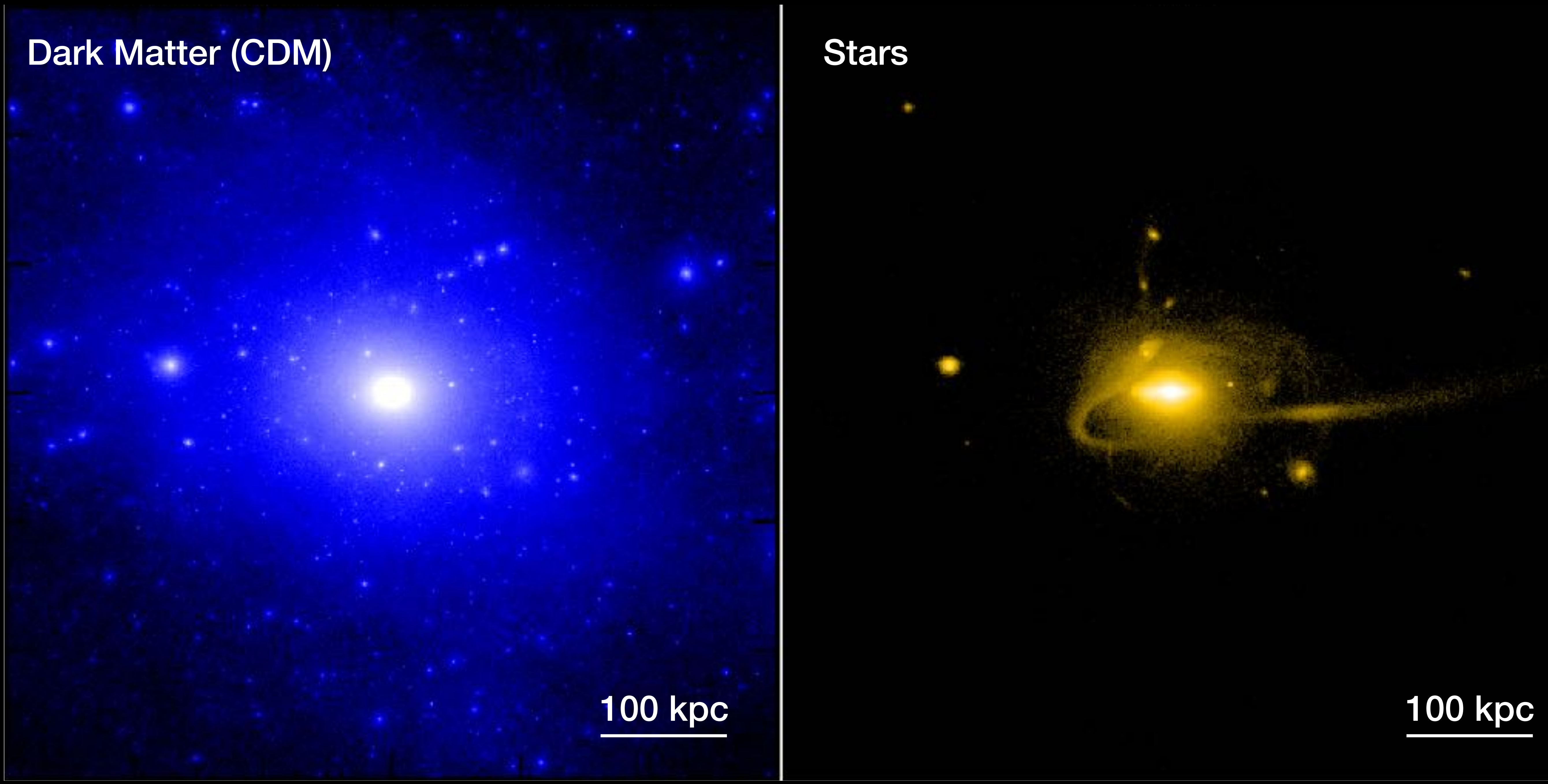
Weighing the Galaxy

Robyn Sanderson

ISSS, Summer 2019

getting oriented: a simulated MW analog



FIRE-2 simulation *m12i*, Wetzel et al 2016

A simple mass estimator - the rotation curve

$$\overrightarrow{F}_{\text{centrip}} = \overrightarrow{F}_{\text{grav}}$$

$\Big\downarrow$ Spherical symmetry

$$\overrightarrow{F} = F_r \hat{r}$$

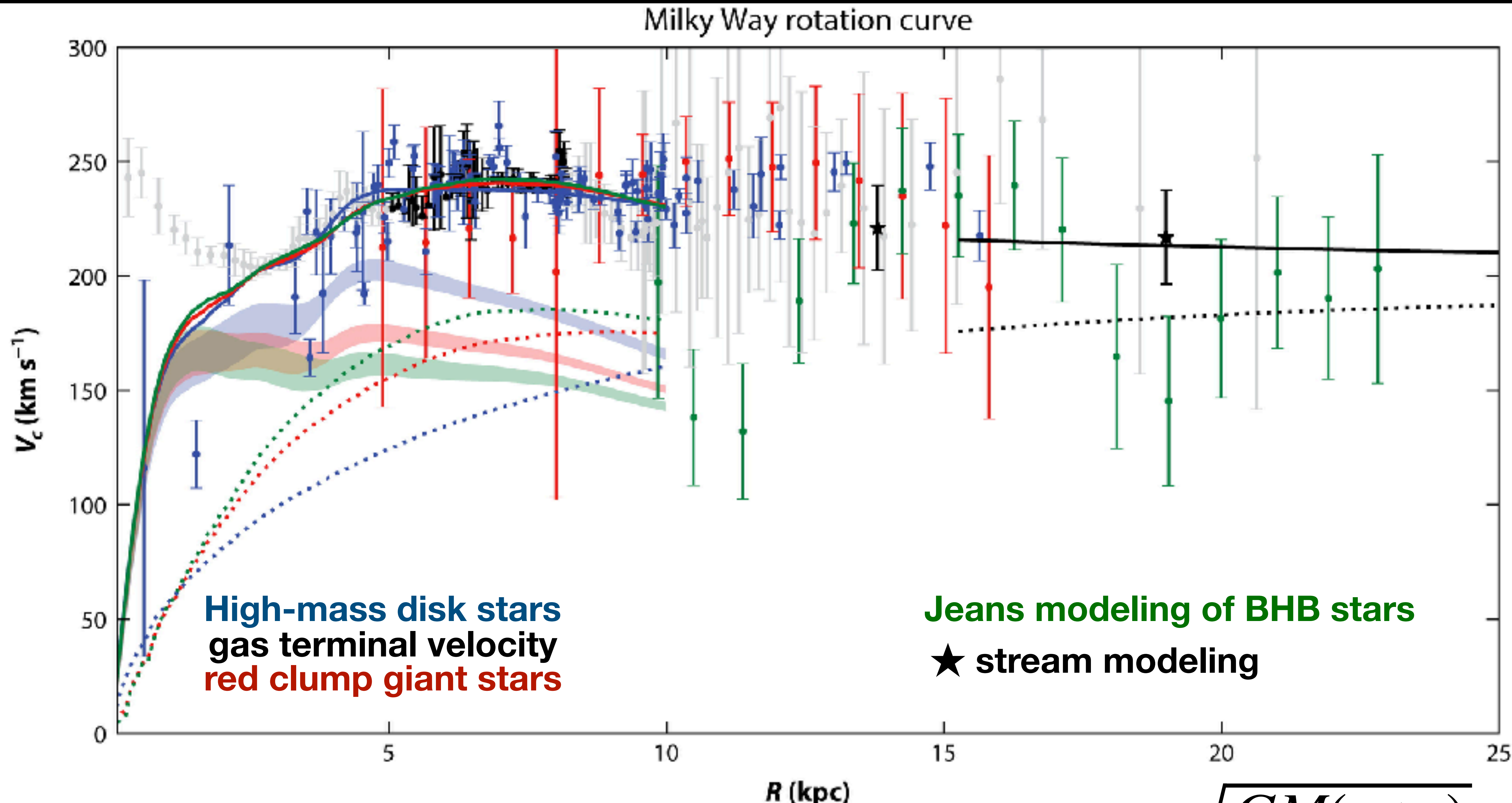
$\Big\downarrow$ Gauss's Law

$$\frac{v_c^2(r)}{r} = \frac{GM(<r)}{r^2}$$

$$\Big\downarrow$$

$$\boxed{v_c(r) = \sqrt{\frac{GM(<r)}{r}}}$$

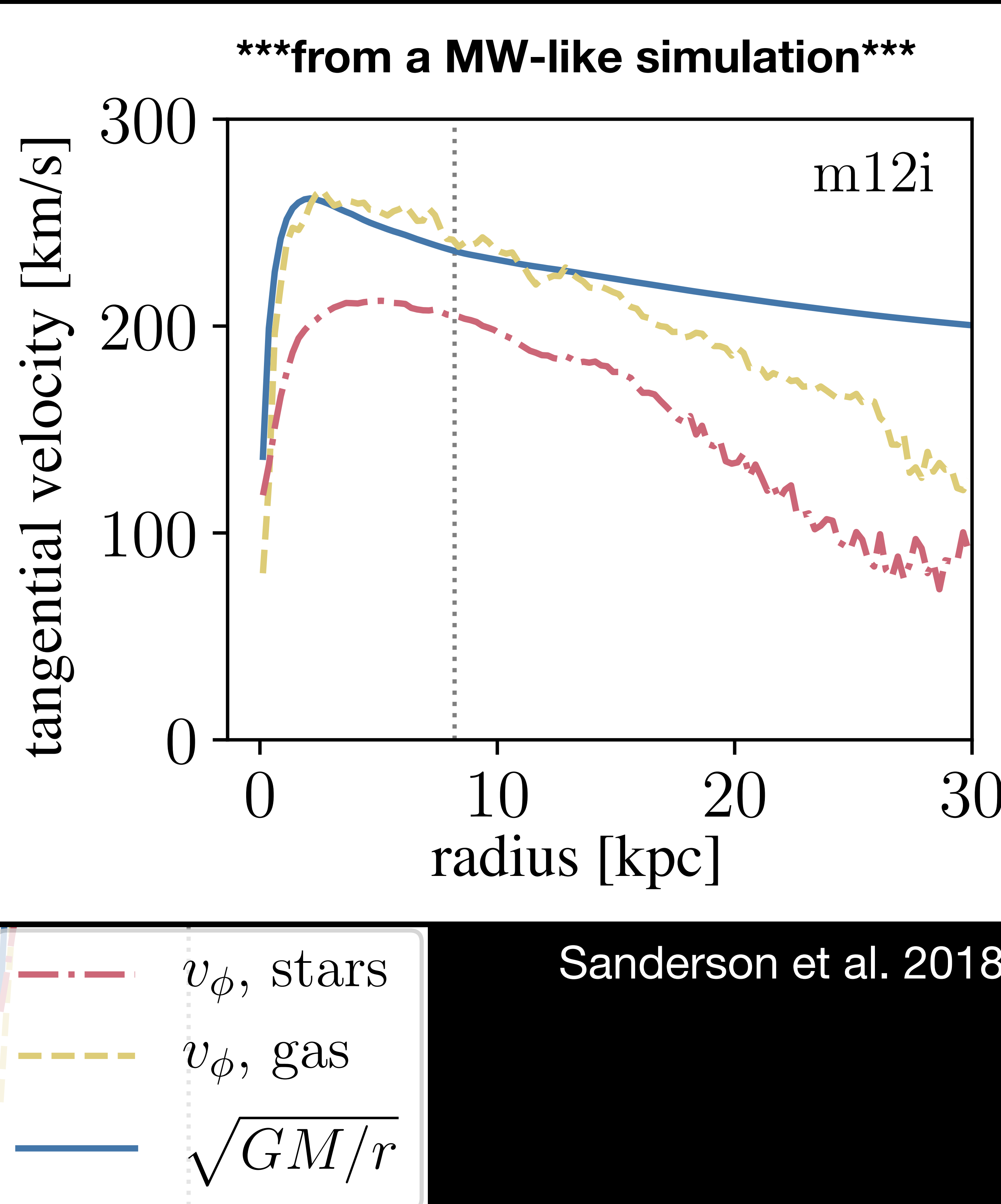
the rotation curve before Gaia



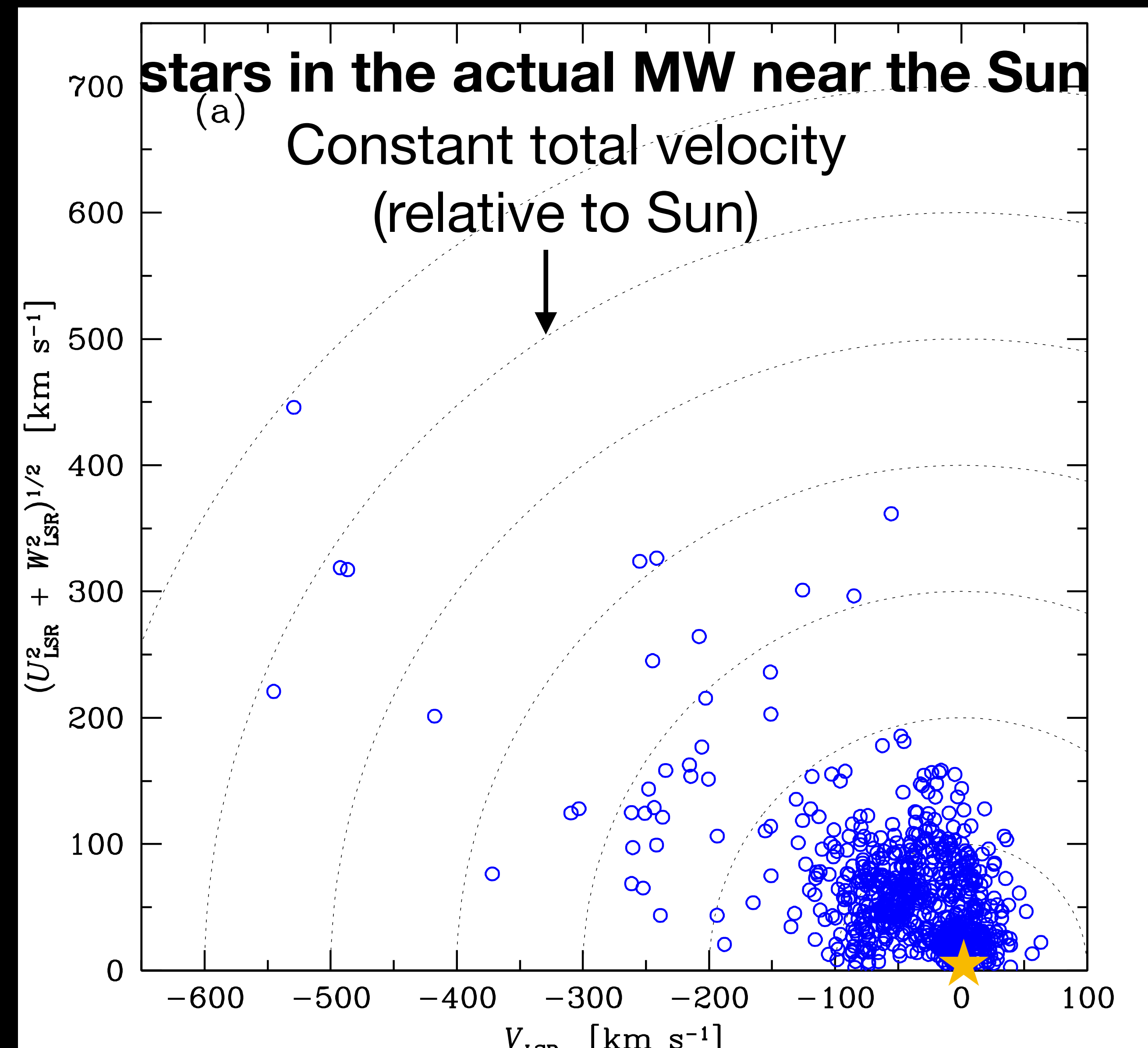
Bland-Hawthorn J, Gerhard O. 2016.
Annu. Rev. Astron. Astrophys. 54:529–96

$$v_c(r) = \sqrt{\frac{GM(<r)}{r}}$$

the rotation curve - not just stellar velocities

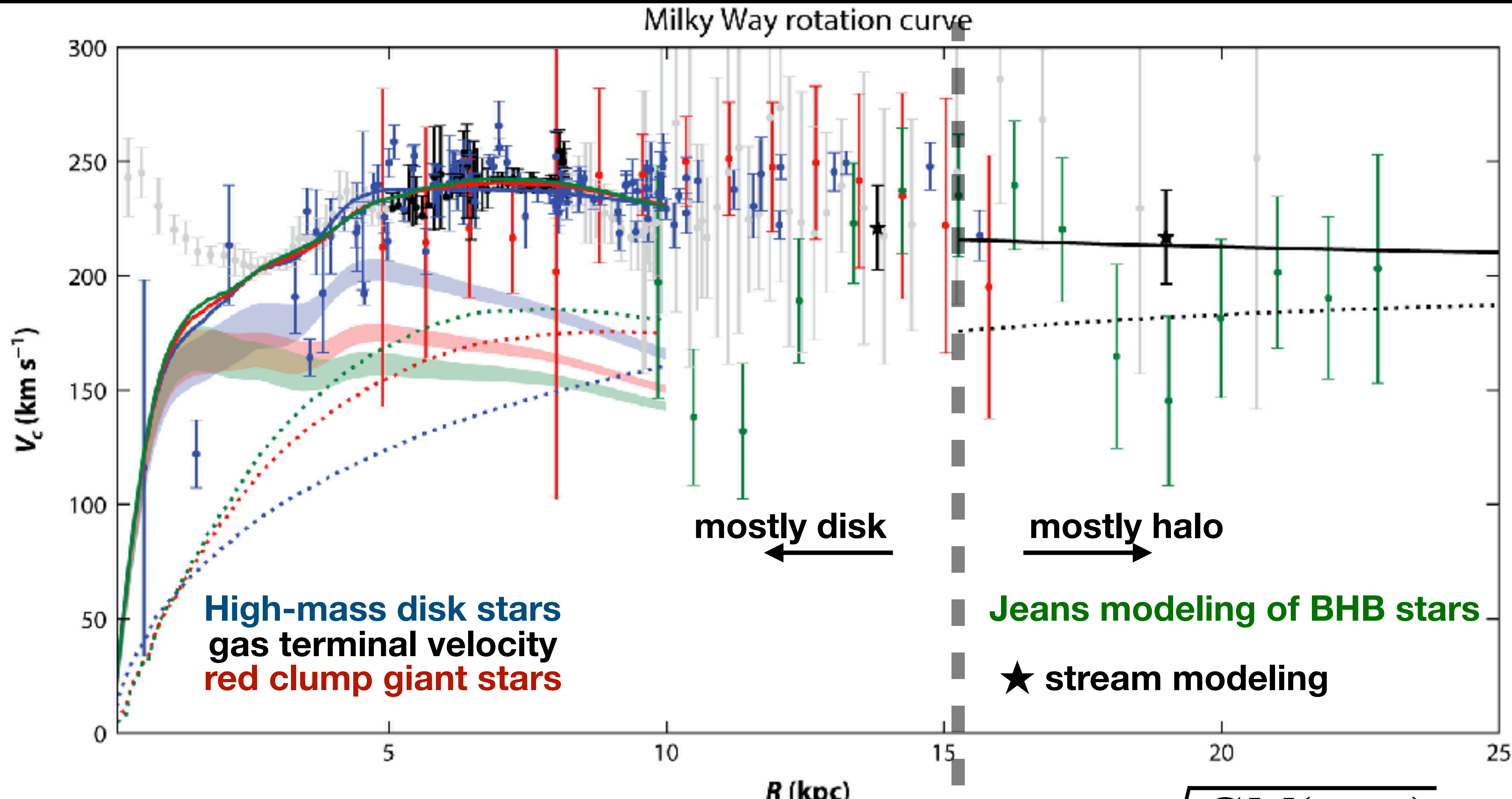


Sanderson et al. 2018



Bensby et al. 2014

the rotation curve before Gaia



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$$v_c(r) = \sqrt{\frac{GM(<r)}{r}}$$

Jeans analysis - a [not quite as] simple mass estimator

Ingredients:

* Collisionless Boltzmann Equation (stars are independent)
* Poisson Equation (relation between mass and gravitational force)

$$\nu(\vec{x}) \equiv \int d^3\vec{v} f(\vec{x}, \vec{v}) \quad n(\vec{x}) = N\nu(\vec{x})$$

phase-space distribution function (DF)

$$\overset{\textit{steady state}}{\cancel{\frac{\partial f}{\partial t}}} + \frac{\partial f}{\partial \vec{x}} \cdot \underbrace{\boxed{\frac{\partial H}{\partial \vec{v}}}}_{=\vec{v}} - \overbrace{\boxed{\frac{\partial H}{\partial \vec{x}}}}^{=-\vec{\nabla}\Phi} \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

CBE, assuming

$$H = \frac{1}{2}v^2 + \Phi(\vec{x})$$

$$\boxed{\vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \frac{\partial \Phi}{\partial \vec{x}} \cdot \frac{\partial f}{\partial \vec{v}} = 0}$$

Take moments of this with respect to velocity to get Jeans eqs.

$$\nabla^2\Phi = 4\pi G\rho(\vec{x}) \quad \rho(\vec{x}) \sim m_* N\nu(\vec{x})$$

Poisson's Equation

Jeans analysis - a [not quite as] simple mass estimator

- Start with collisionless Boltzmann equation
- Multiply by some power of velocity & integrate [over velocity]
- Not generically closed
- To close, make assumptions about the distribution function and/or system symmetries
- for the halo, spherical symmetry & steady state (equilibrium) often assumed

$$\frac{d(\nu\overline{v_r^2})}{dr} + \nu\left(\frac{d\Phi}{dr} + \frac{2\overline{v_r^2} - \overline{v_\theta^2} - \overline{v_\phi^2}}{r}\right) = 0$$

$$\beta \equiv 1 - \frac{\overline{v_\theta^2} + \overline{v_\phi^2}}{2\overline{v_r^2}}$$

$$\frac{d(\nu\overline{v_r^2})}{dr} + 2\frac{\beta}{r}\nu\overline{v_r^2} = -\nu\boxed{\frac{d\Phi}{dr}}$$

this term tells us M(<r) via Poisson eq.

$$\nu \equiv \int d^3\vec{v}\, f(\vec{x}, \vec{v})$$

$$n(\vec{x}) = N\nu(\vec{x})$$

need to count tracers

$$\sigma_{ij}^2 \equiv \overline{v_i v_j} - \overline{v_i}\,\overline{v_j}$$

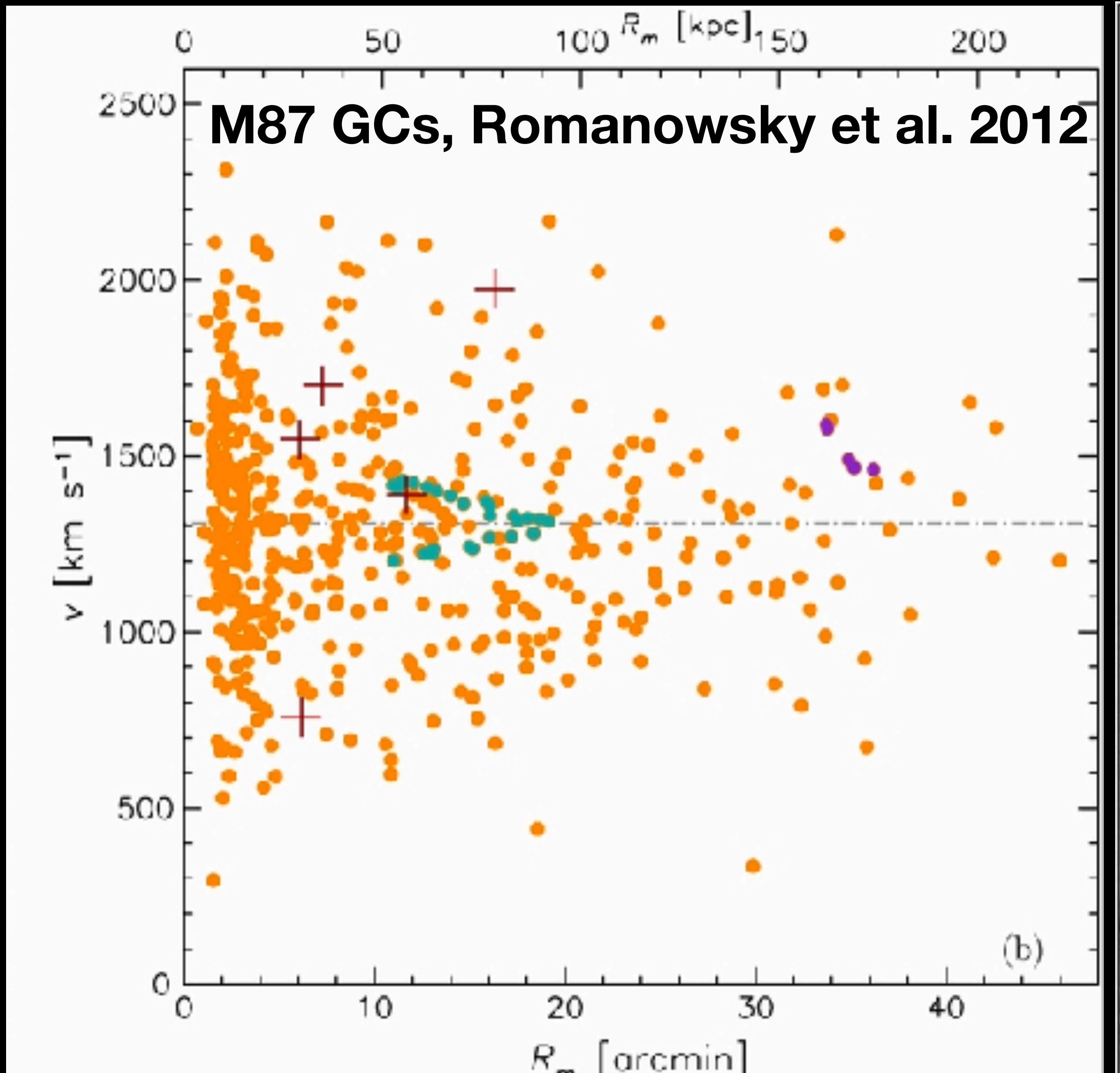
e.g. $\overline{v_i^2} \equiv \sigma_i^2 + (\cancelto{0}{\overline{v_i}})^2$

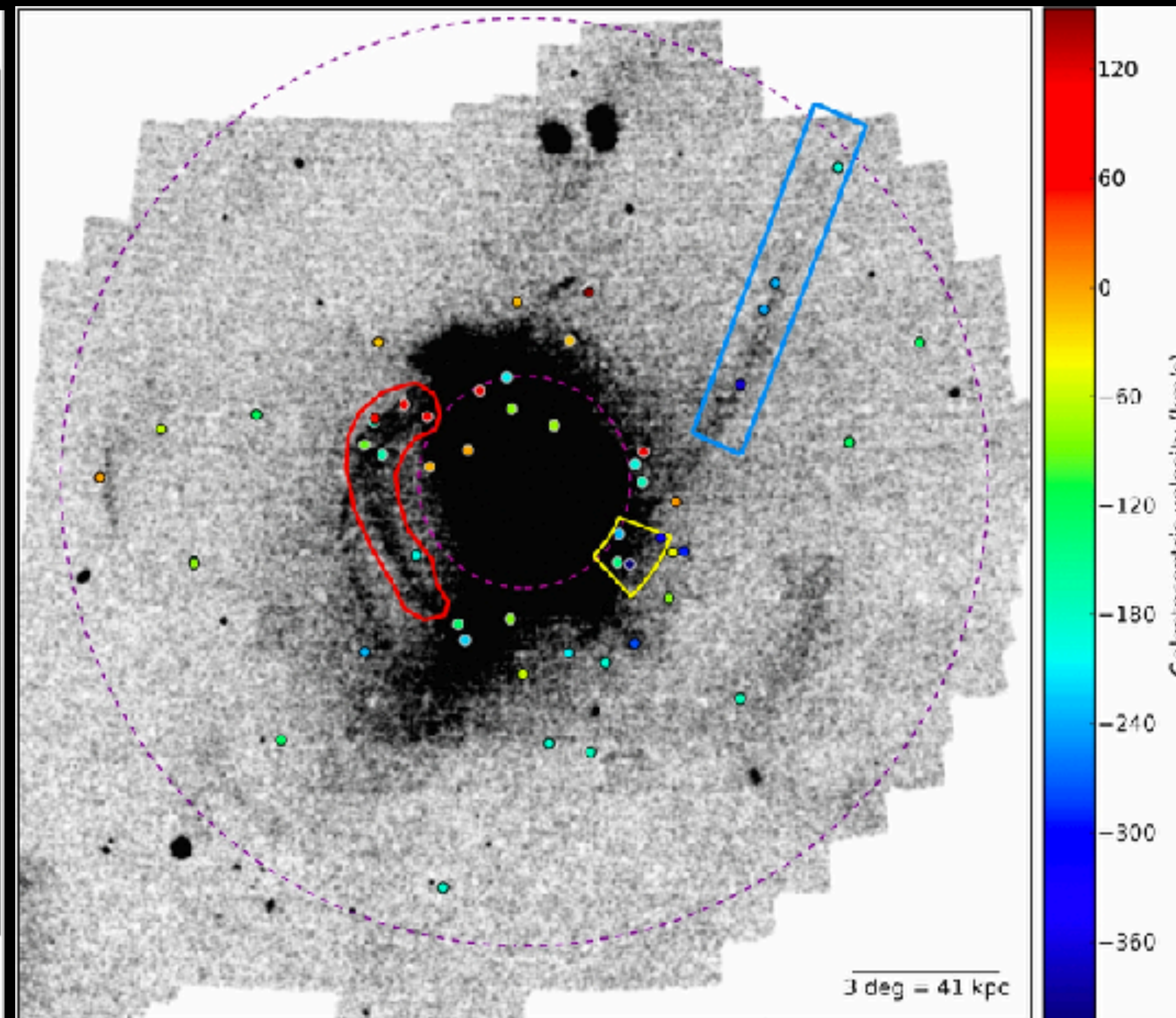
(for θ,ϕ: non-rotating)

we actually measure:

- σ_{los} (before Gaia) and maybe β (with HST or Gaia)
- n(r) and [noisily] dn/dr for some subsample of tracers

Jeans analysis - what is in equilibrium?





M31 GCs, Veljanoski et al. 2013

- What tracers are in equilibrium?
  - Globular clusters?
  - Satellite galaxies?
  - Stars?
- How well do we know β?
- How well do we know ν?

$$\frac{d(\nu\overline{v_r^2})}{dr} + 2\frac{\beta}{r}\nu\overline{v_r^2} = -\nu\frac{d\Phi}{dr}$$

Jeans analysis - what is in equilibrium?

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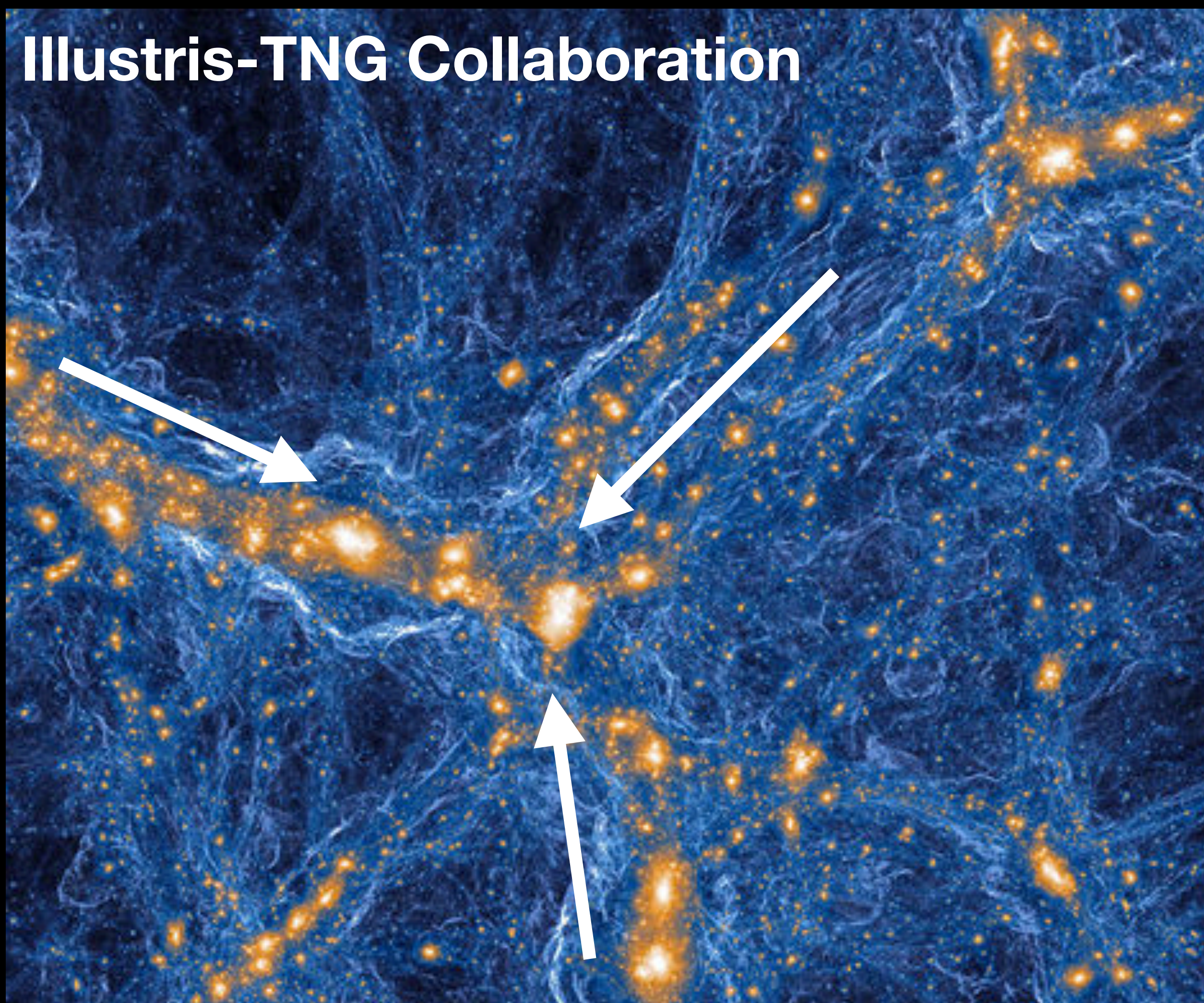

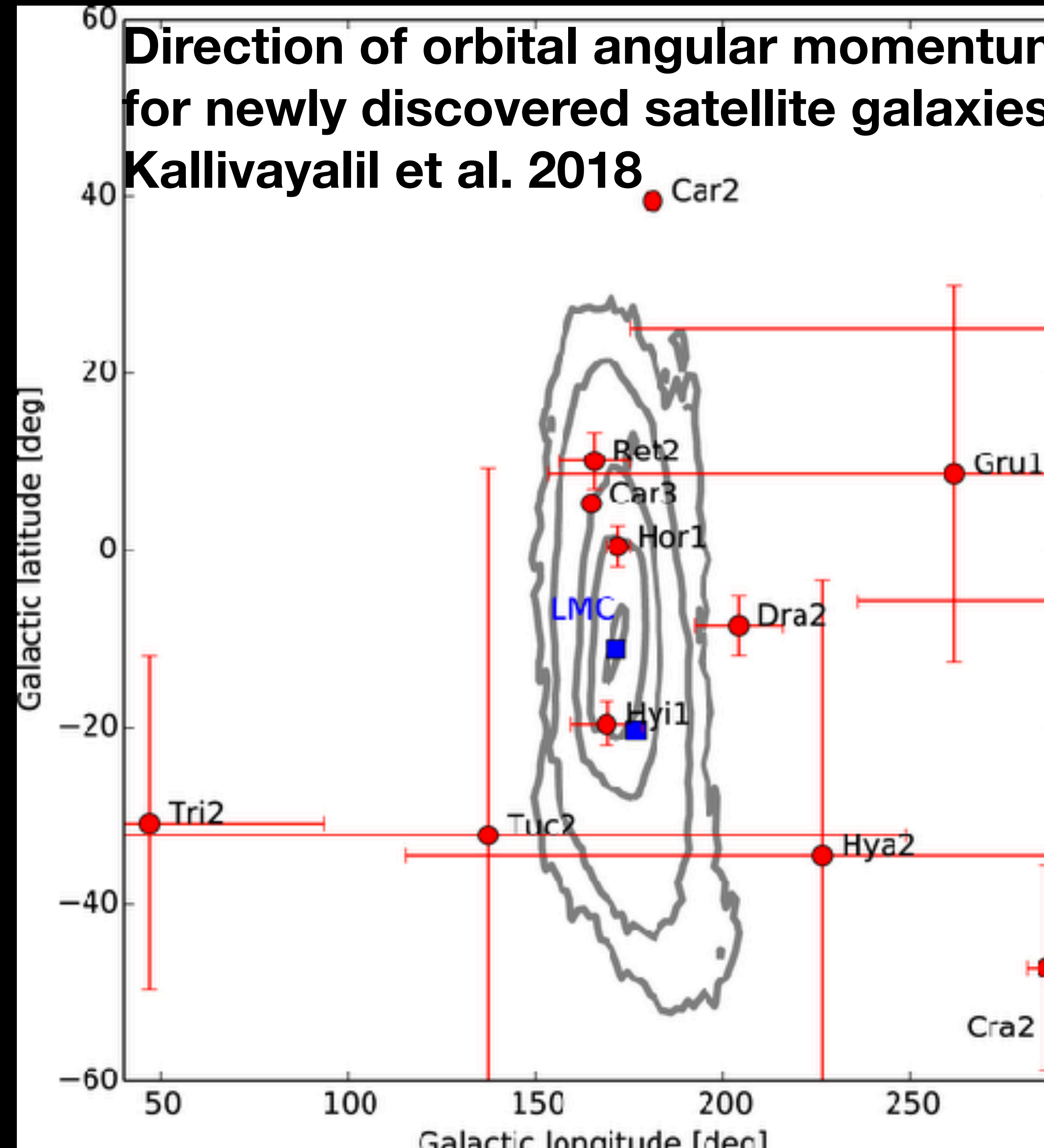

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Jeans analysis - what is in equilibrium?

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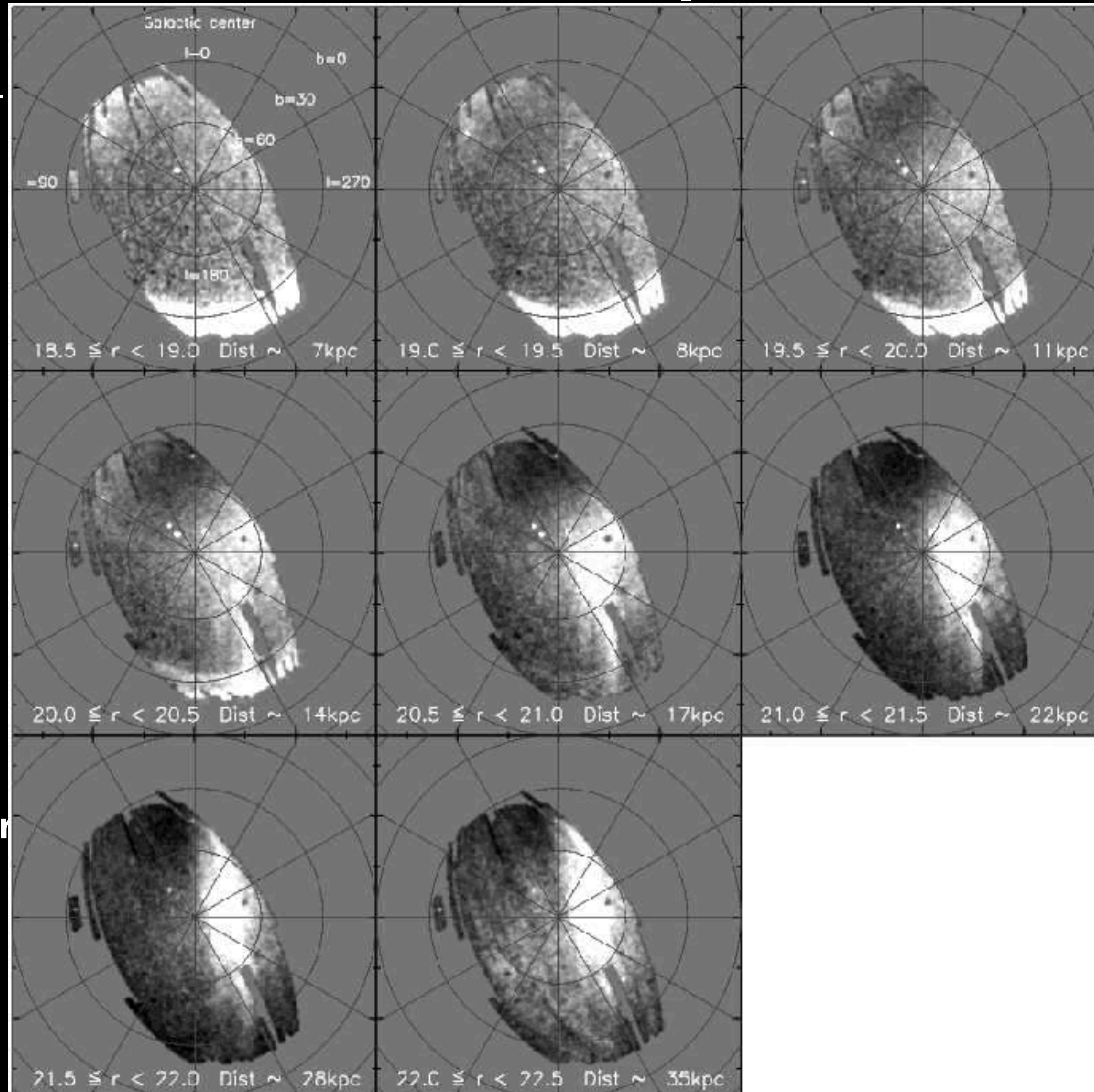

SDSS view of stellar halo at different distances: density-smooth model (Bell et al. 2008)

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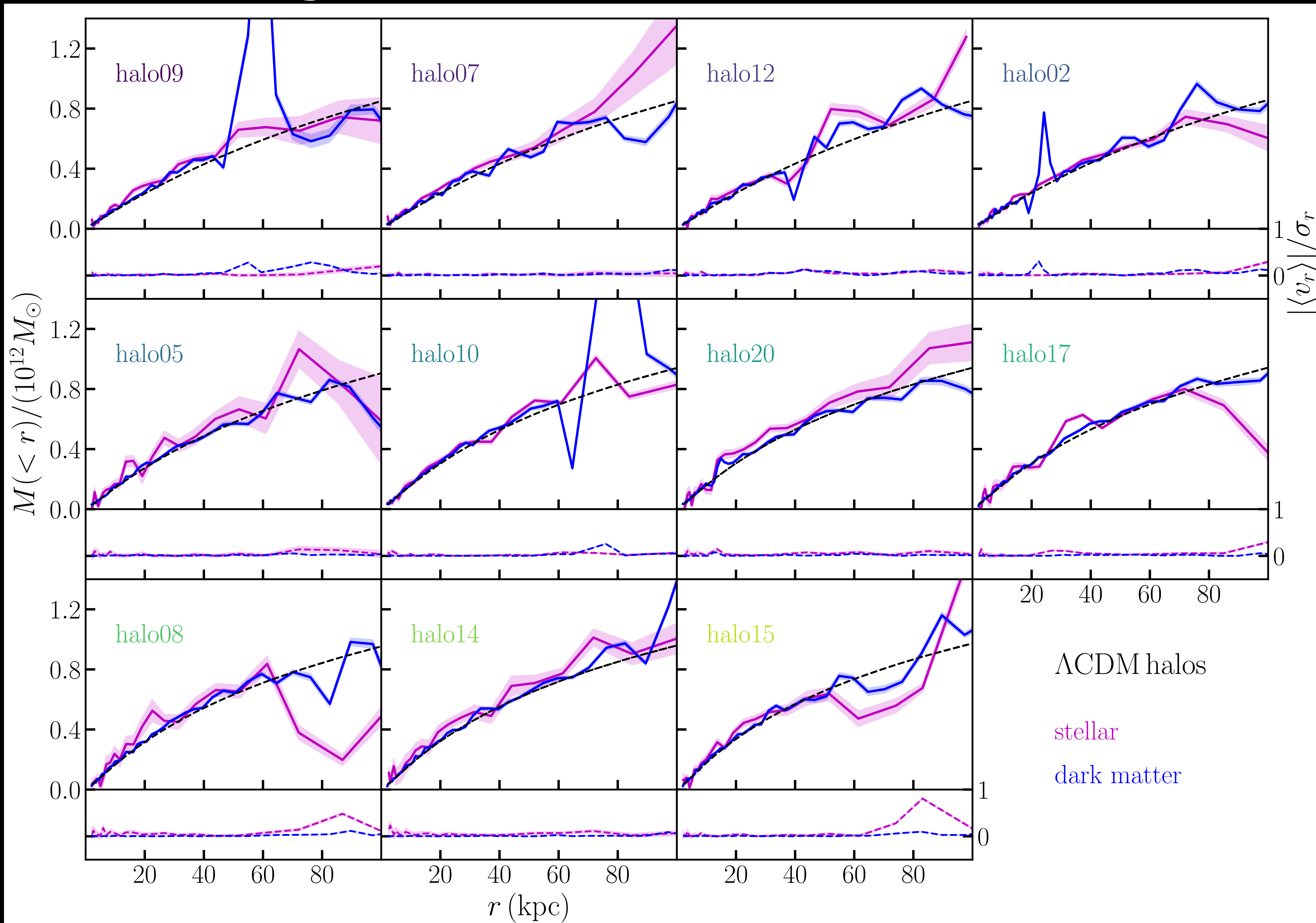
Jeans analysis - what is in equilibrium?



Kafle et al. 2018

- What tracers are in equilibrium?
  - Globular clusters?
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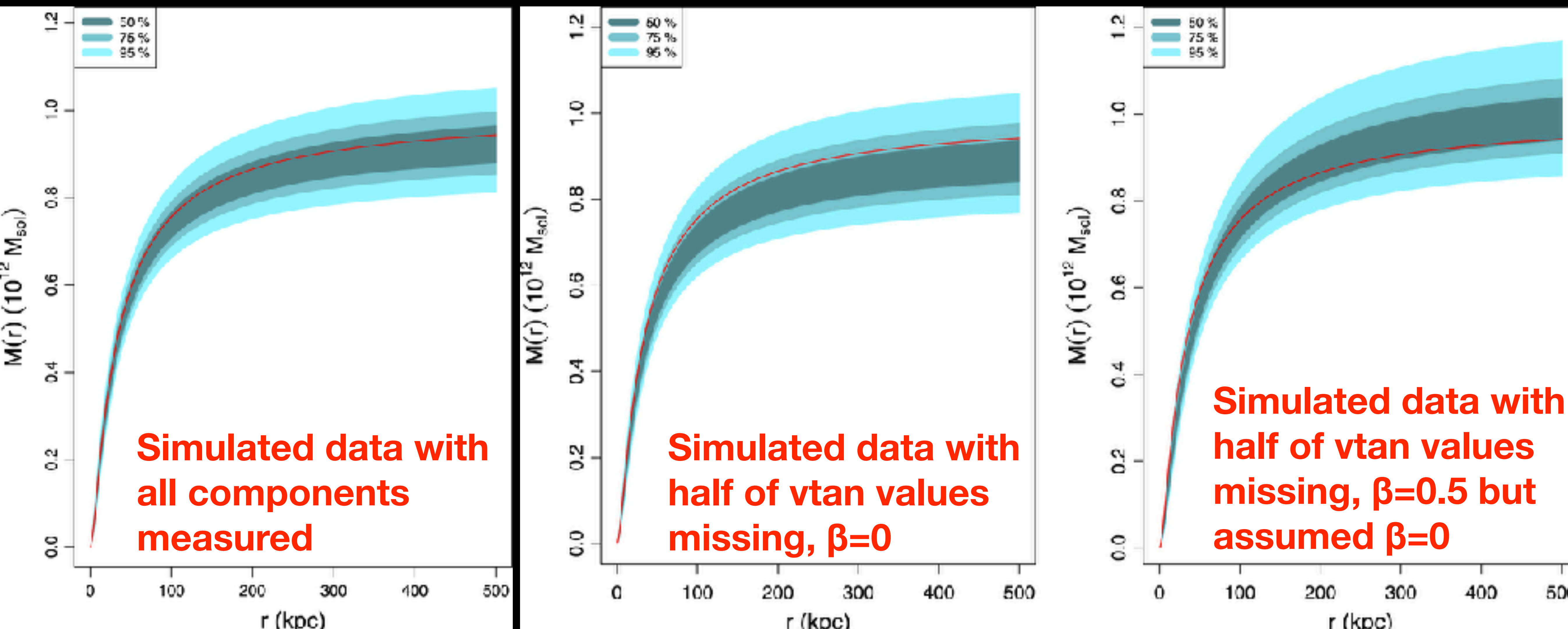
  - Stars?

$$\frac{d(\nu\overline{v_r^2})}{dr} + 2\frac{\beta}{r}\nu\overline{v_r^2} = -\nu\frac{d\Phi}{dr}$$

Jeans analysis - the mass-anisotropy degeneracy

$$\frac{d(\nu\overline{v_r^2})}{dr} + 2\frac{\beta}{r}\nu\overline{v_r^2} = -\nu\frac{d\Phi}{dr}$$



Eadie & Harris 2017

- What tracers are in equilibrium?
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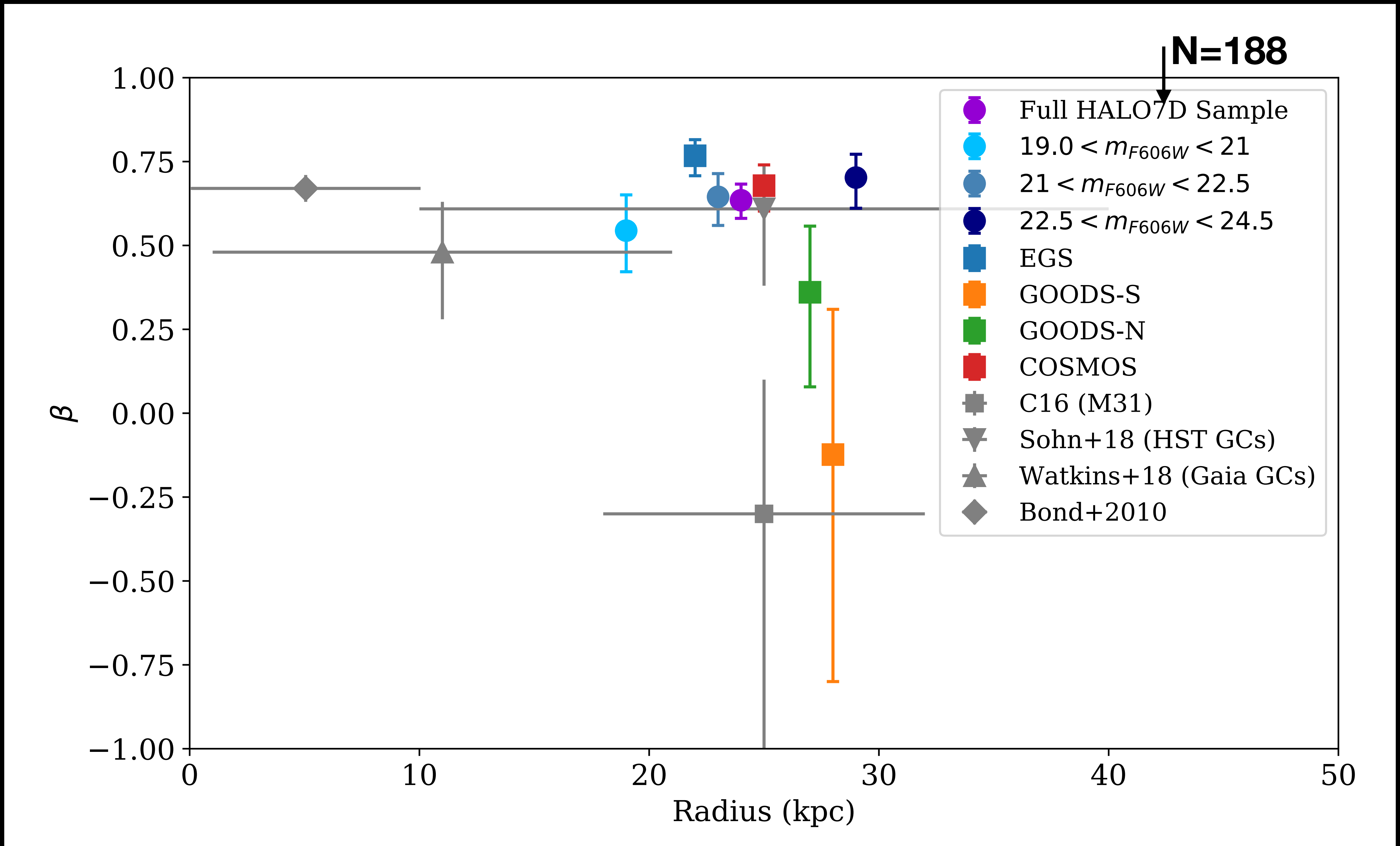
Jeans analysis - the mass-anisotropy degeneracy



Cunningham et al. 2019

- What tracers are in equilibrium?
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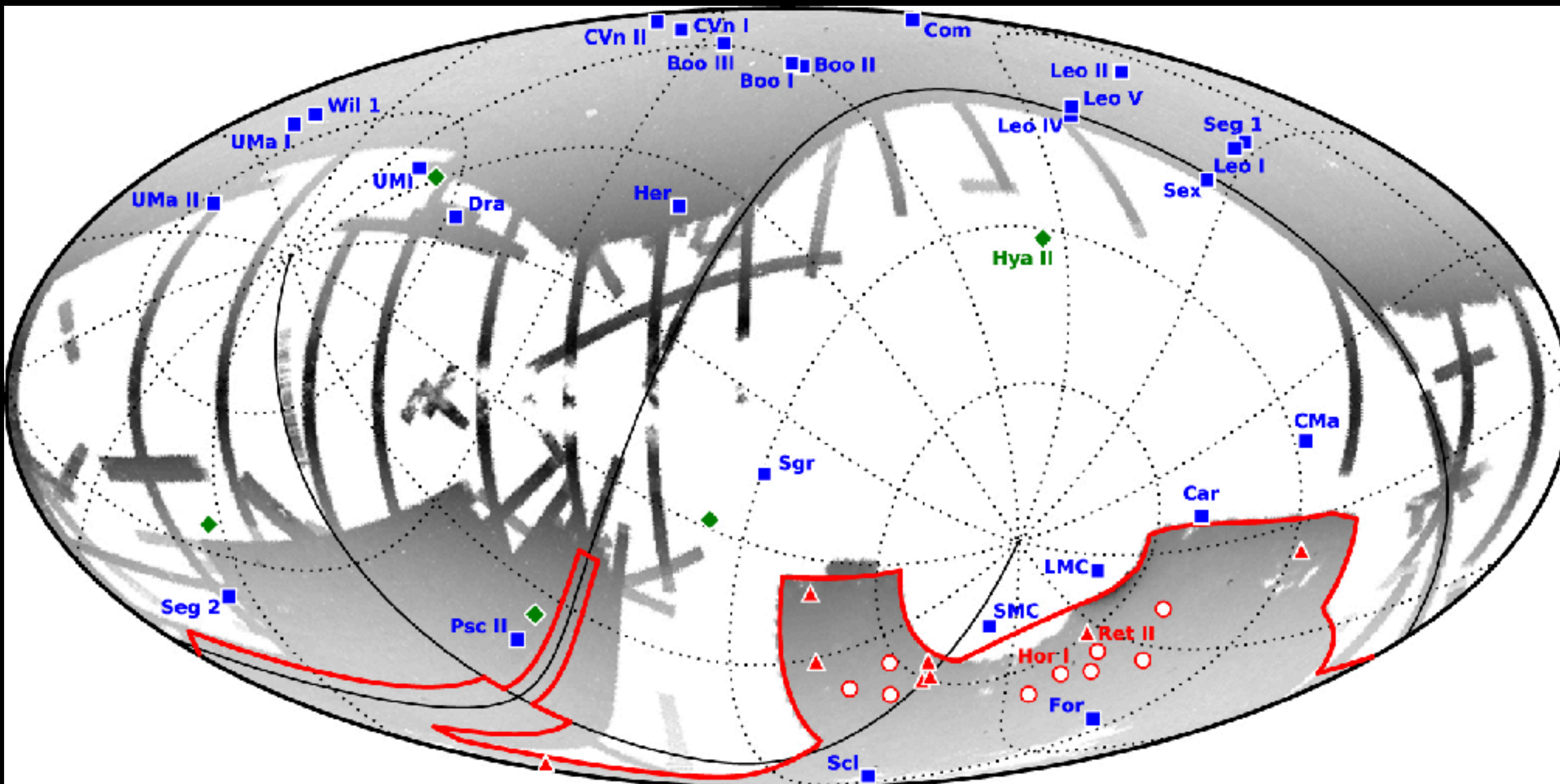
$$\frac{d(\nu\overline{v_r^2})}{dr} + 2\frac{\beta}{r}\nu\overline{v_r^2} = -\nu\frac{d\Phi}{dr}$$

Jeans analysis - the selection function



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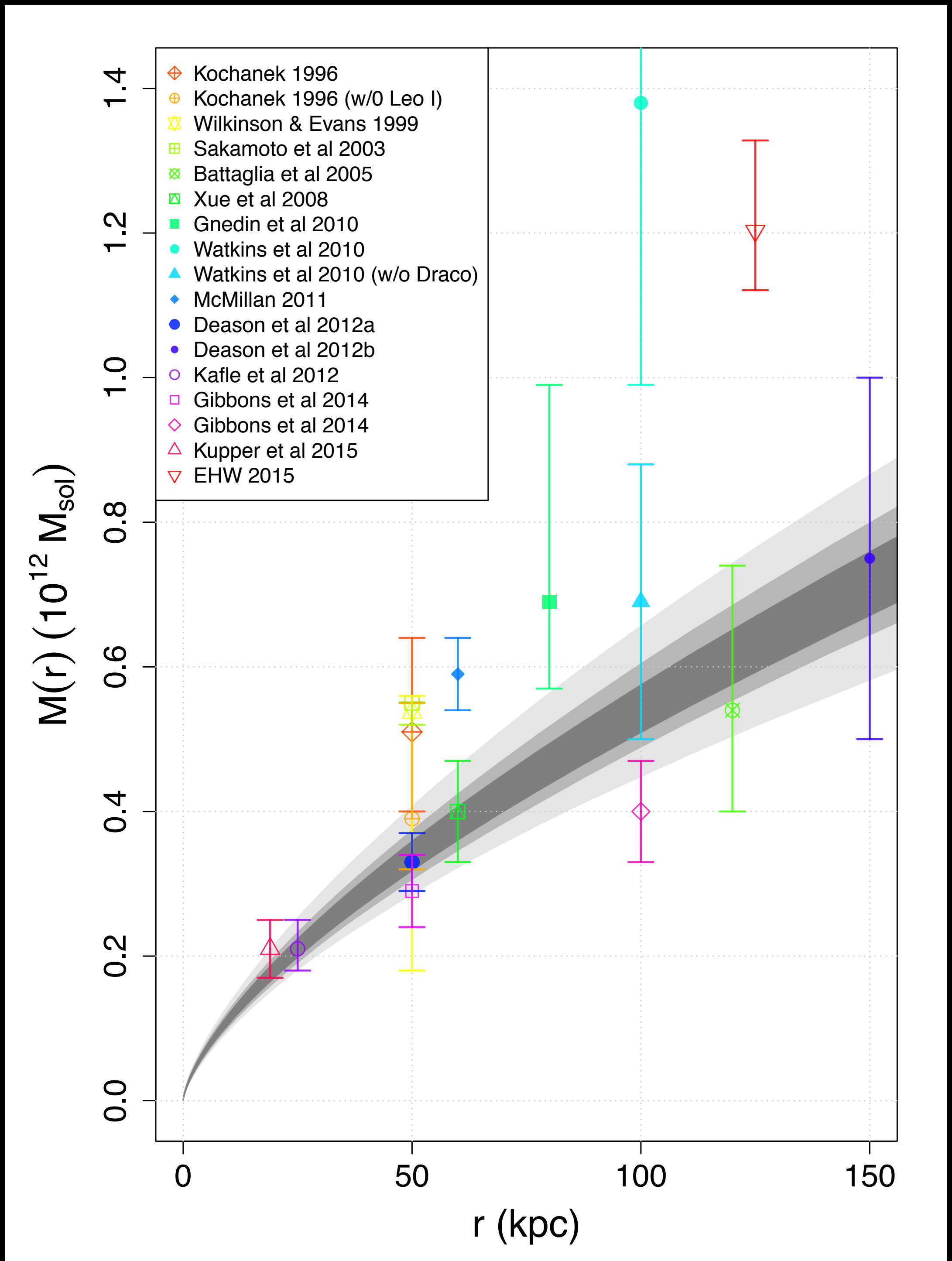
$$\frac{d(\nu\overline{v_r^2})}{dr} + 2\frac{\beta}{r}\nu\overline{v_r^2} = -\nu\frac{d\Phi}{dr}$$

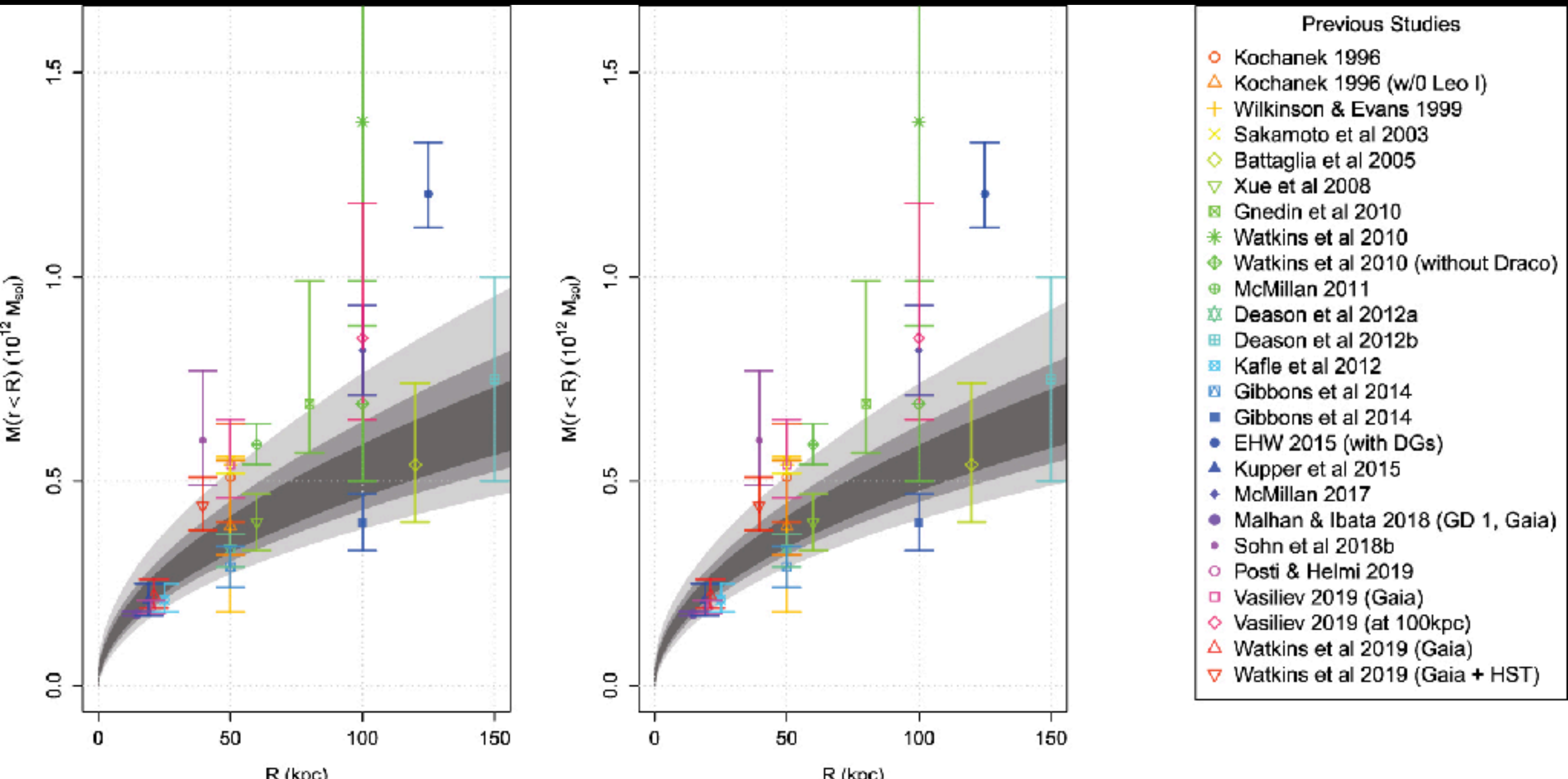
Galactic positions of known satellite galaxies + surveyed regions of sky (as of 2015) Drlica-Wagner et al. 2015 (DES collab)

example of Jeans analysis before & after Gaia



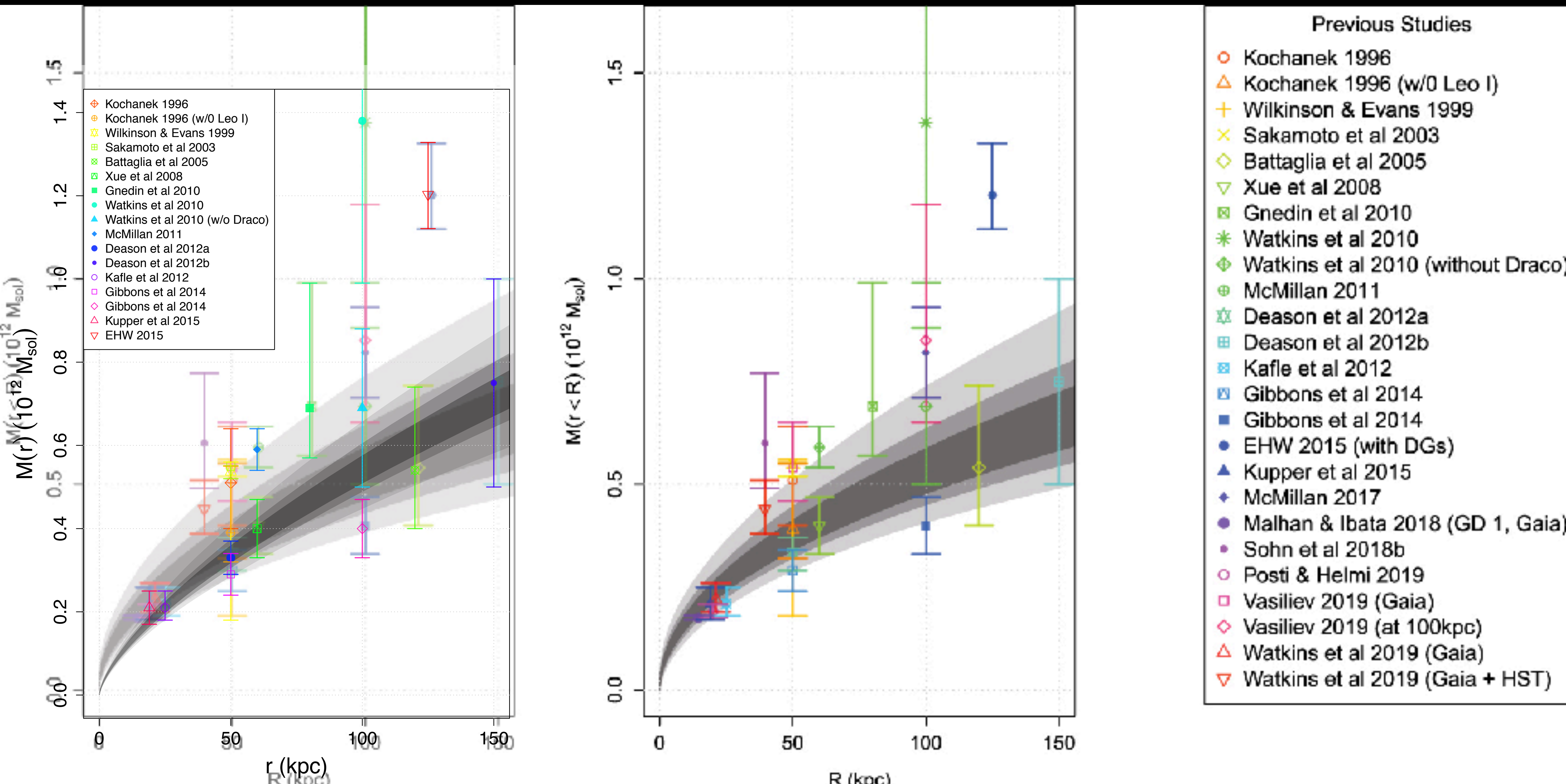
Eadie & Harris 2016

example of Jeans analysis before & after Gaia



Eadie & Juric 2019

example of Jeans analysis before & after Gaia



Eadie & Juric 2019

Stream models: non-equilibrium mass estimation
some key scaling relations

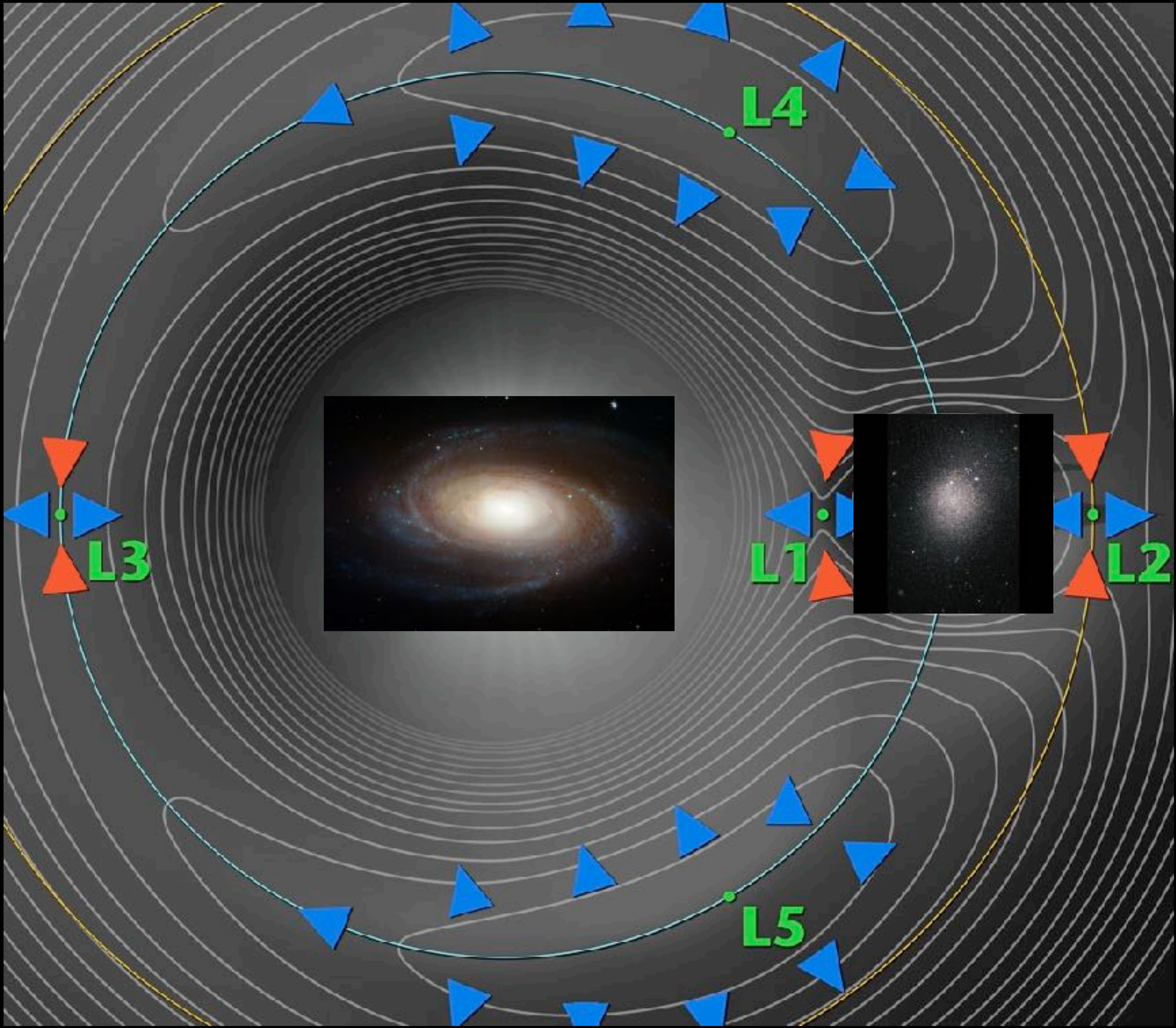
dynamical time $\Omega = \sqrt{\frac{GM}{R_0^3}} \quad t_{\rm dyn} = \frac{2\pi}{\Omega}$

At 10 kpc: t ~ 300 Myr
At 100 kpc: t ~ 3 Gyr

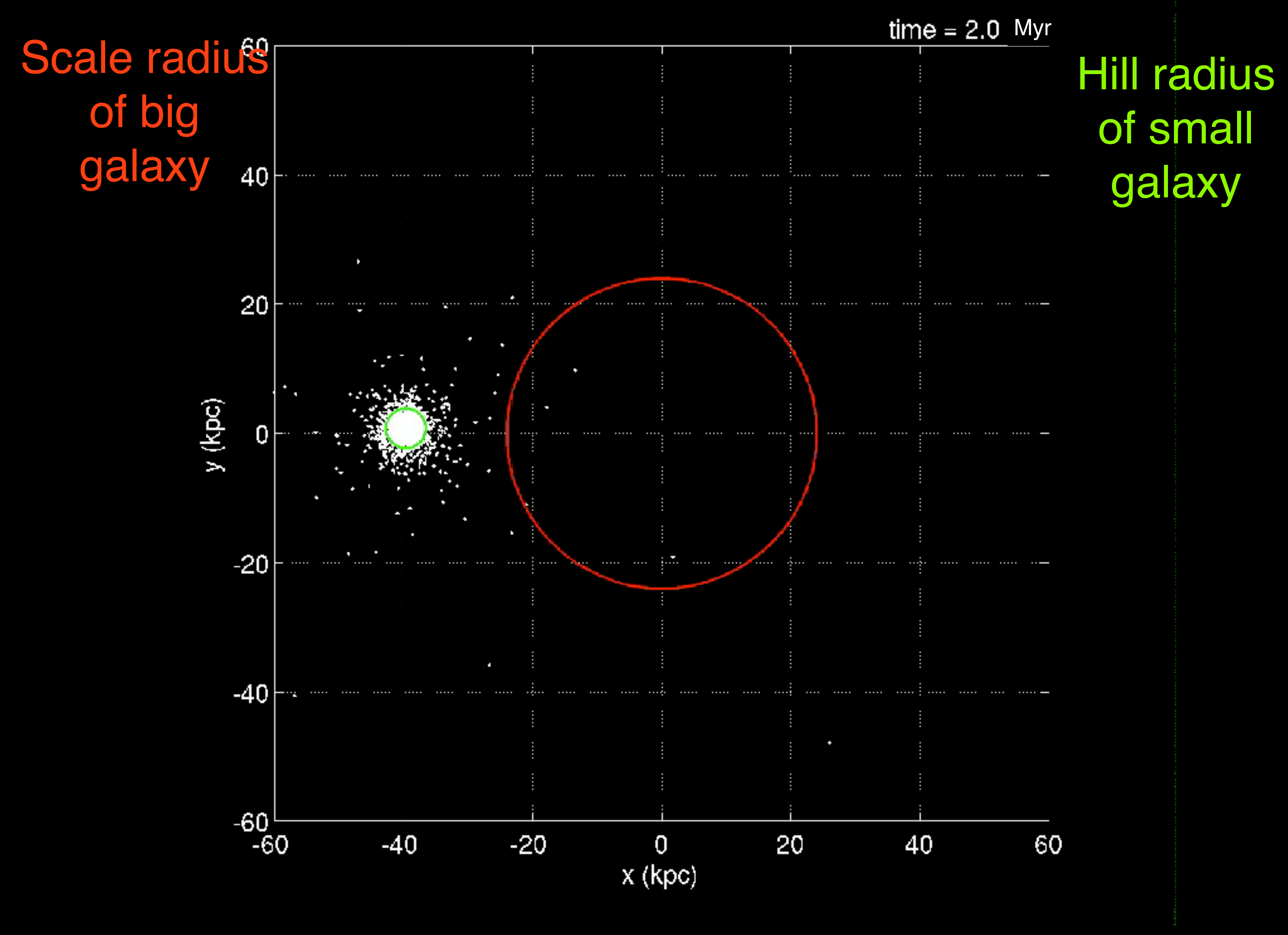
Tidal (Hill) radius [based on 3-body approximation] $r_t = \left(\frac{m}{3M}\right)^{1/3} R_0$

initial phase-space distribution $\frac{r_t}{R_0} \sim \left(\frac{m}{M}\right)^{1/3} \sim \frac{\sigma}{v_{\rm orb}} \quad \frac{\delta E}{E_{\rm orb}} \sim \left(\frac{m}{M}\right)^{1/3}$

See also Johnston 1998, Johnston et al. 1999



Credit: NASA / WMAP Science Team



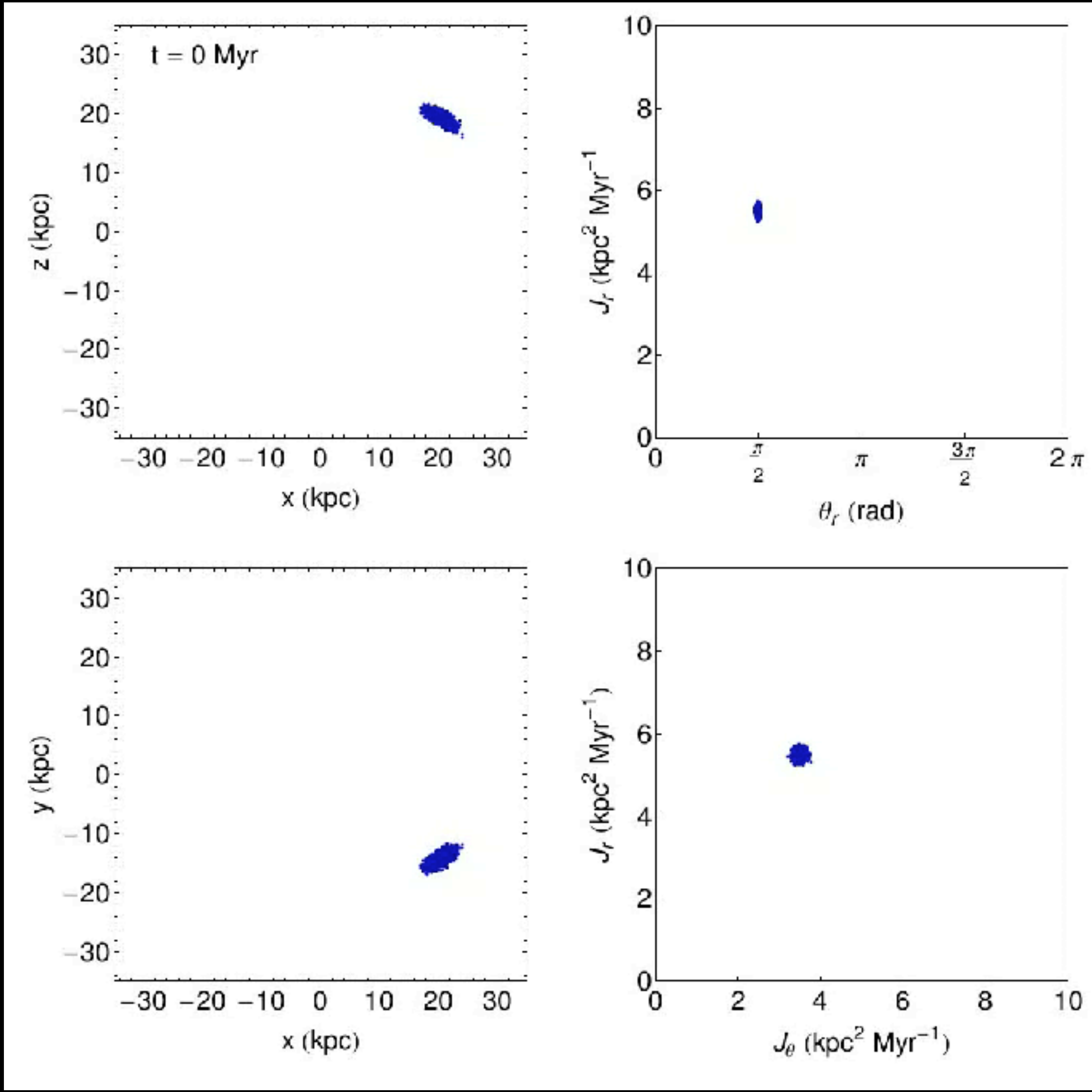
Scale radius of big galaxy
Hill radius of small galaxy
time = 2.0 Myr
y (kpc)
x (kpc)
60
40
20
0
-20
-40
-60
-60
-40
-20
0
20
40
60

Review of action-angle variables

$$(p, q) \leftrightarrow (J, \theta)$$

$$J = \oint p \; dq$$

$$\theta(t) = \Omega t + \theta_0$$



Stream models: non-equilibrium mass estimation

$$r_t = \left(\frac{m}{3M}\right)^{1/3} R_0 \qquad \Omega = \sqrt{\frac{GM}{R_0^3}} \qquad t_{\rm dyn} = \frac{2\pi}{\Omega} \sim T_{\rm orb}$$

$$\frac{r_t}{R_0} \sim \left(\frac{m}{M}\right)^{1/3} \sim \frac{\sigma}{v_{\rm orb}} \qquad \frac{\delta E}{E_{\rm orb}} \sim \left(\frac{m}{M}\right)^{1/3}$$

$$\boxed{\frac{\delta J}{J}\delta\theta_0 \sim \left(\frac{m}{M}\right)^{1/3} \qquad \delta\theta_i(t \gg t_0) = \frac{\partial^2 H}{\partial J_i \partial J_j}\delta J_j t}$$

scaling of length and width with time

phase-wrapping time $\delta\theta_i(t) \sim 2\pi$

phase-mixing time $\delta\theta_i(t) \sim 2\pi N_*$

See also Johnston 1998, Helmi & White 1999

Stream models: non-equilibrium mass estimation

scaling of length and width with time

$$\frac{\delta J}{J}\delta\theta_0 \sim \left(\frac{m}{M}\right)^{1/3}$$

$$\delta\theta_i(t \gg t_0) = \boxed{\frac{\partial^2 H}{\partial J_i \partial J_j}}\delta J_j t$$

spreading depends on potential and its symmetries

in a spherical logarithmic potential,

$$\Phi = v_c^2 \ln(r) + \Phi_0$$

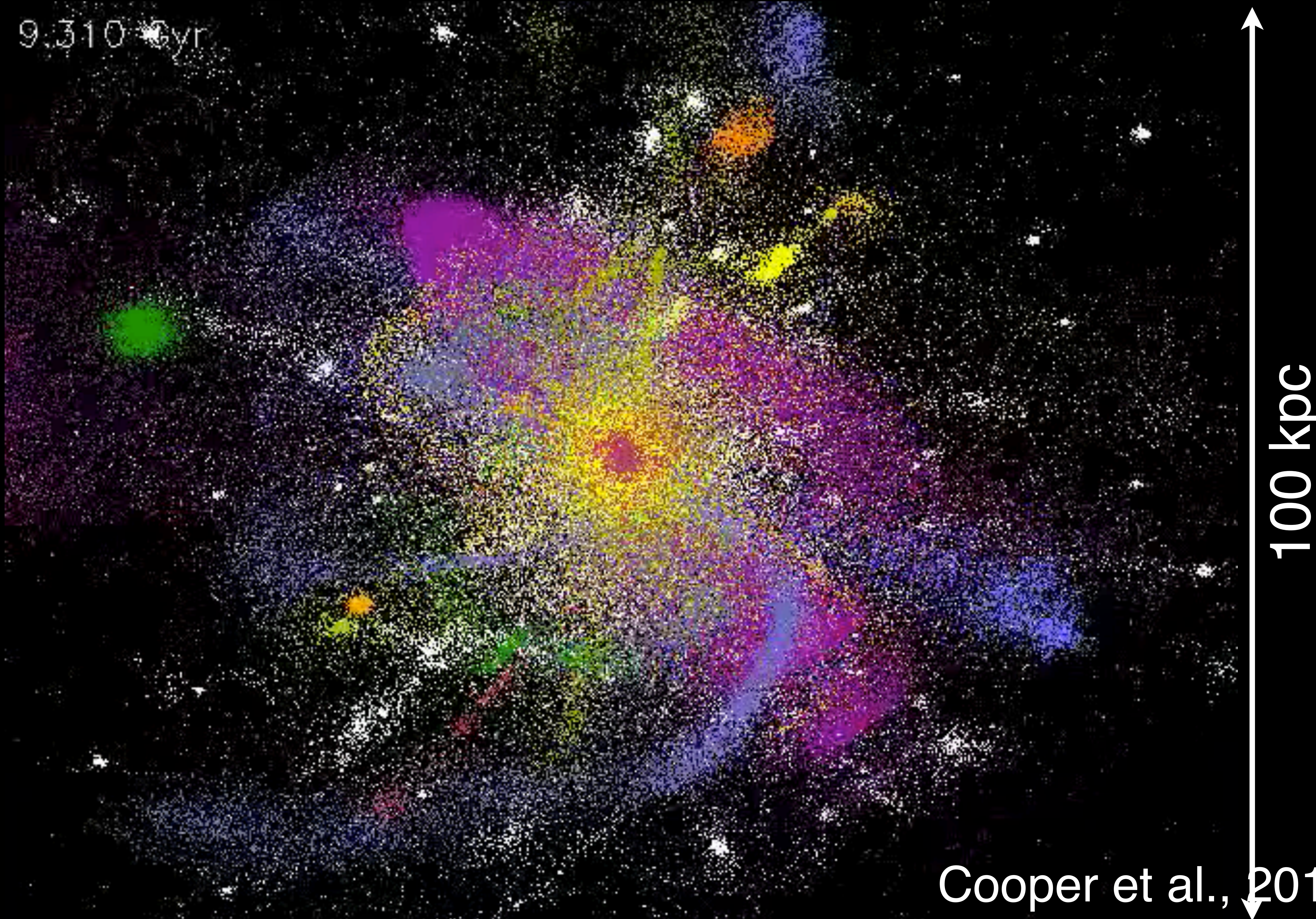
$$T_{\rm orb}(E + \delta E) = \exp\left(\frac{\delta E}{v_c^2}\right)$$

phase-wrapping time $\delta\theta_i(t) \sim 2\pi$

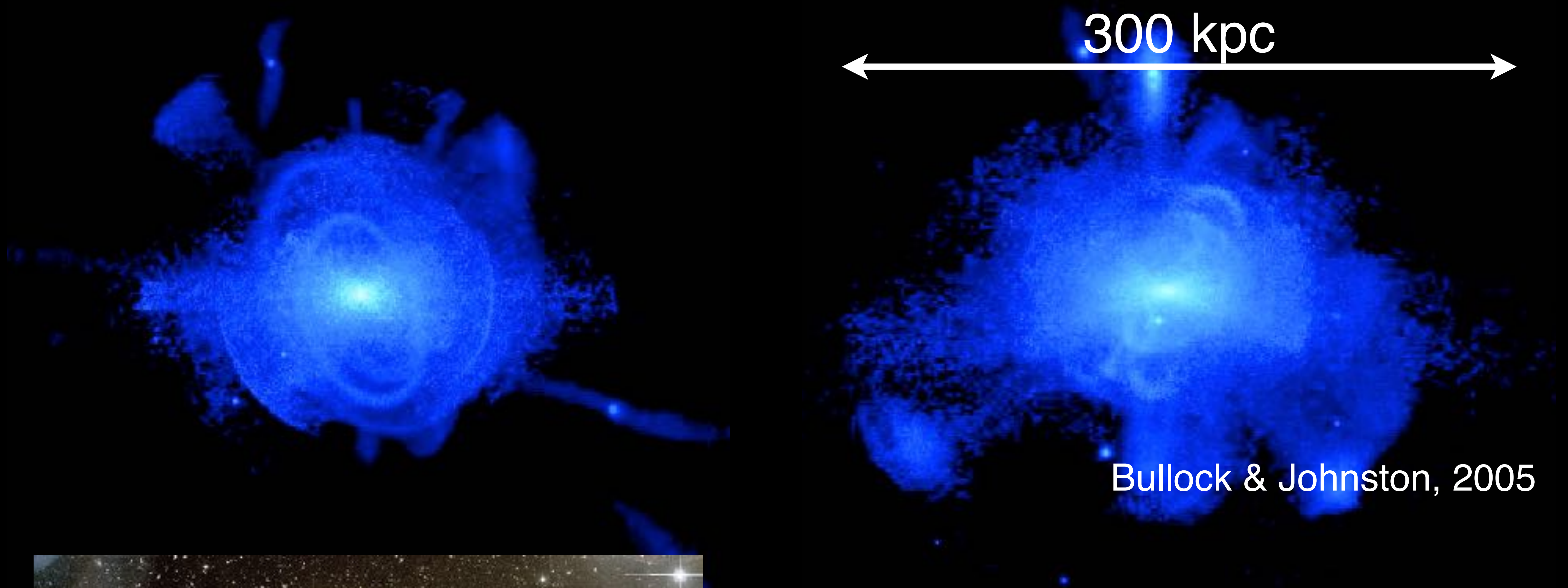
$$T_{2\pi} = \frac{T_{\rm orb}}{1 - \exp[-(2Gm/R_0 v_c^2)^{1/3}]}$$ in a spherical logarithmic potential

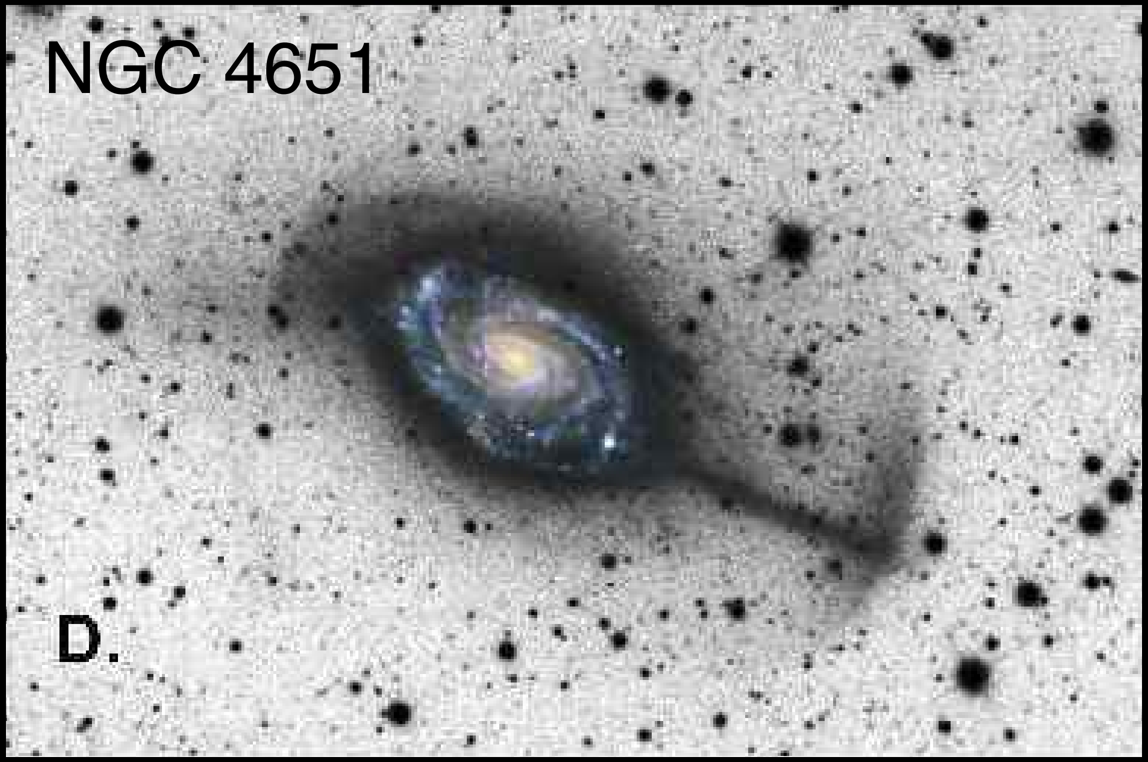
phase-mixing time $\delta\theta_i(t) \sim 2\pi N_*$

See also Johnston 1998, Helmi & White 1999



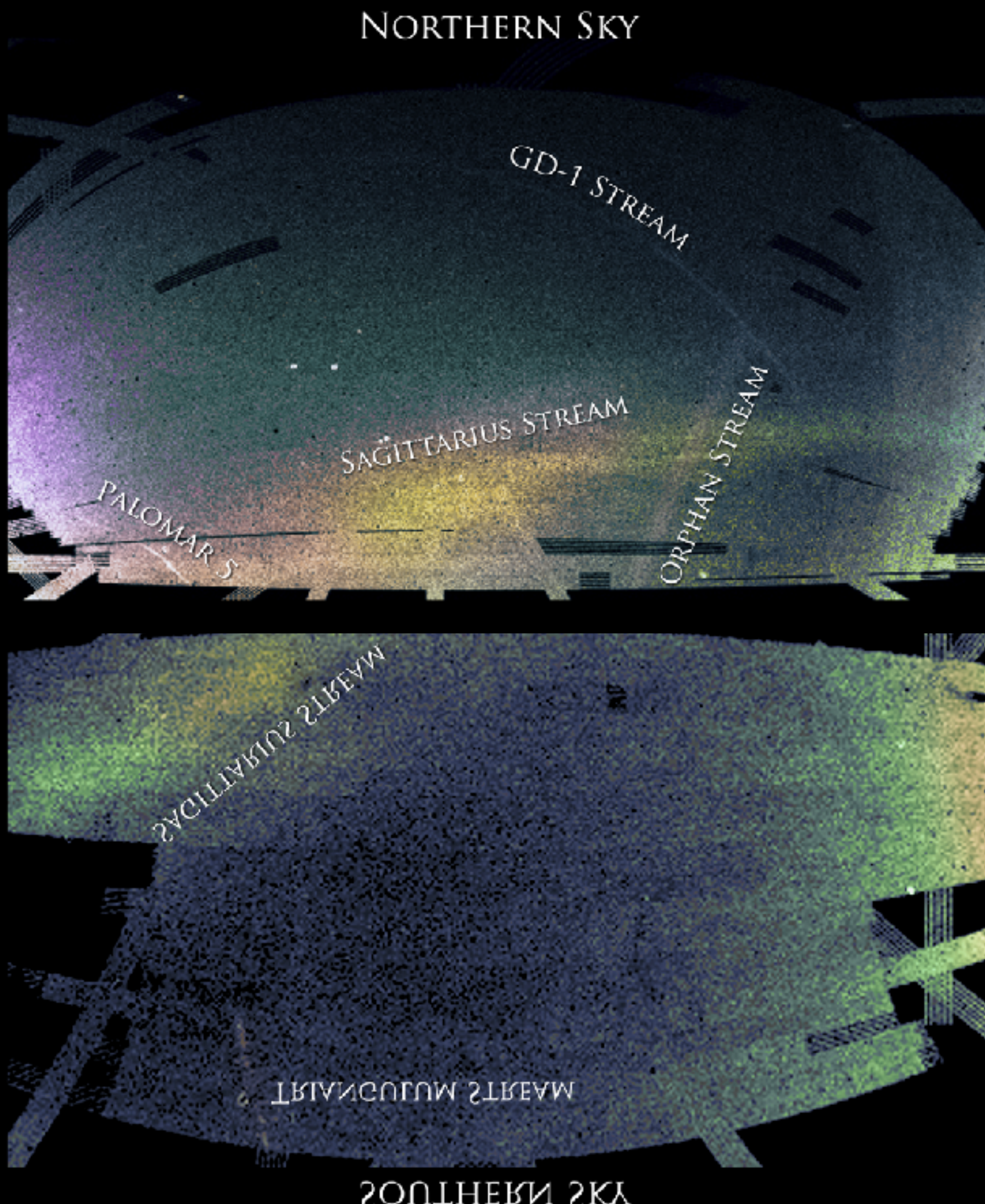
Cooper et al., 2010
Helmi et al., 2011





Martinez-Delgado et al.

We know about plenty of streams in the MW (20-50 depending on who you ask!)



Bonaca, Giguère, & Geha (SDSS DR8)

techniques for modeling tidal streams

- Orbit integration
- Fast stream approximation
- N-body modeling
- Clustering methods

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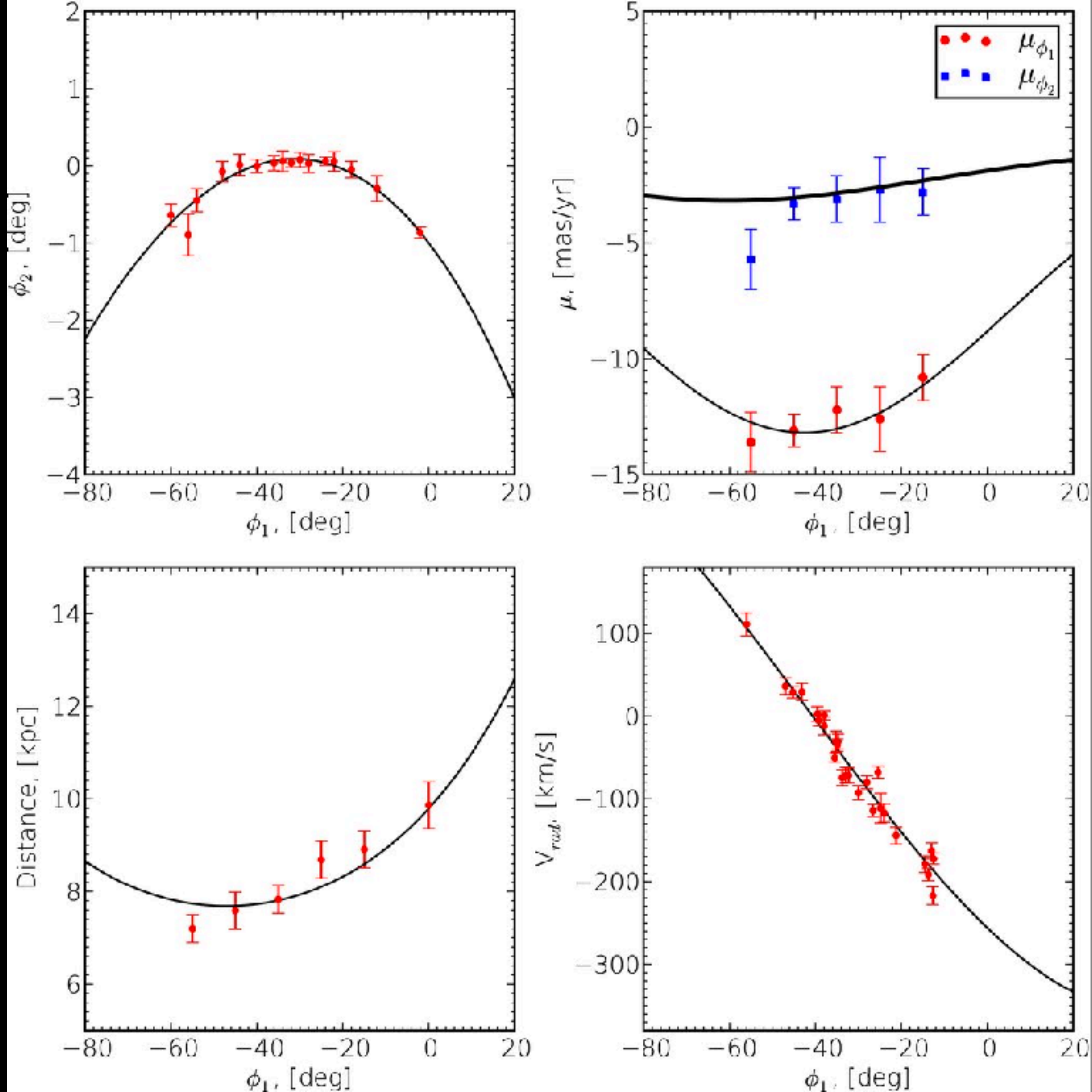
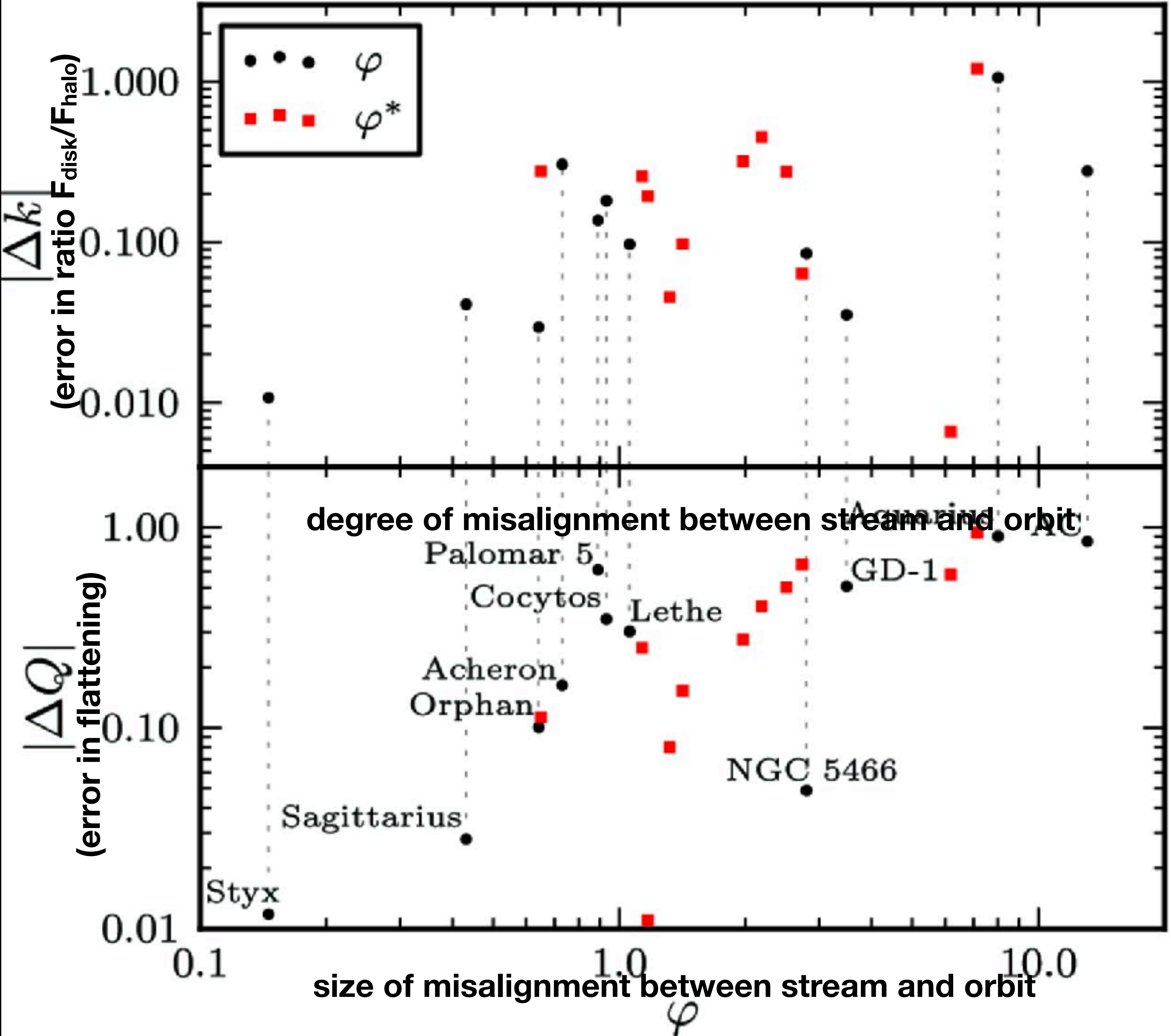


GD-1, Koposov, Rix, & Hogg 2010



Sanders & Binney 2015

techniques for modeling tidal streams

- Orbit integration
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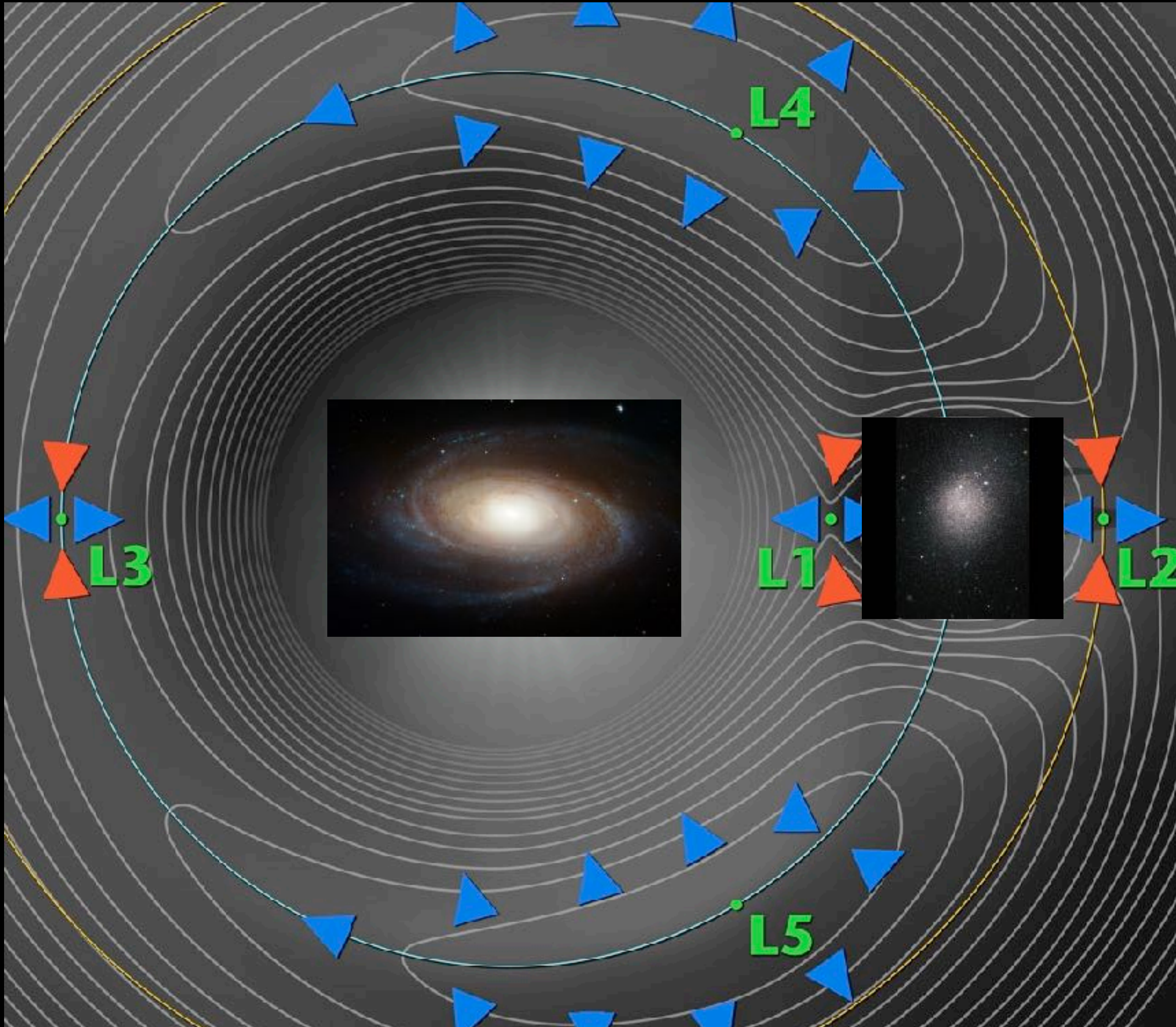
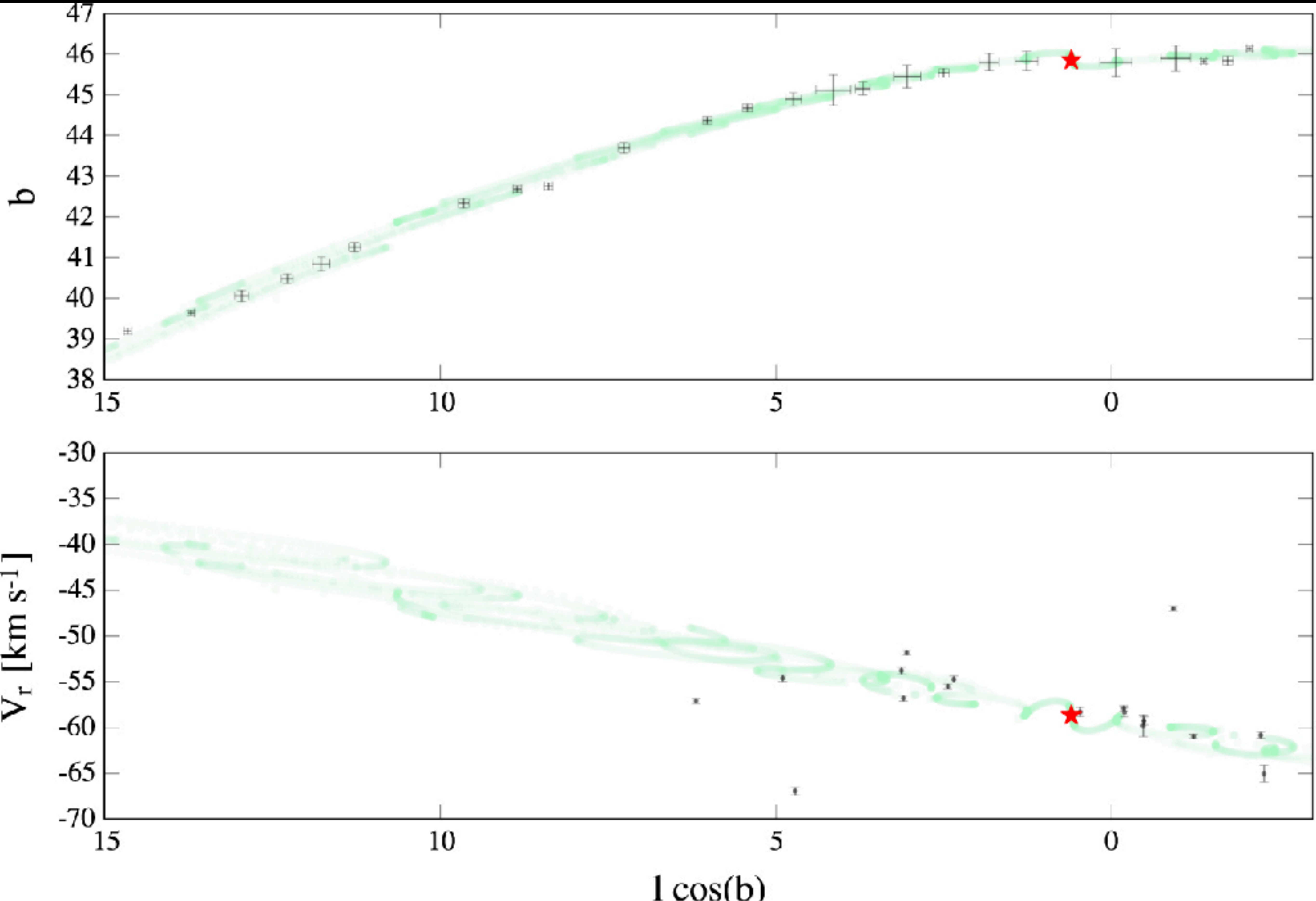
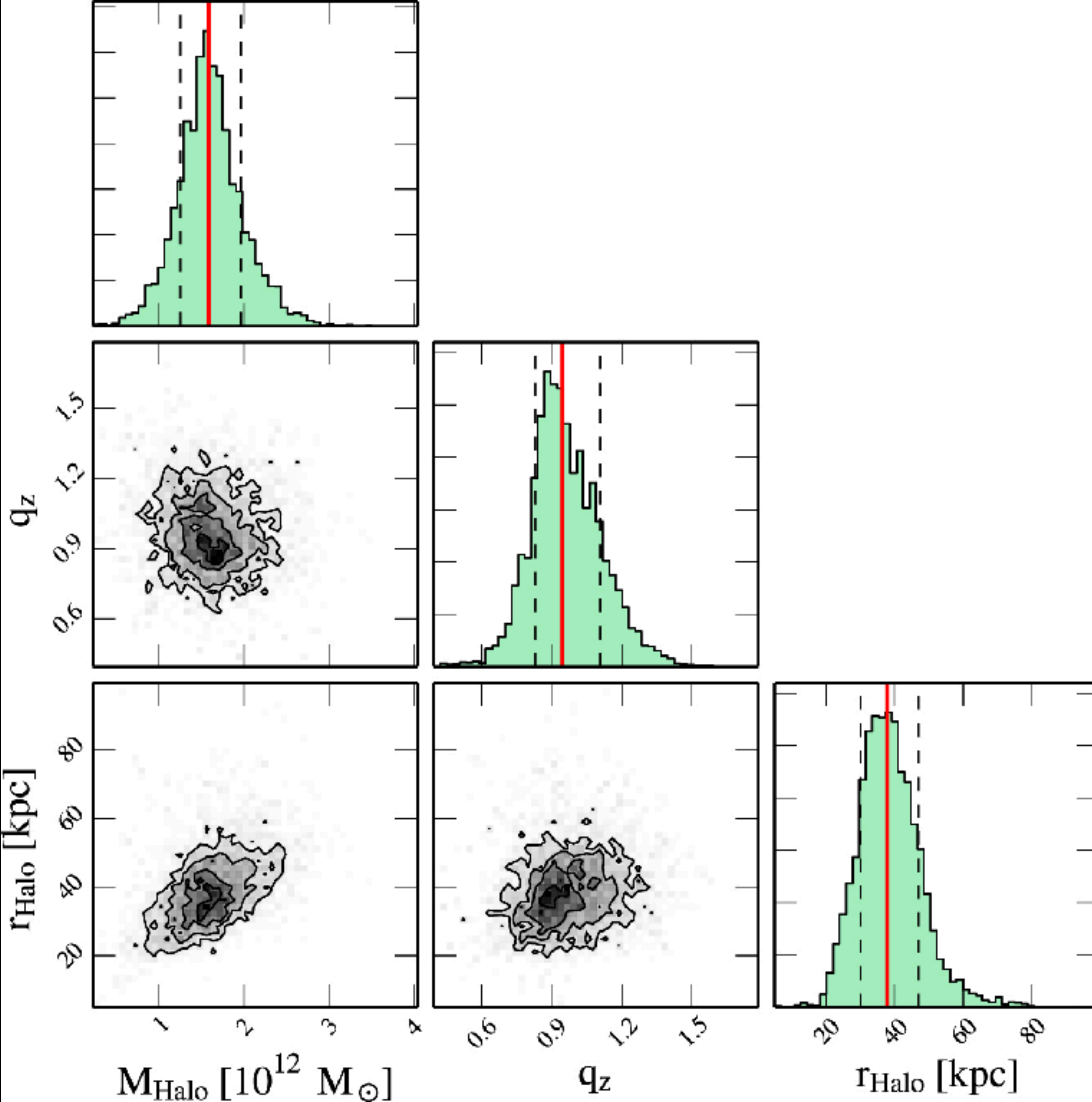


Credit: NASA / WMAP Science Team



Palomar 5, Küpper et al. 2015



Palomar 5, Küpper et al. 2015

techniques for modeling tidal streams

- Orbit integration
- Fast stream approximation
- N-body modeling
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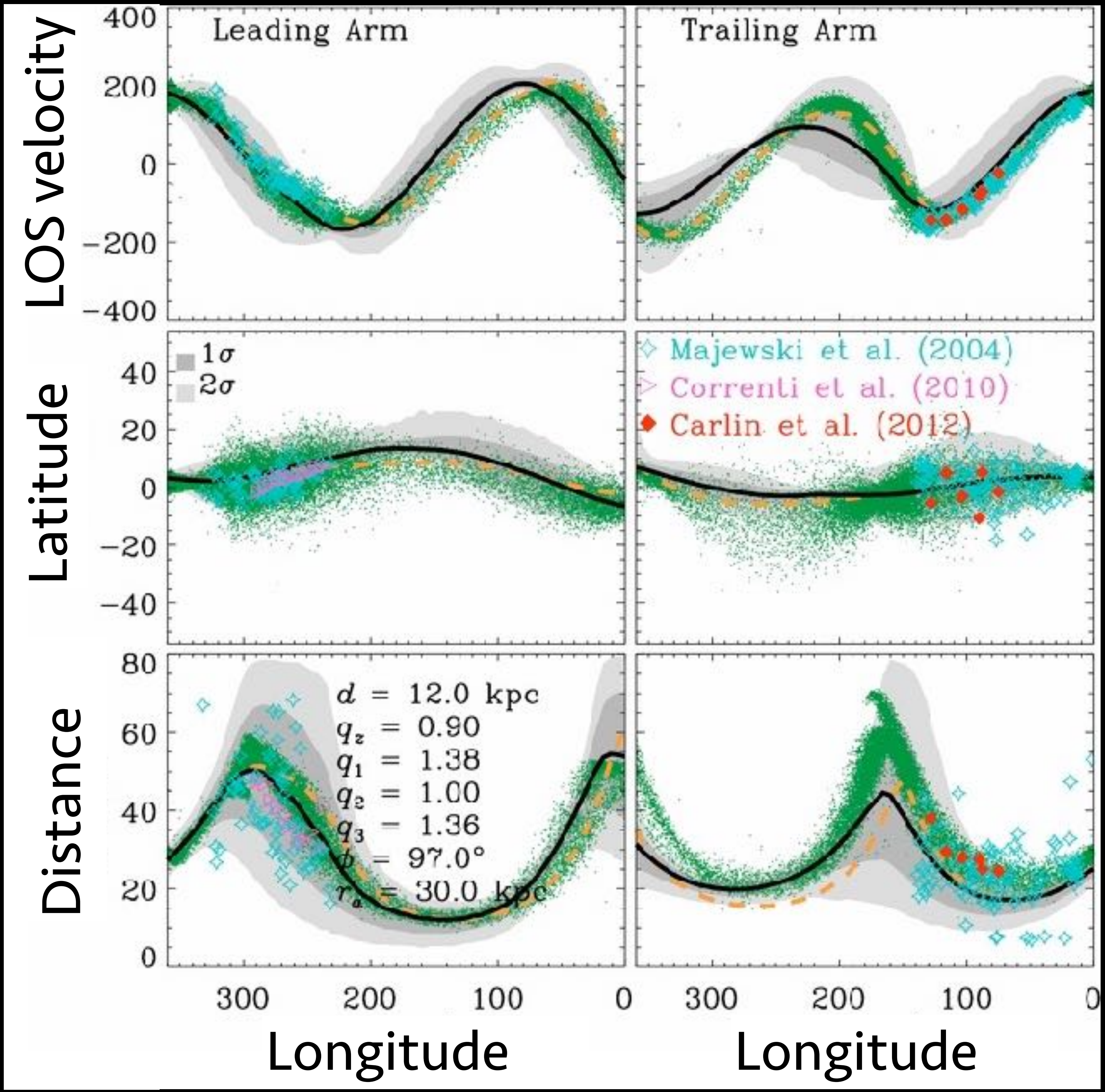
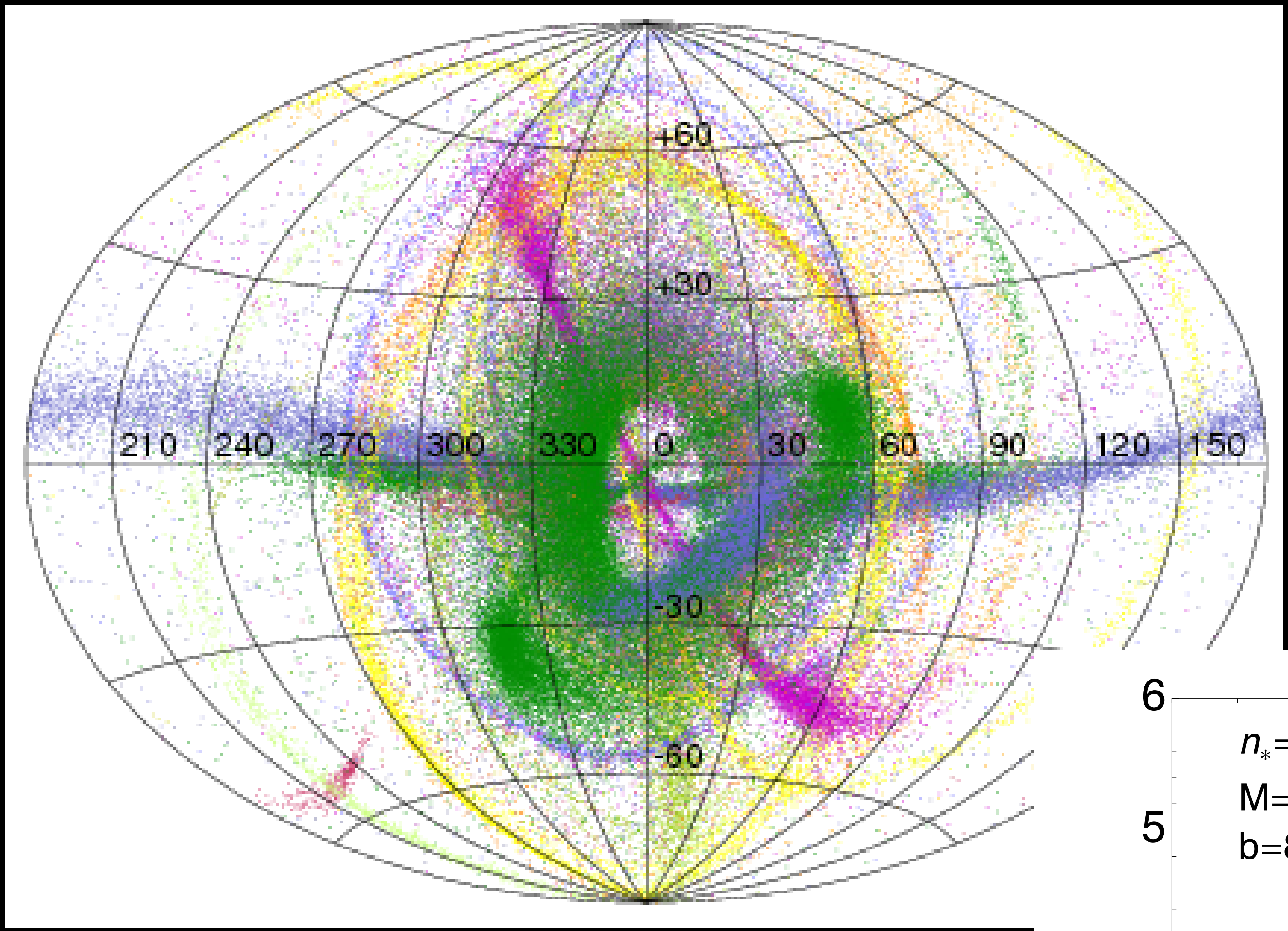
Sagittarius Stream in triaxial dark matter halo:
Vera-Ciro & Helmi 2013;
Law & Majewski 2010



techniques for modeling tidal streams

- Orbit integration
- Fast stream approximation
- N-body modeling
- Clustering methods

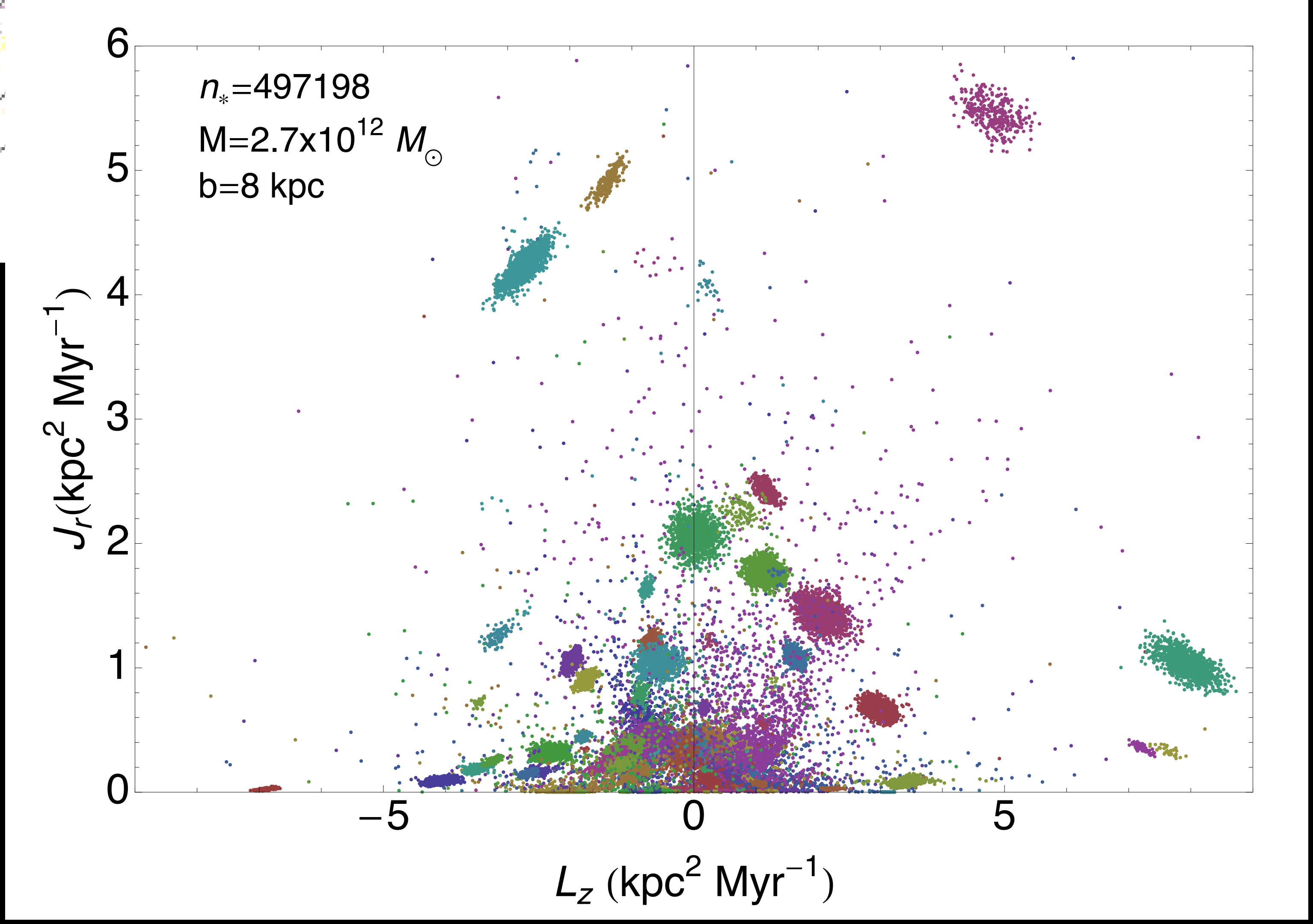
The accreted stellar halo is clumpy in action space



View in Galactic coordinates

Sanderson, Helmi, & Hogg 2015

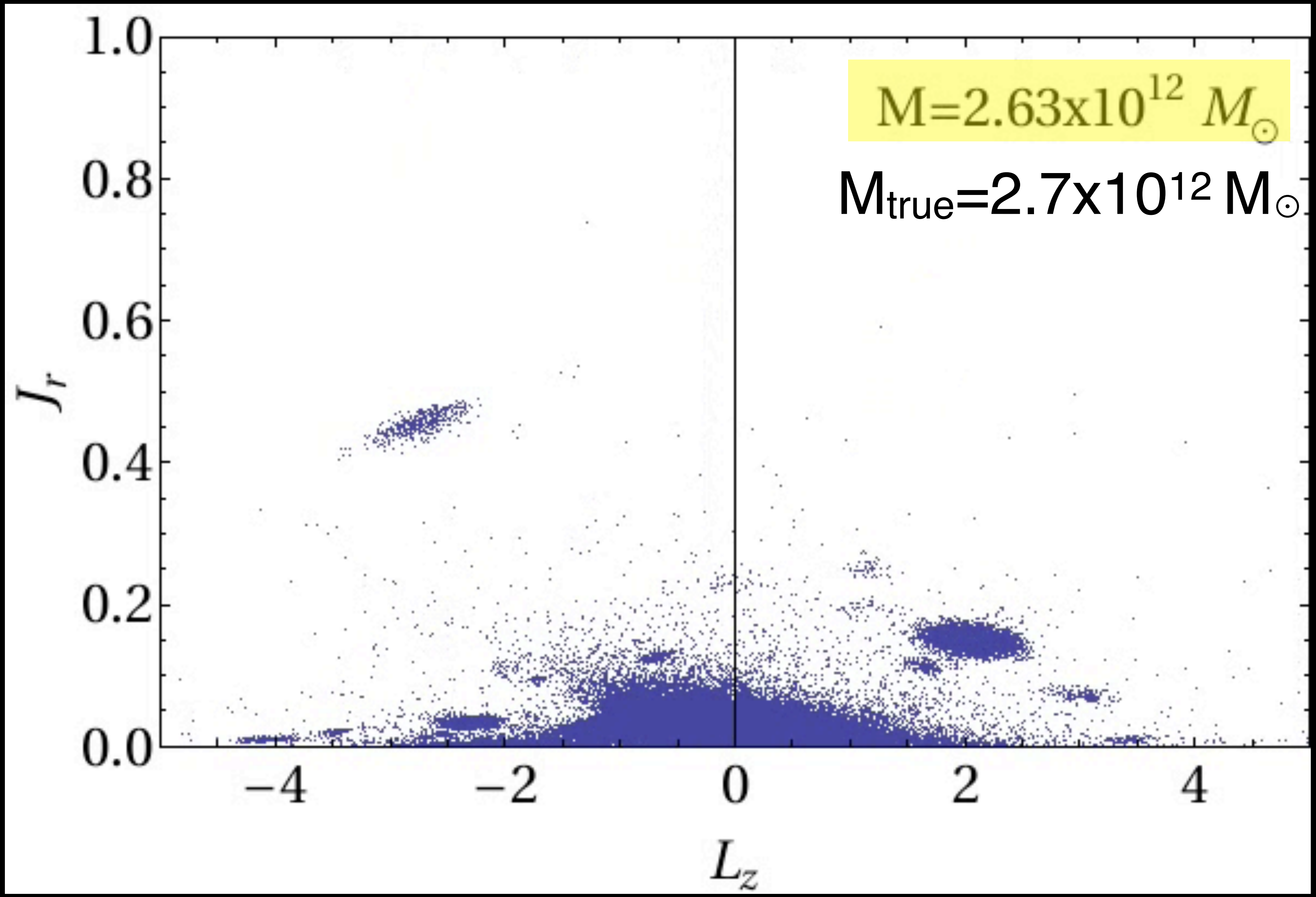
View in action space
(using correct potential)



Actions are most clustered in the correct potential

$$J_r = \frac{GM}{\sqrt{-2E}} - \frac{1}{2}\left(L + \sqrt{L^2 + 4GMb}\right)$$

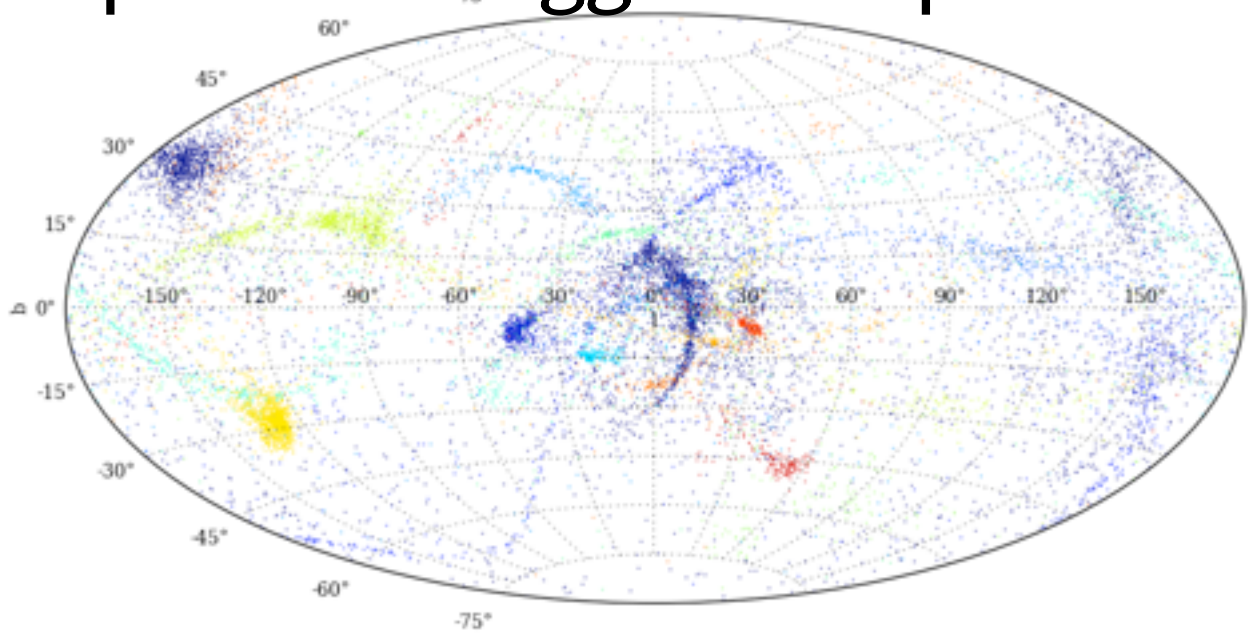
Potential parameters
Observations
Both



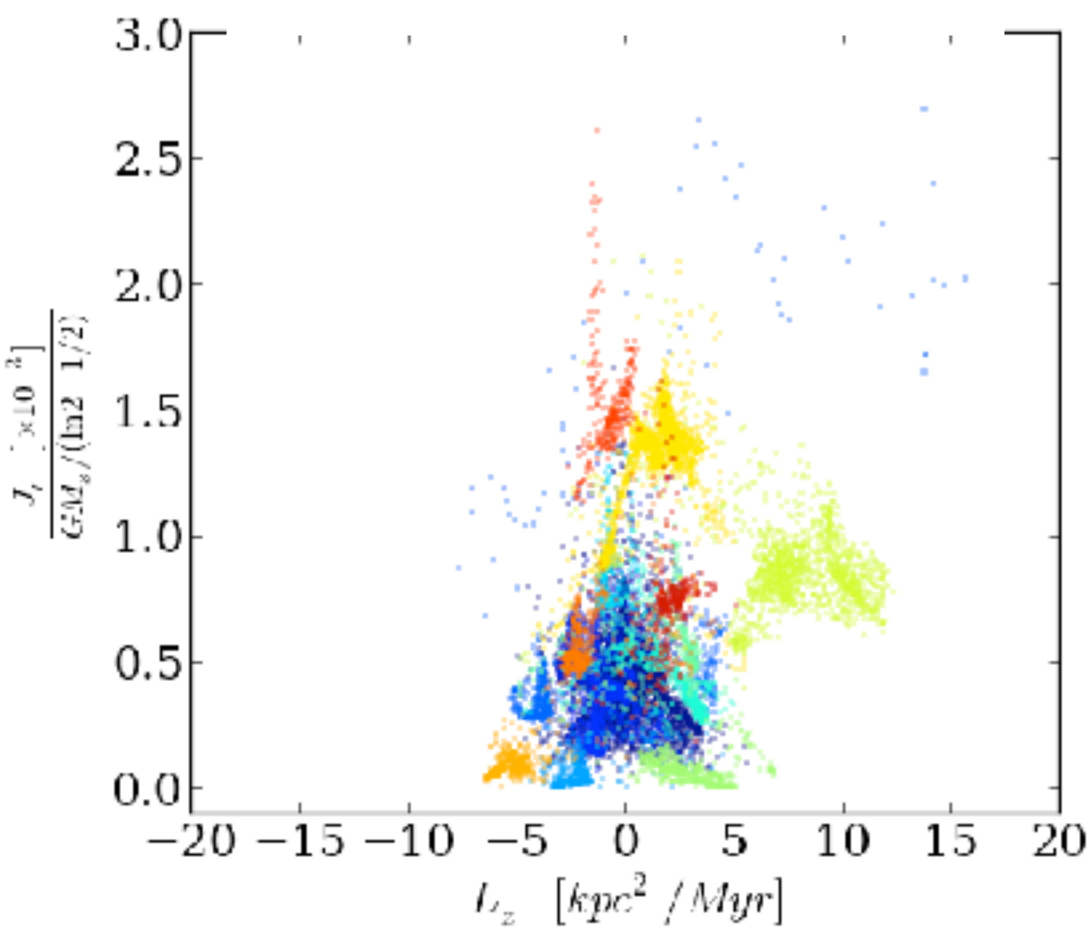
Sanderson, Helmi, & Hogg 2015

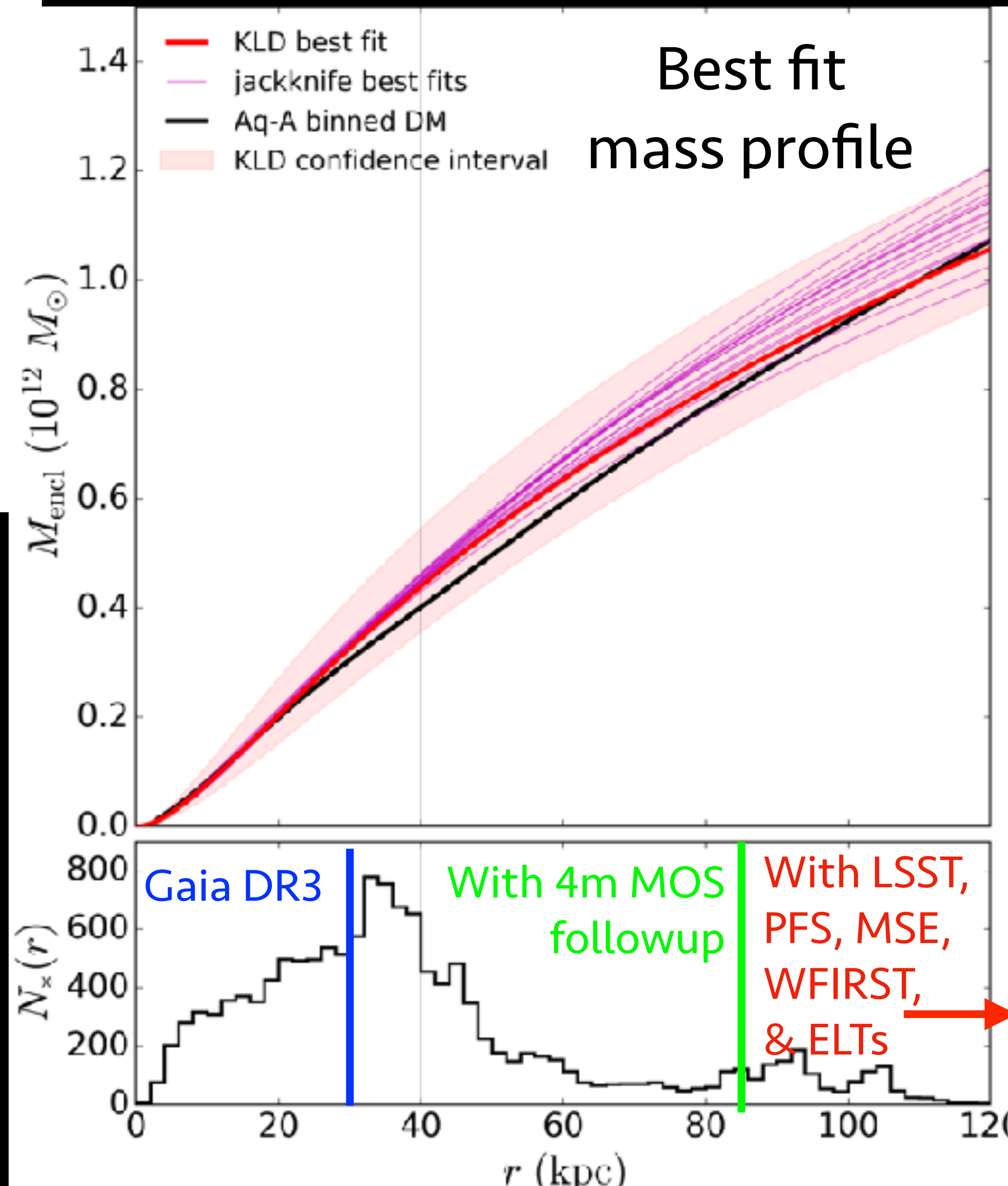
The stellar halo constrains the MW's gravitational potential

Aquarius A tagged star particles



Best spherical NFW fit





Sanderson et al. 2017a

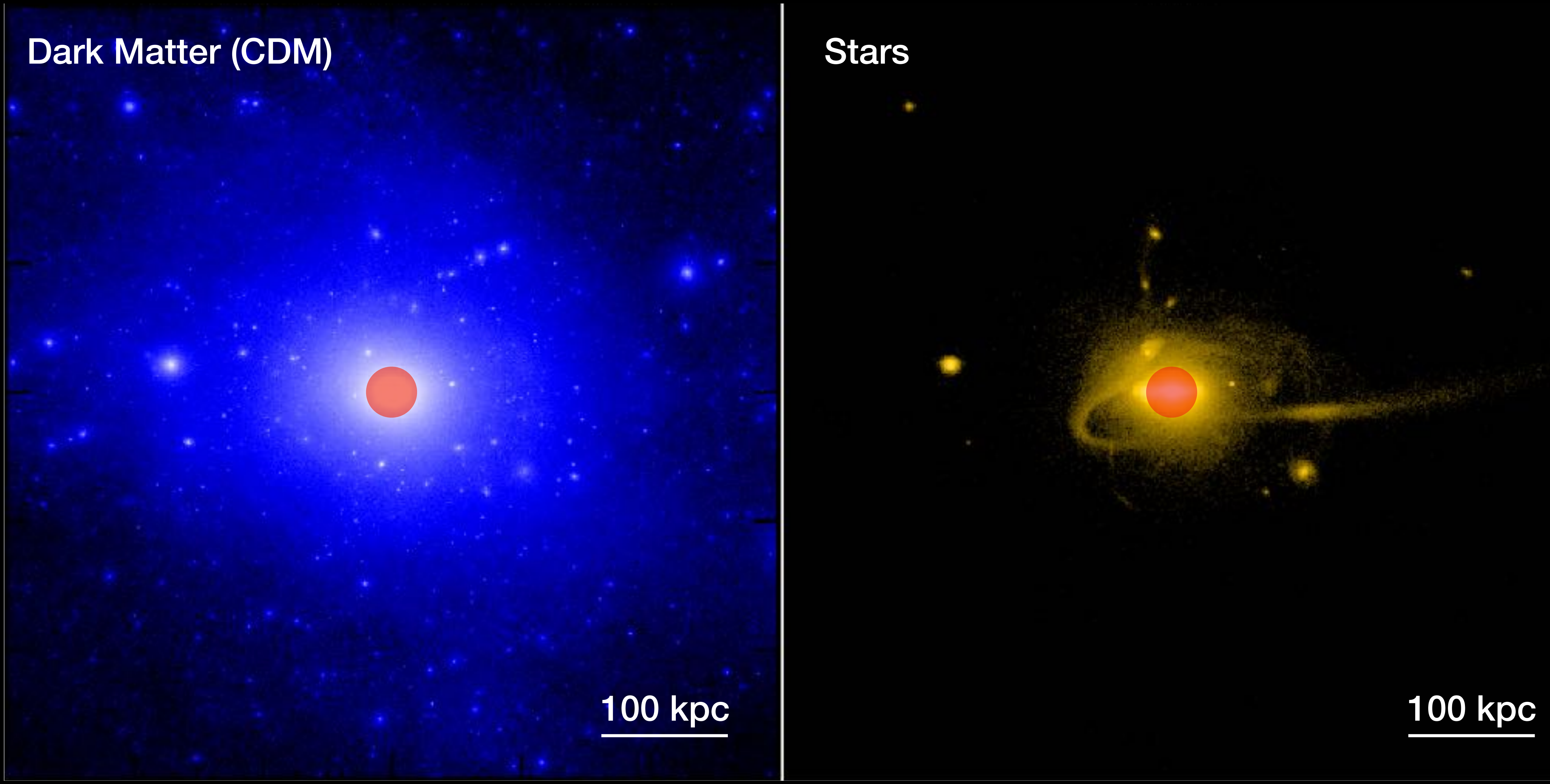
techniques for modeling tidal streams

- Orbit integration (e.g. Koposov, Rix, & Hogg 2010)
  - Fastest but inaccurate (see e.g. Sanders & Binney 2015)
  - can be used to search for streams (e.g. Malhan & Ibata 2018)
- Fast stream approximation (see e.g. Küpper et al 2015)
  - Many methods available (see papers by Fardal, Bonaca, Sanders, Bovy, …)
  - better approximation than orbit integration
  - More free parameters: need assumptions about progenitor, stripping rate
- N-body modeling (e.g. Law & Majewski for Sgr, Fardal et al. for M31 giant stream)
  - Most physically realistic, but computationally demanding (see https://milkyway.cs.rpi.edu/)
  - Above methods used to narrow down large parameter space
- Clustering methods (Peñarrubia+2012, Magorrian 2014, Sanderson+2015)
  - membership not required (but helpful); prior is that stars are accreted
  - fit many streams simultaneously, but can be derailed by one large contributor
  - need 6D data for stars in sample

where are we now?

The MW's mass & shape are well constrained in the inner ~20 kpc…

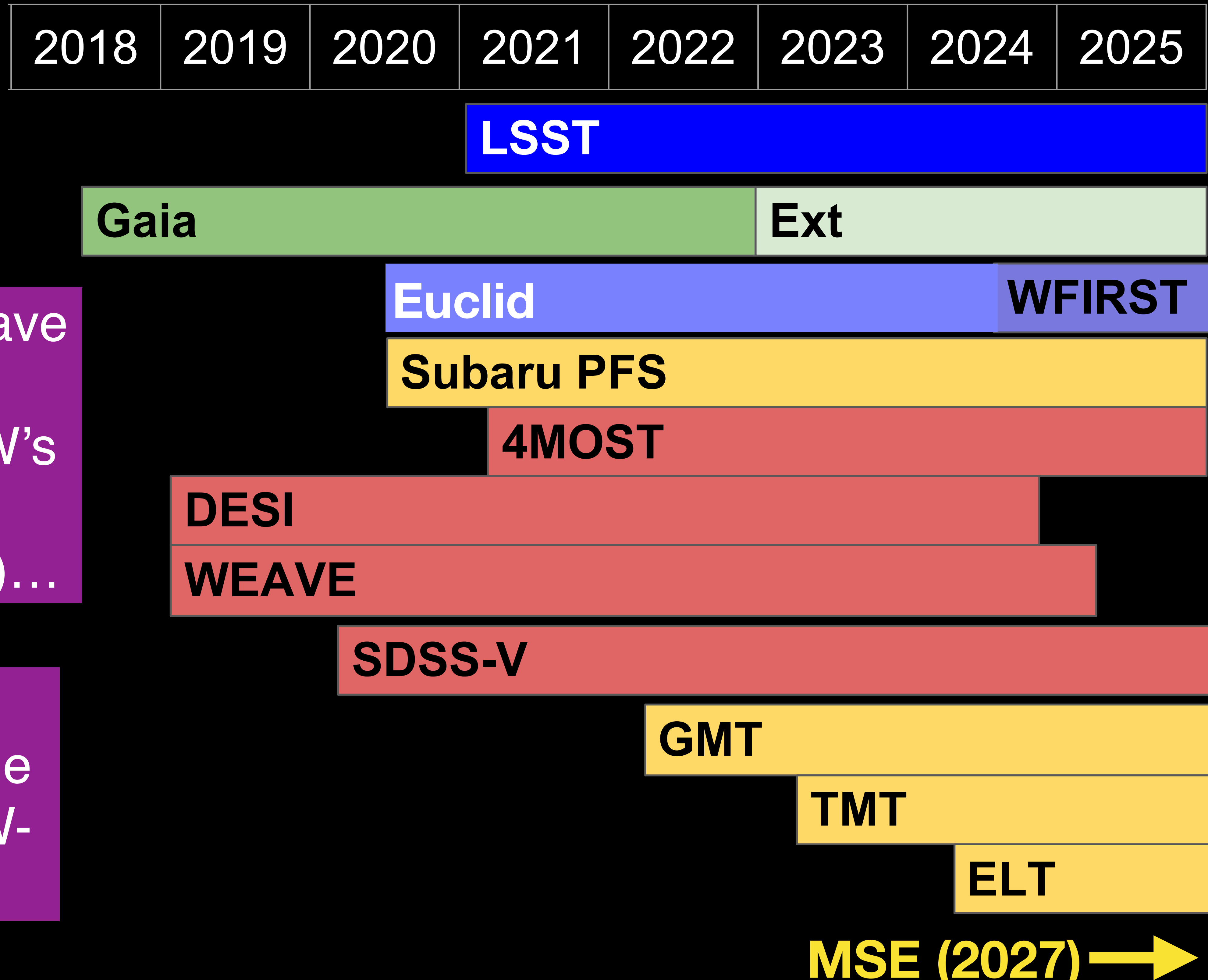
Wegg, Gerhard, & Bieth 2019
Posti & Helmi 2019



…which is probably less than a tenth the radius of the DM halo.

where are we headed?

Gaia is only the beginning



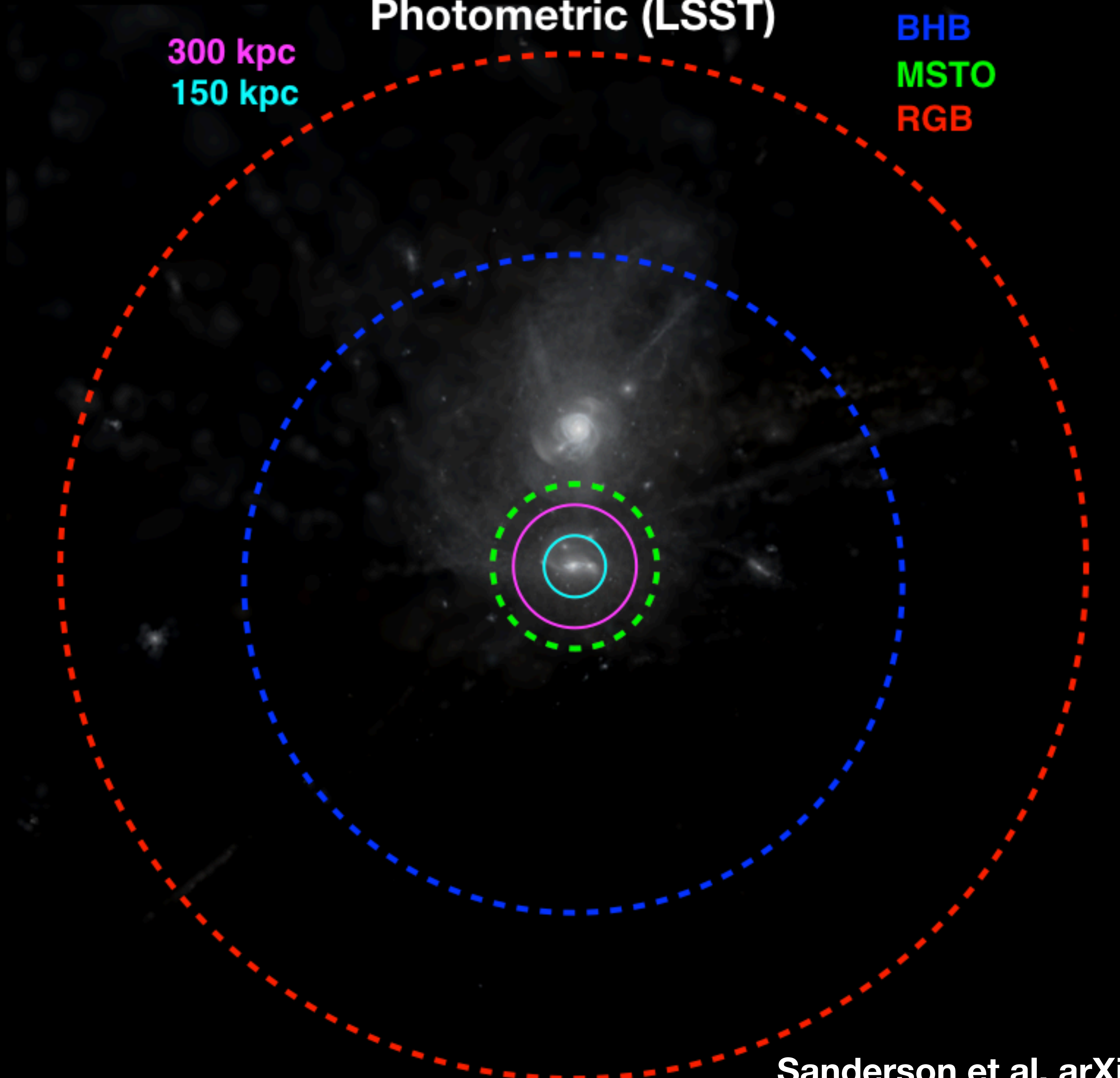
By 2028, we will have 6+D information for stars to the MW's virial radius and beyond (~300 kpc)…

..and resolved stellar maps of the ~100 nearest MW-like galaxies

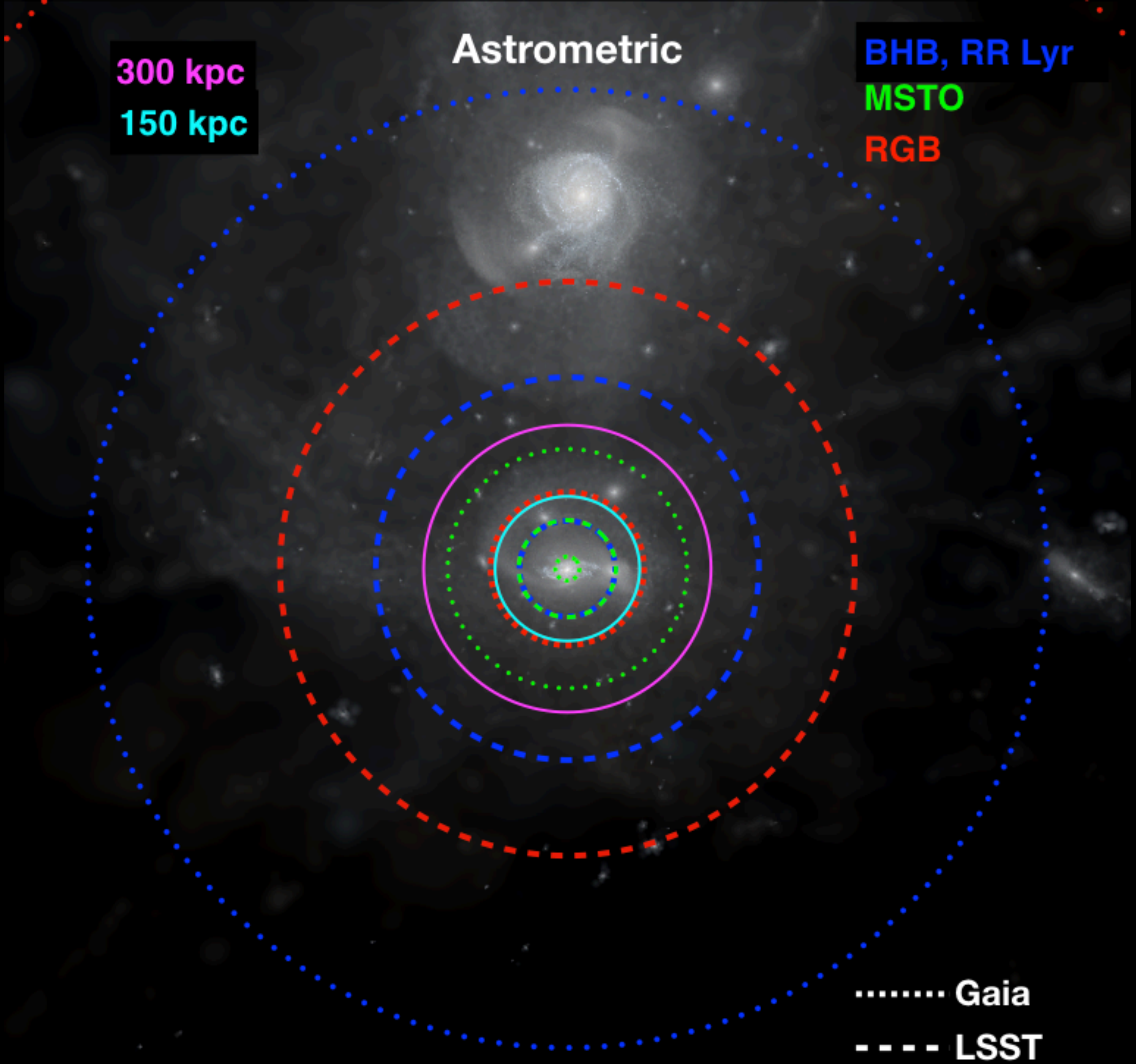
Astrometric + spectroscopic
Photometric + astrometric

Spectroscopic: <4-m class
Spectroscopic: >4-m class

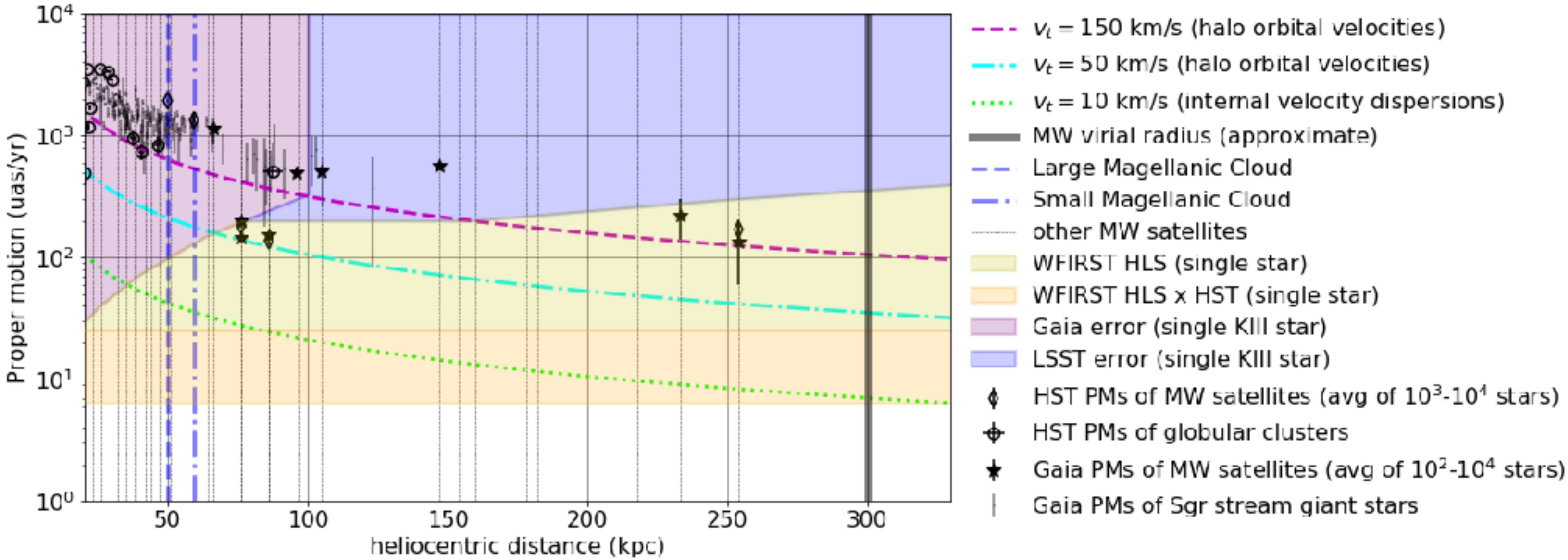
Photometric (LSST)



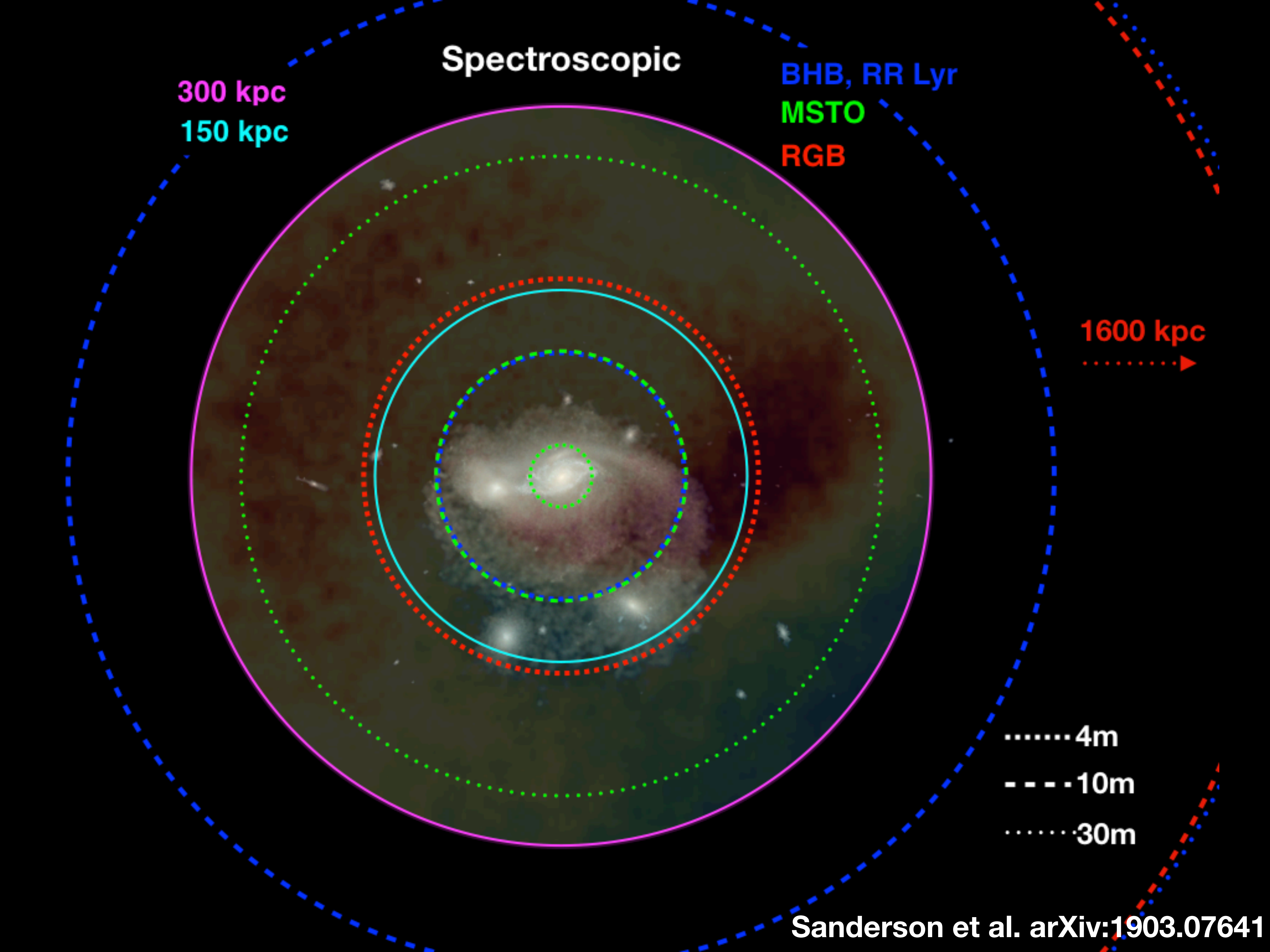
Sanderson et al. arXiv:1903.07641



Sanderson et al. arXiv:1903.07641



Sanderson et al. arXiv:1903.07641


Spectroscopic
300 kpc
150 kpc
BHB, RR Lyr
MSTO
RGB
1600 kpc
4m
10m
30m
Sanderson et al. arXiv:1903.07641

To learn more about action-angle variables…

- Binney & Tremaine, Galactic Dynamics (2008 edition) chapter 3 has an introduction. Your institute may give access to the electronic version.

- Goldstein, Poole & Safko, Classical Mechanics (2002 edition) chapters 8-10 focus on the mathematical physics of the action transformation

- Wilma Trick's talk at the recent KITP conference on Gaia has an intuitive introduction to actions based on the epicyclic approximation

- Helmi & White, 1999 discusses how stellar streams evolve in action space

- McGill & Binney, 1990 and subsequent papers discuss how to compute actions and angles for generic gravitational potentials