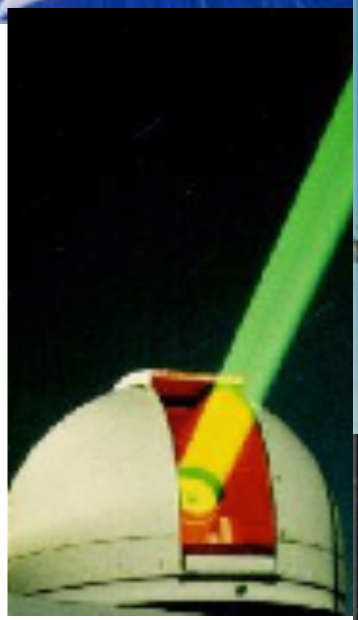
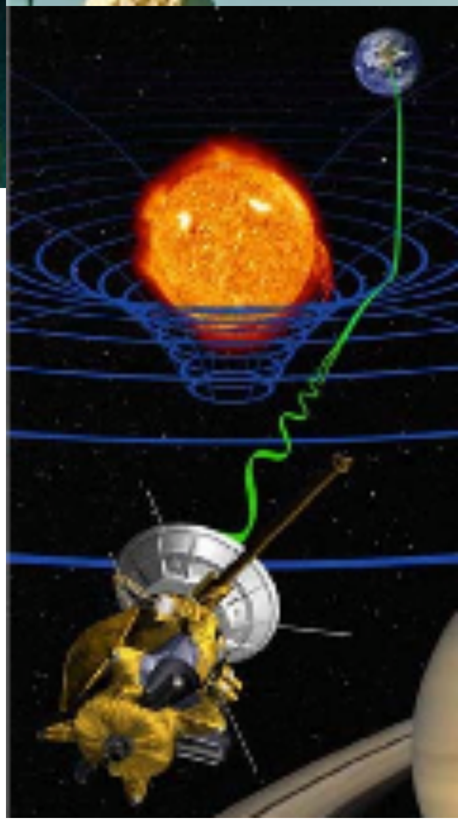
# Ray-tracing in Solar System
## How to solve the null geodesic equations

C. Le Poncin-Lafitte - LNE-SYRTE, Paris Obs.

# Motivations







## Ground & space geodesy accuracy is increasing:

LLR & SLR → From cm to mm
GALILEO

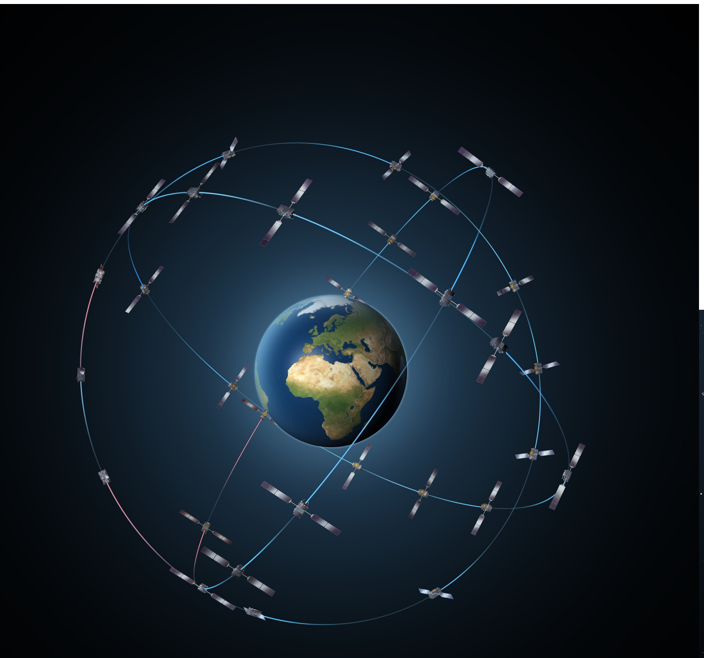
Gravity Probe A to ACES/Pharao → factor 80 on Grav. Redshift

## Ground & space astrometry:

Gaia, Gravity → from milli to micro-arcsecond

## Navigation of interplanetary probes :

Cassini Experiment, use of Ka Band
MORE Experiment on BepiColombo → factor 10 on Doppler
JUNO Experiment 2016, JUICE towards 2030




Need to describe light propagation in relativistic framework

- one can solve null geodesic
- one can introduce new tools

and define properly the observables !

# Light propagation is crucial in the modeling of Sol. Sys. observations

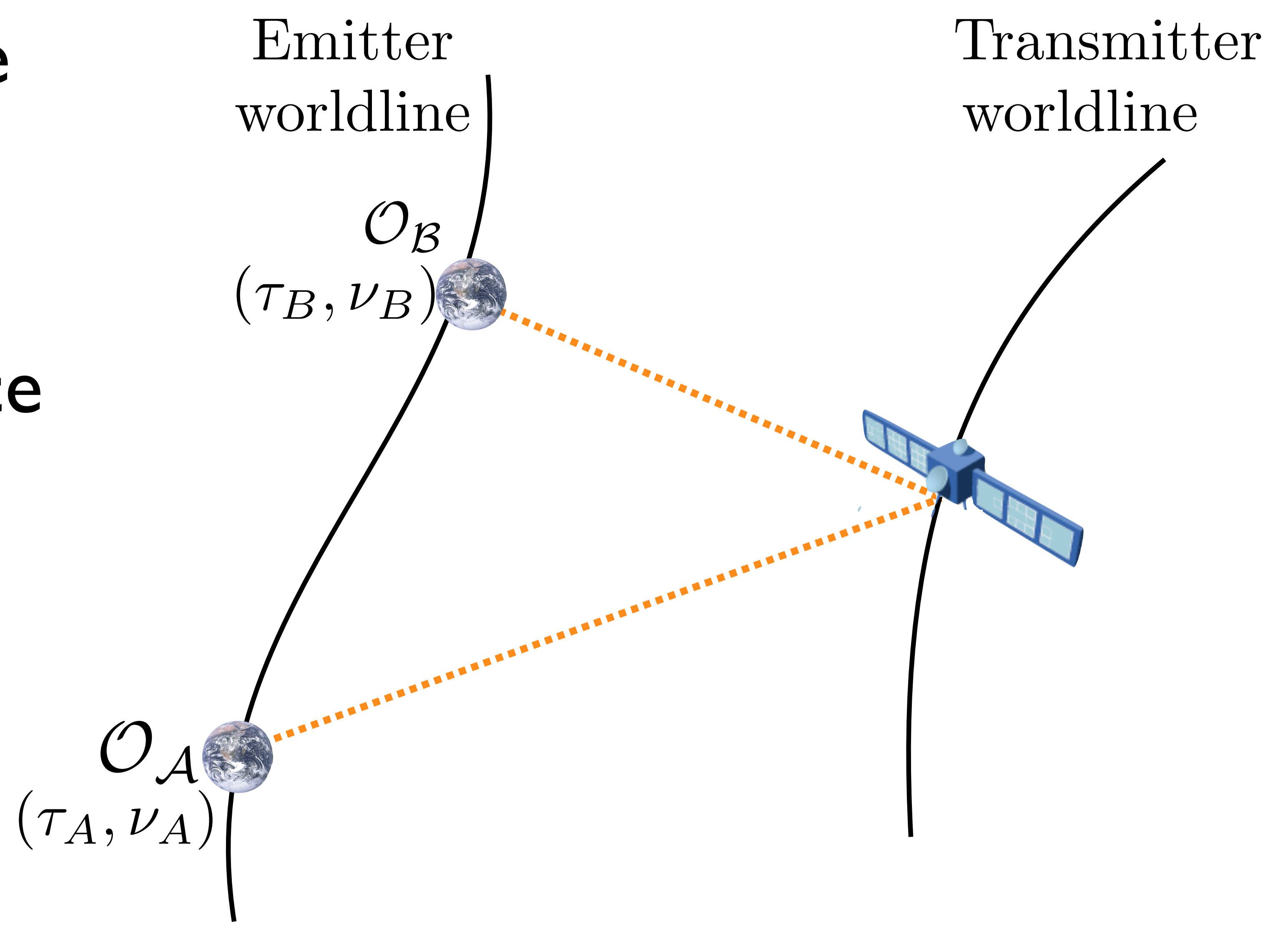
## 1) Range observable

- Difference in proper time

$$\mathrm{Range} = c(\tau_B - \tau_A)$$

- Depends on the difference in coord. time (amongst other parameters)

$$t_B - t_A$$



# Light propagation is crucial in the modeling of Sol. Sys. observations

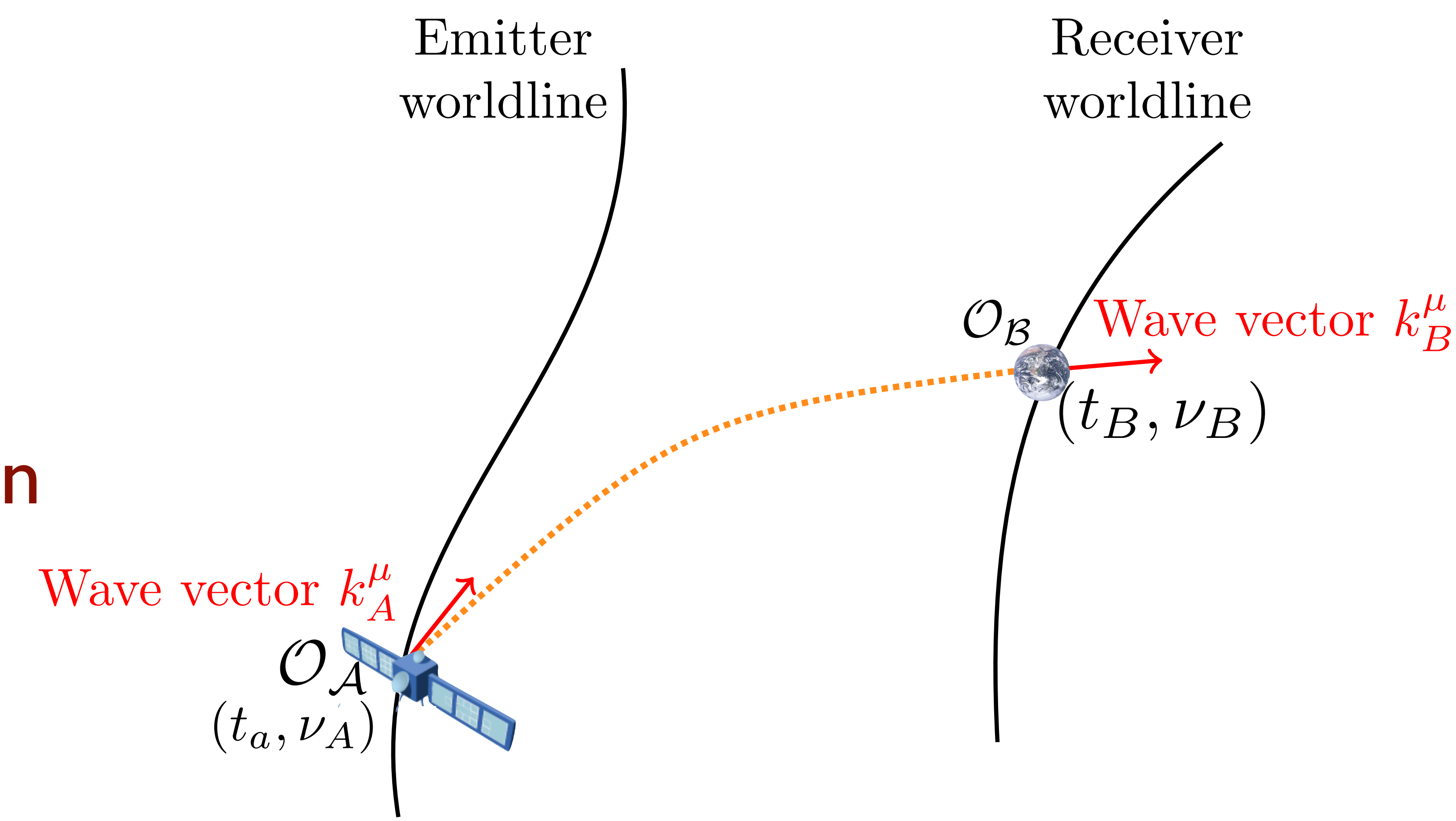
## 2) Doppler observable

- Ratio of proper frequency $D = \frac{\nu_B}{\nu_A} = \left(\frac{d\tau}{dt}\right)_A \left(\frac{d\tau}{dt}\right)_B^{-1} \boxed{\frac{k_0^B}{k_0^A}} \frac{1+\beta^i_B \hat{k}_i^B}{1+\beta^i_A \hat{k}_i^A}$

with $\beta^i = v^i/c$ and

$$\boxed{\hat{k}_i = \frac{k_i}{k_0}}$$

- Wave vector at emission and reception needed



# Light propagation is crucial in the modeling of Sol. Sys. observations

## 3) Astrometric observables

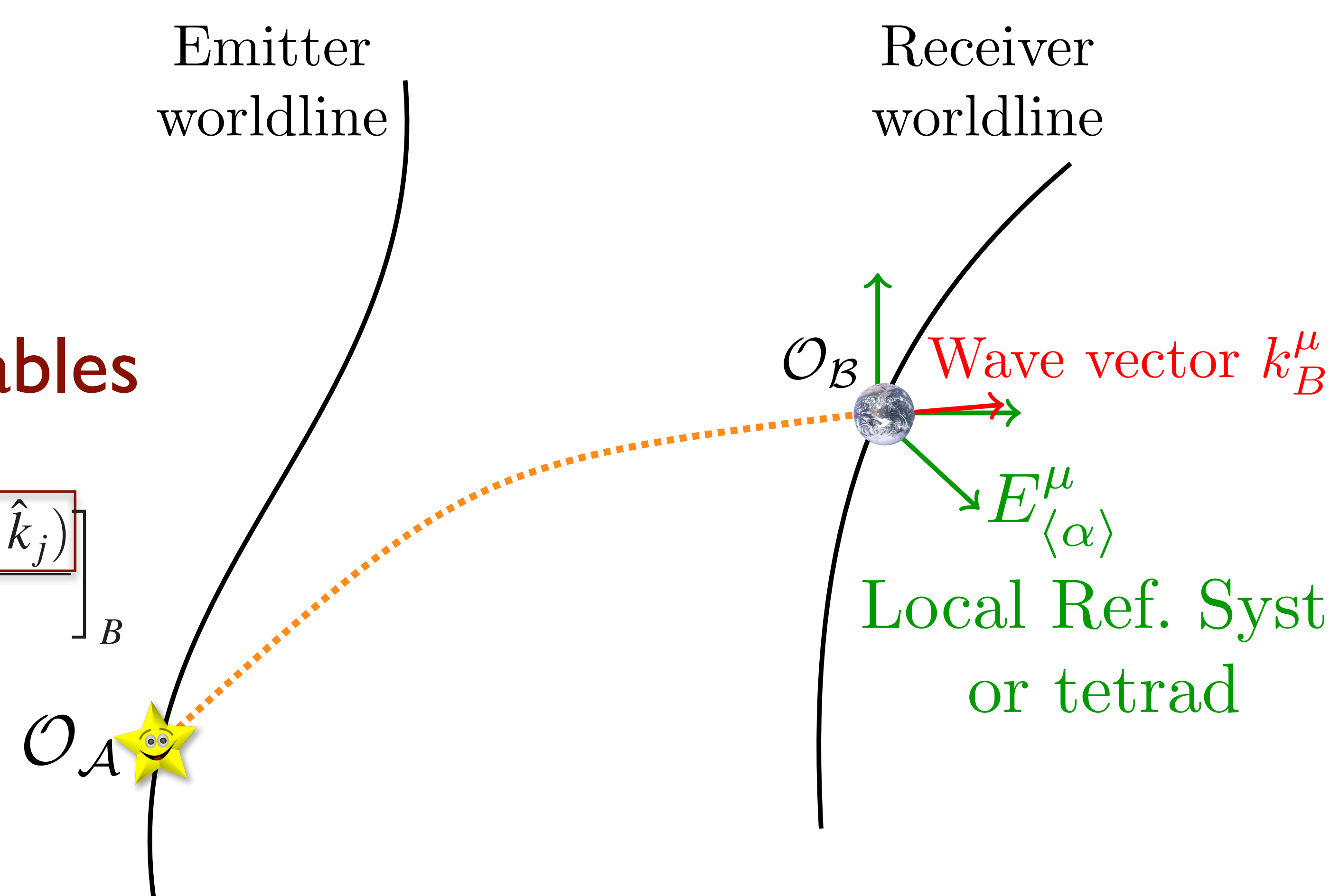
Direction of observation of the light ray in a local reference system (or tetrad)

$$n^{\langle i \rangle} = -\frac{E^0_{\langle i \rangle} + E^j_{\langle i \rangle}\boxed{\hat{k}^B_j}}{E^0_{\langle 0 \rangle} + E^j_{\langle 0 \rangle}\boxed{\hat{k}^B_j}}$$

## 4) Differential astrometric observables

$$\sin^2\frac{\phi}{2} = -\frac{1}{4}\left[\frac{(g_{00}+2g_{0k}\beta^k+g_{kl}\beta^k\beta^l)g^{ij}\boxed{(\hat{k}'_i-\hat{k}_i)(\hat{k}'_j-\hat{k}_j)}}{(1+\beta^m\underline{\hat{k}_m})(1+\beta^l\underline{\hat{k}_l{}'})}\right]_B$$

Angle between 2 incoming light rays




Wave vector at reception needed only

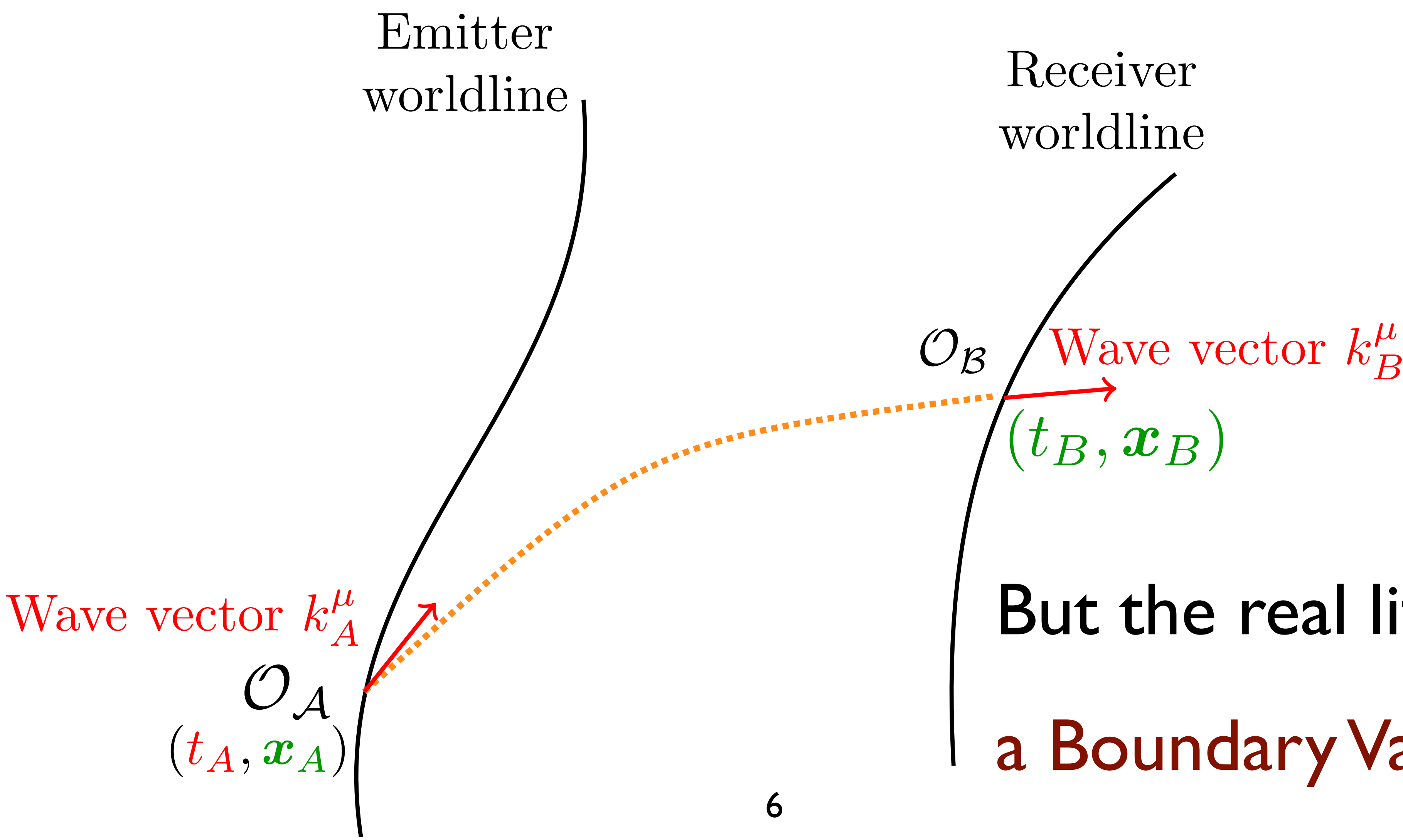
# How to determine the light propagation ?

- At the geometric optics approximation: photons follow null geodesics

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0 \qquad k^\mu k_\mu = 0$$

an Initial Value Problem

with $k^\mu = \frac{dx^\mu}{d\lambda}$ the tangent vector




But the real life :

a Boundary Value Problem

# Methods to solve the null geodesic eqs.

- Full numerical integration of the null geodesic eqs. with a shooting method

see A. San Miguel, Gen. Rel. Grav. 39, 2025, 2007
MT Crosta et al, CQG, 32, 165008, 2015

- Exact analytical solution for some metrics: Schwarzschild and Kerr (solution with Jacobian/Weierstrass elliptic functions)

see for example: de Jans, Mem. de l'Ac. Roy. de Bel., 1922
B. Carter, Com. in Math. Phys. 10, 280, 1968
A. Cadez, U. Kostic, PRD 72, 104024, 2005
A. Cadez, et al, New Astr. 3, 647, 1998

- Analytical solutions for weak gravitational field:
    - 1 pM Schwarzschild metric

      see E. Shapiro, PRL 13, 26, 789, 1964
    - moving monopoles at 1pM order

      see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999
      S. Klioner, A & A, 404, 783, 2003
      MT Crosta, CQG, 28, 235013, 2011
    - static extended bodies with multipolar expansion at 1pM

      see S. Kopeikin, J. of Math. Phys., 38, 2587
      S. Zschocke, PRD 92, 063015, 2015
    - 2 pM Schwarzschild metric

      see G. Richter, R. Matzner, PRD 28, 3007, 1983
      S. Klioner, S. Zschocke, CQG 27, 075015, 2010

- Use of the eikonal equation:
    - perturbative solution for spherically symmetric space-time

      see for example N. Ashby, B. Bertotti, CQG 27, 145013, 2010

# … and the Time Transfer Functions

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

- The Time Transfer Functions - TTF - are defined by

$$t_B - t_A = \mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) \qquad t_B - t_A = \mathcal{T}_e(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B)$$

- The TTF is solution of an eikonal equation well adapted to a perturbative expansion

- The derivatives of the TTF are of crucial interest since

$$\hat{k}_i^A = c\frac{\partial \mathcal{T}_r}{\partial x_A^i} \qquad \hat{k}_i^B = -c\frac{\partial \mathcal{T}_r}{\partial x_B^i}\left[1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1} \qquad \frac{k_0^B}{k_0^A} = 1 - \frac{\partial \mathcal{T}_r}{\partial t_B}$$

Range, Doppler, astrometric observables can be written in terms of the TTF and its derivatives

# Synge's World Function as TTF progenitor

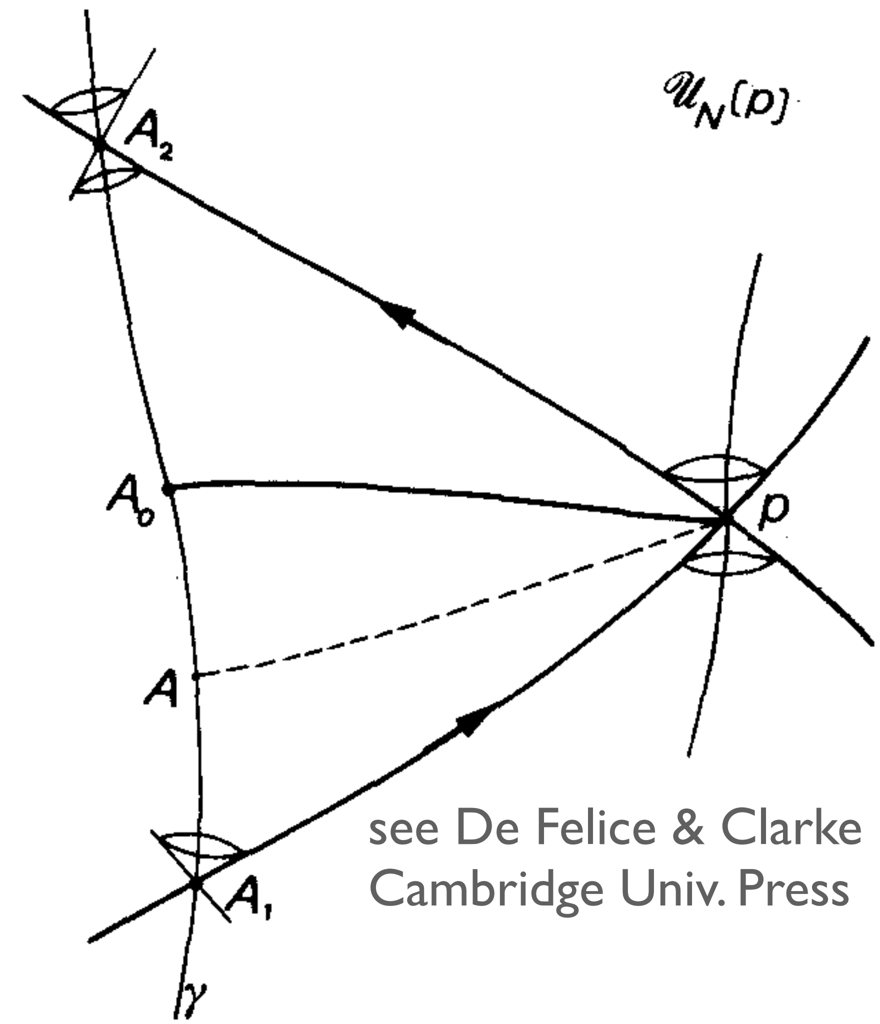
see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

Suppose the existence of two event-points $x_A$ and $x_B$ on a manifold. We assume that they are located in a convex neighbourhood in such a way that they are connected by a unique geodesic.

One can define a Synge's World Function between $x_A$ and $x_B$ (Ruse 1931, Synge 1931, 1964)

$$\Omega\left(x_A, x_B\right) = \frac{\epsilon_{AB}}{2} \int_0^1 g_{\mu\nu}(x^\alpha(\lambda)) \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} d\lambda\,,$$

where $\lambda$ is an affine parameter, $\epsilon_{AB} = -1, 0, 1$



see De Felice & Clarke Cambridge Univ. Press

Very difficult to determine it… Schwarzschild (Buchdhal 1979).
But an iterative Post-Minkowskian expansion has been found

# Synge's World Function as TTF progenitor

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

**World Function property I:** Hamilton-Jacobi equations

$$\frac{1}{2}\,g^{\alpha\beta}(x_A)\frac{\partial\Omega}{\partial x_A^\alpha}(x_A,x_B)\frac{\partial\Omega}{\partial x_A^\beta}(x_A,x_B)=\Omega(x_A,x_B)\,,$$

$$\frac{1}{2}\,g^{\alpha\beta}(x_B)\frac{\partial\Omega}{\partial x_B^\alpha}(x_A,x_B)\frac{\partial\Omega}{\partial x_B^\beta}(x_A,x_B)=\Omega(x_A,x_B)\,.$$

**World Function property II:** Tangent vectors at $x_A$ and $x_B$

$$\left(g_{\mu\nu}\frac{dx^\nu}{d\lambda}\right)_A=-\frac{\partial\Omega}{\partial x_A^\mu}(x_A,x_B)\,,\qquad\left(g_{\mu\nu}\frac{dx^\nu}{d\lambda}\right)_B=\frac{\partial\Omega}{\partial x_B^\mu}(x_A,x_B)\,.$$

**World Function property III:** particular case of light rays

$$\epsilon_{AB}=0\qquad\Leftrightarrow\qquad\Omega\left(x_A,x_B\right)=0$$

# Synge's World Function as TTF progenitor

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

Let us introduce the emission TTF as follows

$$\Omega\left(x_A^0, \boldsymbol{x}_A, x_A^0 + c\mathcal{T}_e(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B), \boldsymbol{x}_B\right) \equiv 0$$

If we differentiate with respect to $x_A^0$, $x_A^i$ and $x_B^i$

$$c\,\frac{\partial\Omega}{\partial x_B^0}(x_A, x_B)\,\frac{\partial\mathcal{T}_e}{\partial x_B^i}(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B) + \frac{\partial\Omega}{\partial x_B^i}(x_A, x_B) = 0\,,$$

$$\frac{\partial\Omega}{\partial x_A^i}(x_A, x_B) + c\,\frac{\partial\Omega}{\partial x_B^0}(x_A, x_B)\,\frac{\partial\mathcal{T}_e}{\partial x_A^i}(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B) = 0\,,$$

$$\frac{\partial\Omega}{\partial x_A^0}(x_A, x_B) + \frac{\partial\Omega}{\partial x_B^0}(x_A, x_B)\left[1 + \frac{\partial\mathcal{T}_e}{\partial t_A}(t_A, \boldsymbol{x}_A, \boldsymbol{x}_B)\right] = 0\,,$$

Same reasoning on reception TTF

$$\Omega\left(x_B^0 - c\mathcal{T}_r(t_B, \boldsymbol{x}_A, \boldsymbol{x}_B), \boldsymbol{x}_A, x_B^0, \boldsymbol{x}_B\right) \equiv 0\,.$$

# Fundamental properties of TTF's

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

## It leads to the fundamental theorem for TTF

$$\left(\frac{k_i}{k_0}\right)_B = -c\,\frac{\partial \mathcal{T}_e}{\partial x^i_B} = -c\,\frac{\partial \mathcal{T}_r}{\partial x^i_B}\left[1-\frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1}, \quad \left(\frac{k_i}{k_0}\right)_A = c\,\frac{\partial \mathcal{T}_e}{\partial x^i_A}\left[1+\frac{\partial \mathcal{T}_e}{\partial t_A}\right]^{-1} = c\,\frac{\partial \mathcal{T}_r}{\partial x^i_A},$$

$$\frac{(k_0)_B}{(k_0)_A} = \left[1+\frac{\partial \mathcal{T}_e}{\partial t_A}\right]^{-1} = 1-\frac{\partial \mathcal{T}_r}{\partial t_B}.$$

## But how to calculate a TTF ?

I. First calculate the world function, then apply $\Omega\left(x_A, x_B\right)$ is equal to 0 and use a Lagrange inversion (2004)

II. Realize that $[g^{\mu\nu}k_\mu k_\nu]_{x_A/x_B} \equiv 0$, so (2008)

$$\Rightarrow g^{00}(x^0_B - c\mathcal{T}_r, \boldsymbol{x}_A) + 2c\, g^{0i}(x^0_B - c\mathcal{T}_r, \boldsymbol{x}_A)\frac{\partial \mathcal{T}_r}{\partial x^i_A} + c^2\, g^{ij}(x^0_B - c\mathcal{T}_r, \boldsymbol{x}_A)\frac{\partial \mathcal{T}_r}{\partial x^i_A}\frac{\partial \mathcal{T}_r}{\partial x^j_A} = 0$$

$$g^{00}(x^0_A + c\mathcal{T}_e, \boldsymbol{x}_B) - 2c\, g^{0i}(x^0_A + c\mathcal{T}_e, \boldsymbol{x}_B)\frac{\partial \mathcal{T}_e}{\partial x^i_B} + c^2\, g^{ij}(x^0_A + c\mathcal{T}_e, \boldsymbol{x}_B)\frac{\partial \mathcal{T}_e}{\partial x^i_B}\frac{\partial \mathcal{T}_e}{\partial x^j_B} = 0.$$

TTF is a <u>dedicated World Function</u> to light ray.
General Post-Minkowskian expansions are possible

# Post-Minkowskian expansion of the TTF

see P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

- A pM expansion of the TTF: $\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \dfrac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}_r^{(n)}$

- Computation with an iterative procedure involving integrations over a straight line between the emitter and the spatial position of the receiver !

- Example at 1 pM: $\mathcal{T}_r^{(1)} = \dfrac{R_{AB}}{2c} \displaystyle\int_0^1 \left[ g_{(1)}^{00} - 2N_{AB}^i g_{(1)}^{0i} + N_{AB}^i N_{AB}^j g_{(1)}^{ij} \right]_{z^\alpha(\lambda)} d\lambda$

  with $z^\alpha(\lambda)$ the straight Mink. null path between em. and rec.

- Main advantages:

  - analytical computations relatively easy

  - very well adapted to numerical evaluation

# Analytical results in Schwarzschild space-time

see B. Linet and P. Teyssandier, CQG 30, 175008, 2014
P. Teyssandier, 2014, arXiv: 1407.4361

- A "simplified" iterative method has been developed for static spherically symmetric geometry

$$ds^2 = \left(1 - 2\frac{m}{r} + 2\beta\frac{m^2}{r^2} - \frac{3}{2}\beta_3\frac{m^3}{r^3} + \dots\right)dt^2 - \left(1 + 2\gamma\frac{m}{r} + \frac{3}{2}\epsilon\frac{m^2}{r^2} + \frac{1}{2}\gamma_3\frac{m^3}{r^3} + \dots\right)d\boldsymbol{x}^2$$

- In GR: $\gamma = \beta = \epsilon = \beta_3 = \gamma_3 = 1$

- A pM expansion of the TTF: $\mathcal{T} = \dfrac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}^{(n)}$

and the corresponding derivatives have been computed up to the 3rd pM order

# Analytical results in Schwarzschild space-time

- A pM expansion of the TTF: $\mathcal{T} = \dfrac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}^{(n)}$

$$\mathcal{T}^{(1)} = \frac{(1+\gamma)m}{c} \ln \frac{r_A + r_B + |\boldsymbol{x}_B - \boldsymbol{x}_A|}{r_A + r_B - |\boldsymbol{x}_B - \boldsymbol{x}_A|}$$

see E. Shapiro, PRL 13, 26, 789, 1964

$$\mathcal{T}^{(2)} = \frac{m^2}{r_A r_B} \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c} \left[ \kappa \frac{\arccos \boldsymbol{n}_A . \boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} - \frac{(1+\gamma)^2}{1 + \boldsymbol{n}_A . \boldsymbol{n}_B} \right]$$

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
S. Klioner, S. Zschocke, CQG 27, 075015, 2010

$$\mathcal{T}^{(3)} = \frac{m^3}{r_A r_B} \left( \frac{1}{r_A} + \frac{1}{r_B} \right) \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c(1 + \boldsymbol{n}_A . \boldsymbol{n}_B)} \left[ \kappa_3 - (1+\gamma)\kappa \frac{\arccos \boldsymbol{n}_A . \boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} + \frac{(1+\gamma)^3}{1 + \boldsymbol{n}_A . \boldsymbol{n}_B} \right]$$

see B. Linet and P. Teyssandier, CQG 30, 175008, 2014

with

$$\kappa = 2 + 2\gamma - \beta + \frac{3}{4}\epsilon$$

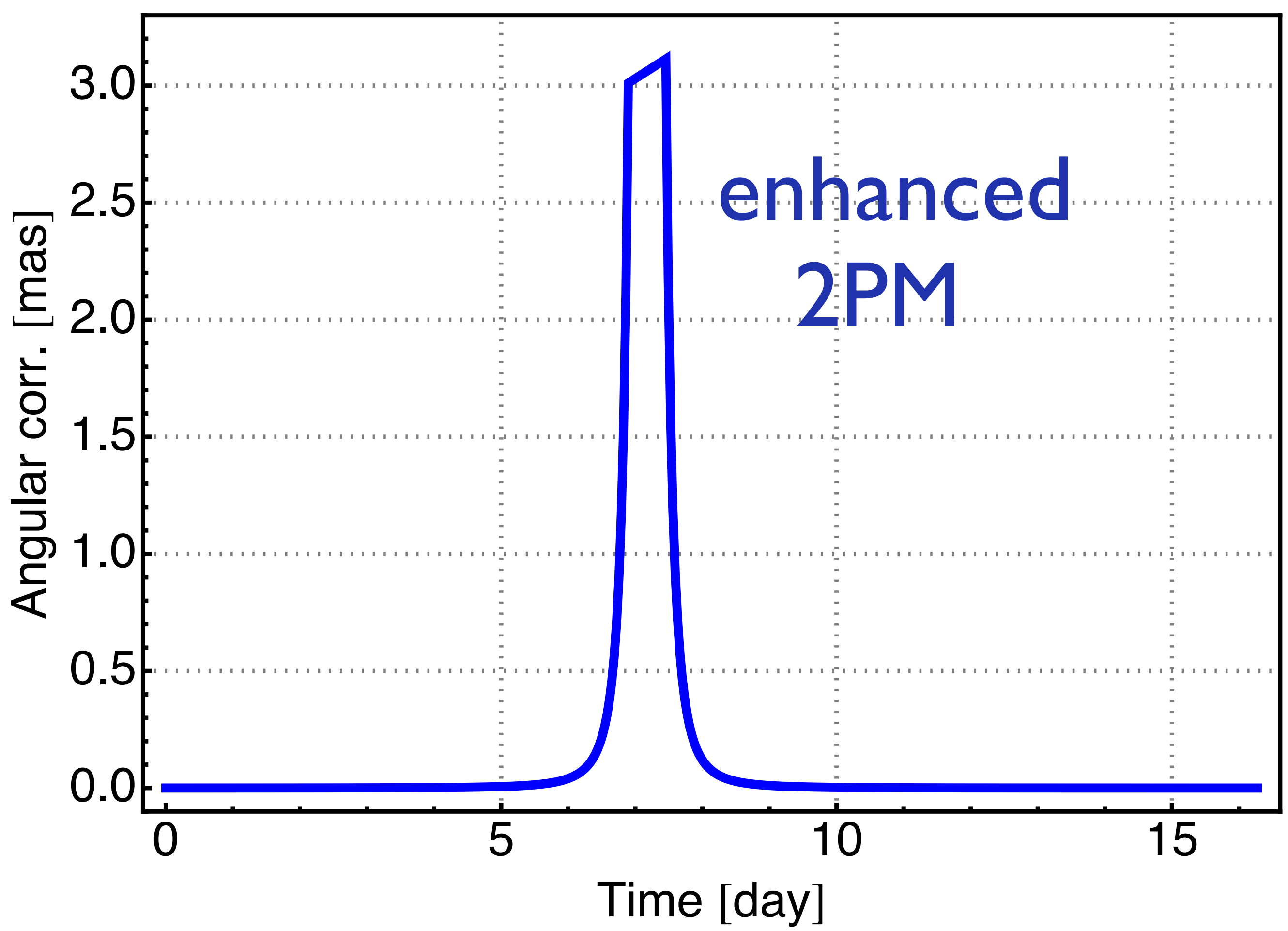
$$\kappa_3 = 2\kappa - 2\beta(1+\gamma) + \frac{1}{4}(3\beta_3 + \gamma_3)$$

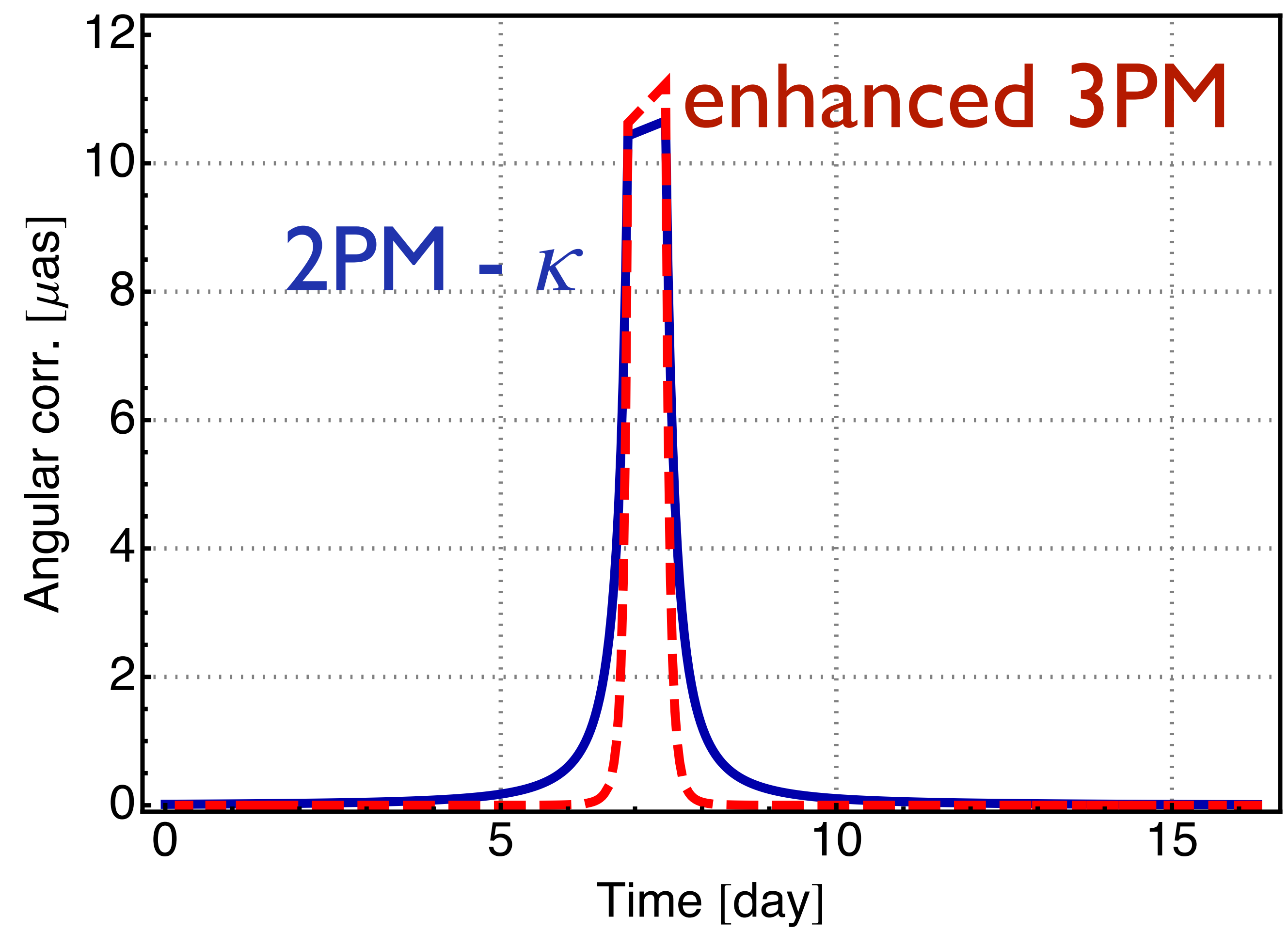
and

$$\boldsymbol{n}_{A/B} = \frac{\boldsymbol{x}_{A/B}}{r_{A/B}}$$

# Is it necessary to go to the 3rd order?

- In a conjunction geometry, at each order n, there are enhanced terms proportional to $(1+\gamma)^n$
- Ex. with light deflection for Sun grazing rays: AGP space mission (old GAME). Expected accuracy: $\mu$as
  $\Rightarrow$ 3pM term needed




see A. Hees, S. Bertone, C. Le Poncin-Lafitte, PRD 89, 064045, 2014
P. Teyssandier, B. Linet, proceedings of JSR 2013, arXiv:1312.3510

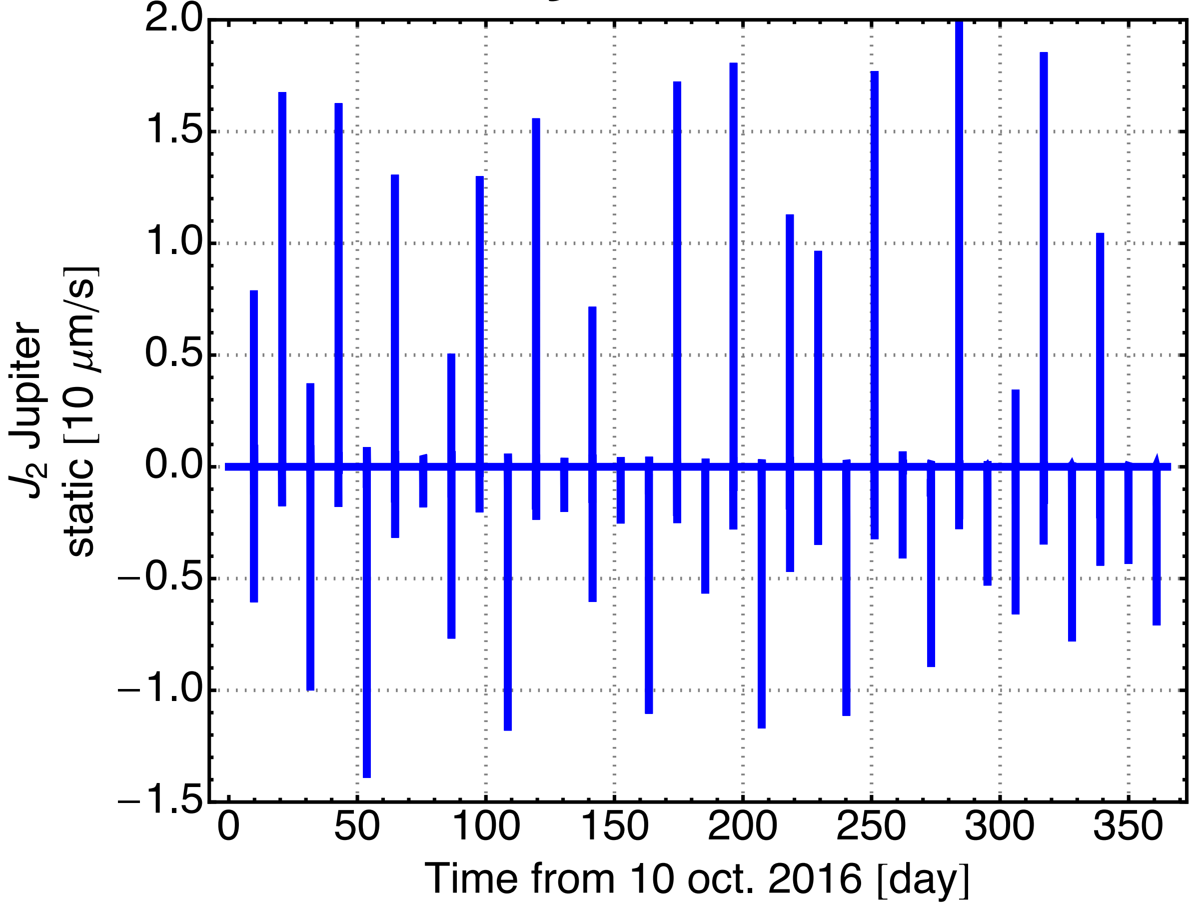
# Analytical result around axisymmetric bodies

- Influence of all the multipole moments $J_n$ from the grav. potential

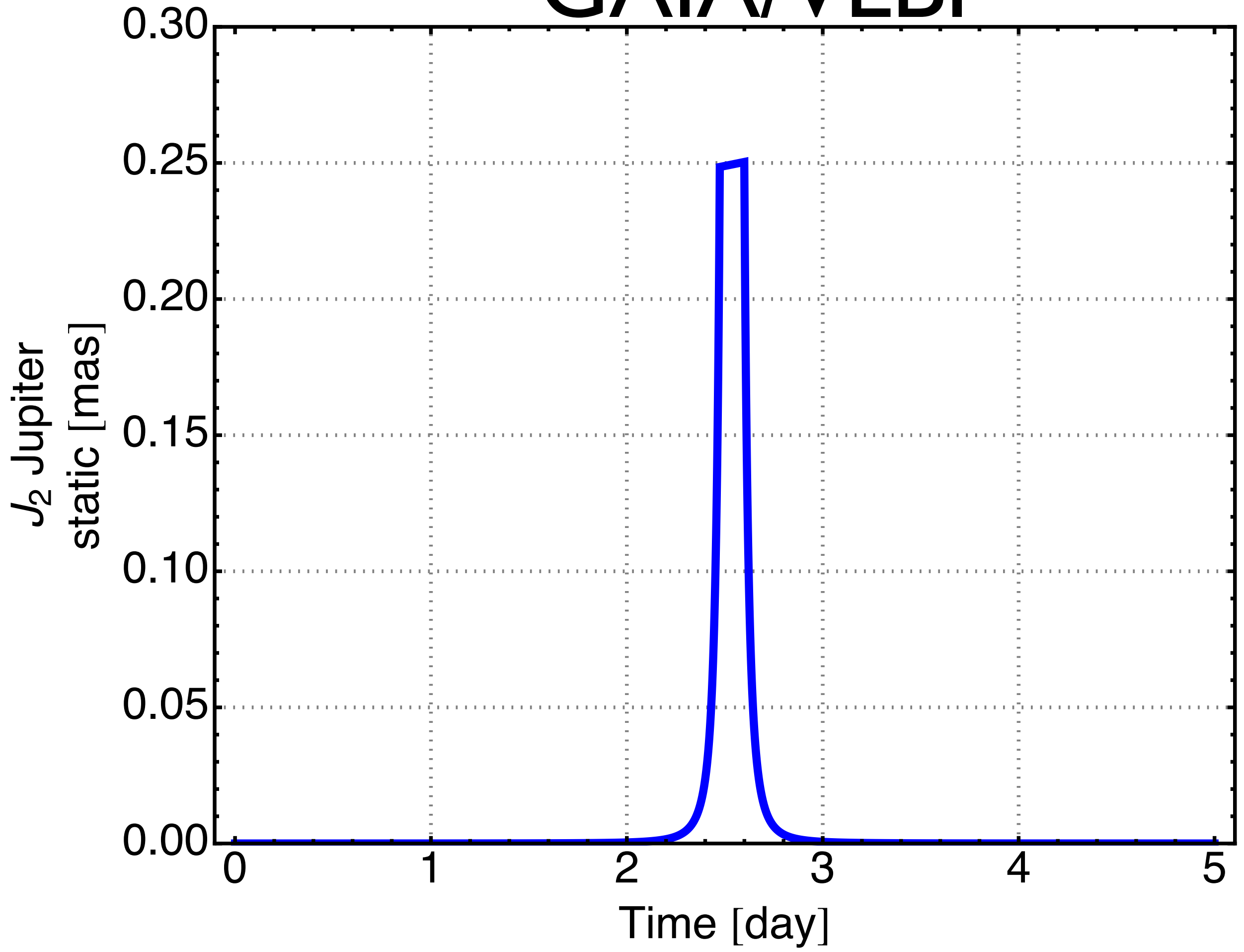
see C. Le Poncin-Lafitte, P. Teyssandier, PRD 77, 044029, 2008 for a computation with the TTF
or S. Kopeikin, J. of Math. Physics 38, 2587, 1997 for another approach

- Influence of Jupiter $J_2$ on the JUNO Doppler (1 $\mu$m/s accuracy)

JUNO




GAIA/VLBI




- terms important for the data analysis for these missions

see Hees, Bertone, Le Poncin-Lafitte, PRD 90, 084020, 2014

# What happens if the body is moving ?

see Hees, Bertone, Le Poncin-Lafitte, PRD 90, 084020, 2014

- At first pM order, the TTF for uniformly moving bodies can easily be derived from the TTF generated by a static body

$$\Delta(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \gamma(1 - \boldsymbol{N}_{AB}.\boldsymbol{\beta})\tilde{\Delta}(\boldsymbol{R}_A + \gamma\boldsymbol{\beta}R_{AB}, \boldsymbol{R}_B)$$

TTF in the moving case

static TTF

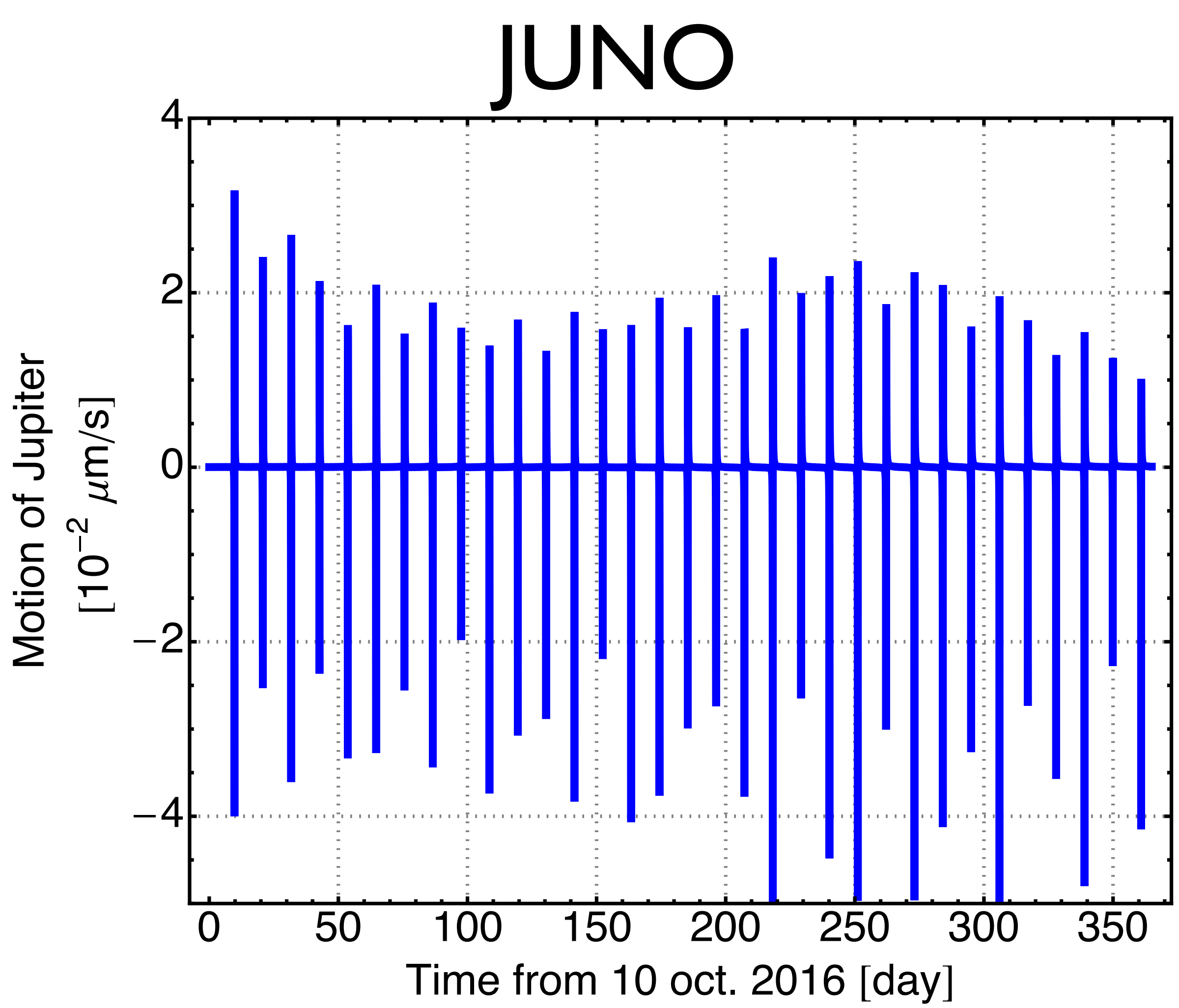
with $\boldsymbol{\beta} = \boldsymbol{v}/c, \quad \gamma = (1 - \beta^2)^{-1/2}$

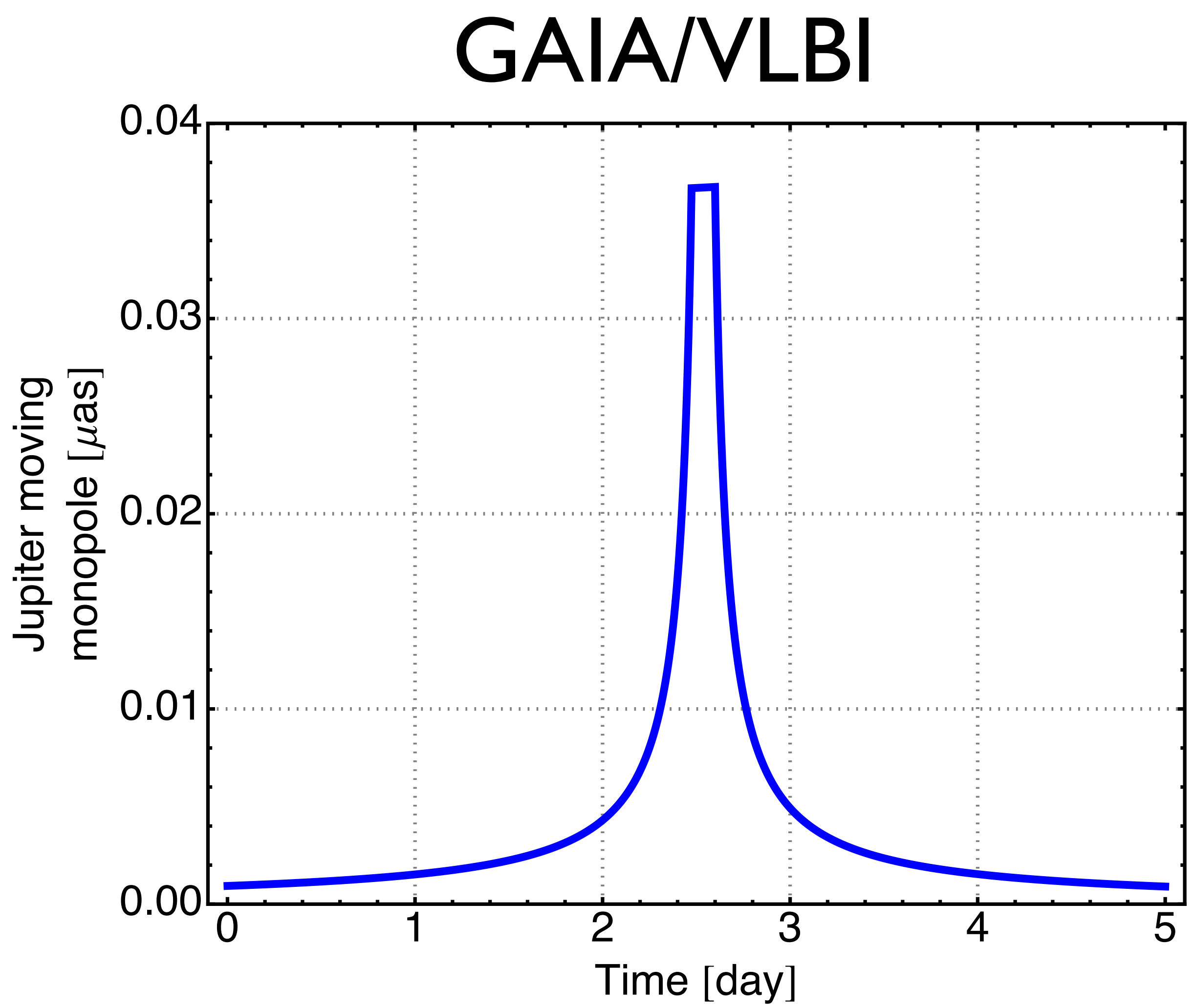
and $\boldsymbol{R}_X$ depends on $\boldsymbol{x}_X, \boldsymbol{\beta}$

- All the analytical results computed for a static source can be extended in the case of a uniformly moving source

# Ex.: motion of Jupiter

- Influence of Jupiter velocity on the JUNO Doppler (1 $\mu$m/s




- depend highly on the orbit geometry: conjunction and $\boldsymbol{\beta}.\boldsymbol{N}_{AB}$
- In particular: should be reassessed for JUICE orbit

see Hees, Bertone, Le Poncin-Lafitte, PRD 90, 084020, 2014

# Numerical evaluation of the TTF

- Iterative procedure involving integrals over a straight line: appropriate for numerical evaluation
- At 1pM order: a simple integral to evaluate

$$\mathcal{T}^{(1)} = \int_0^1 m\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

$$\frac{\partial \mathcal{T}^{(1)}}{\partial x^i_{A/B}} = \int_0^1 m_{A/B}\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

- At 2pM order: a double integral to evaluate

$$\mathcal{T}^{(2)} = \int_0^1 \int_0^1 n\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

$$\frac{\partial \mathcal{T}^{(2)}}{\partial x^i_{A/B}} = \int_0^1 \int_0^1 n_{A/B}\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(2)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta,\gamma\delta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

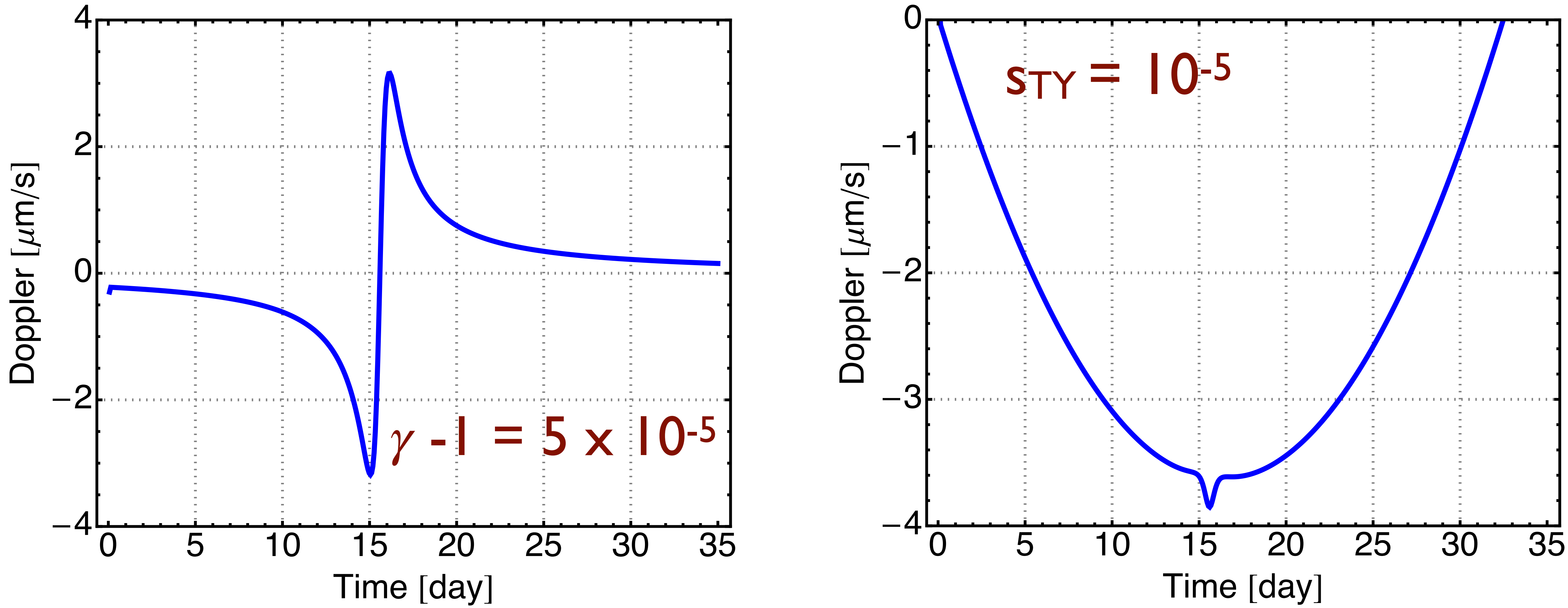
- Numerically efficient ; useful when no analytical solution can be found

see Hees, Bertone, Le Poncin-Lafitte, PRD 89, 064045, 2014

# Numerical evaluation of the TTF

- Numerical evaluation appropriate to evaluate effects due to alternative theories of gravitation
- Example: Doppler for 30 days of Cassini tracking between Jupiter and Saturn ("$\gamma$ experiment")
- Effect of the $\gamma$ PPN and of Standard Model Extension $s_{TY}$ on

for SME, see Q. Bailey and A. Kostelecky, PRD 74, 045001, 2006




see A. Hees, et al, CQG 29, 235027, 2012

# Einstein, Eddington 1919

## Complicated situation

Einstein prediction is published during WW1 in a german journal Annalen den Physik

However, the paper goes through the no man's land up and barbed wire up to England where Sir Eddington obtained the paper.

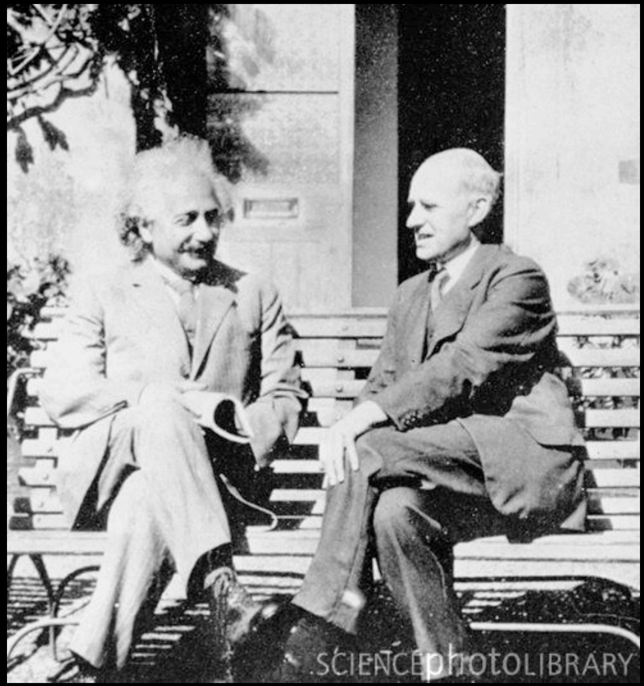



Photo 1930.
Crédit: Royal Astronomical Society

*Eddington (1882- 1944) holds the Professor's Chair Plumian Experimental Astronomy Cambridge from 1912 succeeding G. Darwin. Pacific, he refuses to participate in the war of 14-18. After a thorough study of Einstein's work, he sets out on an expedition off the coast of Africa to observe a total solar eclipse on May 29, 1919*


1. Several poor quality photographic plates
2. Disastrous weather conditions

One plate gaves 1.7”. Eddington kept it


22 November 1919 edition
of The Illustrated London News
Crédit: Royal Astronomical Society

# Testing Relativity since 1919

- How to test the form of the metric/the Einstein field equations ? Two frameworks widely used so far:

## I) Parametrized Post-Newtonian Formalism[1]

- powerful phenomenology making an interface between theoretical development and experiments
- metric parametrized by 10 dimensionless coefficients
- $\gamma$ and $\beta$ whose values are1 in GR

$$ds^2 = (1 + 2\phi_N + 2\beta\phi_N^2 + \dots)dt^2 - (1 - 2\gamma\phi_N + \dots)d\vec{x}^2$$

## II) Fifth force formalism[2]

- modification of Newton potential of the form of a Yukawa potential

$$\phi(r) = \frac{GM}{c^2 r}\left(1 + \alpha e^{-r/\lambda}\right)$$

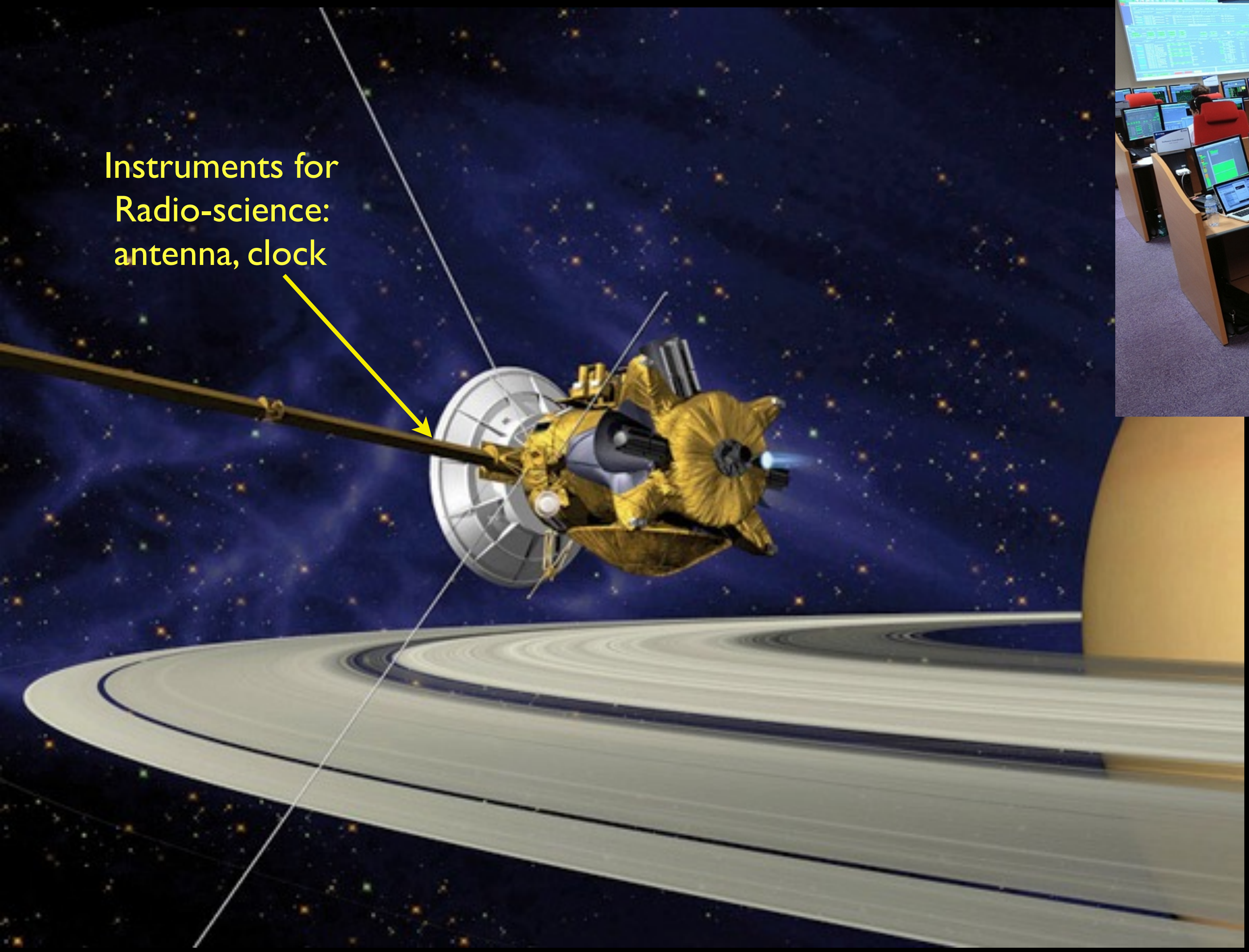
[1] C. Will, LRR, 9, 2006
"Theory and Experiment in Grav. Physics", C. Will, 1993

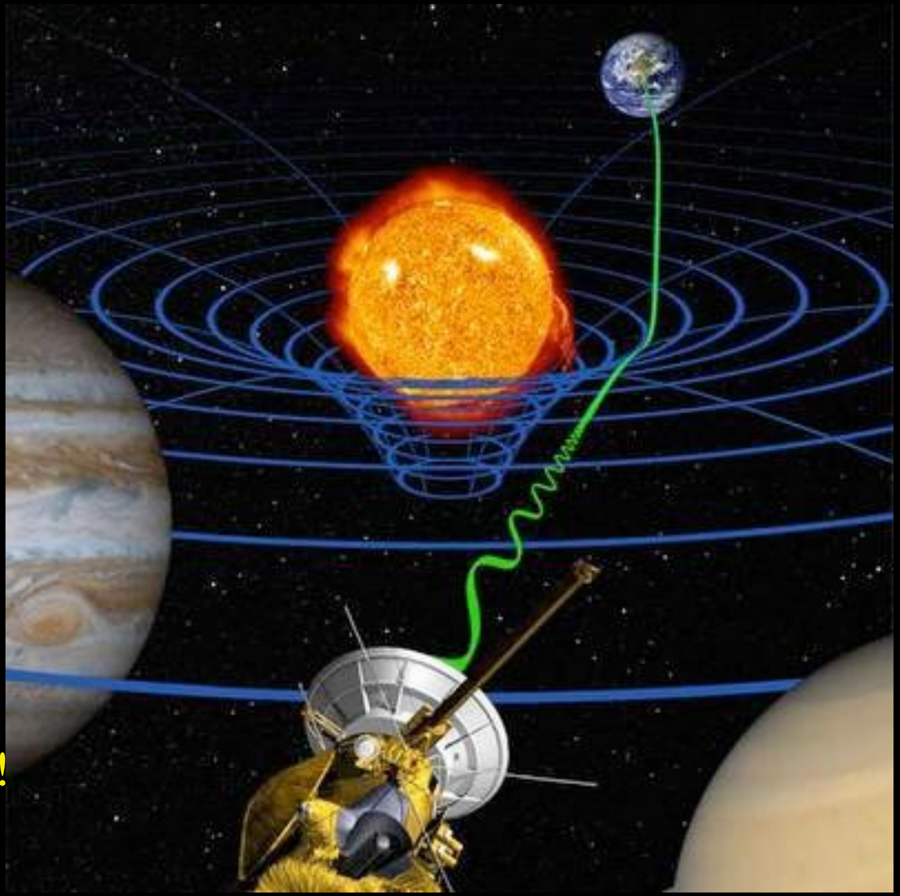
[2] E.G. Adelberger, Progress in Part. and Nucl. Phys., 62/102, 2009
"The Search for Non-Newtonian gravity", E. Fischbach, C. Talmadge, 1998

# PPN parameters and their significance

| Parameter | What it measures, relative to general relativity | Value in GR | Value in scalar tensor theory | Value in semi-conservative theories |
|---|---|---|---|---|
| $\gamma$ | How much space curvature produced by unit mass? | 1 | $(1+\omega)/(2+\omega)$ | $\gamma$ |
| $\beta$ | How "nonlinear" is gravity? | 1 | $1 + \Lambda$ | $\beta$ |
| $\xi$ | Preferred-location effects? | 0 | 0 | $\xi$ |
| $\alpha 1$ | Preferred-frame effects? | 0 | 0 | $\alpha 1$ |
| $\alpha 2$ | | 0 | 0 | $\alpha 2$ |
| $\alpha 3$ | | 0 | 0 | 0 |
| $\zeta 1$ | Is momentum conserved? | 0 | 0 | 0 |
| $\zeta 2$ | | 0 | 0 | 0 |
| $\zeta 3$ | | 0 | 0 | 0 |
| $\zeta 4$ | | 0 | 0 | 0 |

# Radio-Science Experiments



Conjonction between the Earth
And Cassini in September 2003 ⟷ Maximum of relativistic deflection !

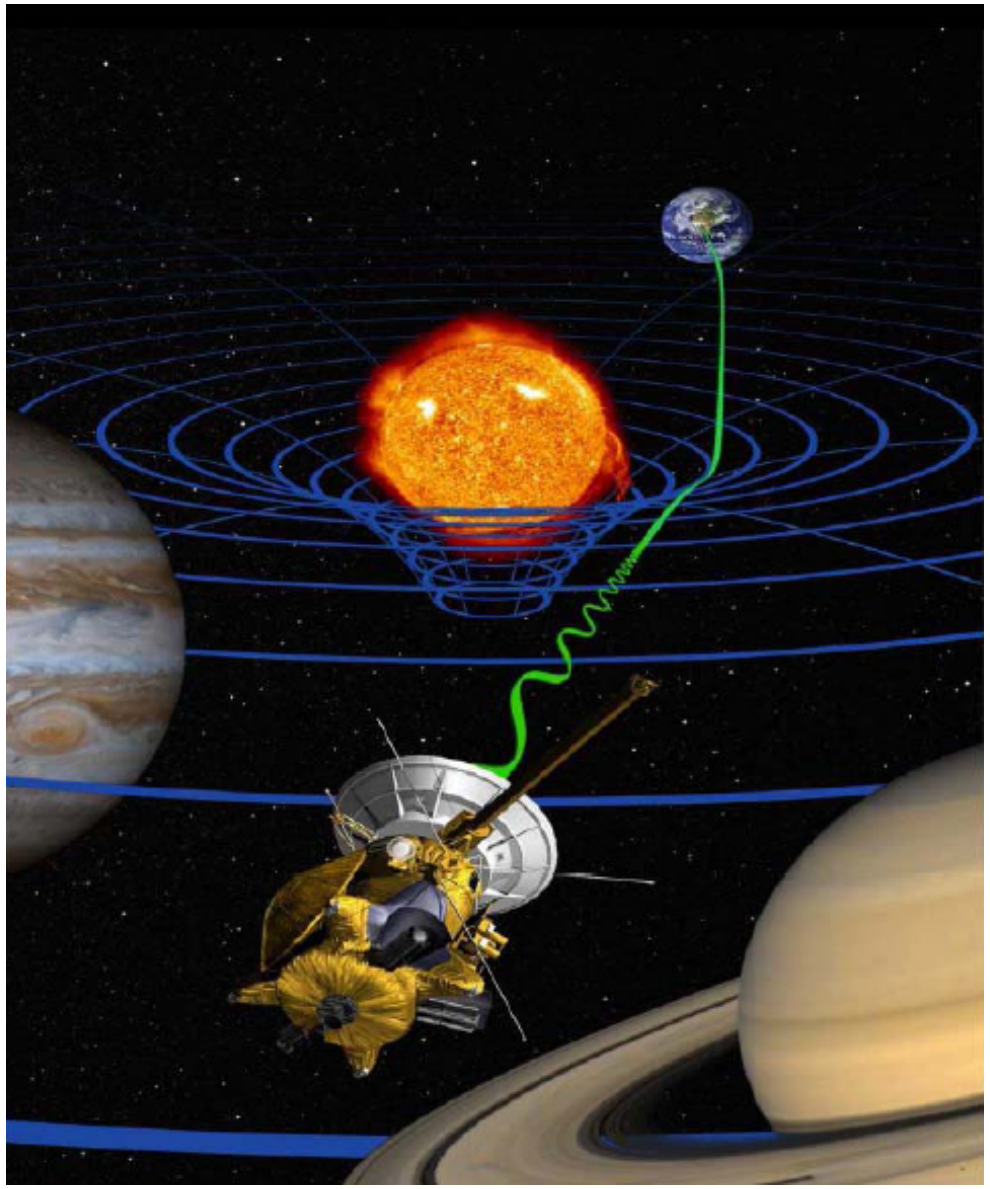
Italian team measured the change in signal frequency with a precision of some $10^{-14}$ in frequency fraction.

**Relativity is then correct at 0.002%**

Bertotti *et al.* 2003, *Nature,* **425,** 374

# Shapiro effect with Viking Probe

$$\Delta t \approx \frac{2GM(1+\gamma)}{c^3}\left[\ln\left(\frac{4r_A r_B}{r_0^{\ 2}}\right)+1\right]$$



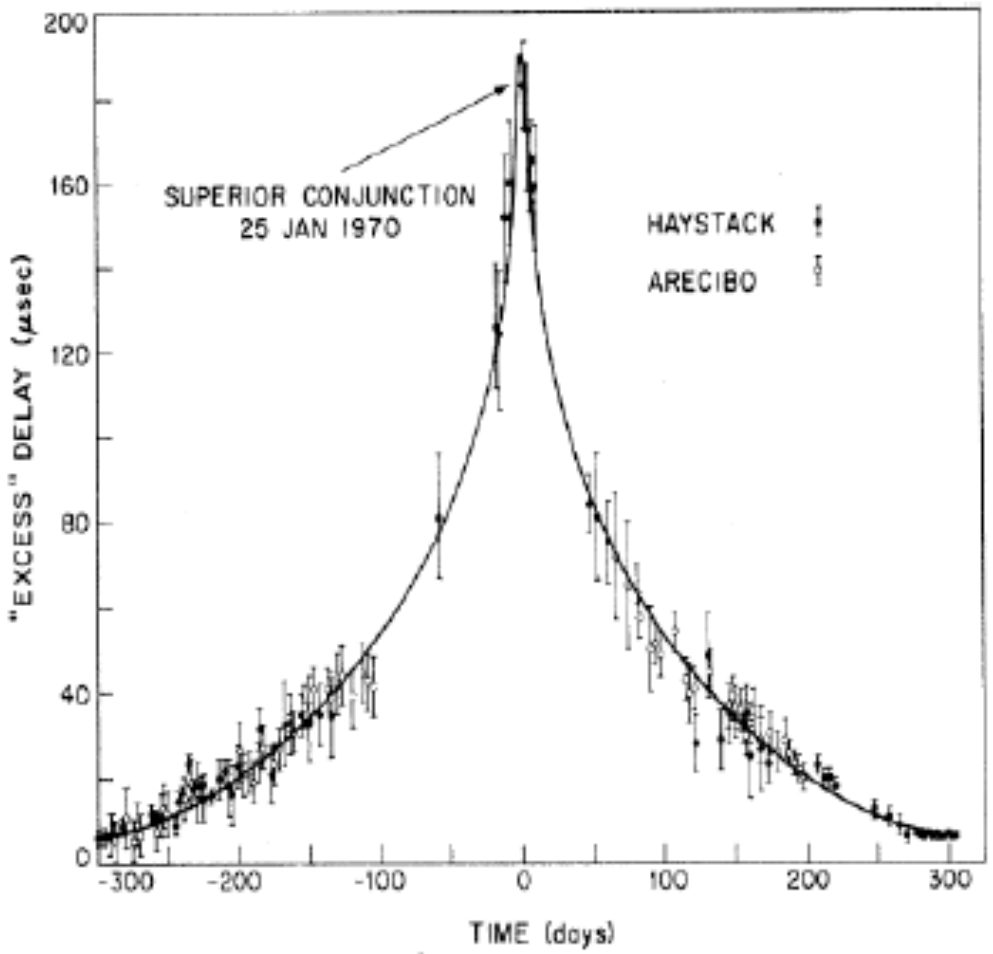
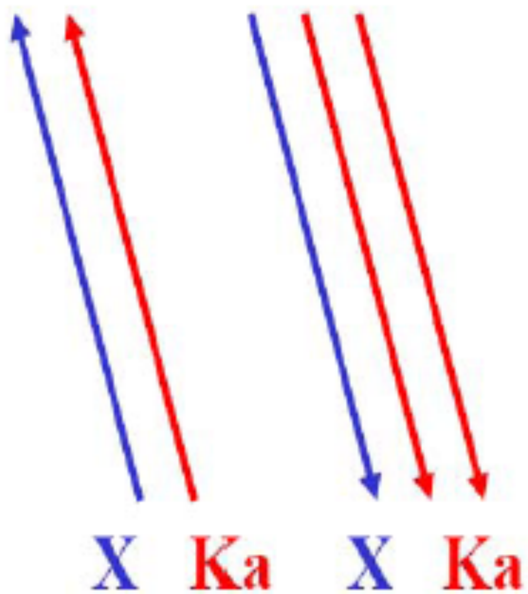
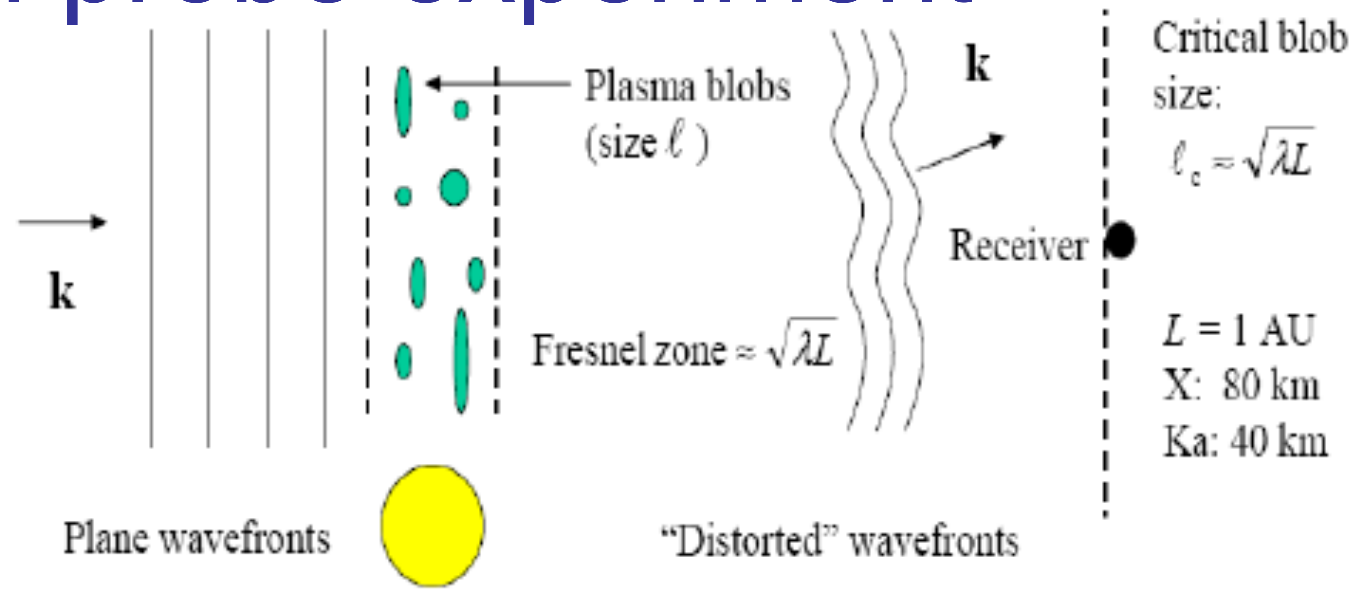



FIG. 1. Typical sample of post-fit residuals for Earth-Venus time-delay measurements, displayed relative to the "excess" delays predicted by general relativity. Corrections were made for known topographic trends on Venus. The bars represent the original estimates of the measurement standard errors. Note the dramatic increase in accuracy that was obtained with the radar-system improvements incorporated at Haystack just prior to the inferior conjunction of November 1970.

*Shapiro, I.I. et al, Phys.Rev.Lett, 26, 1132 (1971)*

# Cassini probe experiment

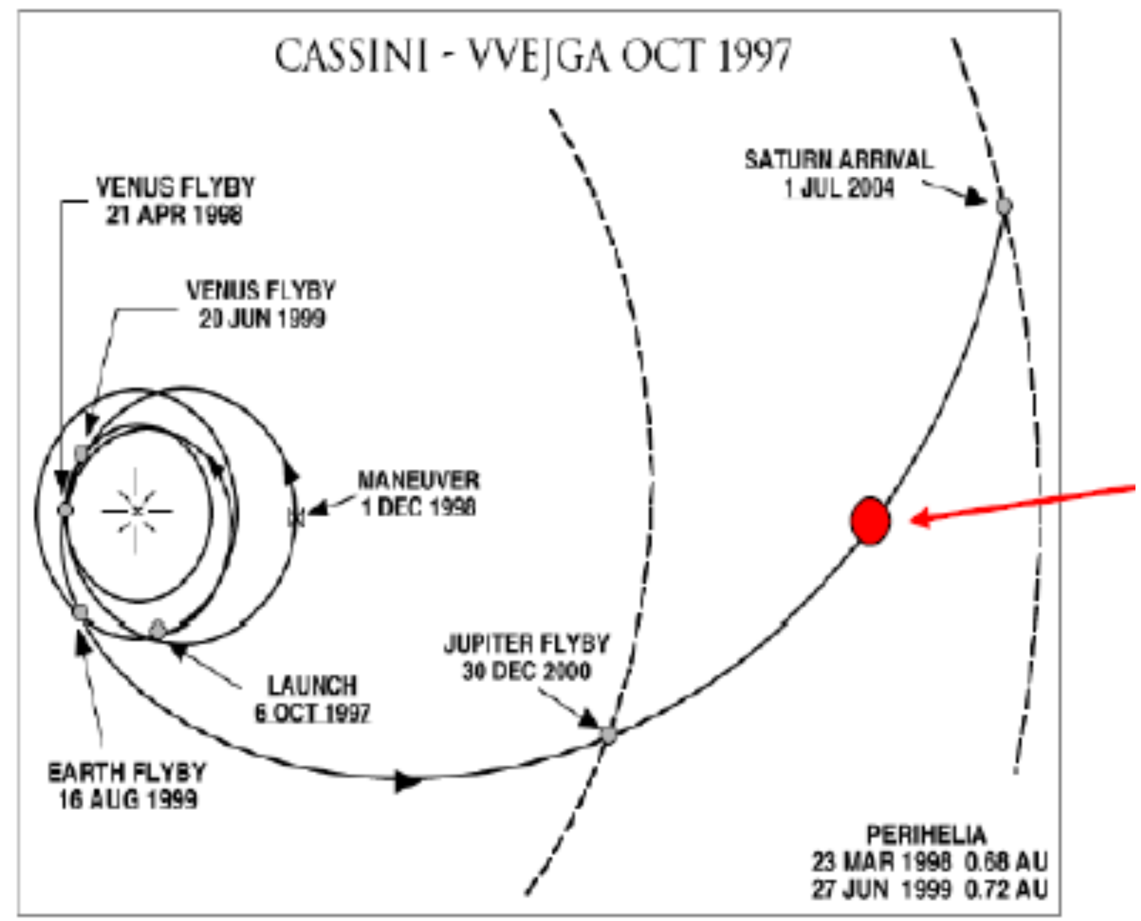


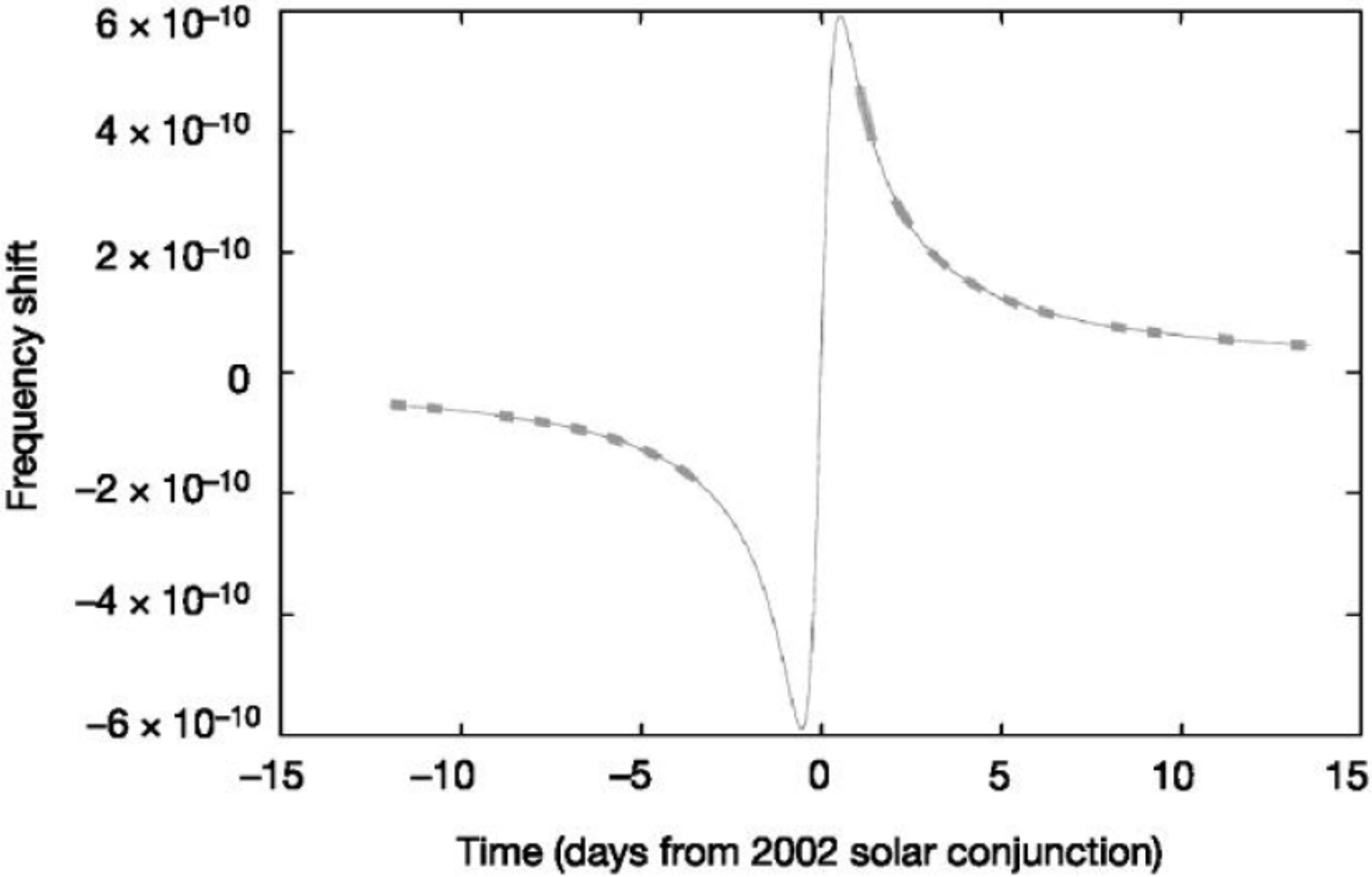

DSS 25 -
Goldstone

Doppler effect

$$\frac{\Delta \nu}{\nu} \approx 4(1+\gamma)\frac{M_{Sol}}{b}\frac{db}{dt}$$




$$\delta\left(\frac{f_r - f_e}{f_0}\right) \approx \frac{d}{dt}\left[2(1+\gamma)\frac{GM}{c^3}\ln\left(\frac{4r_S r_G}{b^2}\right)\right] \approx -4(1+\gamma)\frac{GM}{c^3 b}\frac{db}{dt}$$




$$\boxed{\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}}$$

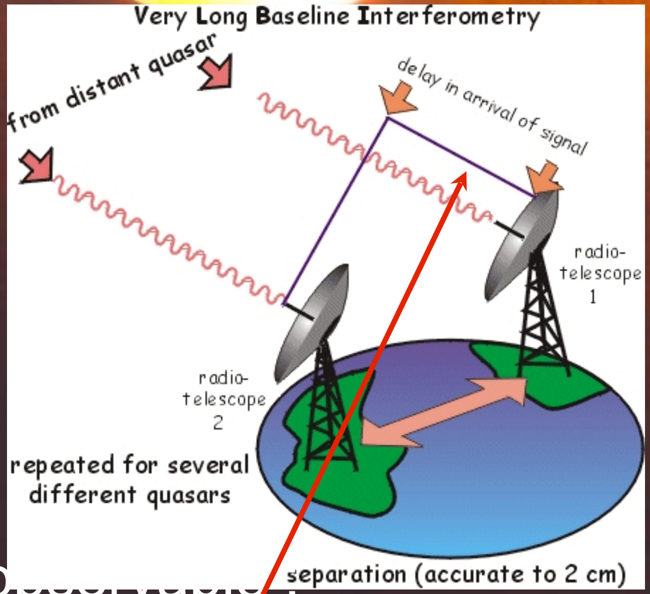
*Bertotti, B. et al, Nature, 425, 374 (2003)*

# Very Long Baseline Interferometry VLBI

Observations of quasars

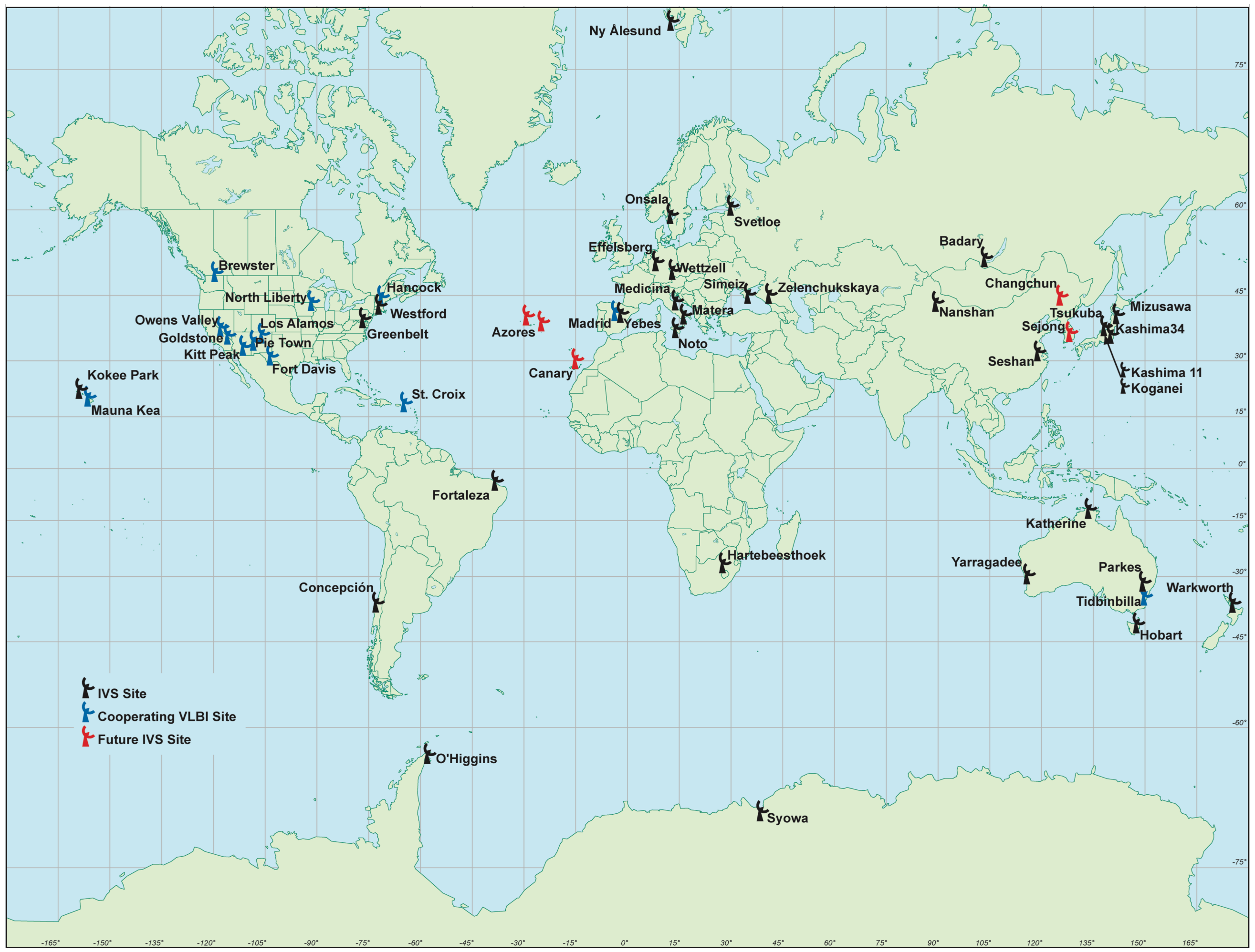
1. Statistically not moving
2. With 2 radiotelescopes, we are able to fix the Earth orientation with respect to quasars

<u>kinematical position of the Earth in space</u>




Observable :
time delay between the reception of signal at the radiotelescopes

# International VLBI service (IVS)



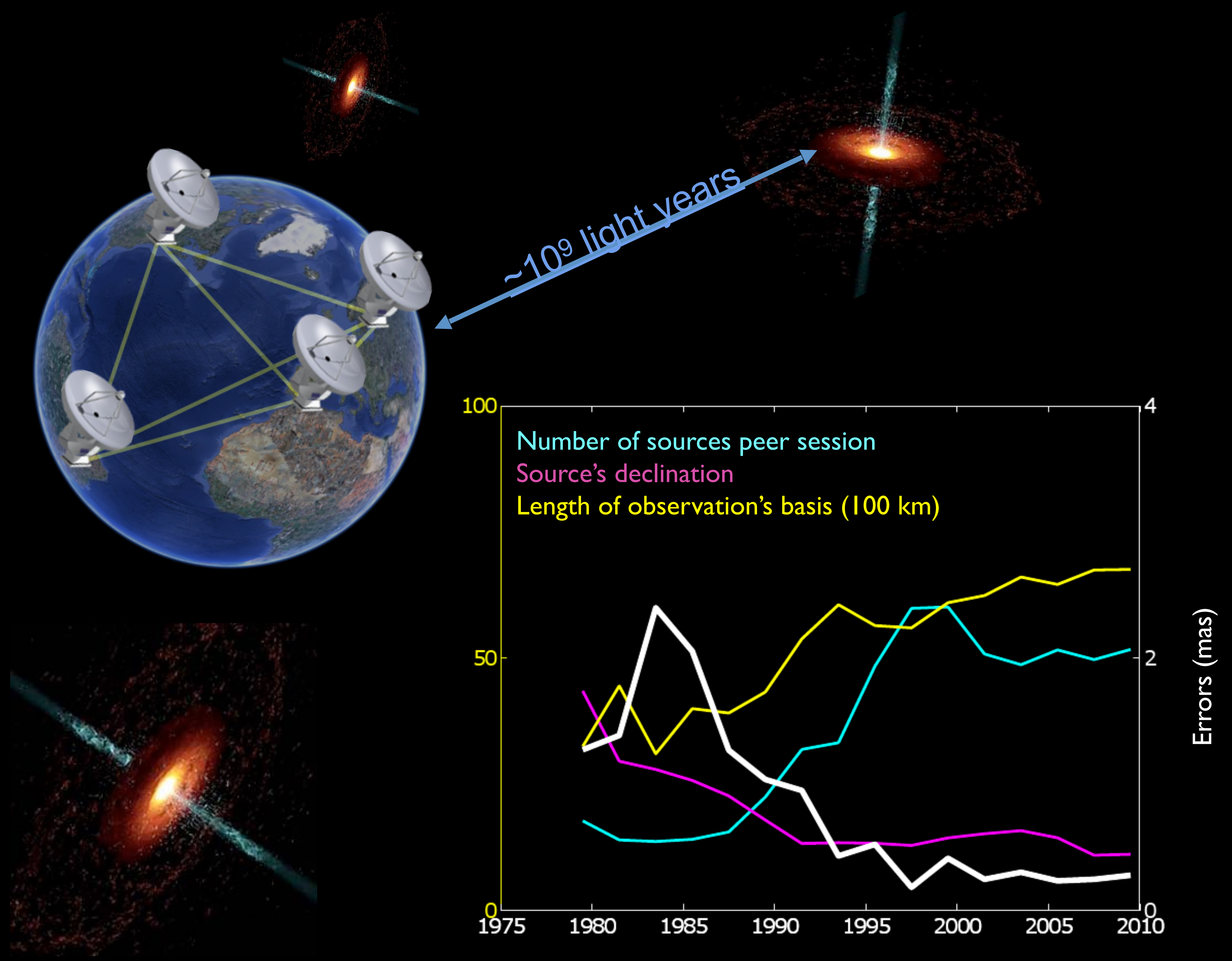

primary goals :
- monitoring the Earth's rotation
- determining reference frames

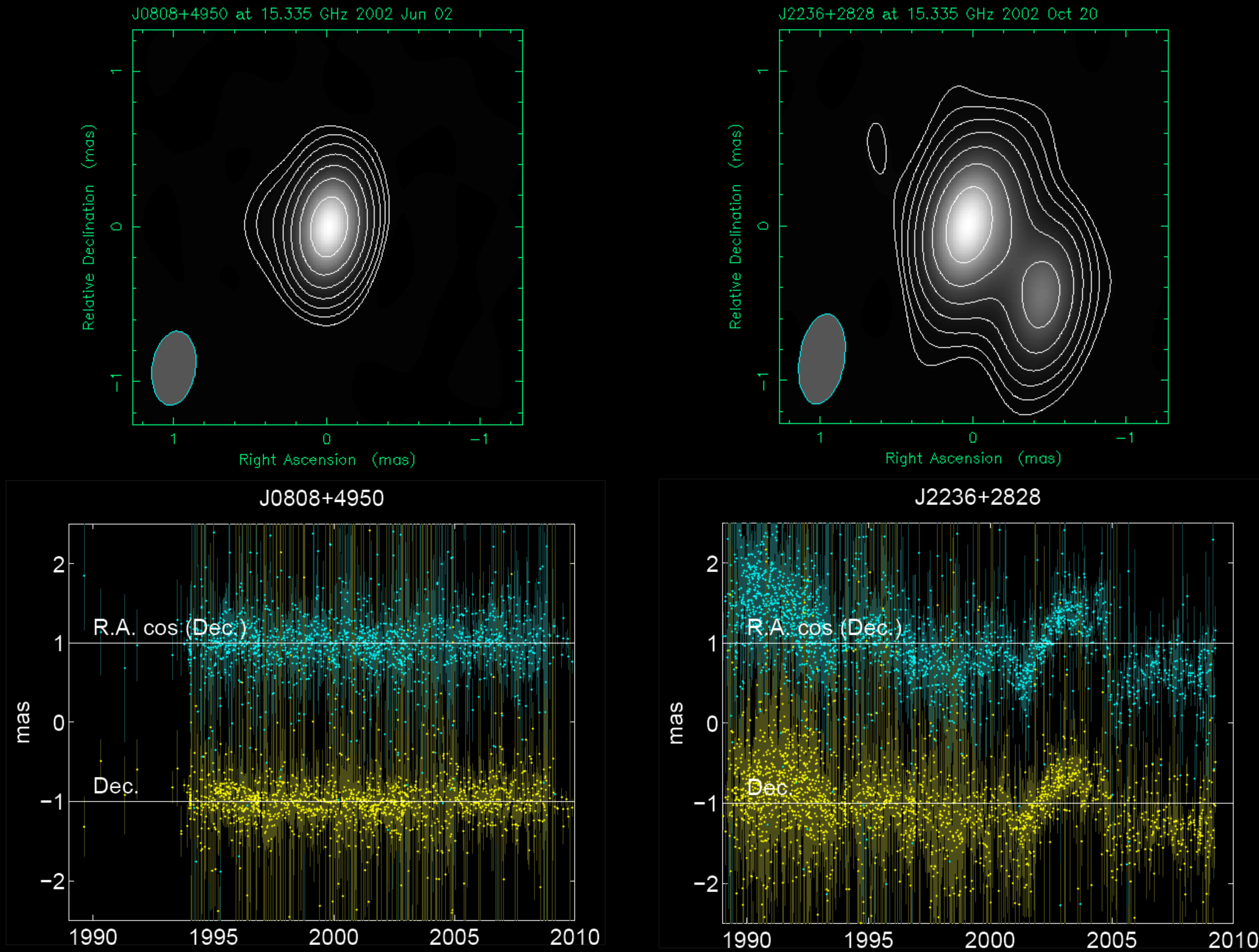
5 data centers and 29 analysis centers

IVS-OPAR @ SYRTE/Obs. Paris
lead : S. Lambert

Use Mark-5 VLBI Analysis Software Calc/Solve.
109 programs, 3680 modules
1.02 million lines of source code
written mainly in Fortran-95

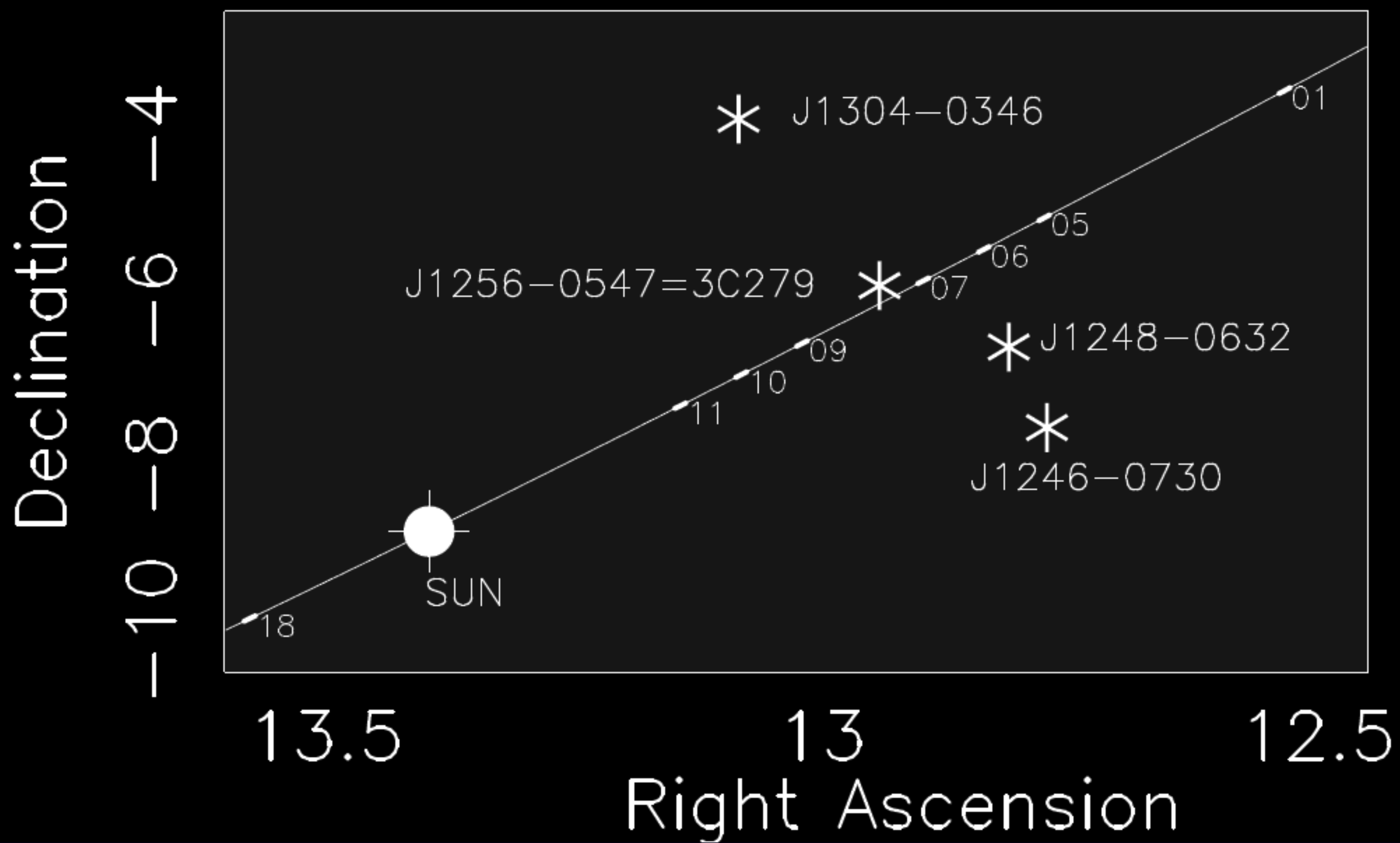
Observation time span : From August 1979 to mid-2016
almost 6000 VLBI 24-hr sessions
(correspondingly 10 million delays)


~10⁹ light years
Number of sources peer session
Source's declination
Length of observation's basis (100 km)
100
50
0
4
2
0
Errors (mas)
1975
1980
1985
1990
1995
2000
2005
2010



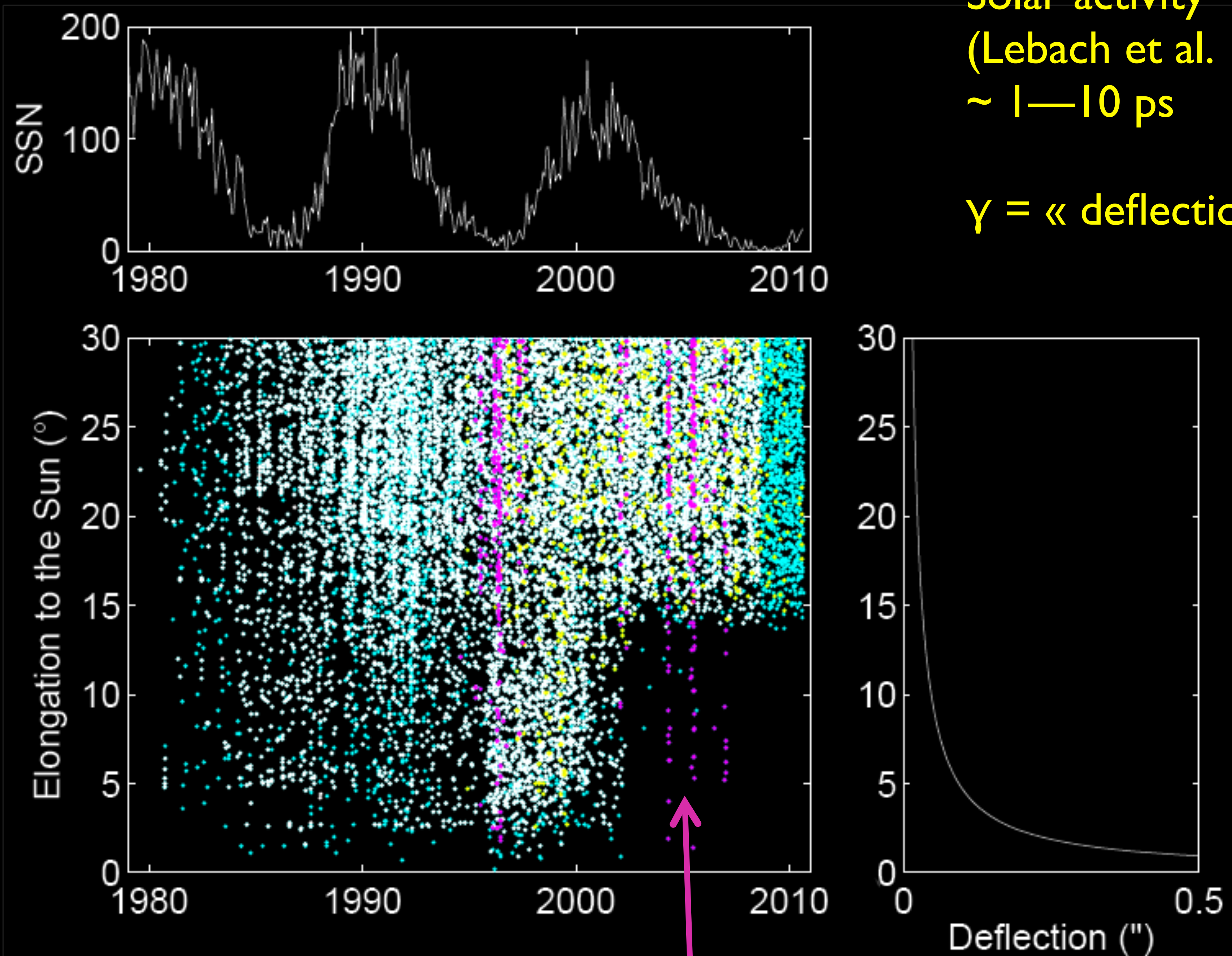
J0808+4950 at 15.335 GHz 2002 Jun 02
Relative Declination (mas)
Right Ascension (mas)
1
0
−1
J2236+2828 at 15.335 GHz 2002 Oct 20
Relative Declination (mas)
Right Ascension (mas)
J0808+4950
R.A. cos (Dec.)
Dec.
mas
2
1
0
−1
−2
1990
1995
2000
2005
2010
J2236+2828
R.A. cos (Dec.)
Dec.
mas

# Fomalont et al. 2009




$\gamma - 1 = 2 \pm 3 \times 10^{-4}$

Lambert & Le Poncin-Lafitte 2009, 2010 : use of the complete geodetic VLBI database




Solar activity = stronger deflection (Lebach et al. 1995)
~ 1—10 ps

γ = « deflection » coefficient

Without VLBA :
$\gamma - 1 = 0.4 \pm 1.4 \times 10^{-4}$

+ RDV :
$\gamma - 1 = -0.7 \pm 1.3 \times 10^{-4}$

+ VCS :
$\gamma - 1 = -0.8 \pm 1.2 \times 10^{-4}$

VLBA ~ 3% of sessions ~ 30% of observations

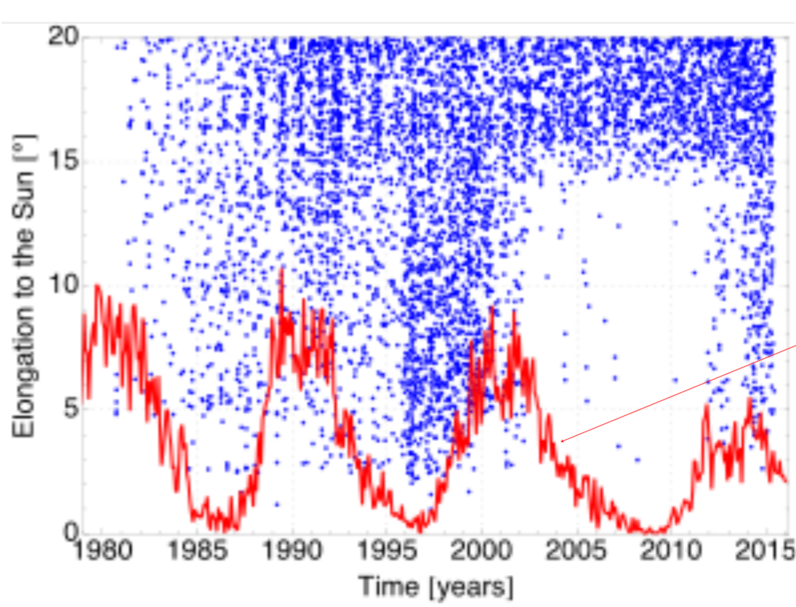
# Modeling SME-VLBI delay & fit

Lambert & CLPL, 2009 and 2011 : determination of PPN Gamma at the level of $10^{-4}$,
1 order of mag below Cassini **but strong statistics & robustess**

First, we derive the VLBI delay in SME from Bailey (2009) :

$$\Delta\tau_{(\mathrm{grav})} = 2\frac{\widetilde{GM}}{c^3}\left(1-\frac{2}{3}\bar{s}^{TT}\right)\ln\frac{r_1+\boldsymbol{k}.\boldsymbol{x}_1}{r_2+\boldsymbol{k}.\boldsymbol{x}_2} + \frac{2}{3}\frac{\widetilde{GM}}{c^3}\bar{s}^{TT}\left(\boldsymbol{n}_2.\boldsymbol{k}-\boldsymbol{n}_1.\boldsymbol{k}\right) .$$

with $\boldsymbol{x}_{1/2}$ positions of stations and $r_{1/2} = |\boldsymbol{x}_{1/2}$ $\quad \boldsymbol{n}_{1/2} = \frac{\boldsymbol{x}_{1/2}}{r_{1/2}}$
and $\boldsymbol{k}$ is the direction of the source.

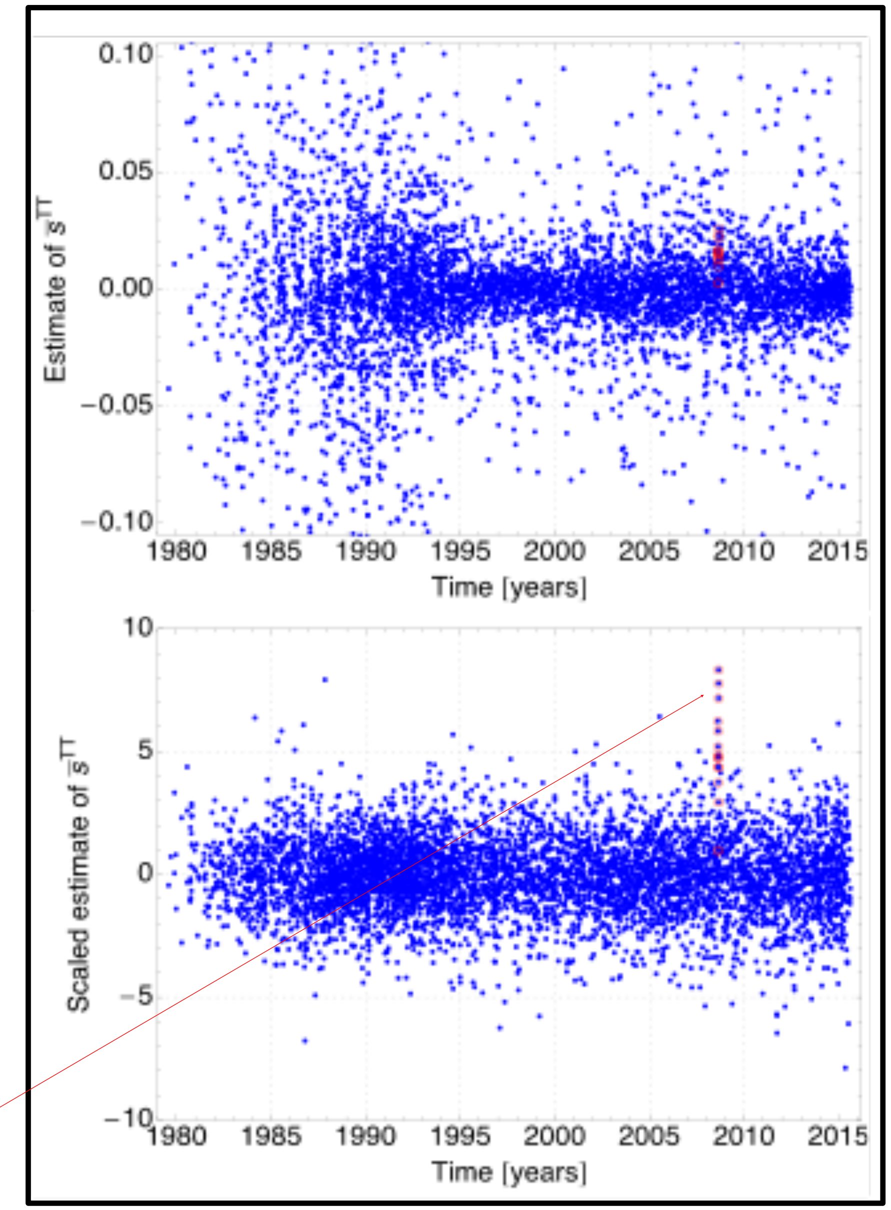



- Modification of CALC with module USERPART. Test with post-fit analysis :

$$\bar{s}^{TT} = (-0.6 \pm 2.1) \times 10^{-8}$$

- 2 & 8 Ghz for solar activity
- 8 Ghz for SME analysis
- Systematics on CONT08 data
 but we kept them.

$$\bar{s}^{TT} = (-5 \pm 8) \times 10^{-5}$$




CLPL, Hees & Lambert, PRD 2016
**arXiv:1604.01663**

# THE PARAMETER $(1+\gamma)/2$