proper motion*, being the equivalent of the tangential proper motion but in the radial direction. To have ζ_0 in mas/yr, V_{r0} is given in Km/s, ϖ_0 in mas, and the Astronomical Unit *A* is expressed in Km yr s^{-1}

- The quantities $\mu_{\alpha*0}$, $\mu_{\delta 0}$, ζ_0 in the previous expression are the component of the space velocity scaled by the inverse distance at epoch T_0 along the vectors of the normal triad at $\mathbf{r}_0$
- The normalization factor can be computed as

$$f = |\vec{r}_0||\vec{r}(t)|^{-1} = [1 + 2\zeta_0 t + (\mu_0^2 + \zeta_0^2)t^2]^{-1/2}$$

where $\mu_0^2 \equiv \mu_{\alpha*0}^2 + \mu_{\delta 0}^2$, from which we obtain the **epoch propagation equation**

$$\vec{r}(t) = [\vec{r}_0(1+\zeta_0 t) + \vec{\mu}_0 t]f$$

- Finally, the spherical coordinates α(t),δ(t) can be easily computed from the components of **r**(t)

First step: (2) Propagation of uncertainties to the observation epoch

- The **rigorous propagation of positional uncertainties** is obtained by computing the 2x2 ***covariance** matrix*

$$C_{\alpha\delta} = J C_{\alpha_0\delta_0} J^T$$

$C_{\alpha 0 \delta 0}$ is the 2x2 covariance matrix of the star's catalog coordinates, and J is the 2x2 **Jacobian** matrix of partial derivatives:

$$J = \begin{pmatrix} \frac{\partial \alpha_*}{\partial \alpha_{*0}} & \frac{\partial \alpha_*}{\partial \delta_0} \\ \frac{\partial \delta}{\partial \alpha_{*0}} & \frac{\partial \delta}{\partial \delta_0} \end{pmatrix} = \begin{pmatrix} p^T p_0 (1+\zeta_0) f - p^T r_0 \mu_{\alpha*0} t f & p^T q (1+\zeta_0) f - p^T r_0 \mu_{\delta 0} t f \\ q^T p_0 (1+\zeta_0) f - q^T r_0 \mu_{\alpha*0} t f & q^T q_0 (1+\zeta_0) f - q^T r_0 \mu_{\delta 0} t f \end{pmatrix}$$

where $t=(T_j-T_0)$ and the suffix 'T' denotes vector transposition.

- A less rigorous, but most **useful formula** can be obtained from the **simplified model** for the propagation of position, which does not take into account variations in proper motion or parallax with time:

$$\alpha = \alpha_0 + \mu_{\alpha*0} t \sec\delta$$
$$\delta = \delta_0 + \mu_{\delta 0} t$$

which gives for the final **α** and **δ variances**

$$\sigma_\alpha^2 = \sigma_{\alpha*0}^2 + \sigma_{\mu_{\alpha*0}}^2 t^2 + 2t C_{\alpha_{*0}\mu_{\alpha*0}}$$
$$\sigma_\delta^2 = \sigma_{\delta 0}^2 + \sigma_{\mu_{\delta 0}}^2 t^2 + 2t C_{\delta_0 \mu_{\delta 0}}$$

Second step: From Equatorial to Tangential Coordinates

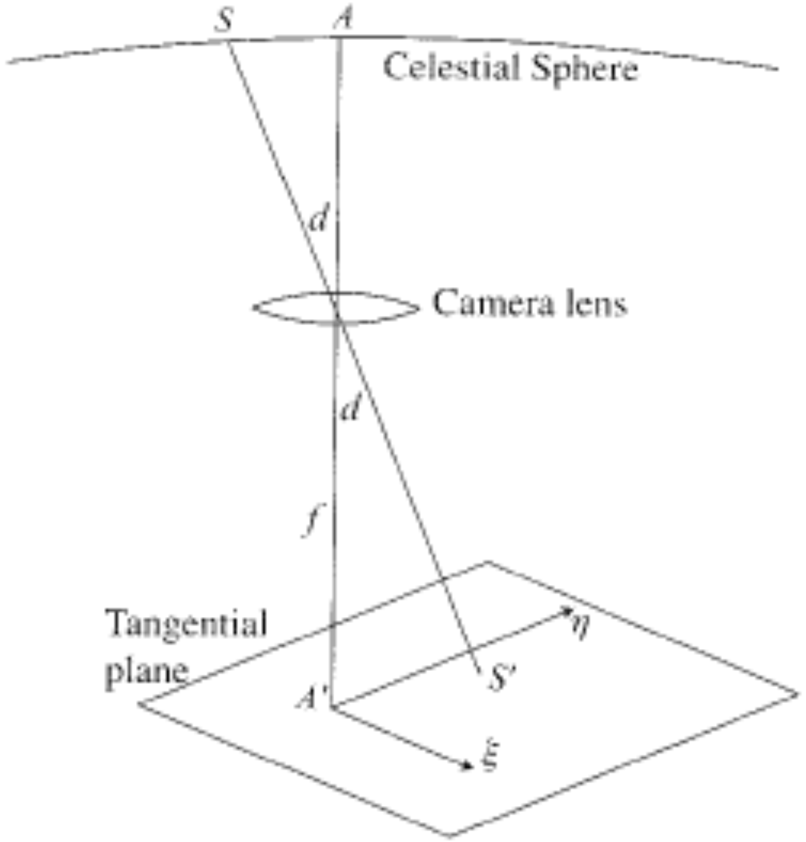
- Let $A(\alpha_0,\delta_0)$ define the direction of the optical axis, and ξ,η be **rectangular Cartesian coordinates** measured from the intersection of the optical axis and the tangent plane A', toward East and toward North respectively ; the *gnomonic projection* mapping the equatorial coorindastes (α,δ) into the so called ***standard coordinates*** (ξ,η) reads

$$\xi = \frac{\cos\delta \sin(\alpha-\alpha_0)}{\sin\delta_0 \sin\delta + \cos\delta_0 \cos\delta \cos(\alpha-\alpha_0)}$$
$$\eta = \frac{\cos\delta_0 \sin\delta - \sin\delta_0 \cos\delta \cos(\alpha-\alpha_0)}{\sin\delta_0 \sin\delta + \cos\delta_0 \cos\delta \cos(\alpha-\alpha_0)}$$

  where (ξ,η) are in radians.
- The **measured coordinates** (x,y), if the detector system is perfectly aligned with the standard plane , are $x=f\xi$, $y=f\eta$ with f equal to the telescope **focal length**
- It can be shown that the ***tangent point error***, due to an imperfect knowledge of the telescope pointing, introduces an error on the calculated standard coordinates given by

$$d\xi = a + c\eta + \xi(a\xi + b\eta)$$
$$d\eta = b - c\xi + \eta(a\xi + b\eta)$$

- On the other hand, a ***misalignment*** of the detector with respect to the ideal focal plane, introduces an error in the measured (x,y) of the kind $x(ax+by)$, $y(ax+by)$
- The second-order terms of both the above effects are known as ***tilt terms***



Third step: Plate Model

- A realistic *plate model* can take the following polynomial form, with $r^2=(x^2+y^2)$

$$\xi = ax + by + c + ex + fy + gx^2 + hxy + qxr^2$$
$$\eta = ay - bx + c' - ey + fx + hy^2 + gxy + qyr^2$$

- The quantities *a, b, c, c′,e,f,g,g′,h,h′,q,q′* are referred to as ***plate constants****:*
  - *a,b,c,c′* represent an **orthogonal transformation** (rotation+translation)
  - *e,f* represent the **affine** part of the transformation (scale change in x, y + non-orthogonality of axes)
  - *g,h* correct for *tilt terms*
  - *q is* the third-order optical distortion term
- These ***observation equations*** relate the **calculated standard coordinates** of the *reference stars* to their ***measured* coordinates** via the *plates constants* which can therefore be *estimated*, provided that a **sufficient number of reference stars** is available on the FOV
  - *residual differential astrometric effects* due aberration and refraction should be evaluated and, if appropriate, incorporated as **second-order** or **third-order** polynomial terms in the above equations
- Once the values of the plate constants have been determined, one can derive the standard coordinates of any anonymous/target star in the FOV
- By then applying the **inverse** of the transformation (ξ,η) → (α,δ) one can obtain the **sky coordinates** of each target star in the *frame* defined by the *reference stars* being used

Principles of Statistical Parameter Estimation

- The *observation equation* for each star takes the form $\vec{O} + \vec{\varepsilon} = F(\vec{p})$, where **O** is the vector of (pseudo-)**observations**, with associated *random* error vector **ε**; *F* is **in general** a **non-linear** function of the **vector of unknown** parameters **p**.
- If an approximate value for the vector **p** is known, the observation equation can be *linearized* as

$$\vec{O} + \vec{\varepsilon} = \mathrm{F}(\overrightarrow{p_0}) + \sum_1^n \frac{dF}{dp_i} \Delta p_i$$

  where we have assumed that the **second order** terms of the Taylor expansion are **negligible** with respect to the measurement errors and therefore no bias is introduced in the estimation process
- Bringing all the constant terms to the left side of the equation, one can rewrite it in the more usual **matrix form**

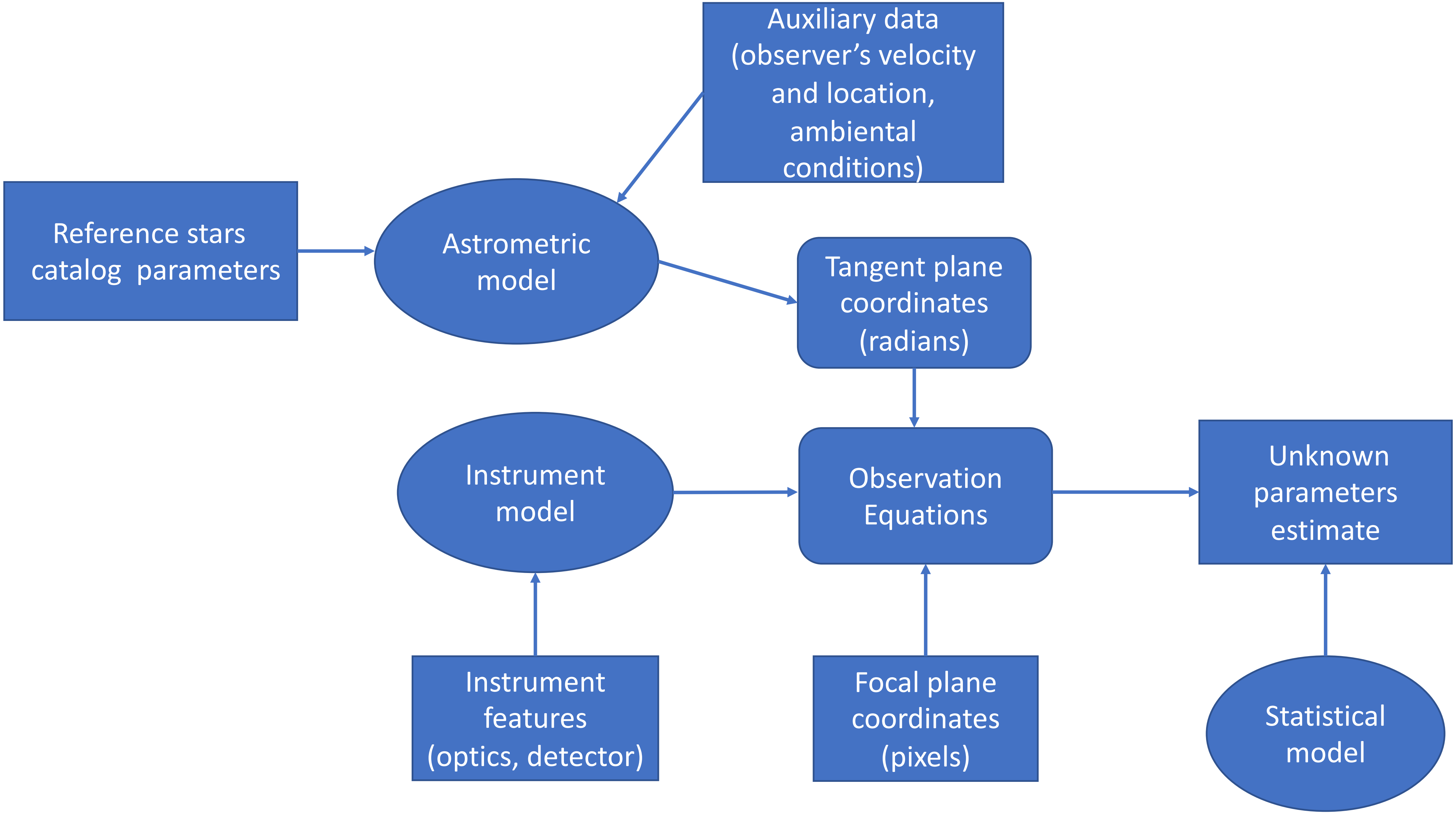
$$\vec{y} + \vec{\varepsilon} = \mathrm{A}\vec{x}$$

  A is the (mxn) ***matrix of coefficients***, where *m* represents the number of observation equations and *n* the number of *parameters* to be estimated; the solution vector **x** represents the **adjustment** to its approximate known value.
- An **optimal** estimation of **x** is obtained by the **method of *least-squares***, which corresponds to minimizing the quadratic form

$$Q = (\mathrm{A}\vec{x} - \vec{y})^T\,(\mathrm{A}\vec{x} - \vec{y}) \equiv \vec{\varepsilon}^T\vec{\varepsilon}$$

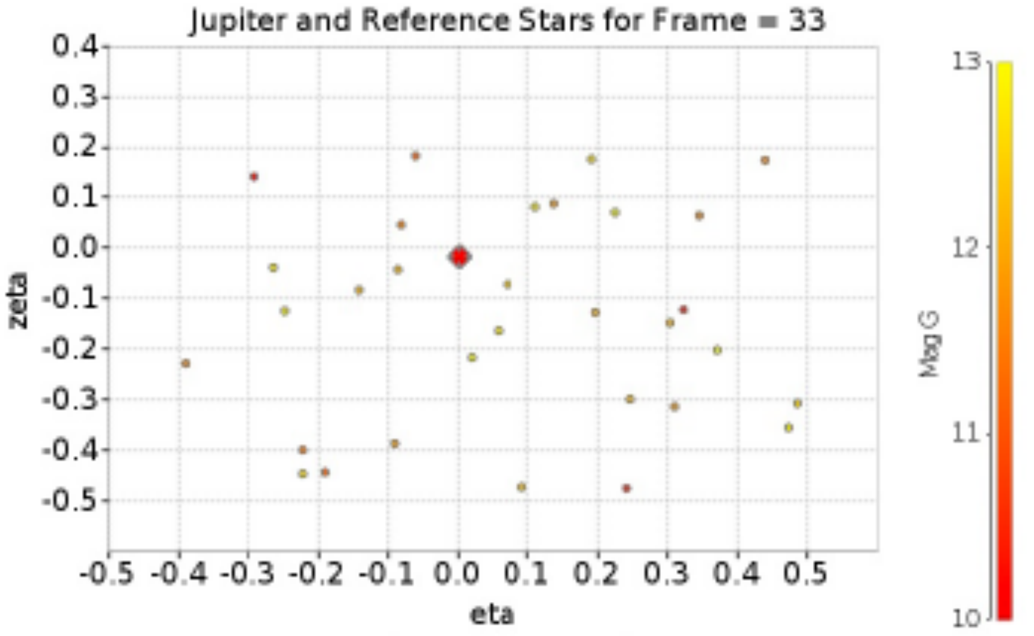
  with respect to **x**, i.e., minimizing the **squared sum of the measurement errors**.
- If the errors are *heteroshedastic*, possibly *correlated, random variables*, the system of equations must be properly weighted → *weighted least-squares theory*

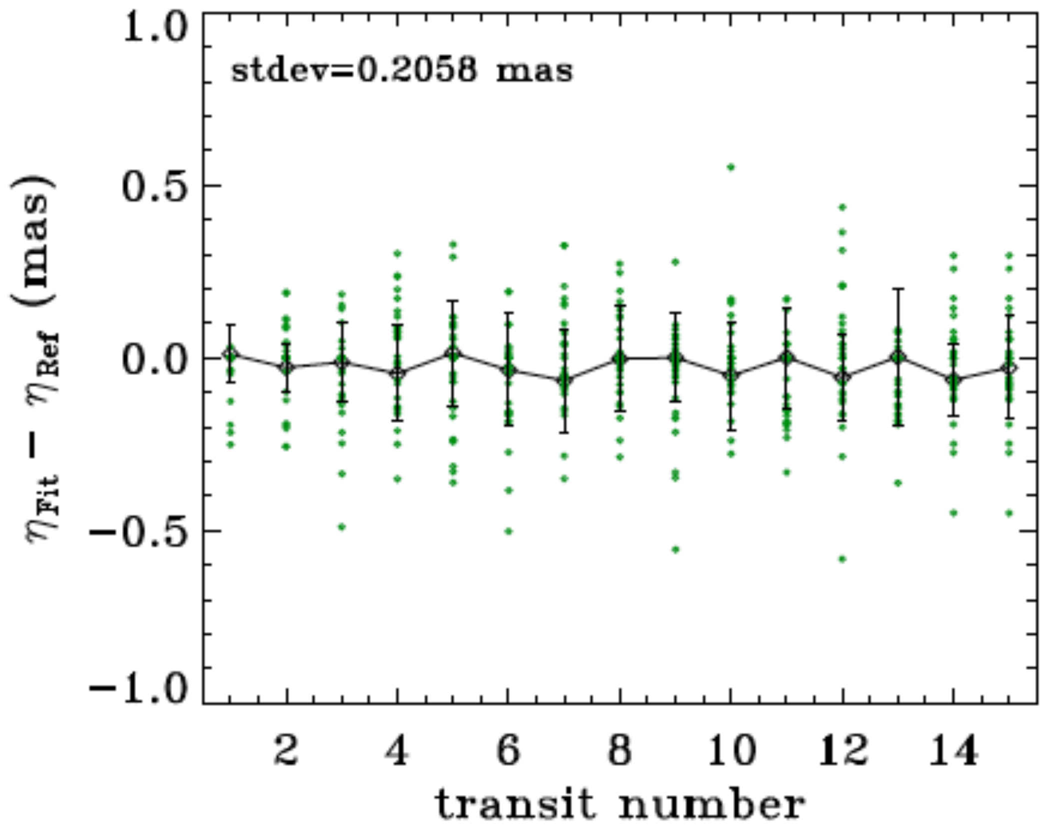
Differential Astrometric Field Treatment: Summary Diagram

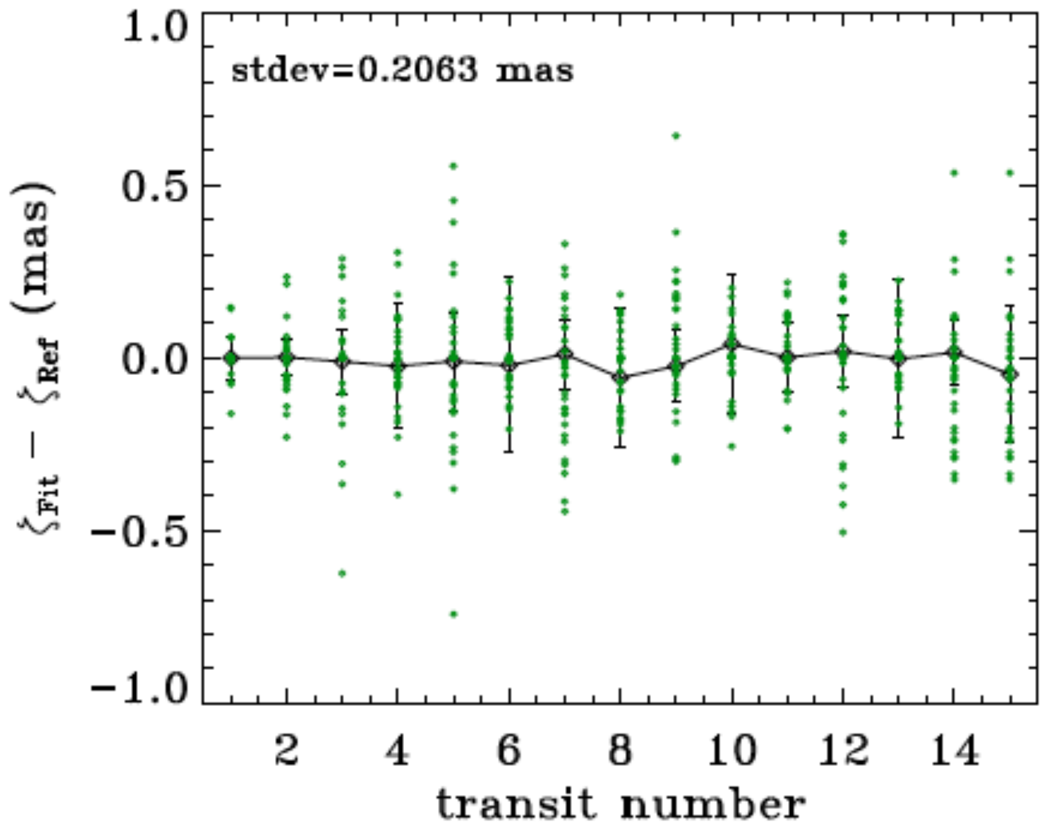


Example: Relativistic Light Deflection from Jupiter in Gaia's measurements

- On-going experiment: detect the Jupiter quadrupole light deflection via Gaia's focal plane data
  → *test of GR predictions*
- Principles of differential astrometry can be used to set up an accurate local reference frame at the micro-arcsecond level
  - Gaia observes in *TDI mode*, so the fundamental observation is the *observing time* t_{obs}, i.e., the time at which the stellar centroid crosses the *fiducial line* of the CCD
  - t_{obs} can be converted into *field angles* $(\eta(t_{ref}),\zeta(t_{ref}))$ at the chosen *reference time* if the satellite *scanning law* is known, i.e., one is able to compute $\dot{\eta}$, $\dot{\zeta}$
  - Each time the satellite FOV scans the same sky area (about 1°) , one can collect the *field angle coordinates* of the *reference stars* and put them on the *tangent plane,* pre-correcting for known astrometric effects
  - Once each *local reference frame* has been set up, the *standard coordinates* of each frame are linked together by means of a *polynomial model* (*plate solution*)
  - The light deflection effect (monopole+quadrupole) from Jupiter on the *target star* is treated as ***residual astrometric signal***







- Along (AL) and across-scan (AC) residuals after least-squares adjustment of 15 overlapping Gaia transits over a **realistically simulated stellar field around Jupiter**.
  - a-priori corrections for proper motion and parallactic effects have been applied
  - no relativistic aberration nor gravitational deflection are included in the simulation
  - observations are error-free
  - satellite attitude errors are of 10 muas/sec around each axis