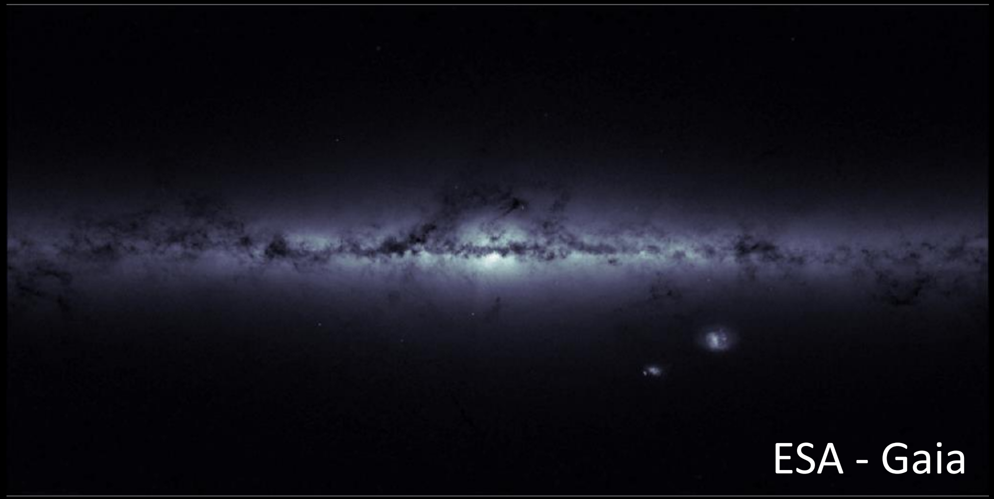
Dynamical modelling techniques

Jason Hunt
Dunlap Fellow
Dunlap Institute for Astronomy & Astrophysics
University of Toronto

Why should I care?

- Milky Way Structure & Dynamics uncertain;
  - R_0=8.35 ± 0.35 kpc
  - M_d=6.43 ± 0.63 x 10^{10} $M_\odot$
  - $R_{d,thin}$=2.6 ± 0.52 kpc
  - $R_{d,thick}$=3.6 ± 0.72 kpc
  - $V_{rot,\odot}$=239 ± 5 kms^{-1}
- Hard to know global picture due to our position and Galactic dust extinction.

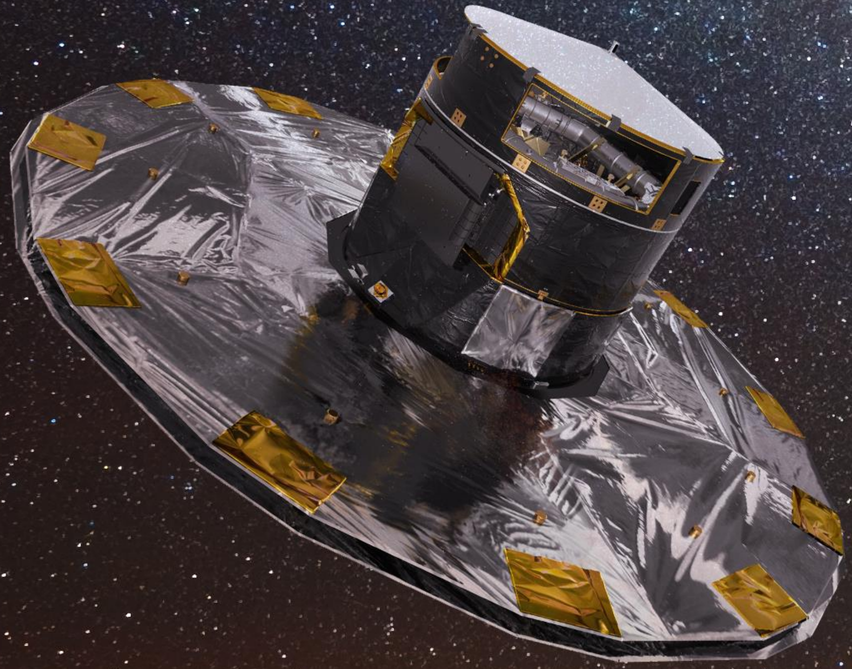
McMillan (2011)

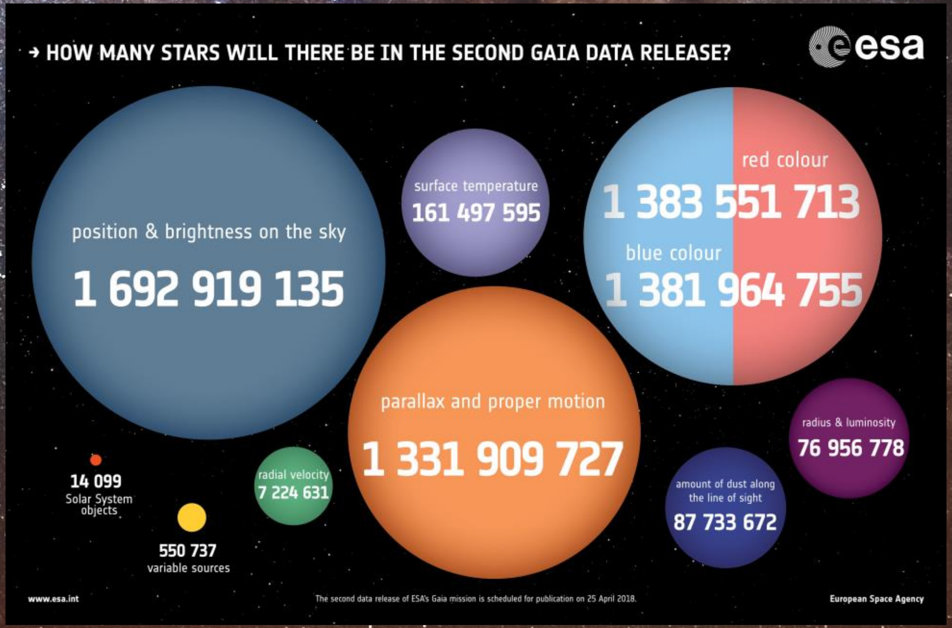


So, we need good surveys to give us more data, but also good models to help us take advantage of that data.

The Gaia mission

- Launched December 2013, ESA cornerstone mission.
- Complemented well by ground based spectroscopic surveys, e.g. APOGEE2, GALAH, LSST.
- .DR2 -> So many stars!





Types of Galaxy model

- Mass models describe the density distribution and the Galactic potential. They do not describe the kinematics.

- Kinematic models specify the density and velocity distributions, but do not require the model to be in a steady state in the Galactic potential.

- Dynamical models specify the density and velocity distribution AND require that the model is consistent within its own potential.

Dynamical models

- There are arguably 5 different dynamical modelling techniques, although where the lines of distinction are drawn can be ambiguous.
    1. Jeans modelling – Solve the Jeans equations (or the CBE)
        - Advantages: Very fast, no need to assume a distribution function
        - Disadvantages: Must bin data to calculate moments, no guarantee there is a DF matching moments, calculation is difficult without assumption of symmetry
    2. Distribution function modelling
    3. Schwarzschild modelling
    4. Torus modelling
    5. N-body modelling

Dynamical models

- There are arguably 5 different dynamical modelling techniques, although where the lines of distinction are drawn can be ambiguous.
    1. Jeans modelling
    2. Distribution function modelling – Fit a DF to the data
        - Advantages: Can model discrete data, theoretically triaxiality is fine.
        - Disadvantages: Must assume DF, if it doesn’t contain the correct solution result will be biased. Assumption of potential required.
    3. Schwarzschild modelling
    4. Torus modelling
    5. N-body modelling

Dynamical models

- There are arguably 5 different dynamical modelling techniques, although where the lines of distinction are drawn can be ambiguous.
    1. Jeans modelling
    2. Distribution function modelling
    3. Schwarzschild modelling – Weight a library of orbits
        - Advantages: Doesn't require knowledge of integrals of motion. In principle not restricted by symmetry (but in practice usually only applied to axisymmetric systems for simplicity).
        - Disadvantages: Assume potential, initial conditions must be chosen carefully (e.g. if orbits not representative, you can’t get a good fit)
    4. Torus modelling
    5. N-body modelling

Dynamical models

- There are arguably 5 different dynamical modelling techniques, although where the lines of distinction are drawn can be ambiguous.
    1. Jeans modelling
    2. Distribution function modelling
    3. Schwarzschild modelling
    4. Torus modelling – Weight a set of orbital tori
        - Advantages: Once orbits are weighted you can recover the DF easily. Tori require fewer numbers to characterise than an orbit library.
        - Disadvantages: Assumes potential, tori must be 'warped' from simple potential. Only regular orbits can be modelled this way.
    5. N-body modelling

Dynamical models

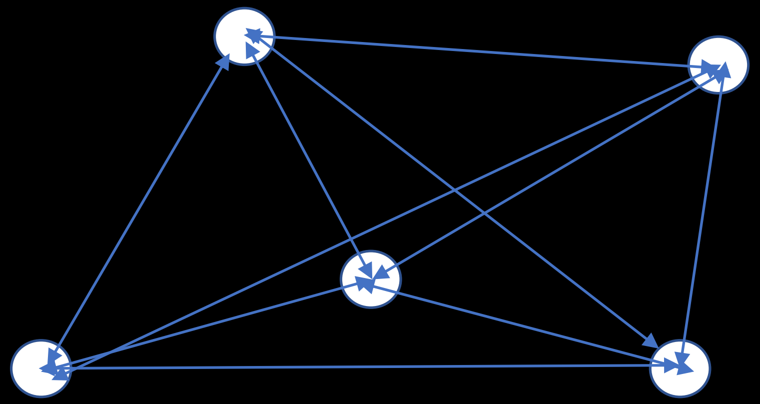
- There are arguably 5 different dynamical modelling techniques, although where the lines of distinction are drawn can be ambiguous.
    1. Jeans modelling
    2. Distribution function modelling
    3. Schwarzschild modelling
    4. Torus modelling
    5. N-body modelling – Evolve a system of particles in their own gravity
        - Advantages: No requirement of symmetry, no need to assume a potential.
        - Disadvantages: It’s hard to actually reproduce a specific galaxy/data because the evolution of the system is not known beforehand.

N-body modelling

- N-body models are based on the gravitational interaction between a collection of ‘N’ bodies.
- Individual particles represent some number of stars, with a given position, velocity & mass. The force is calculated from the particles.
- Can be collisional, or collisionless.

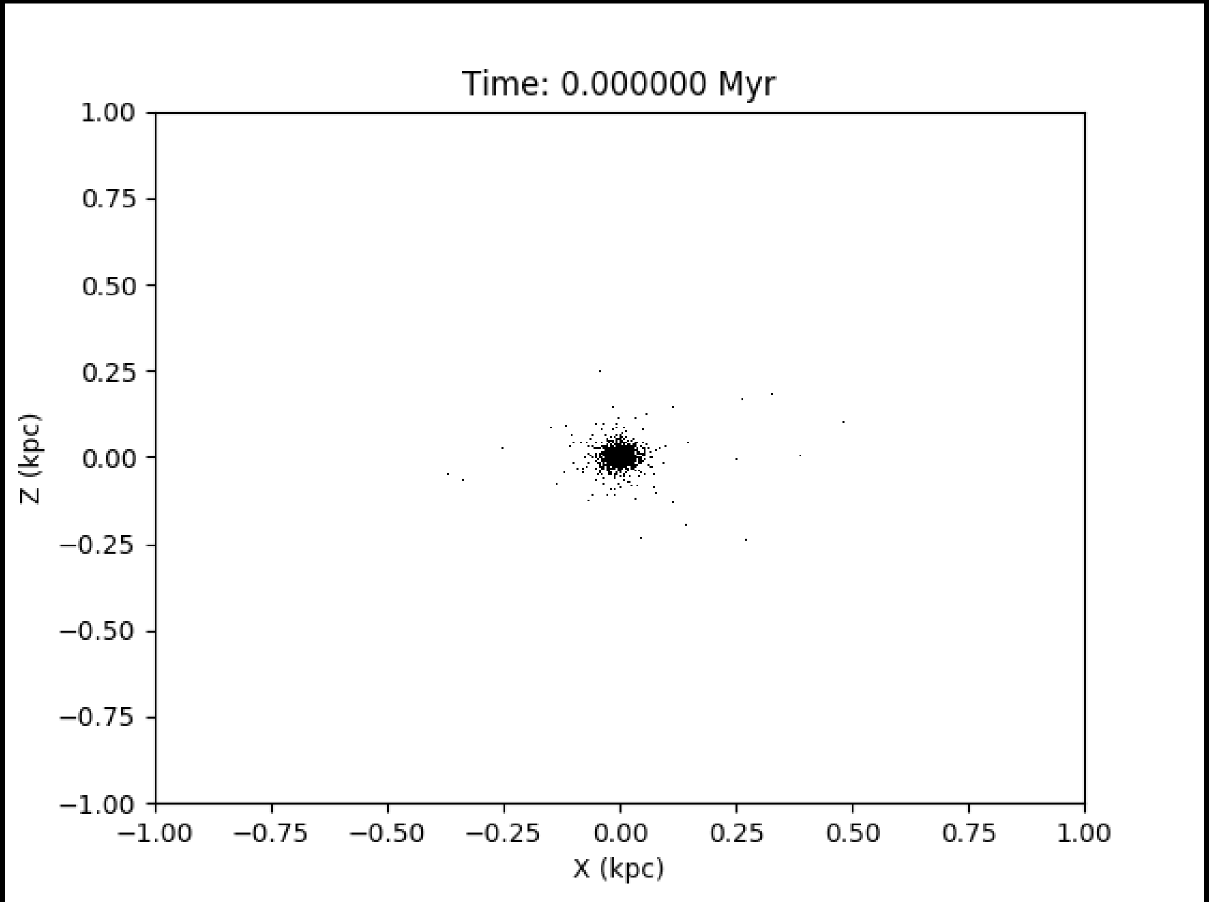
N-body methods

- Direct force calculation:
  - Most accurate
  - Very slow -> scales with $O(N^2)$ particles
  - Good for modelling smaller systems where high accuracy is important.
  - E.g. globular clusters,
  - Example code: Nbody6 (Aarseth)



Example of direct N-body simulation

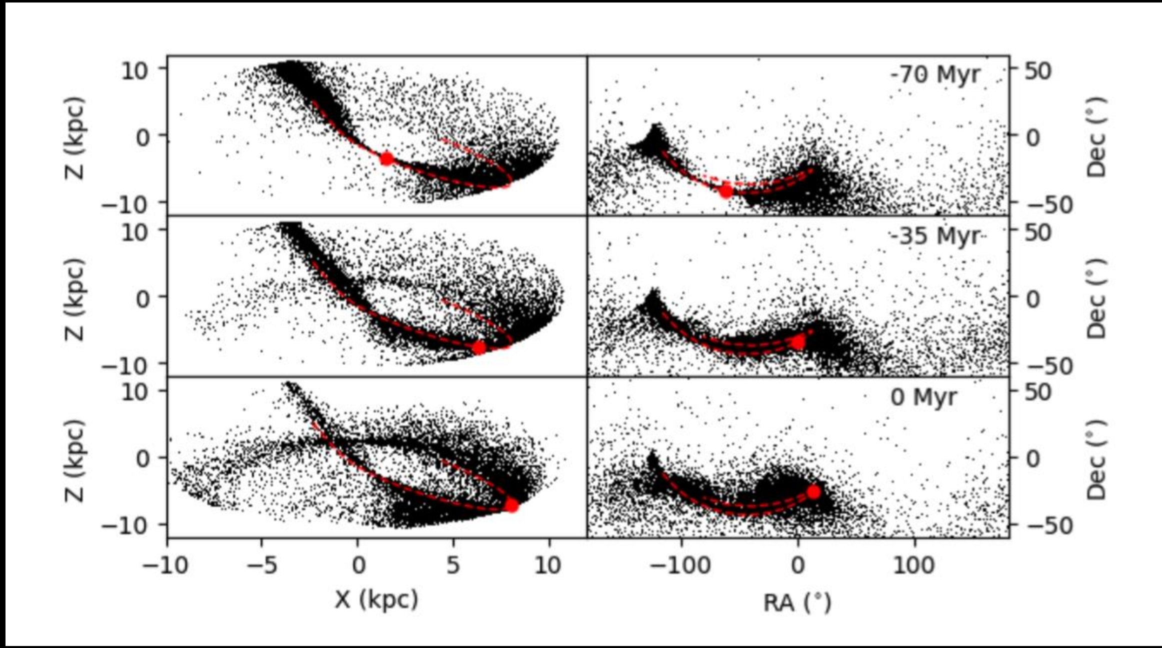
- Direct N-body codes are often used to study the dynamics of star clusters.
- Kaderali et al. perform a direct N-body simulation of NGC288 to explore the extra tidal structure.
- Embedded in a fixed potential galaxy.



Kaderali et al (2019)

Example of direct N-body simulation

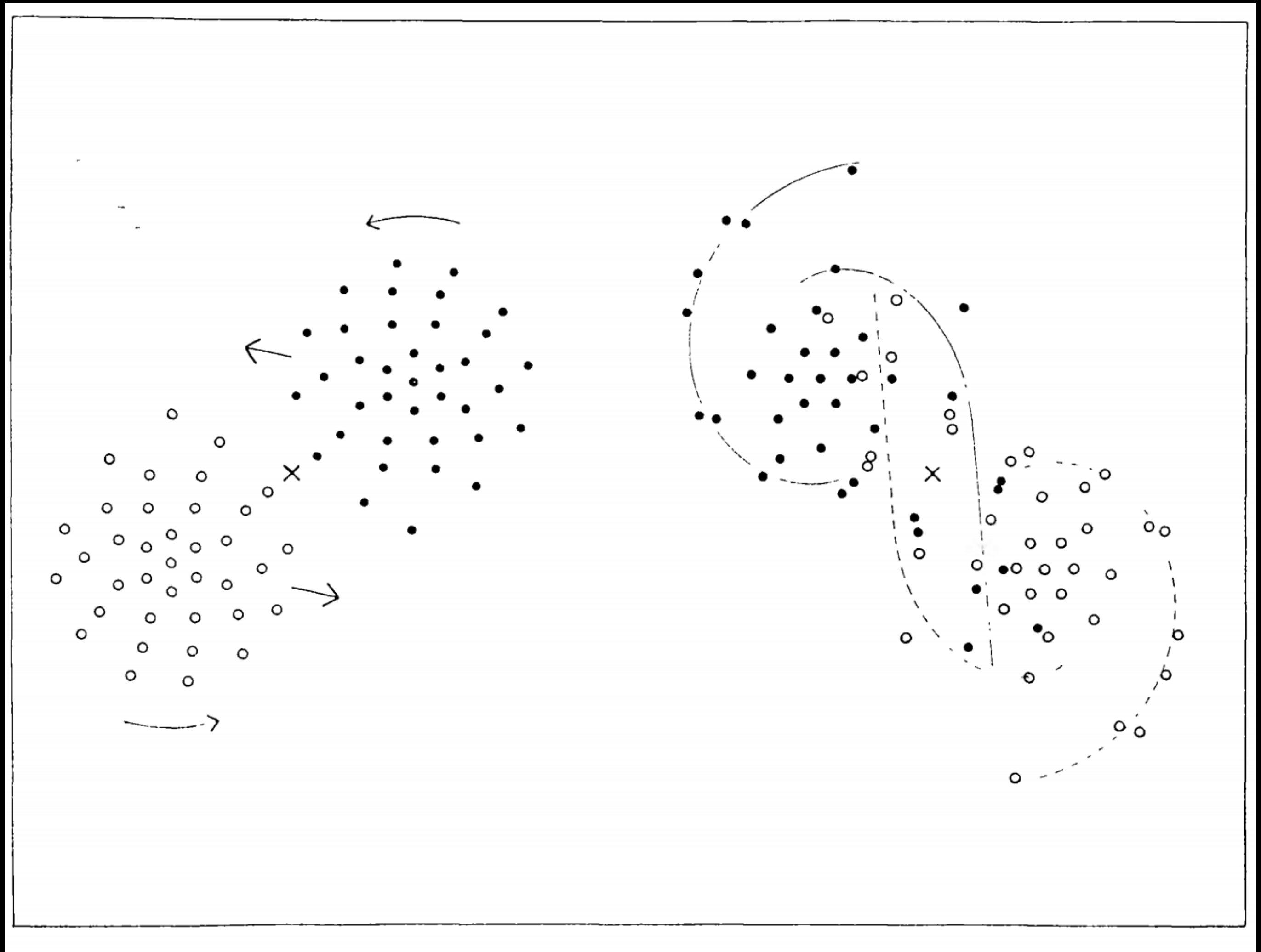
- Direct N-body codes are often used to study the dynamics of star clusters.
- Kaderali et al. perform a direct N-body simulation of NGC288 to explore the extra tidal structure.
- Embedded in a fixed potential galaxy.



Kaderali et al (2019)

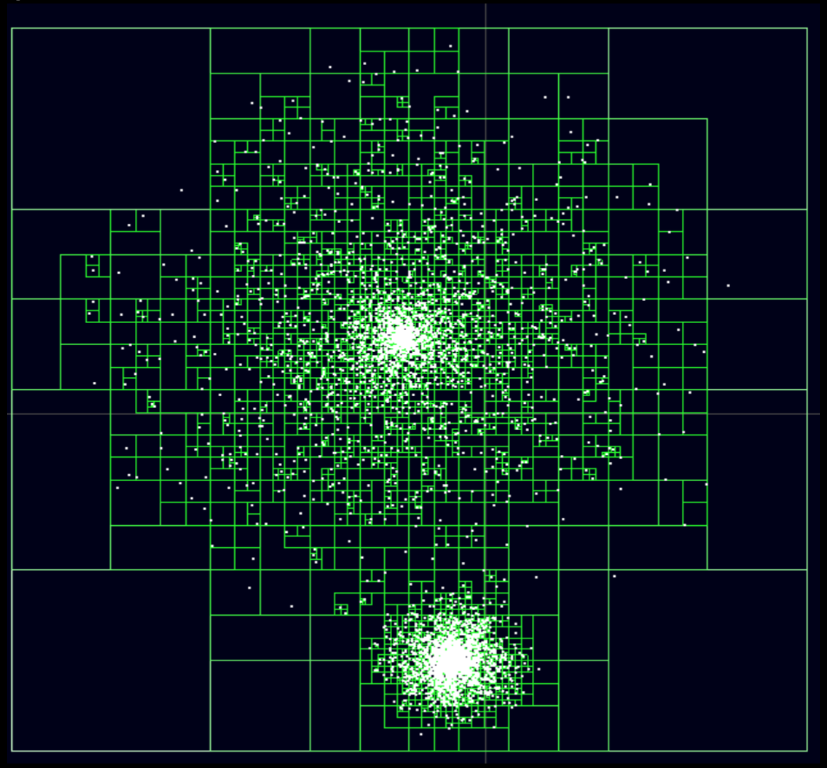
N-body methods

- Direct force calculation:
  - Bonus example: Lightbulbs!
  - Holmberg (1941) used light as a proxy for gravity to study tidal interactions.



N-body methods

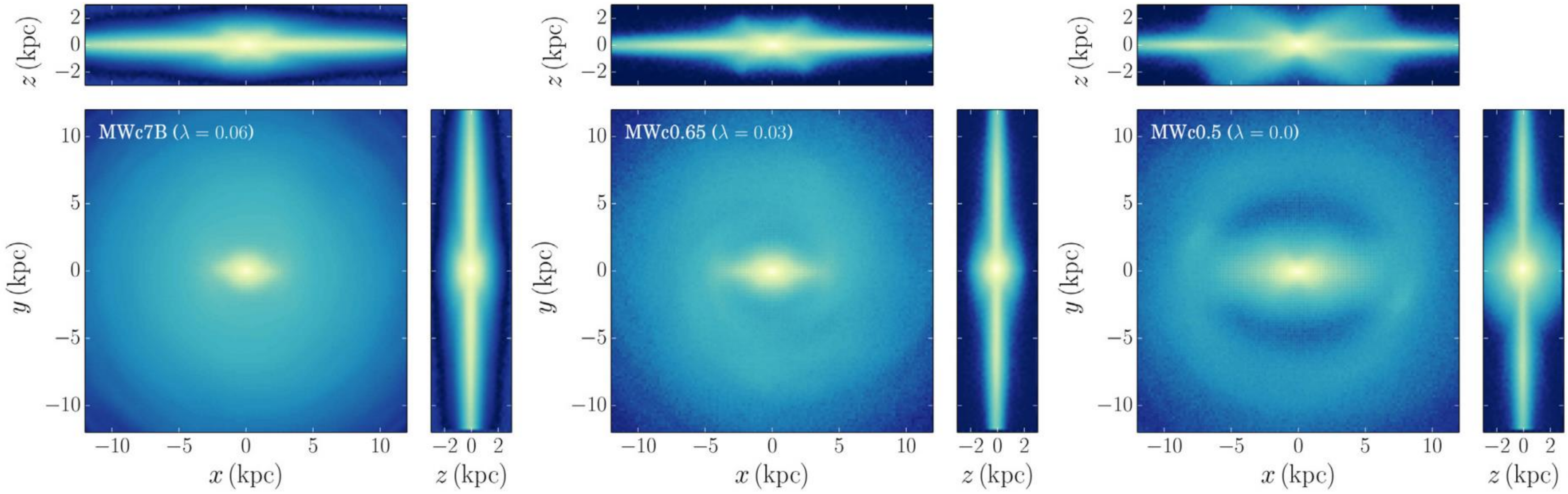
- Tree codes (e.g. Barnes & Hut 1986):
  - Compromise on speed and accuracy of the force calculation
  - Faster than direct method -> scales with Nlog(N) particles
  - Good for modelling (multi)galaxy scale systems
  - Example codes: GADGET (Springel)
- Divide volume into cells
  - Particles nearby treated individually
  - Particles far away use centre of mass of distant cell as a single large particle.
  - Opening angle controls balance between speed & accuracy.



https://beltoforion.de/

Tree code examples - Galaxy

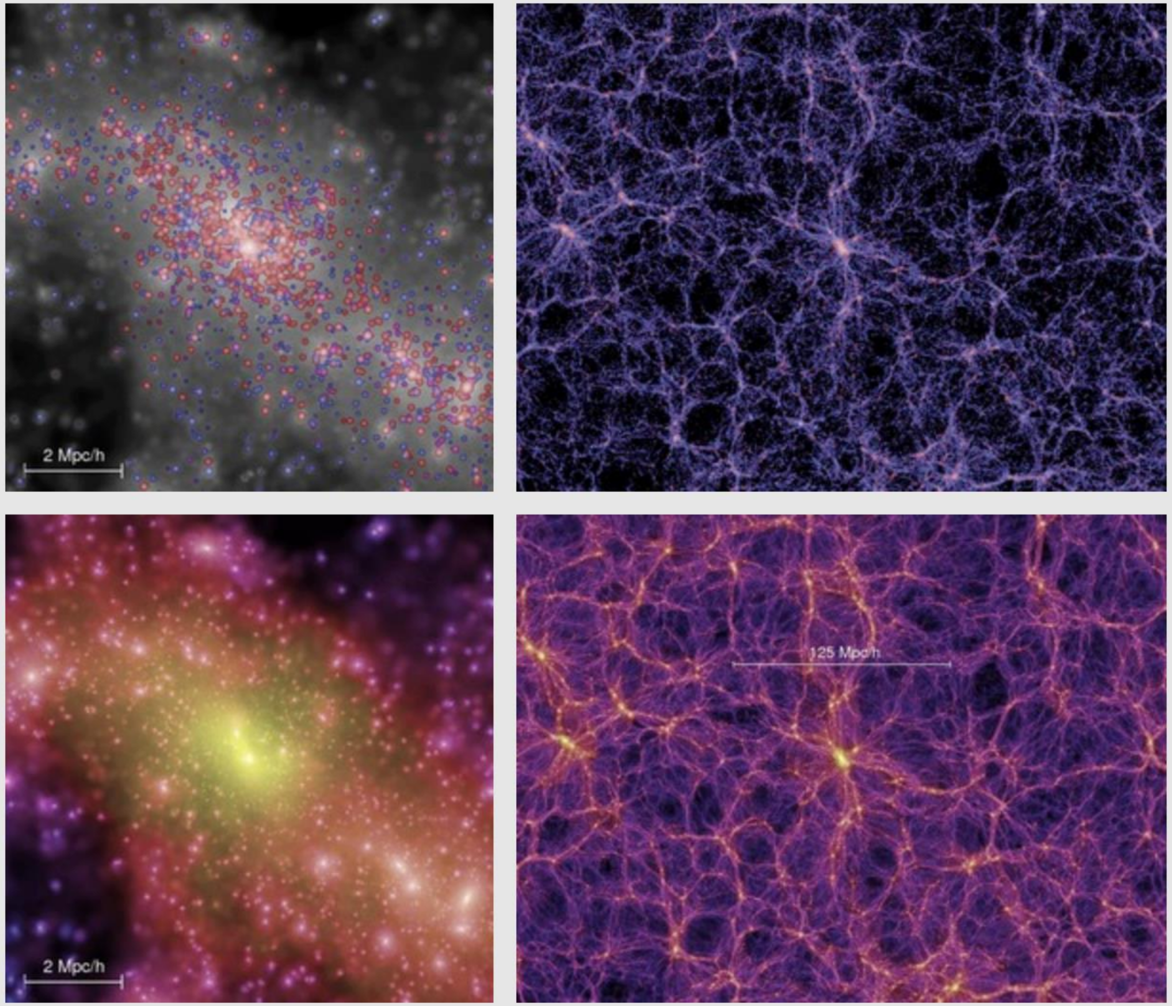
- Fujii et al. use Bonsai (Bédorf et al 2012) to model Milky Way like discs.



Tree code examples - cosmological

The top row of these pictures shows the **galaxy distribution** in the simulation, both on very large scales, and for a rich cluster of galaxies where one can see them individually. The top right panel hence represents the **large-scale light distribution** in the Universe. For comparison, the images in the lower row give the corresponding dark matter distributions.

Click to enlarge the images.



The millennium simulation (Springel et al. 2005)

N-body methods

- Particle mesh codes:
    - Potential calculated over a grid, starting form the density field and solving the Poisson equation.
    - Particles interact with the mean field, not directly with each other.
    - Less accurate over small distances.
        - Adaptive mesh refinement can improve this, e.g. using higher resolution grids where the higher resolution is needed.
    - Faster -> scales with O(N) particles and $O(N_g \log(N_g))$ grid points
    - Good when speed is important, or when small scale interactions are less important. E.g. cosmological simulations.
    - Example code: ENZO (Bryan et al 2014)

N-body methods

- Particle mesh codes
- Refinement can happen over multiple levels, where needed.
- Courses to finer grids are nested.
- Can occur as specified by the user, or when the simulation meets some density criteria.

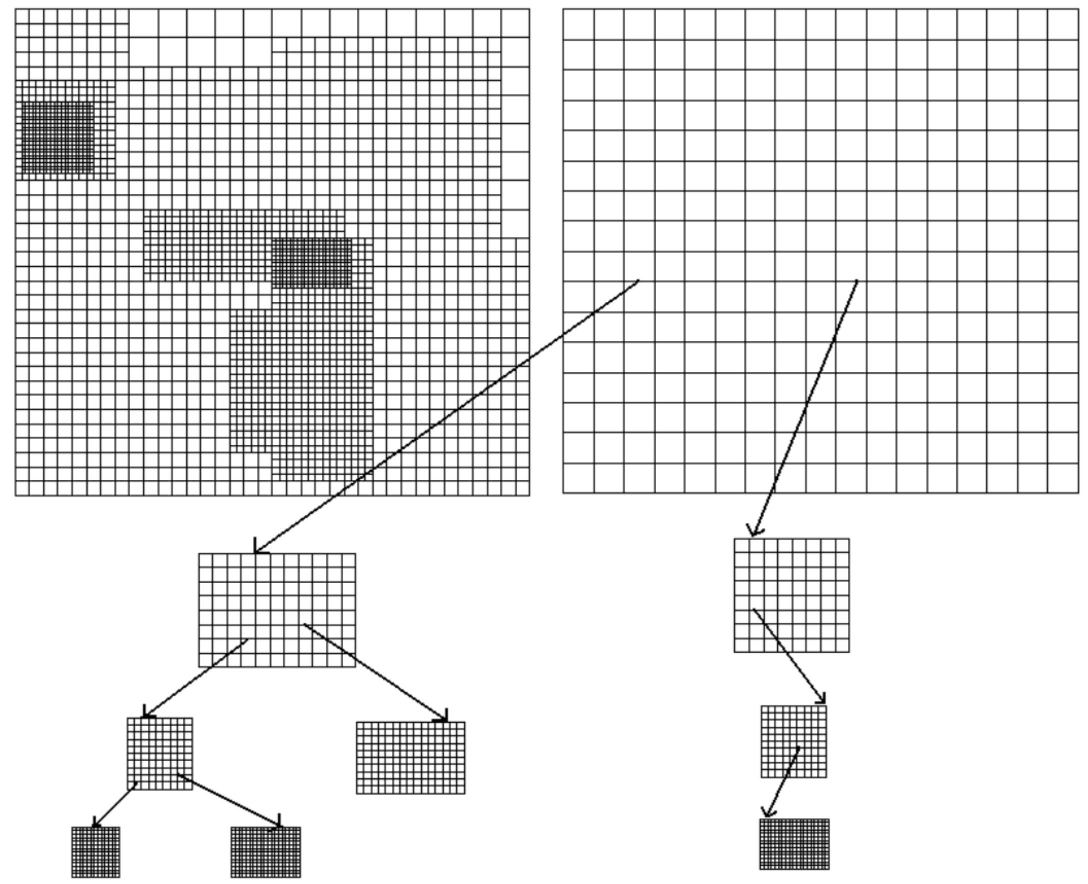


Figure 6.1.: *Adaptive Mesh Refinement (AMR) schemes in ENZO N-body simulation*

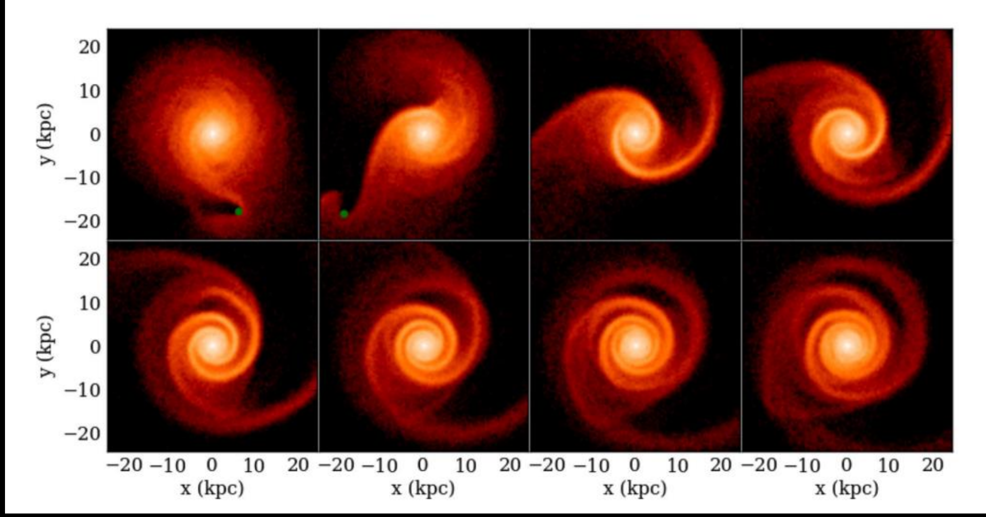
Surajit Paul PhD thesis (2009)

Hydrodynamics

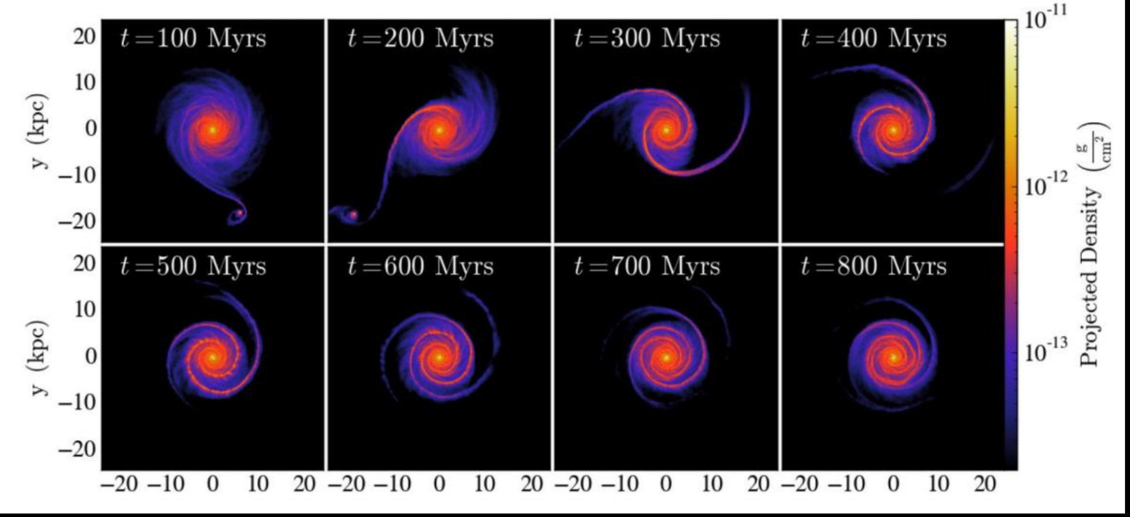
- Of course, galaxies do not merely consist of stars, and many dynamical models contain a gas component.
- These come in both grid based (e.g. ENZO) and Smooth Particle Hydrodynamics (SPH) codes (e.g. GADGET)

Pettitt et al. (2016)
(with GASOLINE)

Stars

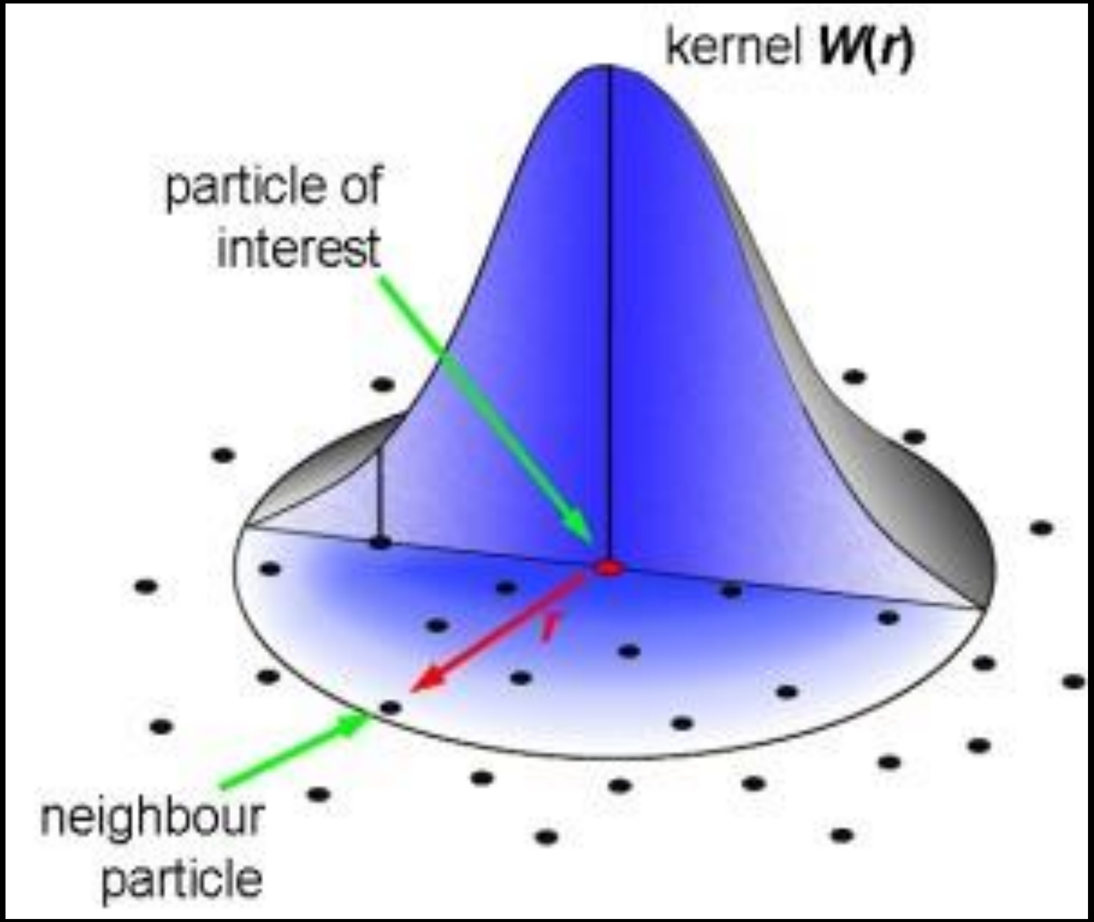


Gas



Smoothed Particle Hydrodynamics (SPH)

- Assign smoothing kernel around each particle.
- Contribution of neighbours is dependent on proximity.
- Creates a smooth density field by the superposition of many kernels.



Application to the Milky Way (+Universe)

- Tailoring models to match data is not always straightforward.
- The Milky Way is not spherical, and is not really axisymmetric.
- And as highlighted by Gaia DR2: It’s not really in equilibrium.
  - E.g. vertical disequilibria, interaction with satellite/dwarf galaxies.

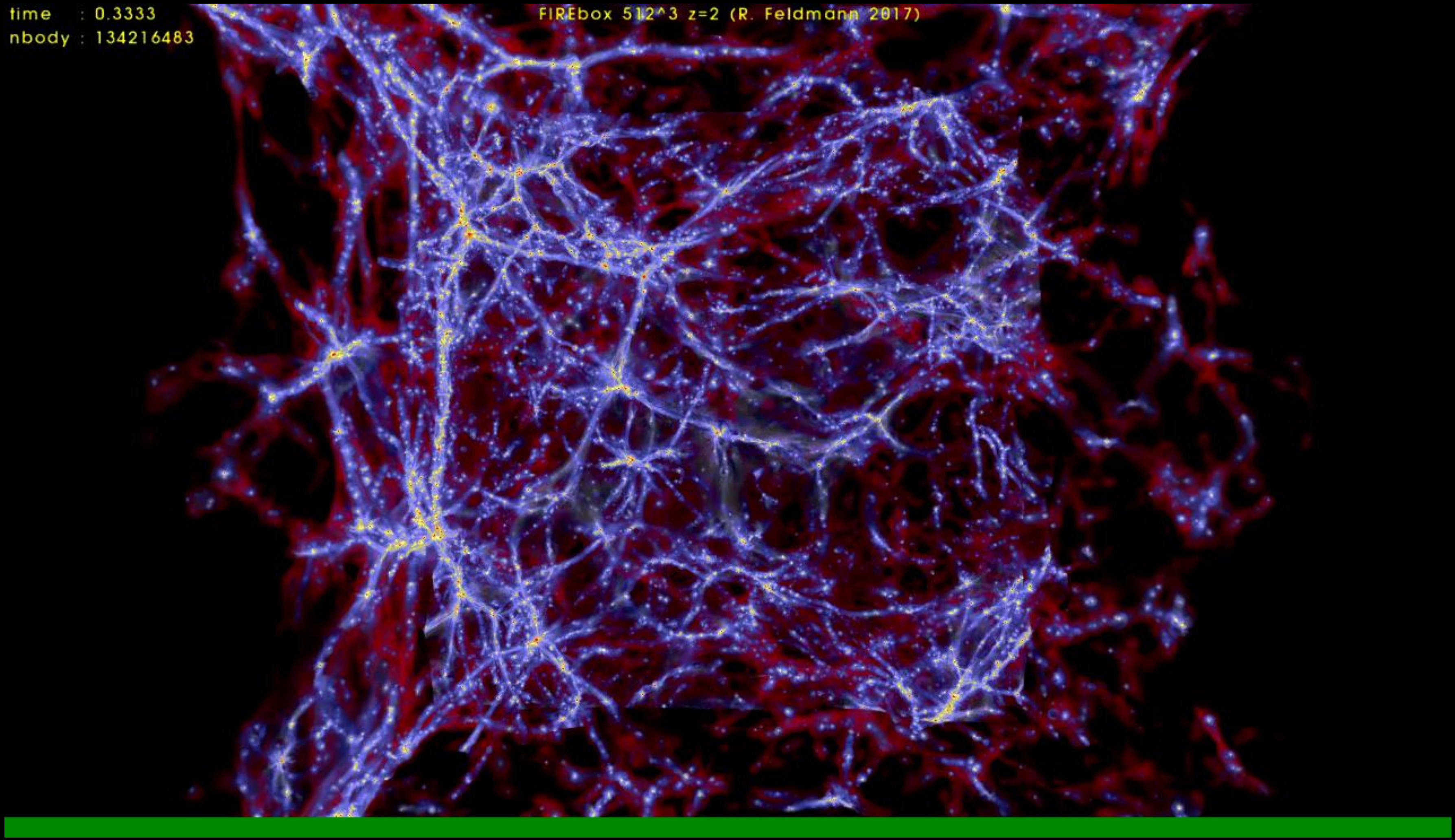
- Nbody/Hydro models are well placed to explore these kind of systems, but as mentioned above it remains difficult to tailor them to a specific outcome.
  - Many different approaches to try and understand the nature of the Galaxy.

1: Cosmological simulations

- By performing very large simulations originating in the early universe, you can track the formation histories of a large number of Milky Way like Galaxies.
- E.g. Millenium, FIRE, Illustris, Auriga, EAGLE and more…

- Advantages: Large samples in which to find MW-like galaxies, with 'realisistic' formation histories
- Disadvantages: Comparatively low resolution, different implementations still have (some) differences in evolution.

E.g. FIRE: Cosmic web



(Feldmann 2017)

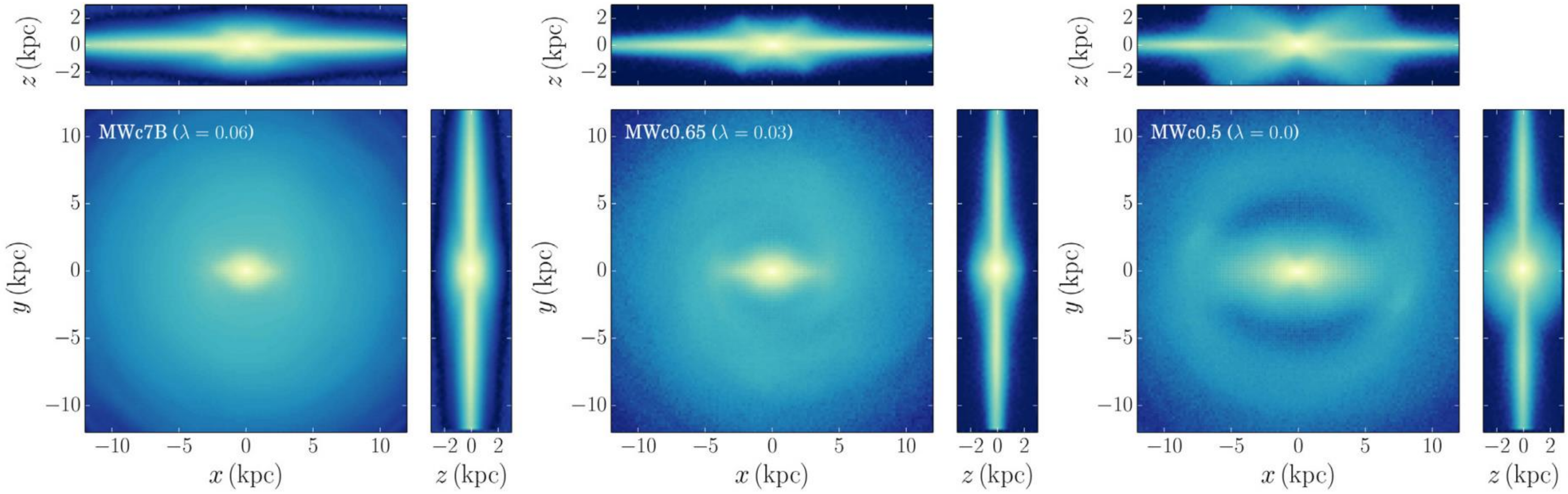
2: Isolated disc simulations

- By only simulating a single galaxy you can achieve much higher resolution, and explore the dynamics and evolution of stars and gas across smaller regions.

- Advantages: High resolution, detailed exploration of dynamical structures such as bars, spiral arms, resonances etc.
- Disadvantages: Only one galaxy to explore. Less realistic formation histories. Unlikely to perfectly represent the MW.

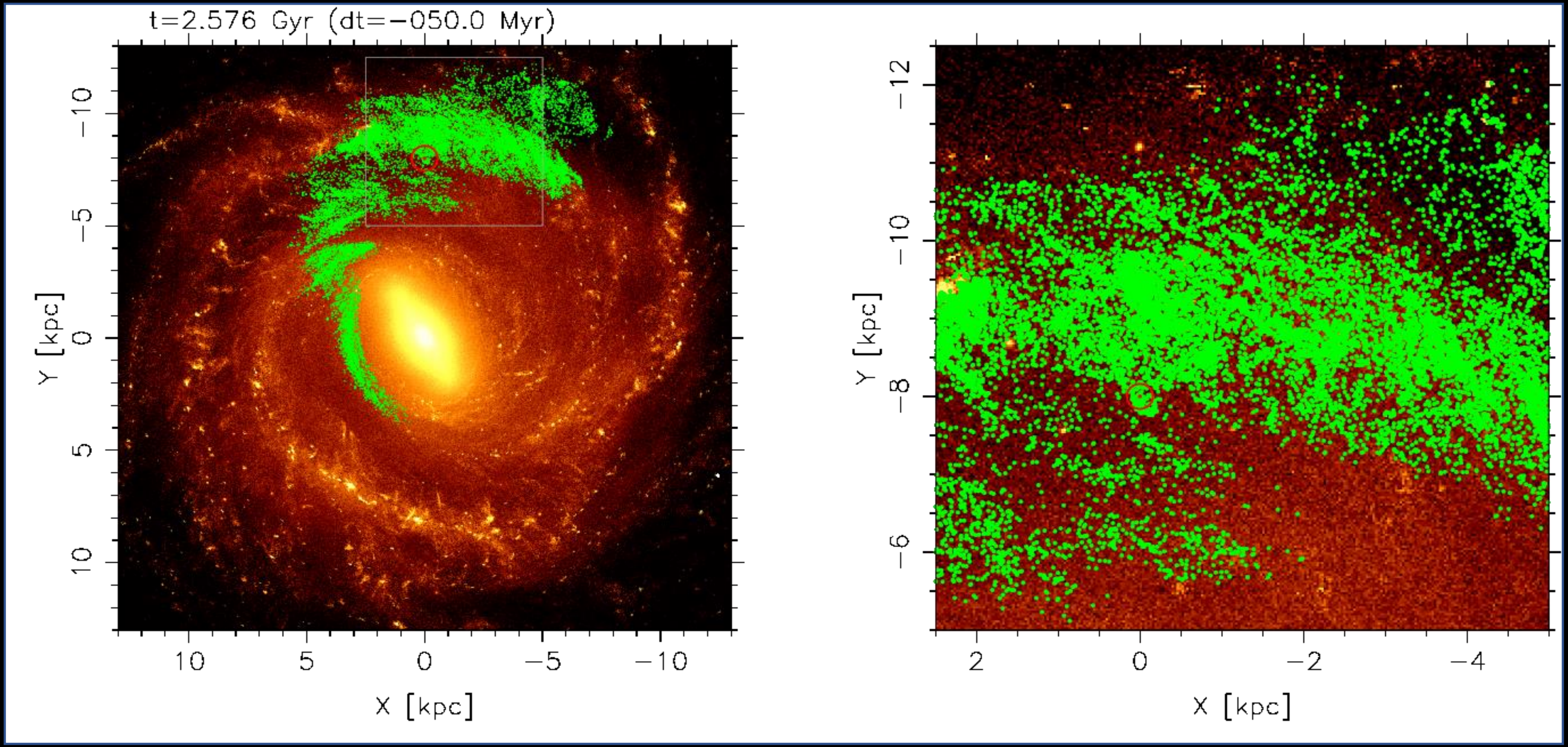
E.g. Bonsai: Isolated discs

- Up to 8 billion particles per galaxy!



Isolated discs: Good for exploring detail

- E.g. trace the motion of stars in various structures.



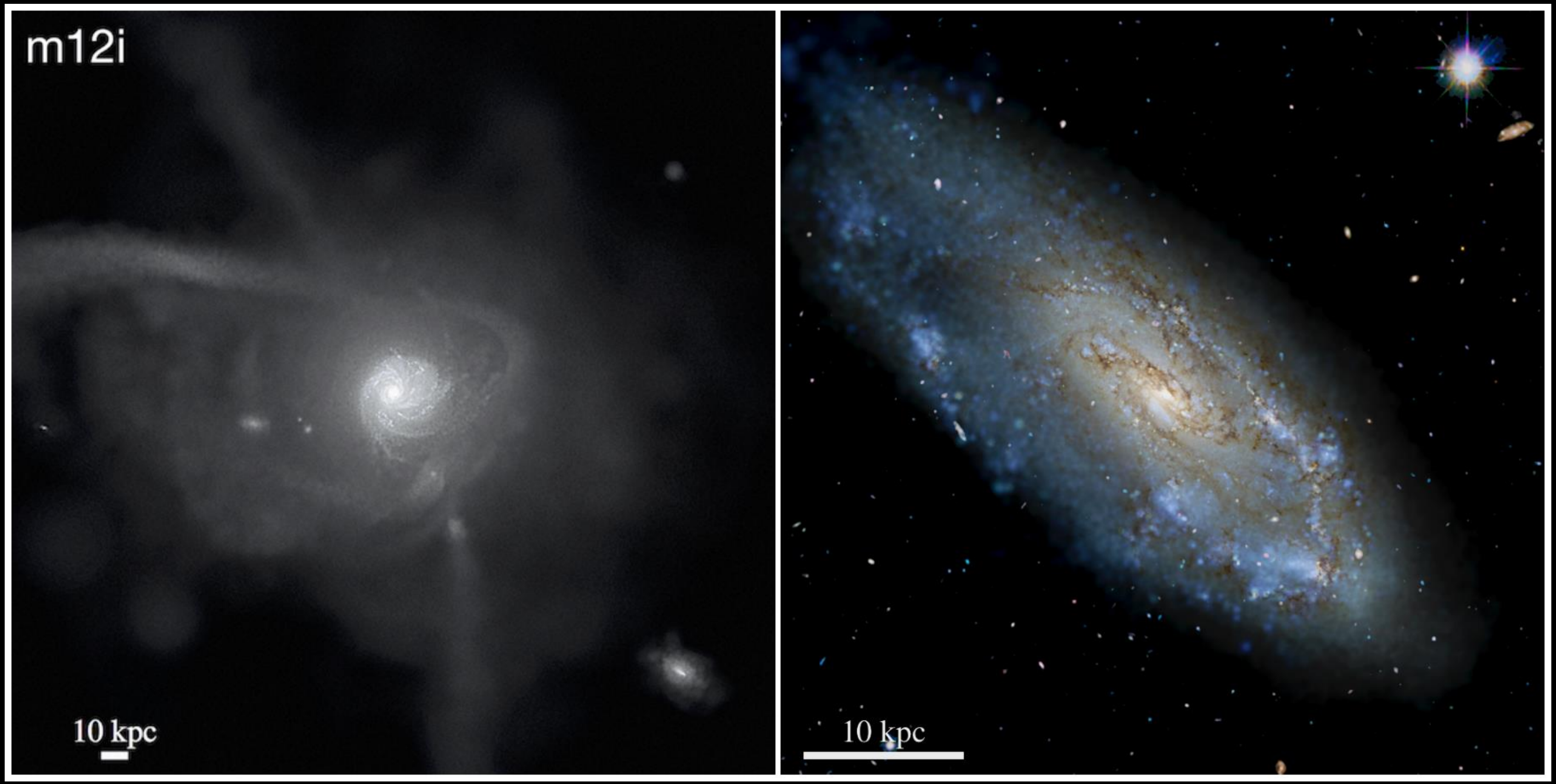
2.5: Zoom simulations

- A compromise solution for the disadvantages of cosmological and isolated galaxy simulations are zoom simulations.

- E.g. you take Milky Way like galaxies from the cosmological simulation, and simulate the appropriate region in high resolution.

- Advantages: Comparatively high res simulations of interesting galaxies, ‘realistsic’ formation histories.
- Disadvantages: Still not perfect MW’s, still lower res than isolated.

E.g. Latte: Zoom in on MW like halo/disc

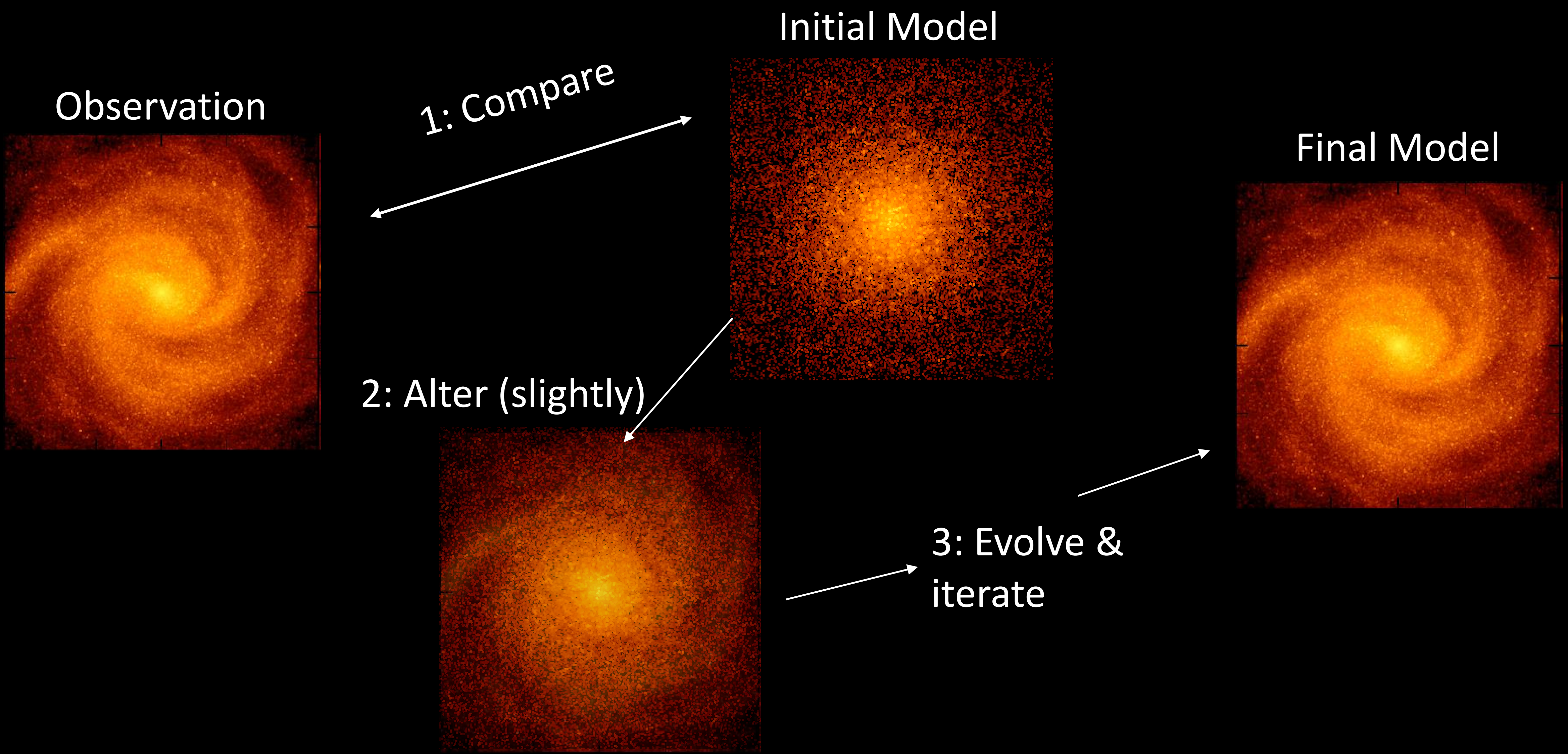


3: Tailoring models – M2M

- Made-to-measure offers a way to tailor a N-body model to some desired specifications during the simulation.
- Can be used to create initial conditions, tailor an N-body model to some distribution function, or even reproduce observational data.

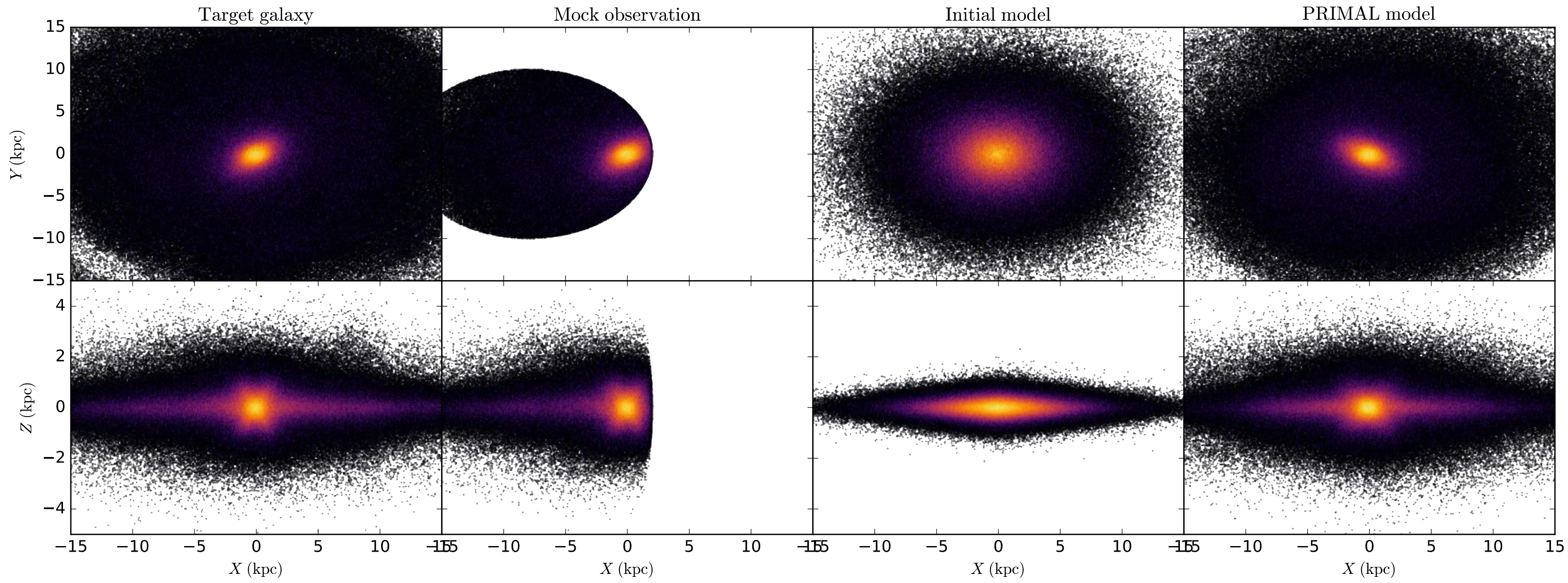
- Advantages: An N-body model with the desired characteristics.
- Disadvantages: No useful evolutionary history, hard to know whether the solution is unique, or correct.

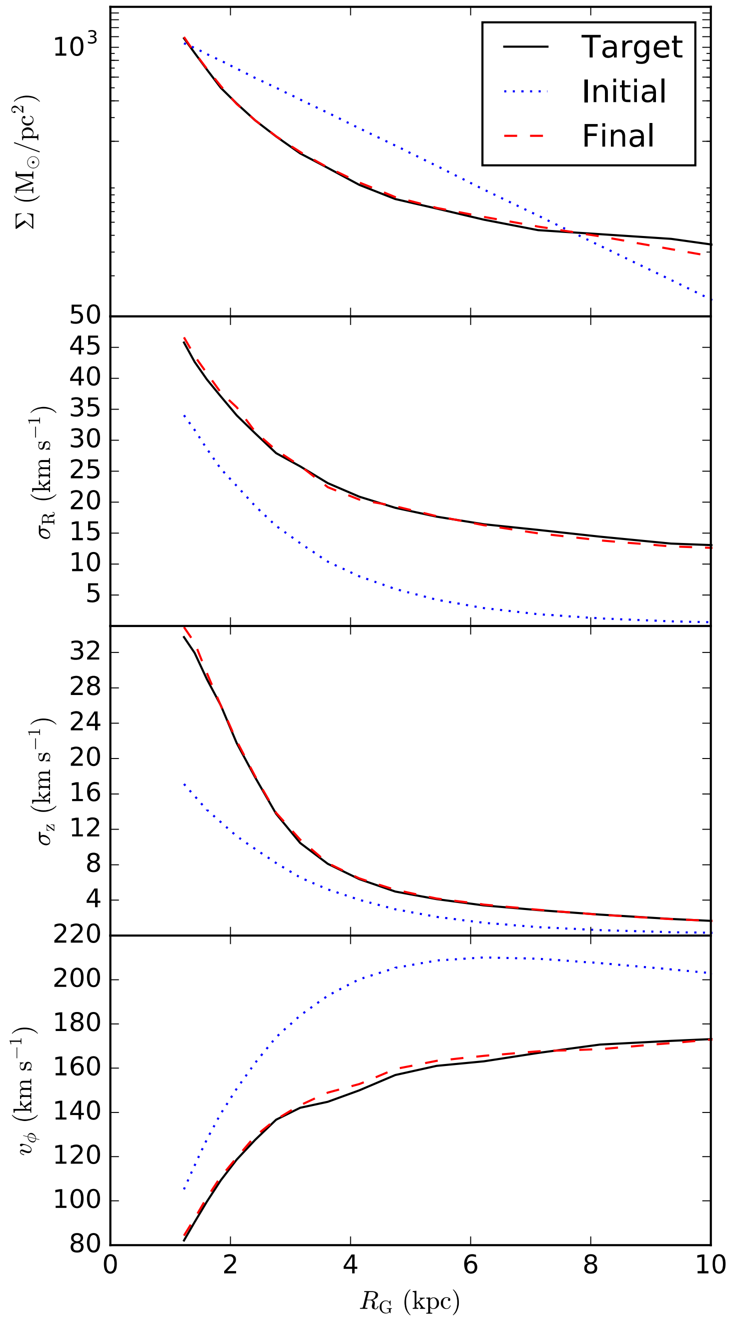
Made-to-Measure: Method



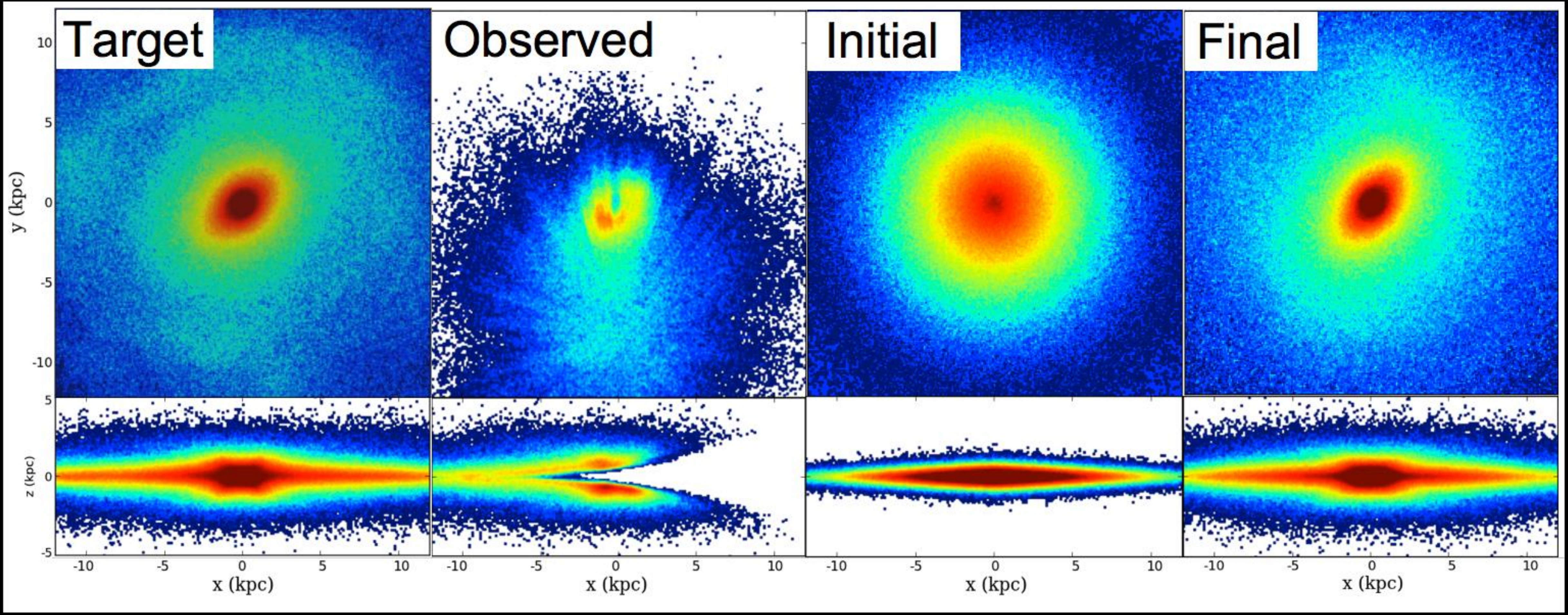
Made-to-Measure: PRIMAL

- Pretty effective at tailoring a ‘poor’ guess at the true galaxy to match good data.
- Harder with error & extinction.



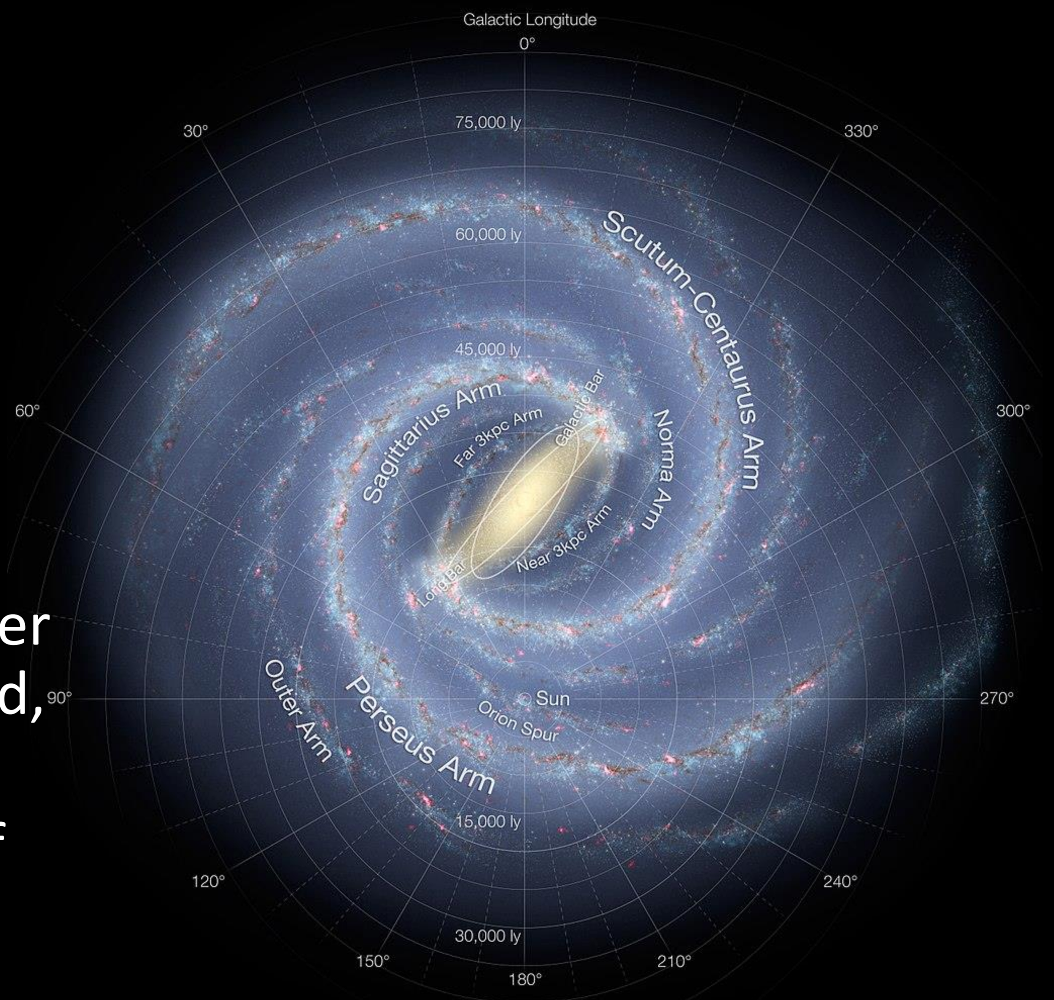


PRIMAL: M0III tracers + error and extinction



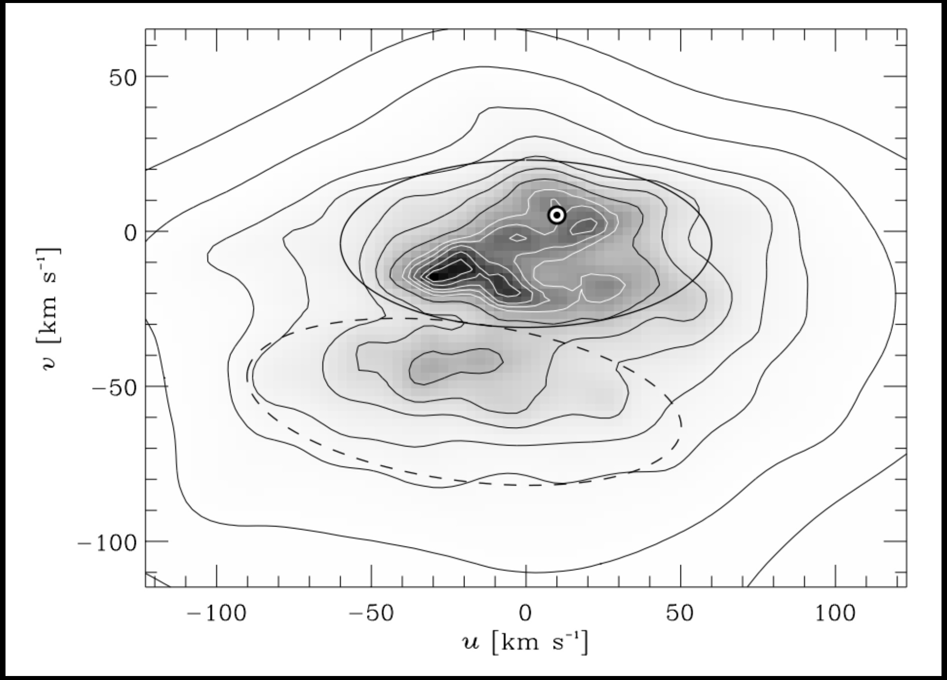
Modelling the Galactic bar and spiral arms

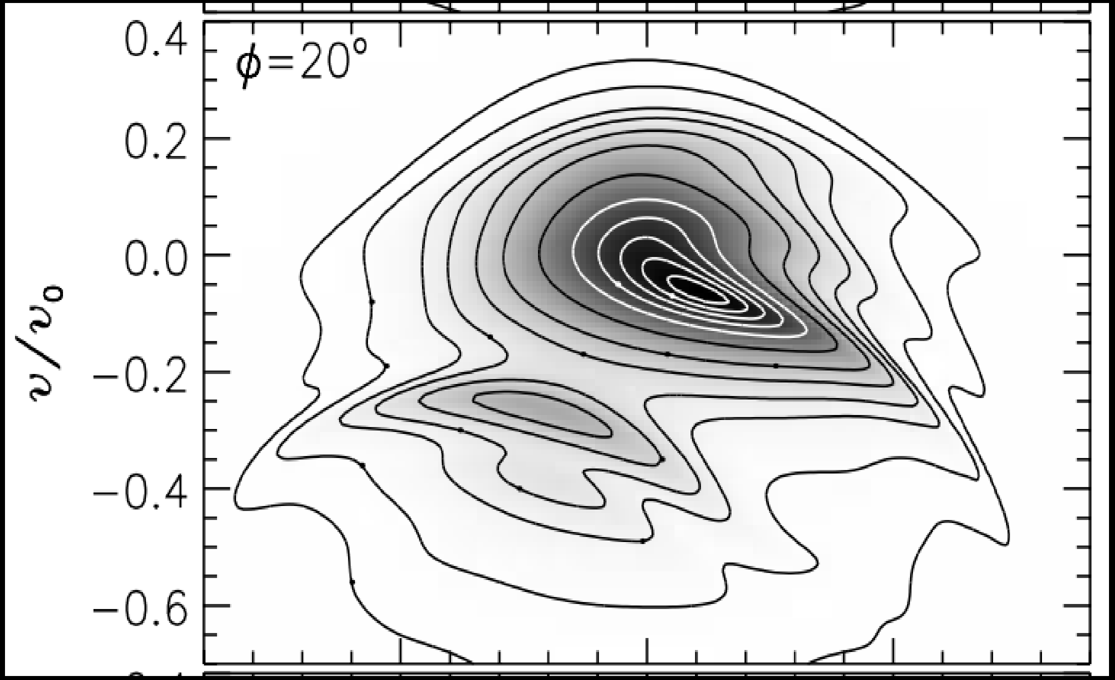
- The Milky Way is known to be a barred galaxy.
  - But there is still some disagreement on length, pattern speed & structure.
- The Milky Way is known to be a spiral galaxy.
  - But there is still disagreement on the number of arms, and their pitch angle, pattern speed, strength, etc.
  - There is also disagreement on the nature of spiral structure itself.



Modelling the Galactic bar – short & fast

- One way of modelling the bar, involves fitting models to observed Solar neighborhood kinematics.
  - E.g the Hercules stream



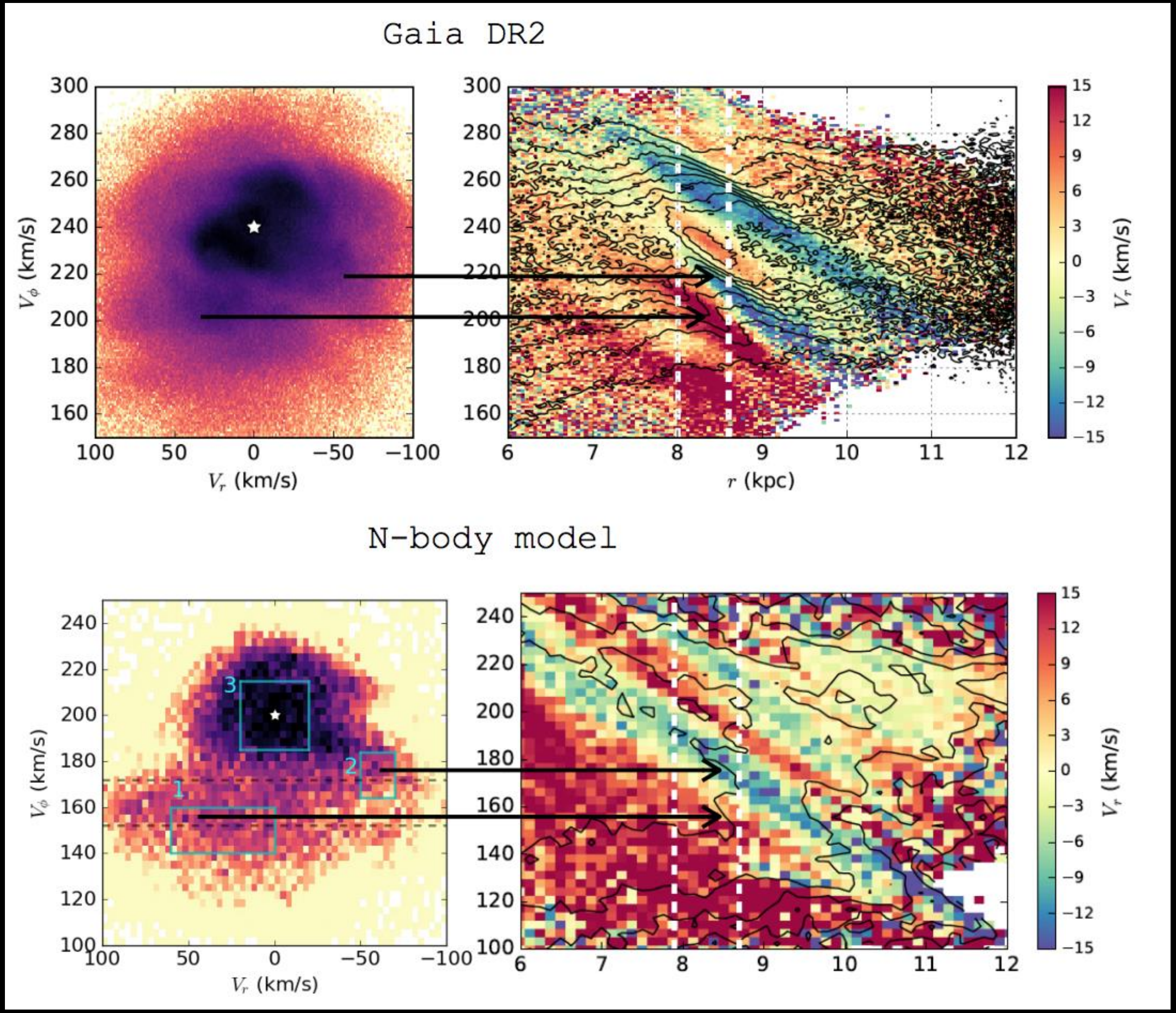


Dehnen (2000)

Modelling the Galactic bar – short & fast

- Fragkoudi et al. (2019) show that a short fast bar in an Nbody isolated disc matches Gaia data.

- Hercules stream and stripes of inwards/outwards moving stars in R-V_Φ plane.



Modelling the Galactic bar – long & slow

- Portail et. al (2017) use the M2M method to model the bar.
- Constraints from a previous mass model (Wegg et al. 2015), and BRAVA, OGLE & ARGOS bulge kinematics.
- They find a longer, slower bar.

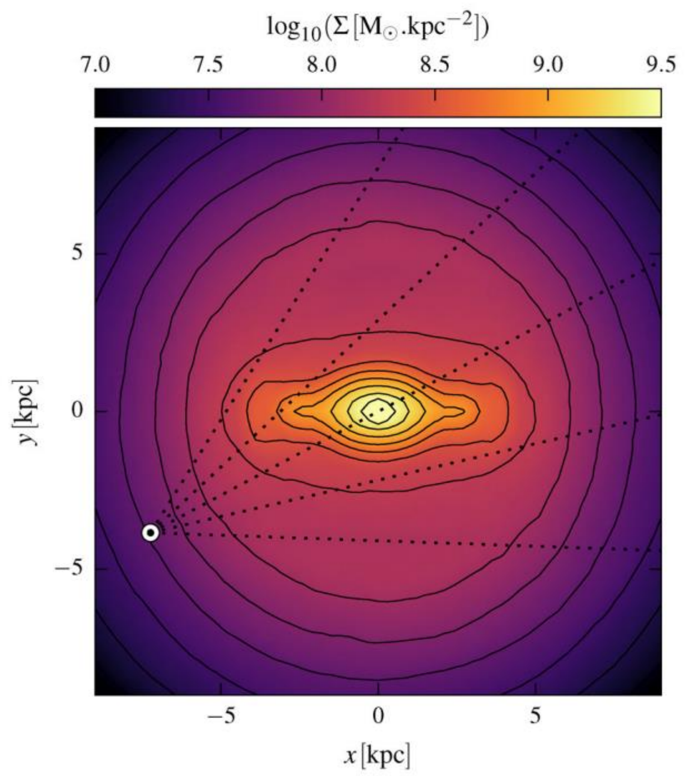
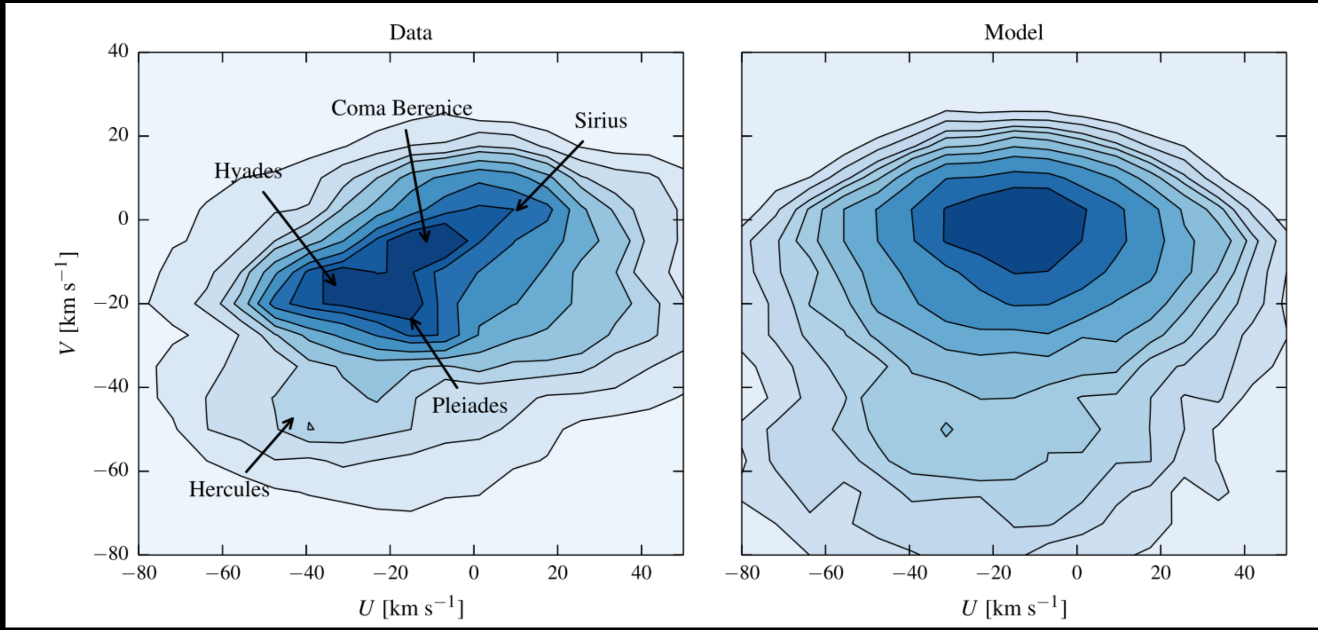
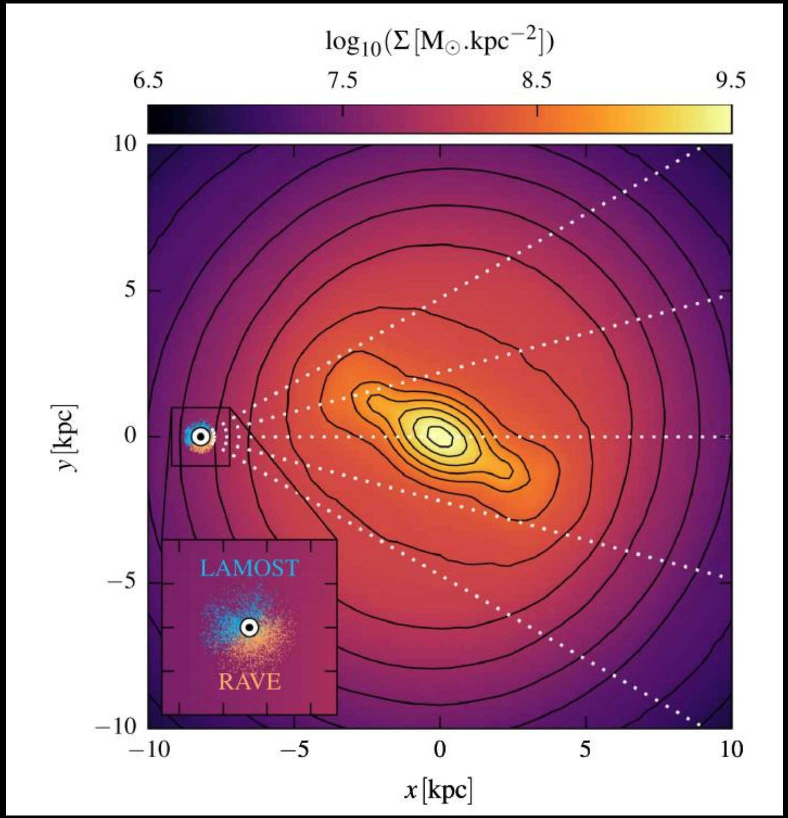


Figure 20. Surface density map of our best model. The bar extends to 5.3 kpc from the centre and rotates at $\Omega_b = 40\,\mathrm{km\,s^{-1}\,kpc^{-1}}$. The dotted lines originating from the Sun (dot symbol) indicate sightlines with galactic longitudes $l = -30°, -15°, 0°, +15°$ and $+30°$.

Modelling the Galactic bar – long & slow

- Also (mostly) reproduces the Hercules stream, and the other groups can be from spirals or other resonances.

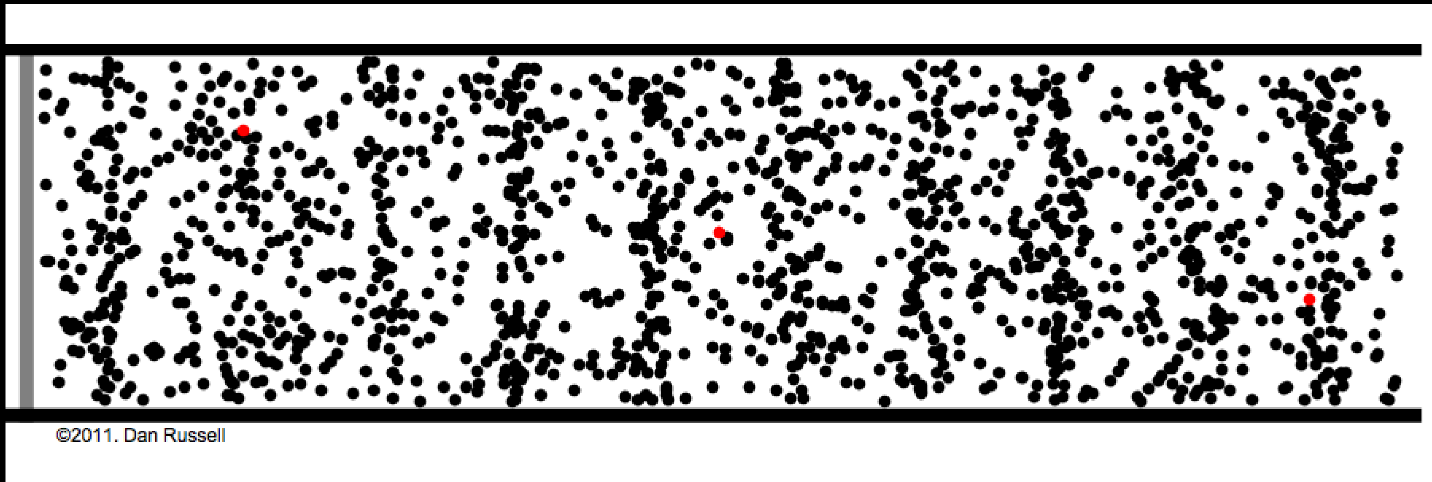


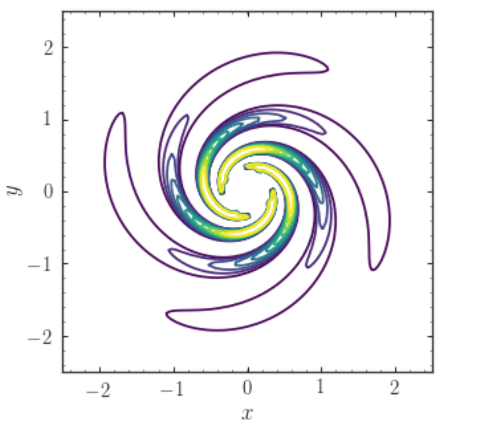


Perez-Villegas et al. (2017)

Modelling the spiral structure – Density Waves

- Stars in the center of Galaxies have shorter orbital periods than stars further out.
- So, if stars move with the spiral arms, they should wind up over time and get disrupted. However, we see lots of ‘Grand design’ spirals.
- This is known as the winding dilemma.
- But if stars move as a wave, they can be long lived (Lin & Shu 1964)

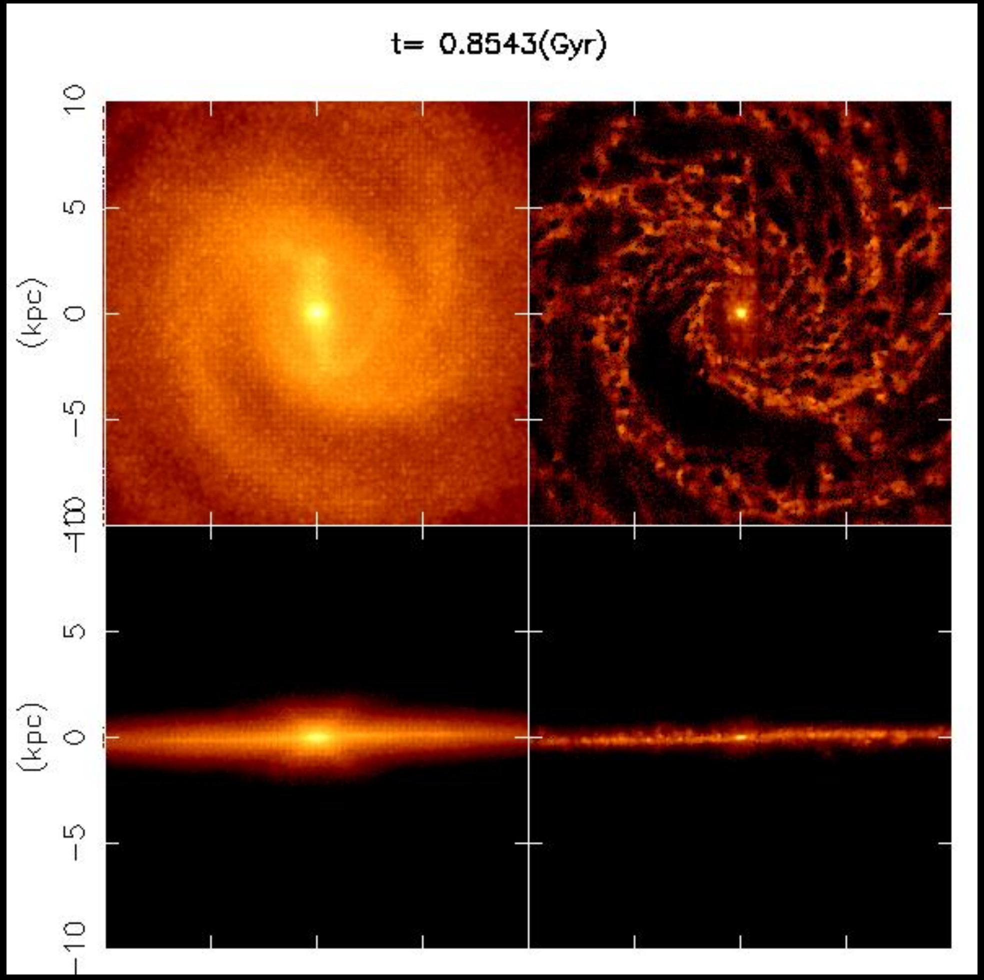




Modelling the spiral structure – Not waves

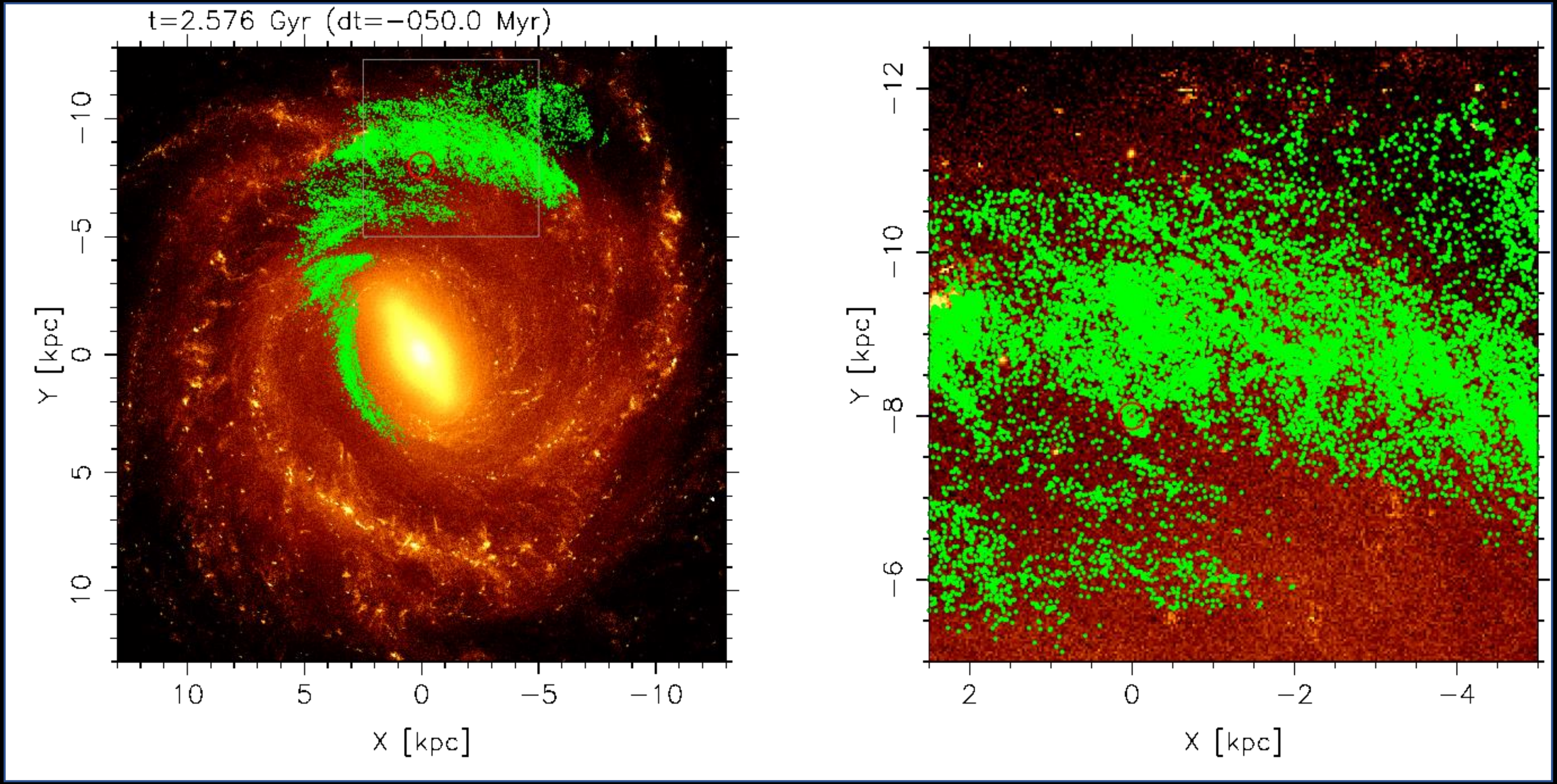
- But, despite what I just said, N-body simulations, which are supposed to represent 'real' dynamics, just don't make density wave spirals.

- Instead they grow and then disrupt, moving mostly with the stars.



R J J Grand

Modelling the spiral structure – Not waves

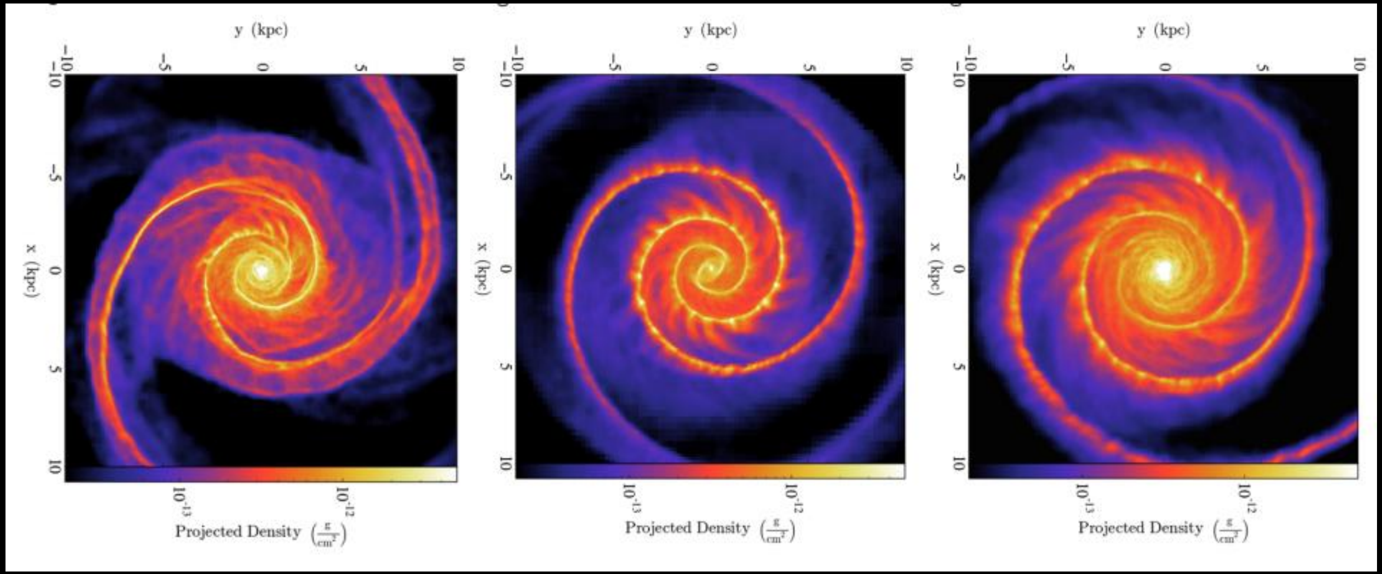


Baba et al. (2017)

This is still an open question

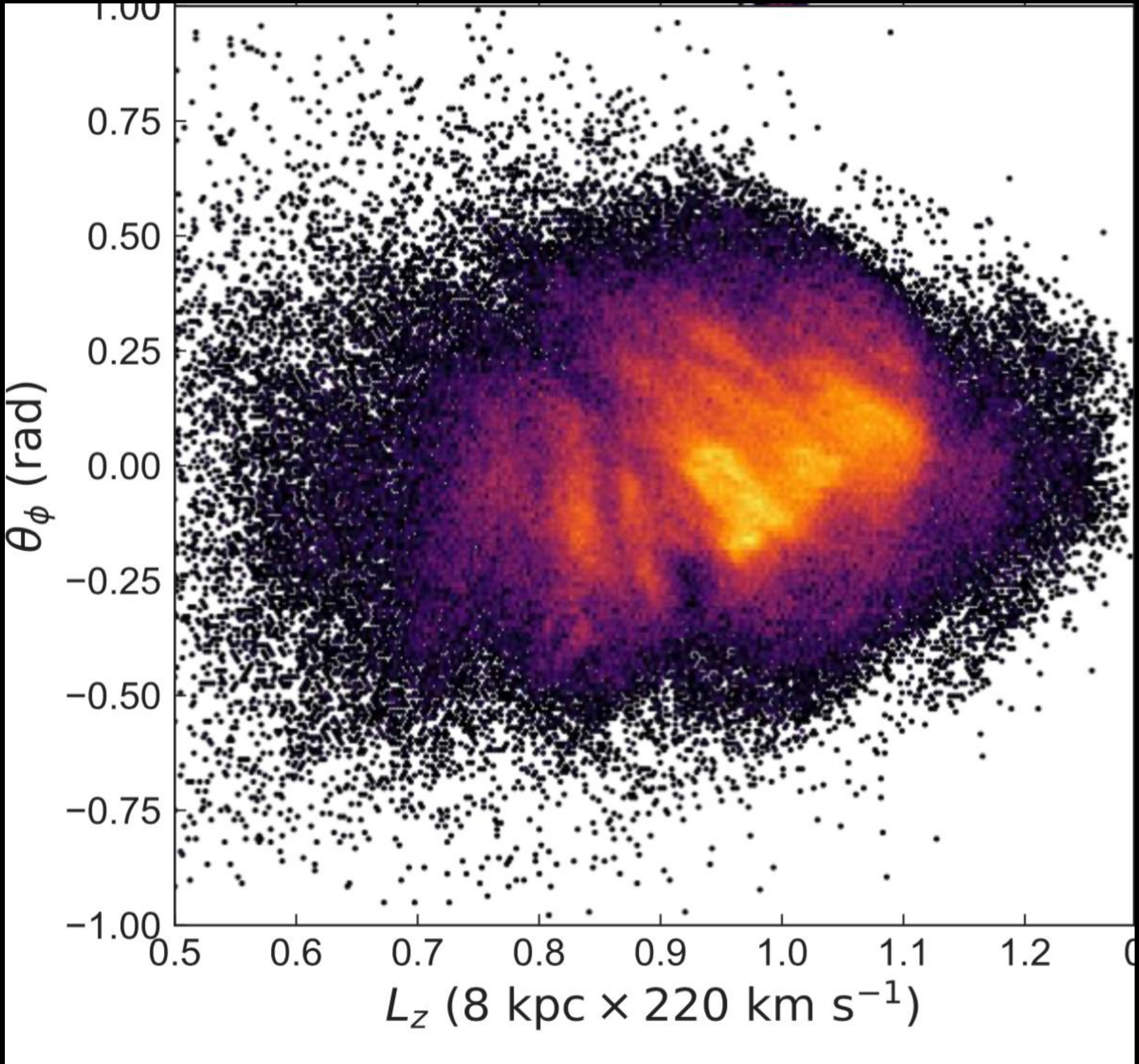
- The transient winding arms can also be modelled as the superposition of multiple modes with a fixed pattern speed (e.g. Sellwood & Carlberg 2014).
- Also, spirals can be tidally driven by satellites.

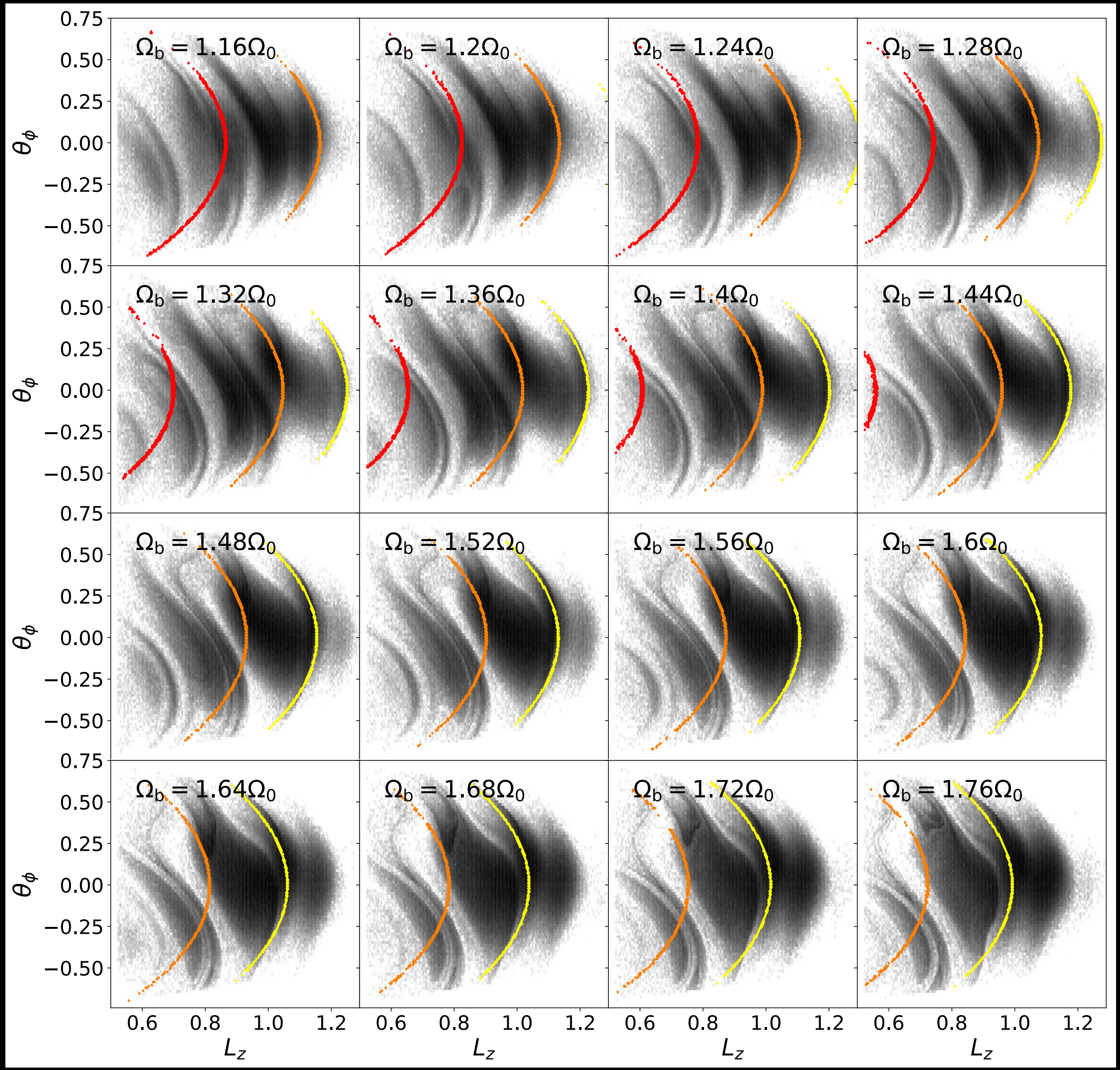
Transient winding | Density wave | Tidal interaction



Pettitt et al. (2016)

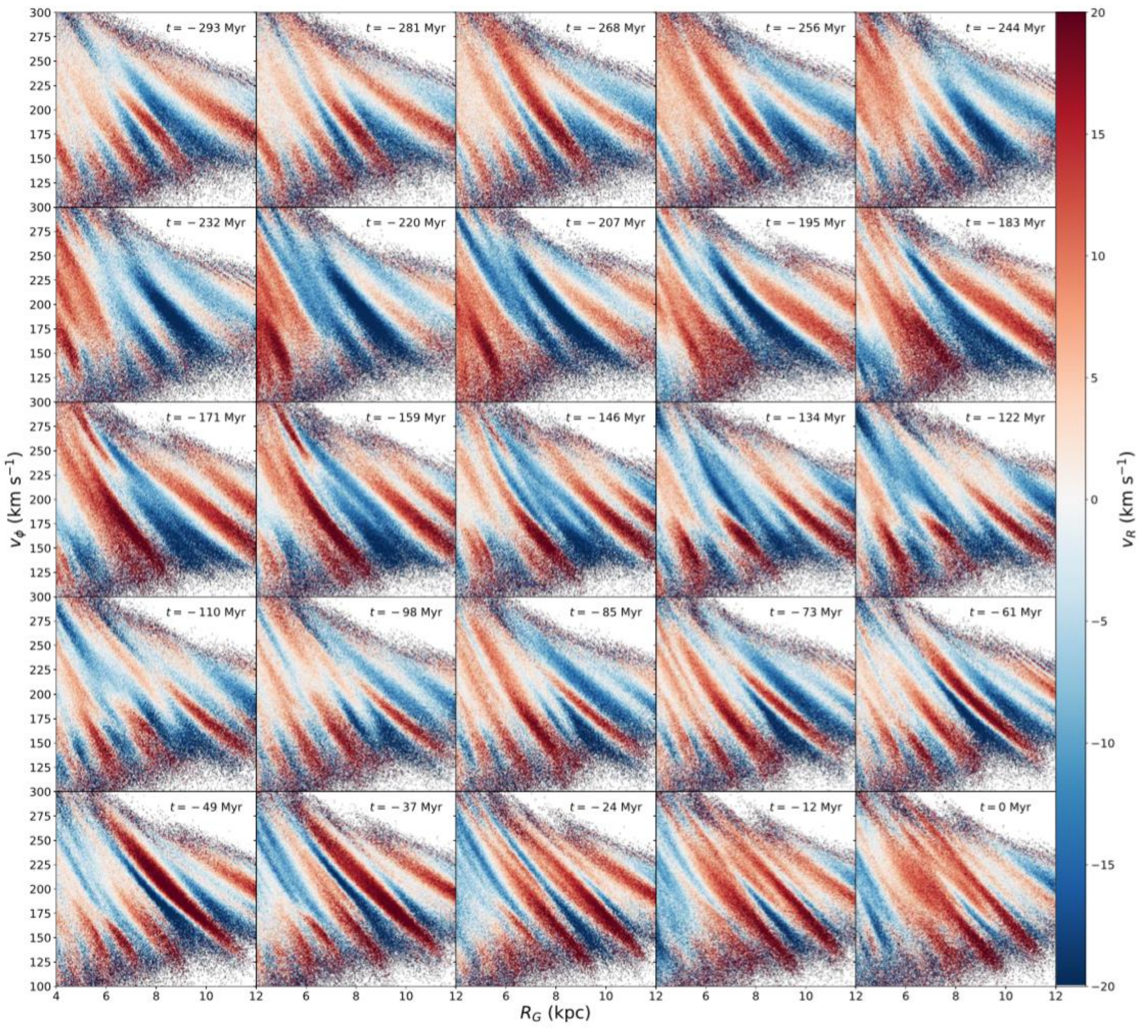
Combining both bars & spirals - Bar change



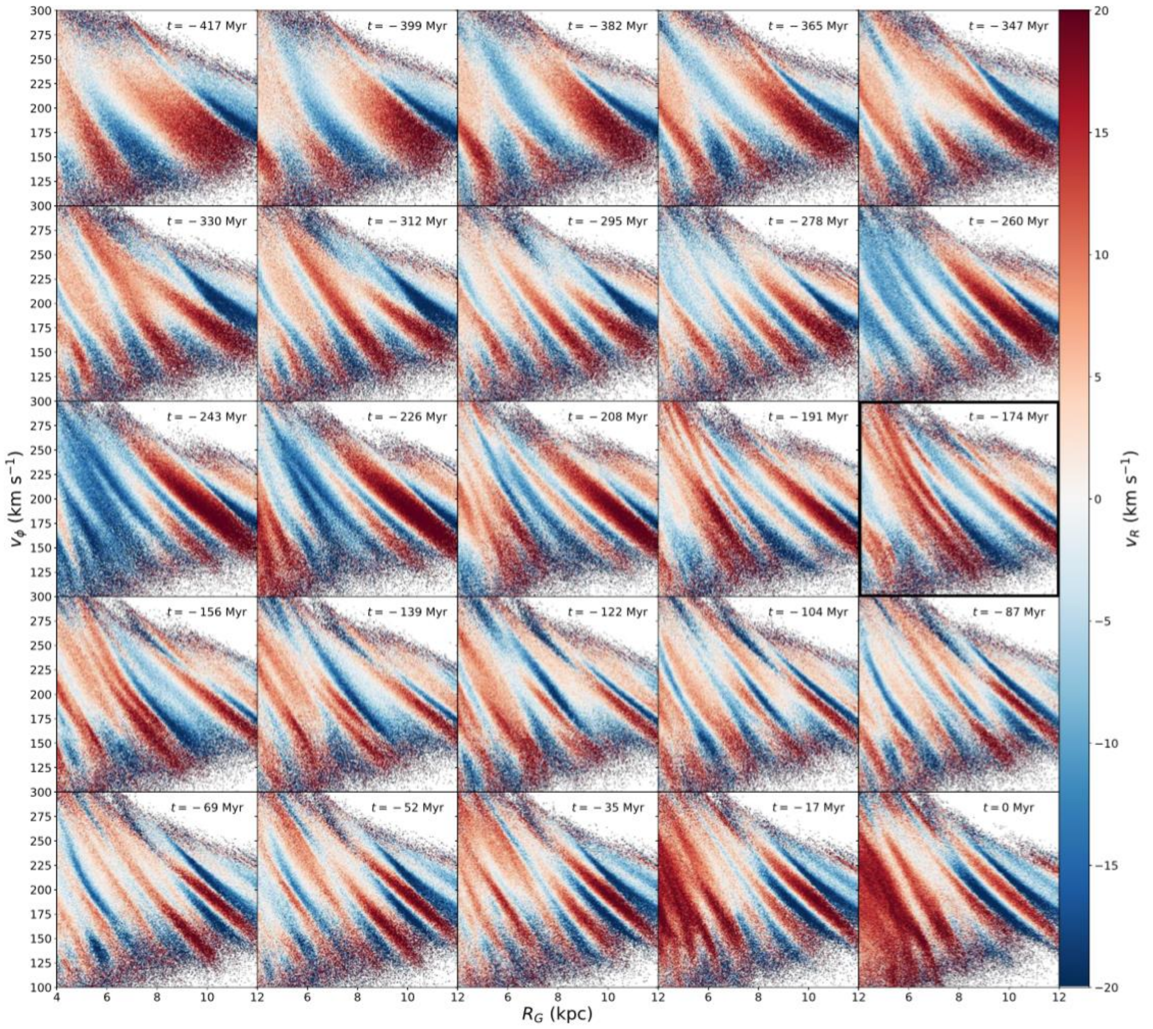


Hunt et al. (2019)

Combining both bars & spirals – Time change



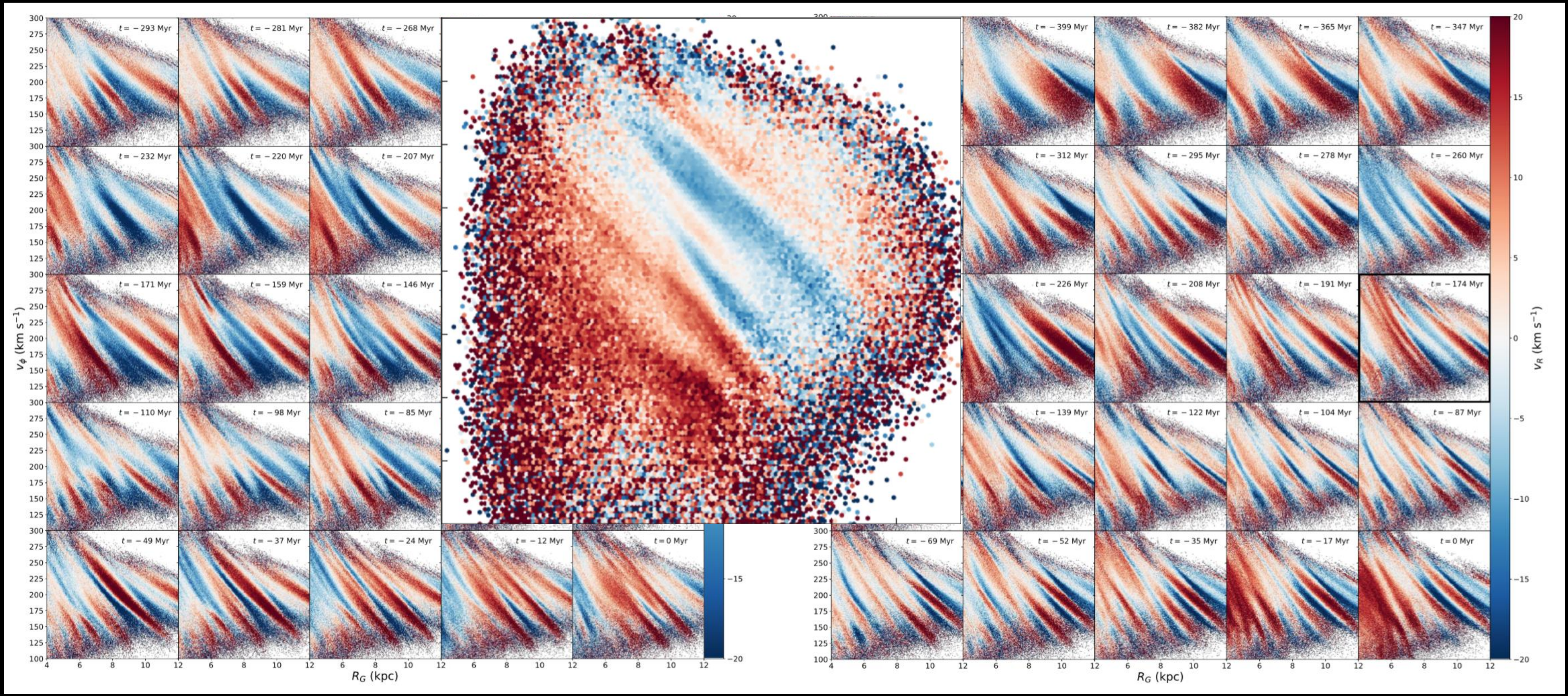
Shot & fast + spirals



Long & slow + spirals

Hunt et al. (2019)

Combining both bars & spirals – Time change



Shot & fast + spirals

Long & slow + spirals

Hunt et al. (2019)

Summary

- Dynamical models help us compensate for observational errors & bias.
- Most methods have some requirement of symmetry or knowledge of the potential.
- N-body methods don't have this restriction but difficult to tailor.
- Used for clusters, galaxies, cosmological sims, with or without gas
- Galaxy sims test bar & spiral structure -> MW structure uncertain.
- Bar: how long/fast? Spirals: How many/what type?
- Not always easy to disentangle.

Tutorial later!

- If you want to follow the galpy tutorial after lunch, please install galpy
- And, astroquery
- They'll be used in the exercise later, but as long as one person per group has it this is ok (in case you can't get it to work!)