

# NUMERICAL MODELING OF RADIATION BELT DYNAMICS

Alexander Drozdov

University of California, Los Angeles, CA, USA

## Acknowledgments

RBSP ECT Team, NASA Omniweb, NCAR Yellowstone, NASA CCMC, GFZ

D. Konrashov, Y. Shprits, Y. Zheng, L. Rastaetter, M. Usanova, G. Reeves, and many others

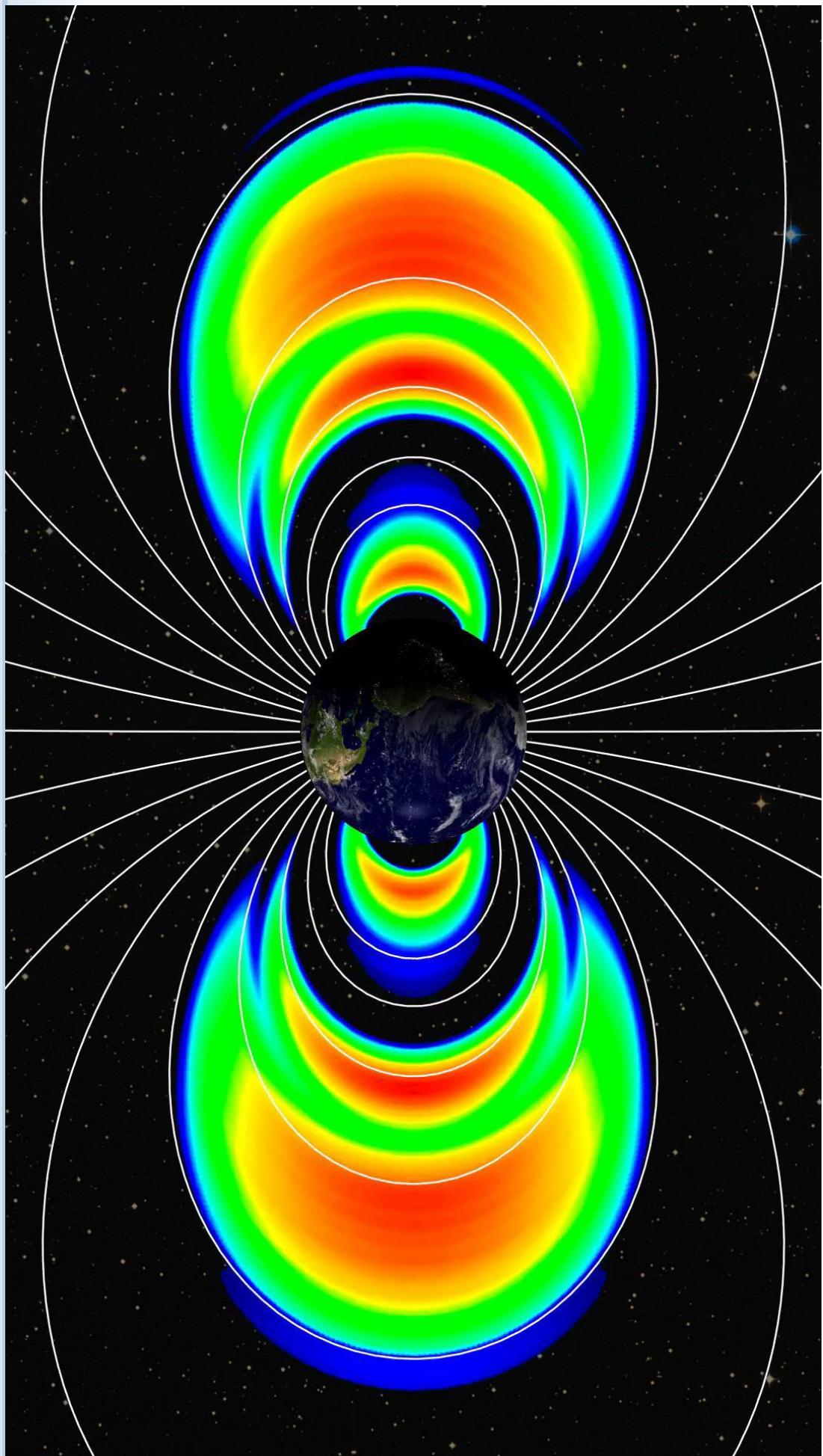
## Outline

---

1. Introduction
2. Fokker-Planck equation
3. From 1D simulation to 3D
4. How can you run diffusion code? (NASA CCMC)
5. 3D simulation on a single grid
6. Practical results
7. 4D simulations

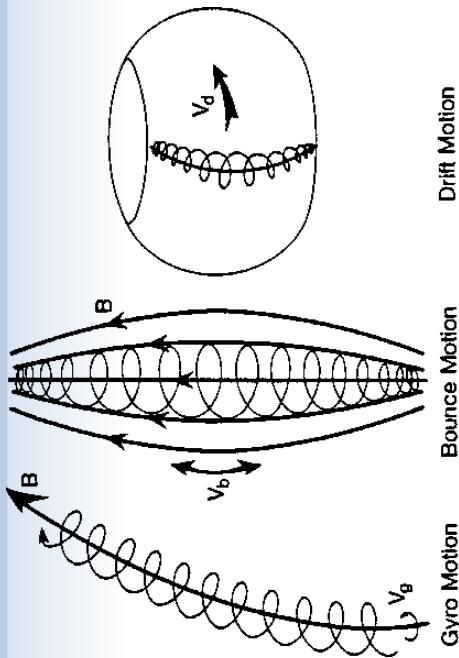
# **INTRODUCTION**

## Radiation Belts



*Image Credit: API, NASA*

# Adiabatic invariants of the particle motion



$$\mu = \frac{p_{\perp}^2}{2m_0B} = \frac{p^2 \sin^2(\alpha)}{2m_0B}$$

- The gyration around field lines
- **The magnetic momentum invariant**
- The bounce motion between magnetic mirror points
- **The longitudinal invariant**

$$J = \oint p_{\parallel} ds = 2 \int_{s_s}^{s_n} p_{\parallel} ds$$

$$K = \frac{J}{\sqrt{8m_0 u}} = \int_{s_s}^{s_n} \sqrt{B_m - B(s)} ds$$

- The periodic azimuthal drift in the non-uniform geomagnetic field
- **The flux invariant**

$$L^* = \frac{2\pi M}{\Phi R_E} \sim \frac{1}{\Phi}$$

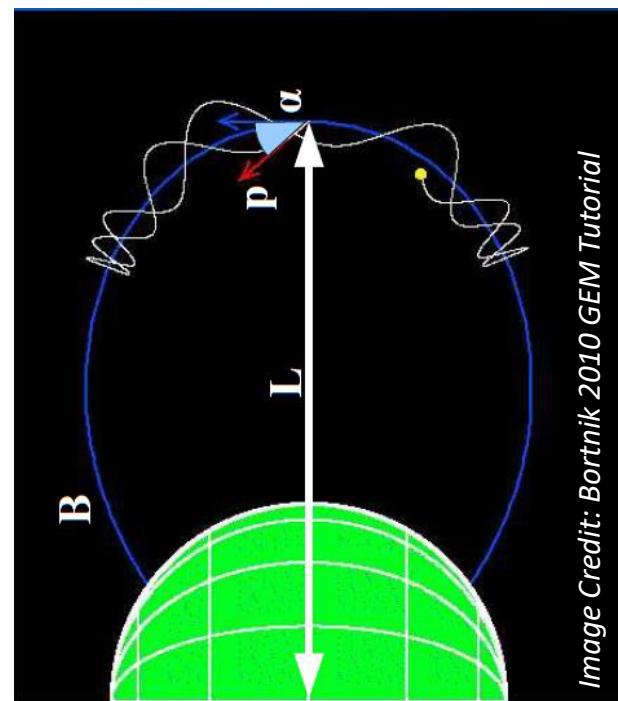
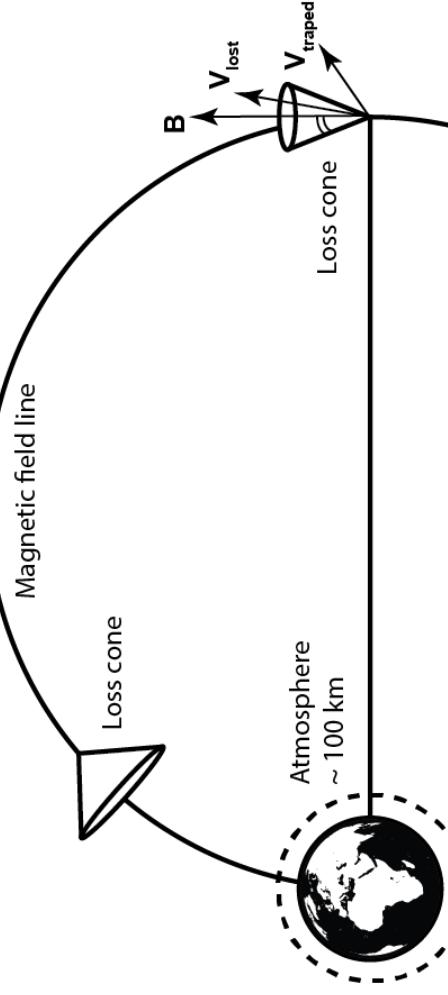
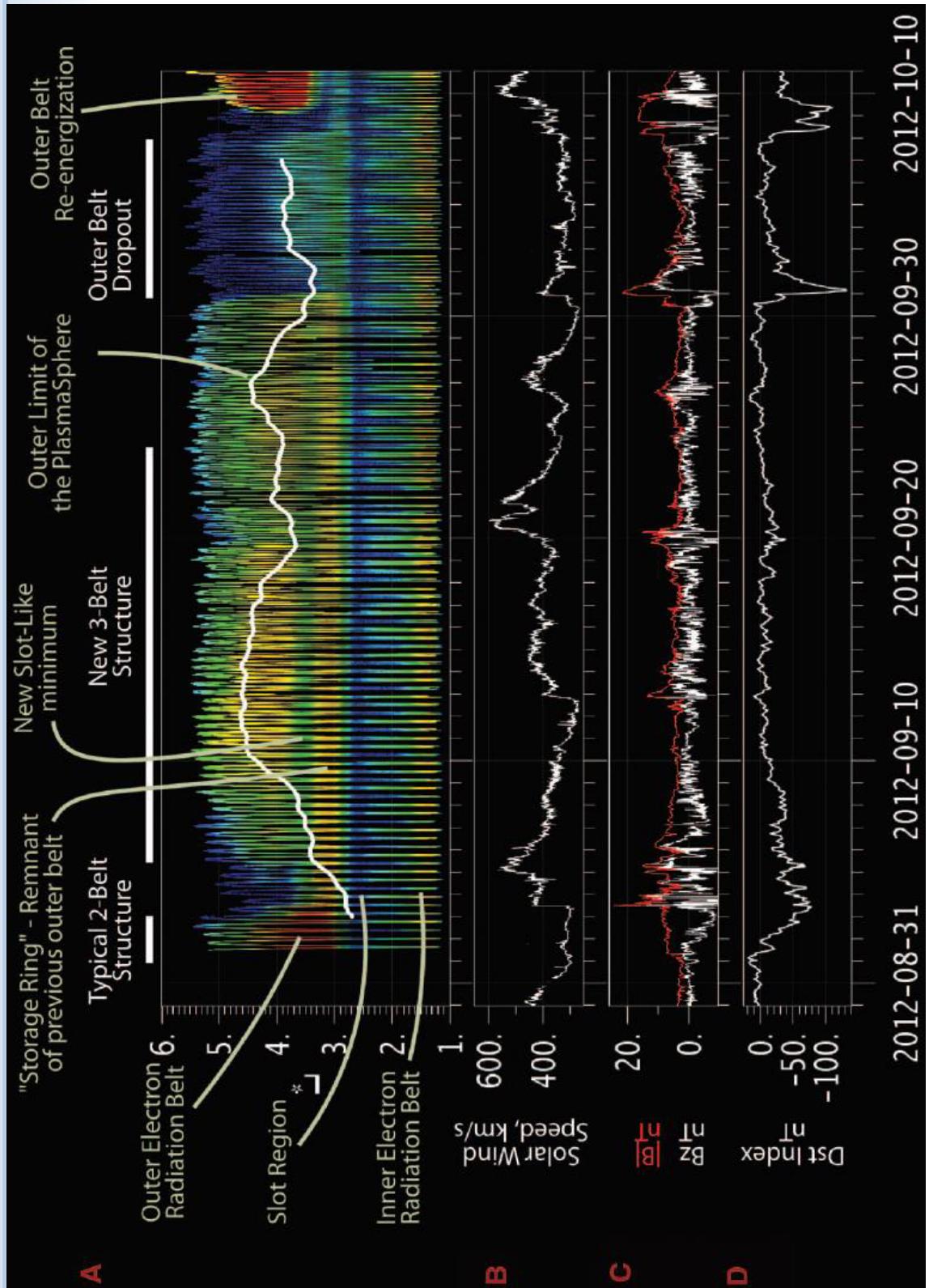


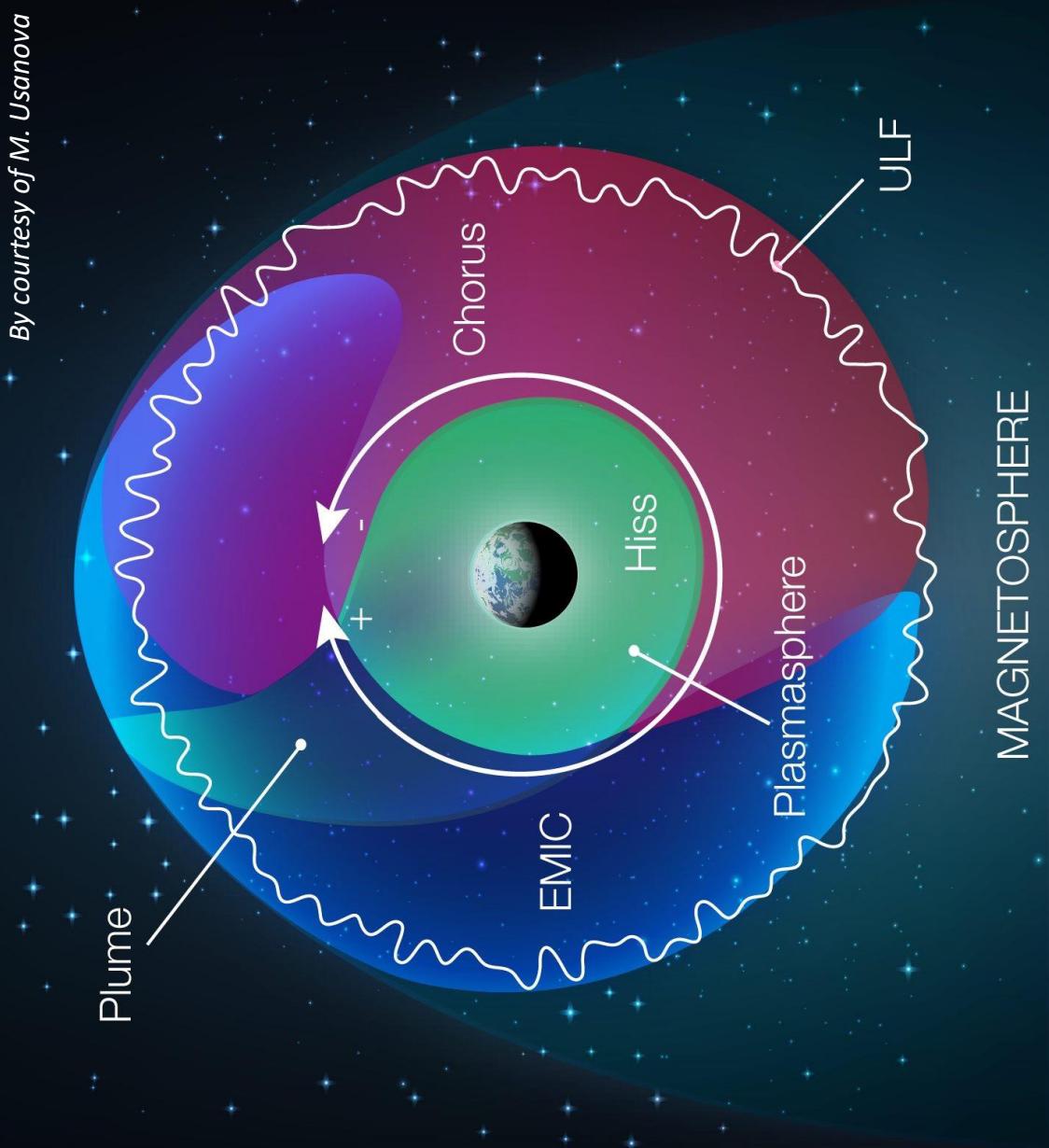
Image Credit: Bortnik 2010 GEM Tutorial



# Radiation belts dynamics

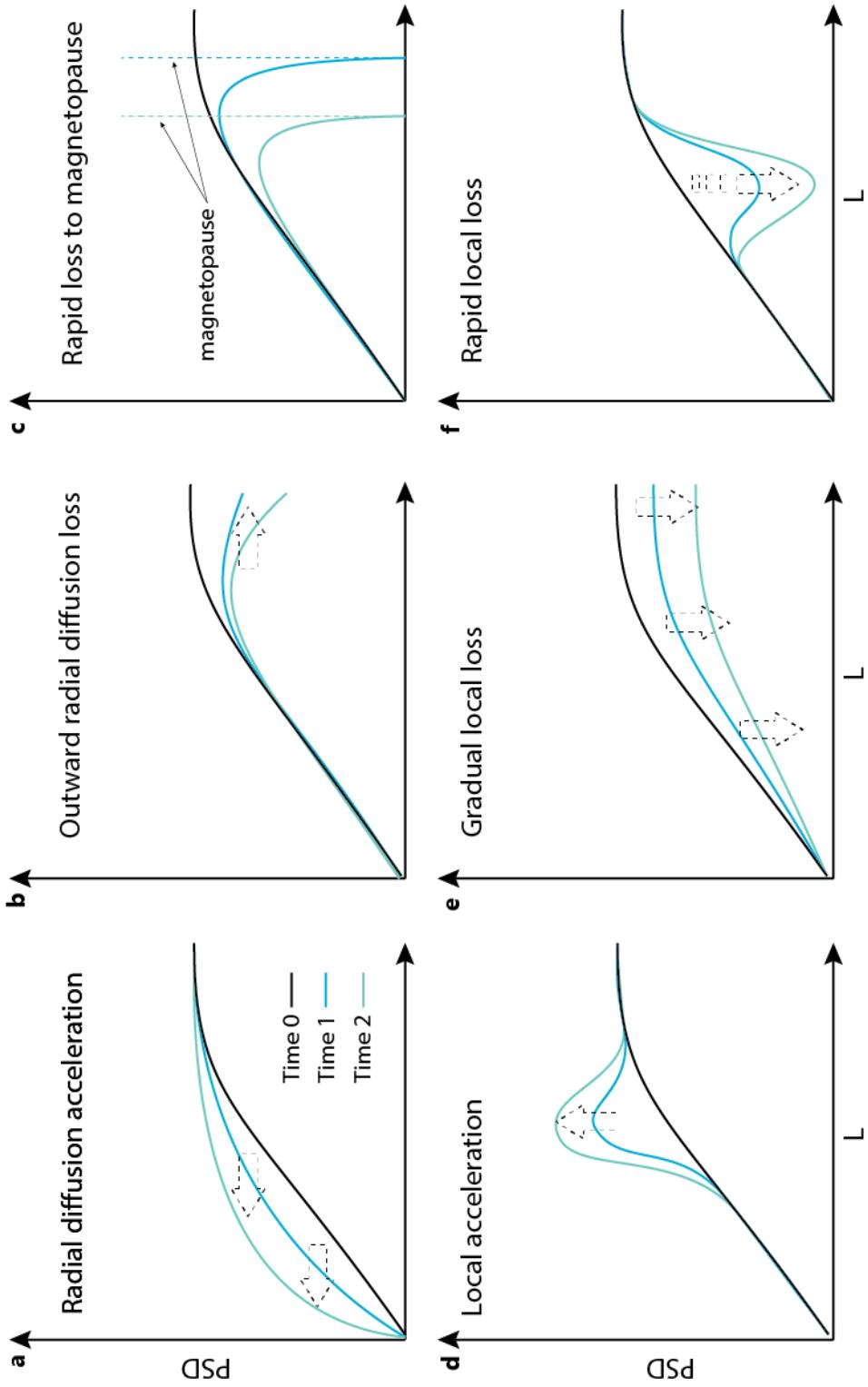


# Waves in magnetosphere



## Phase Space Density

$$f = \frac{dN}{dx \cdot dy \cdot dz \cdot dp_x \cdot dp_y \cdot dp_z} = \frac{dN}{p^2 \cdot dA \cdot dt \cdot d\Omega \cdot dE} \equiv \frac{j}{p^2}$$



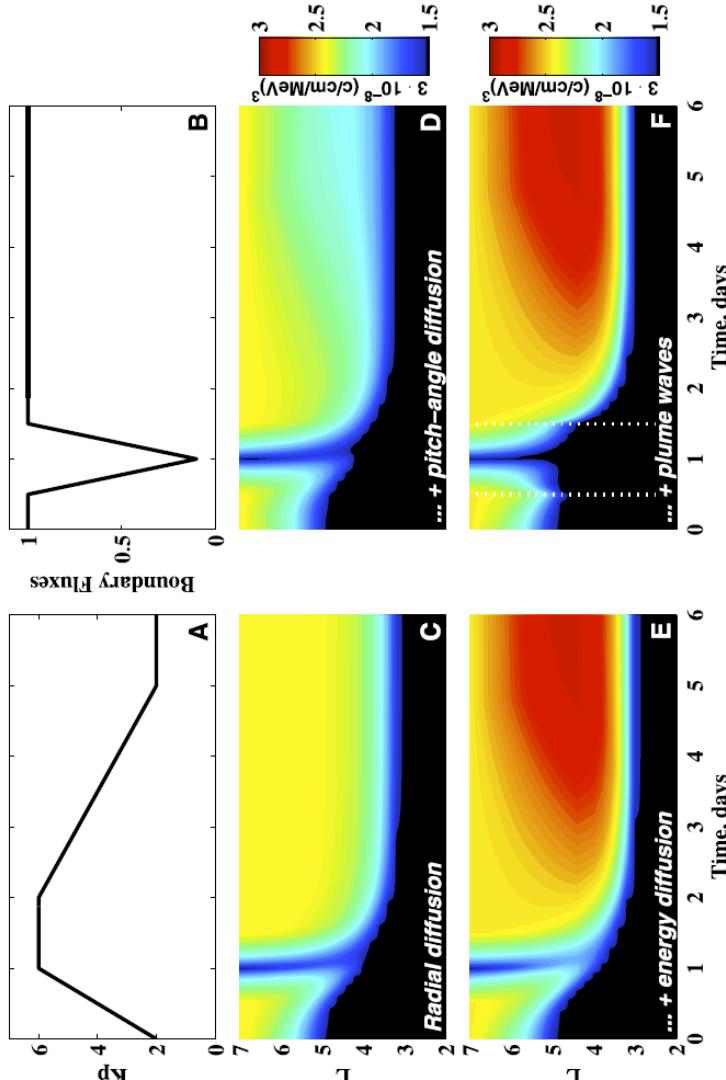
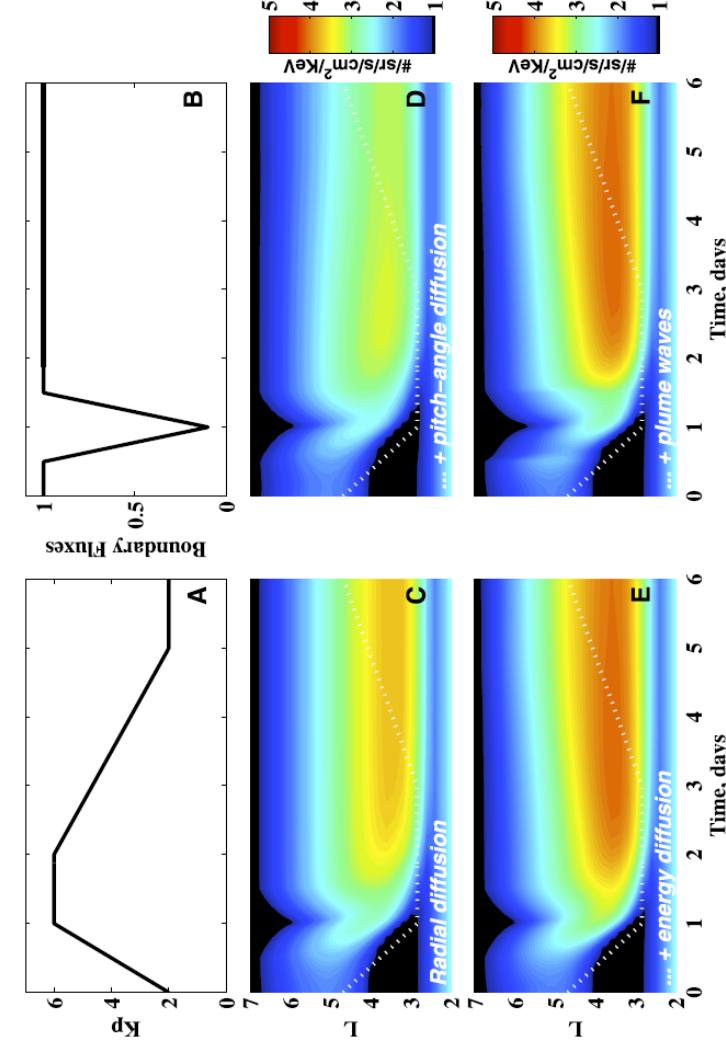
# Flux vs PSD (Idealized geomagnetic storm)

**Flux. Fixed Energy (E) and pitch angle ( $\alpha$ )**

$$J = \frac{dN}{dA \cdot dt \cdot d\Omega \cdot dE}$$

**PSD. Fixed adiabatic invariants  $\mu$  and K**

$$f = \frac{j}{p^2}$$



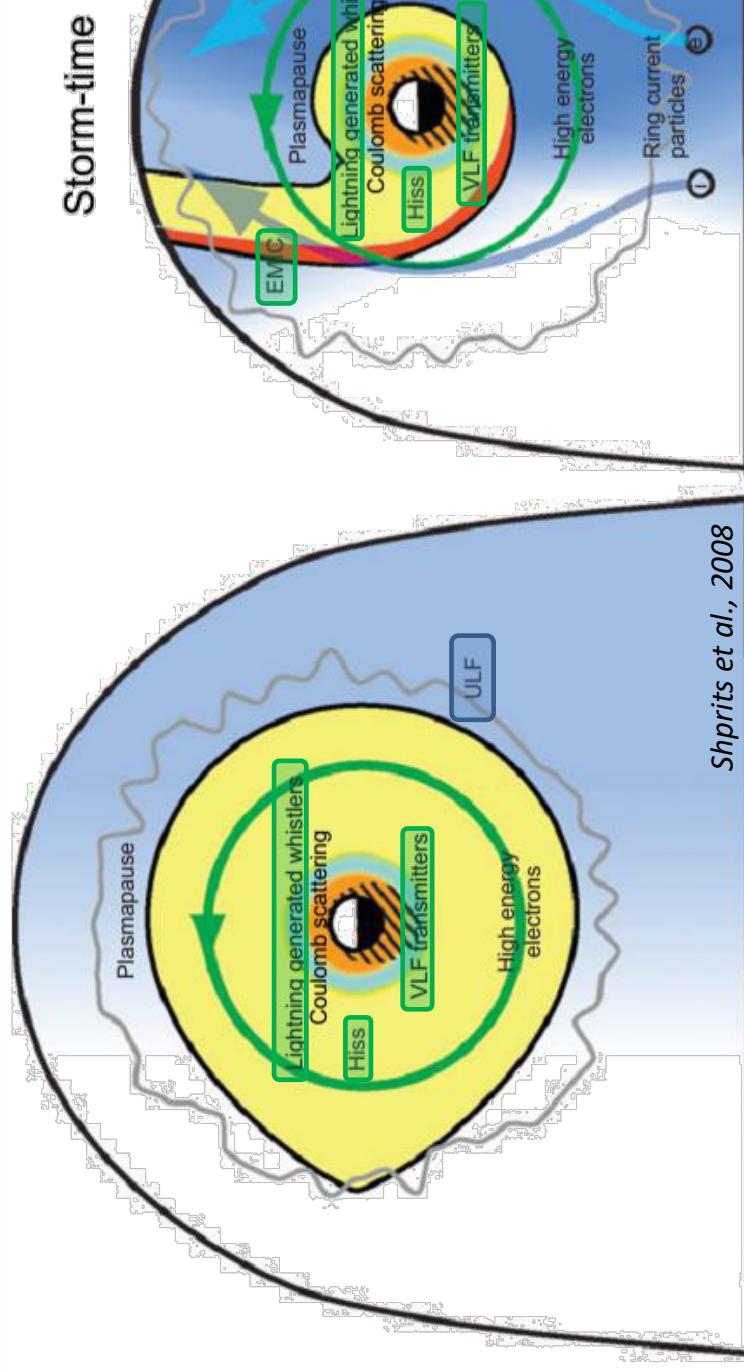
# **FOKKER-PLANCK EQUATION**

# Diffusion codes that solve Fokker-Planck equation

- **Salammbo**, [Beutier and Boscher, 1995]
  - The first radiation belt code capable of solving the 3D Fokker-Planck equation
  - Varotsou et al. [2005] and Horne et al. [2006] added pitch-angle and energy scattering by equatorial chorus waves
- Pitch Angle and Energy Diffusion of Ions and Electrons (**PADIE**) [Glauert and Horne, 2005] and developed to **BAS-RBM** Radiation Belts Code [Glauert et al., 2014].
  - Includes mixed terms.
- Improved atmosphere interactions (**RAW**) model, [Jordanova and Miyoshi, 2005]
  - Convective model with radial diffusion due to the magnetic component of the ULF waves and parameterized losses
- Multidimensional quasi-linear diffusion code, [Albert, 2004], [Albert and Young, 2005]
- Radiation Belt Environment (**RBE**) model [Fok et al. 2008]
  - Magnetospheric differential convection associated with changes in the electric field, electron loss, and acceleration due to wave-particle resonant interaction with equatorial chorus waves in the Tsyganenko and Sitnov [2005] magnetic field model and Weimer [2001] electric field model
- Versatile Electron Radiation Belt (**VERB**) code [Shprits et al., 2008, Subbotin and Shprits, 2009]
  - VERB code is a useful tool for space weather forecasting and nowcasting of the relativistic electron environment. The computational efficiency of the code makes it also appropriate for future use with data assimilation tools to specify the state of the radiation environment and correct imperfect models
- Dynamic Radiation Environment Assimilation Model: **DREAM** [Reeves et al., 2012]
  - **DREAM3D** code [Tu et al., 2013]
  - includes radial, pitch angle, and momentum diffusion and mixed pitch angle-momentum diffusion, which are driven by dynamic wave databases from the statistical CRRES wave data, including plasmaspheric hiss, lower-band, and upper-band chorus
- Storm-time evolution of electron radiation belt (**STEERB**) [Su et al., 2010]
  - Includes coulomb collisions, radial diffusion due to magnetic and electric field perturbations, and local pitch angle, energy, and cross-pitch angle-energy diffusion due to various wave-particle interactions.
- **Salammbo 3-D**, [Varotsou et al., 2008]
  - 3D diffusion using VERB approach
- **VERB-4D**, Shprits et al. [2014]
  - Convection, loss to the asymmetric magnetopause, adiabatic effects, and can use a non-dipole magnetic field.
- Commissariat à l'Energie Atomique **CEVA** code, [Ripoll et al., 2020]

## Fokker-Planck Equation

**Quiet-time**



Shprits et al., 2008

- Phase-space density dynamics
- VERB code
- Radial diffusion
- Local diffusion
- Pitch-angle
- Energy
- Mixed-terms
- Loss into atmosphere or magnetopause

Waves:

- ULF
- Hiss
- Chorus

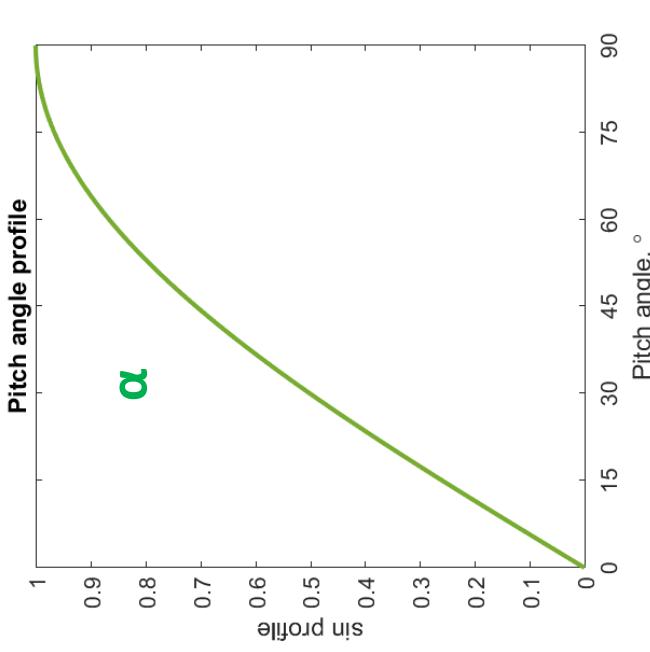
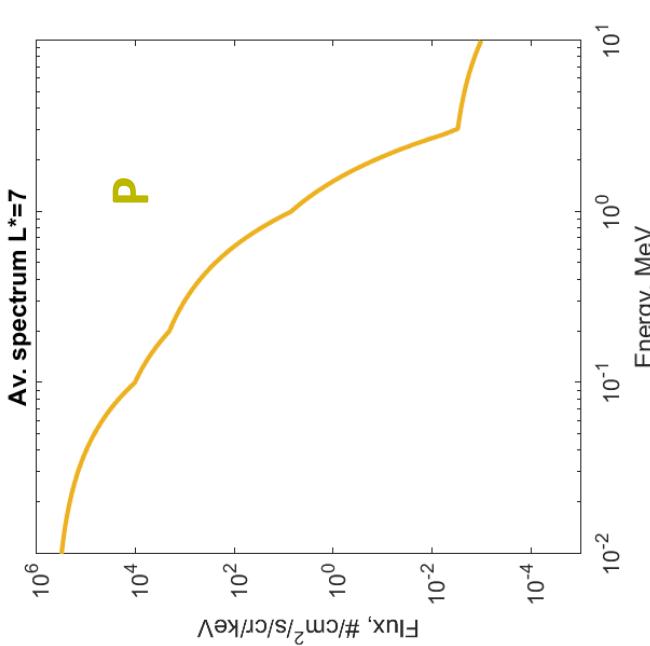
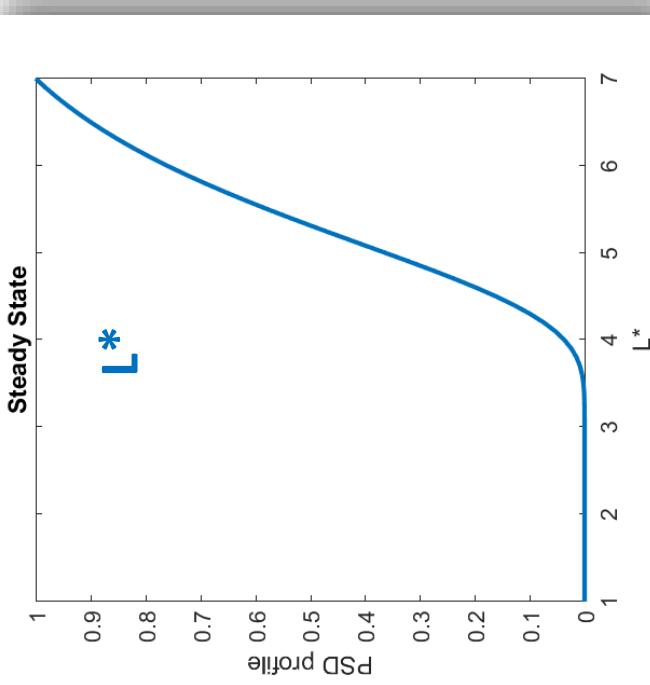
- VLF transmitters
- Lightning whistlers
- EMIC
- Chorus

$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial L^*} \Big|_{\mu_J} \frac{1}{L^{*2}} D_{L^* L^*} \frac{\partial f}{\partial L^*} + \frac{1}{p^2} \frac{\partial}{\partial p} \Big|_{\alpha_0, L} p^2 \left( D_{pp} \frac{\partial}{\partial p} \Big|_{\alpha_0, L} f + D_{p\alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L} f \right) \\ + \frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} \Big|_{p, L} T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L} f + D_{\alpha_0 p} \frac{\partial}{\partial p} \Big|_{\alpha_0, L} f \right) - \frac{f}{\tau}$$

## Initial conditions

$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial \textcolor{blue}{L}^*} \Big|_{\mu,J} \frac{1}{L^{*2}} D_{L^* L^*} \frac{\partial f}{\partial \textcolor{blue}{L}^*} \Big|_{\mu,J} - \frac{f}{\tau_{param}} = 0$$

$\tau_{param}$  is an imitation of local loss processed due to wave-particle interaction with hiss, chorus, etc.



$$\begin{aligned} \frac{\partial f}{\partial t} &= L^{*2} \frac{\partial}{\partial \textcolor{blue}{L}^*} \Big|_{\mu,J} \frac{1}{L^{*2}} D_{L^* L^*} \frac{\partial f}{\partial \textcolor{blue}{L}^*} \Big|_{\mu,J} + \frac{1}{p^2} \frac{\partial}{\partial \textcolor{brown}{p}} \Big|_{\alpha_0, L^*} p^2 \left( D_{pp} \frac{\partial}{\partial \textcolor{brown}{p}} \Big|_{\alpha_0, L^*} f + D_{p\alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} f \right) \\ &\quad + \frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} f + D_{\alpha_0 p} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} f \right) - \frac{f}{\tau} \end{aligned}$$

## Boundary Conditions

- L\***
  - The absence of electrons at the atmospheric level at L = 1
  - PSD = const\*b(t), for L\* upper boundary due to observed variation

$$\frac{\partial f}{\partial q} = 0$$

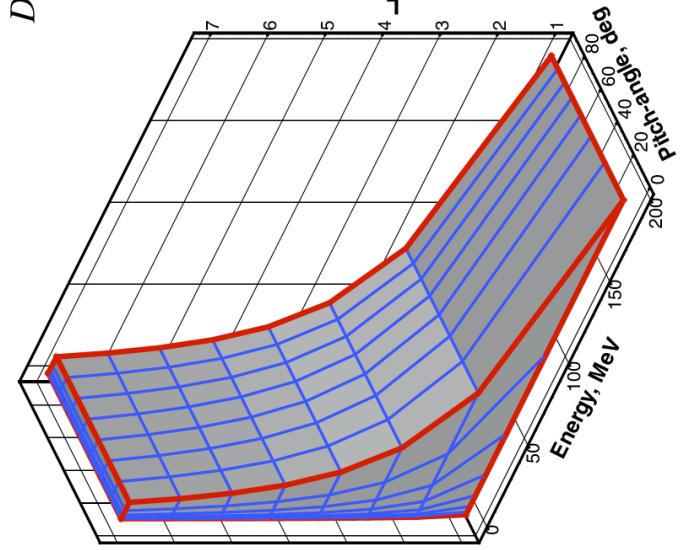
- P,**
- Energy**

- PSD = const, for low-energy electrons to represent a balance between a convective source and losses.
- The absence of very high-energy electrons

$$f(q_0) = 0$$

$$f(q_0) = F_0$$

- Zero gradient at 90° accounts for the flat pitch angle distribution observed at 90°.
- PSD = 0 at 0° simulates the loss at the atmospheric level and surface.



$$\begin{aligned} \frac{\partial f}{\partial t} &= L^{*2} \frac{\partial}{\partial L^*} \Big|_{\mu_J} \frac{1}{L^{*2}} D_{L^* L^*} \frac{\partial f}{\partial L^*} \Big|_{\mu_J} + \frac{1}{p^2} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} p^2 \left( D_{pp} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} f + D_{p\alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} f \right) \\ &\quad + \frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} f + D_{\alpha_0 p} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} f \right) - \frac{f}{\tau} \end{aligned}$$

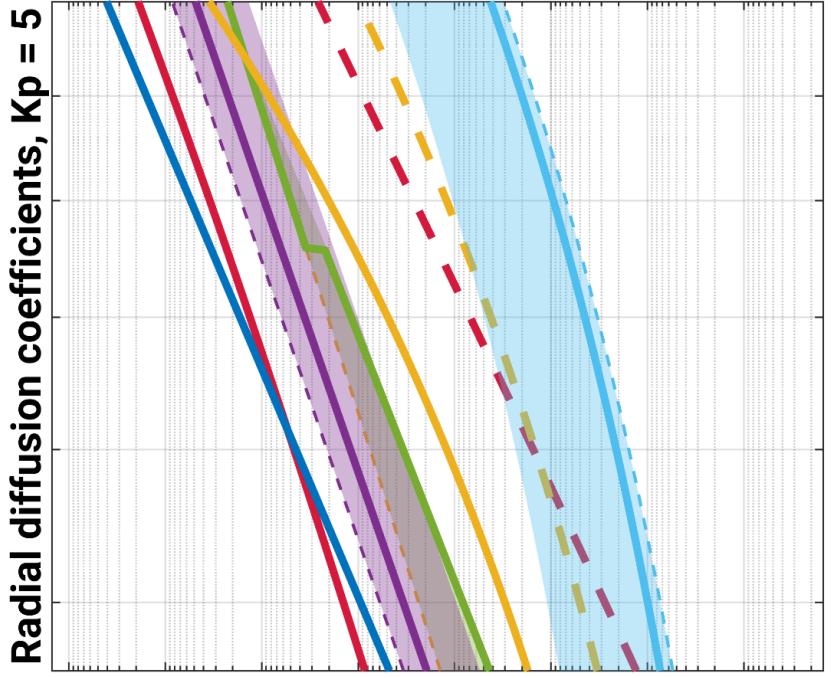
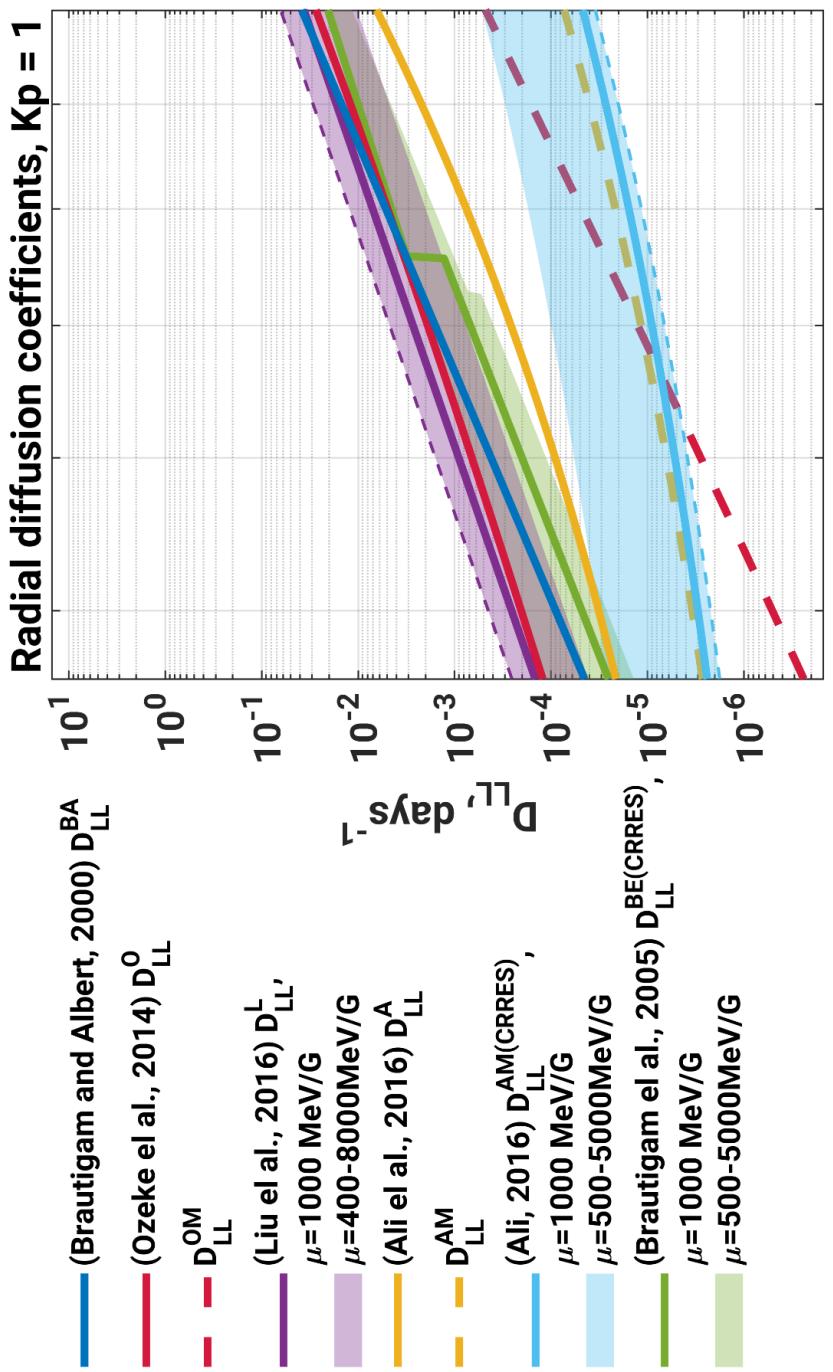
## Diffusion Coefficients in Fokker-Plank Equation

- Radial diffusion
- Local pitch-angle scattering
- Local energy diffusion
- Mixed diffusion
- Losses
- Radial diffusion coefficients
  - Brautigam and Albert, 2000
  - Ozek et al. 2014
  - etc
- Local diffusion coefficients
  - Chorus (day-side, night-side)
  - Hiss,
  - EMIC, etc.
- Loss
  - Loss cone (atmosphere)
  - Magnetopause

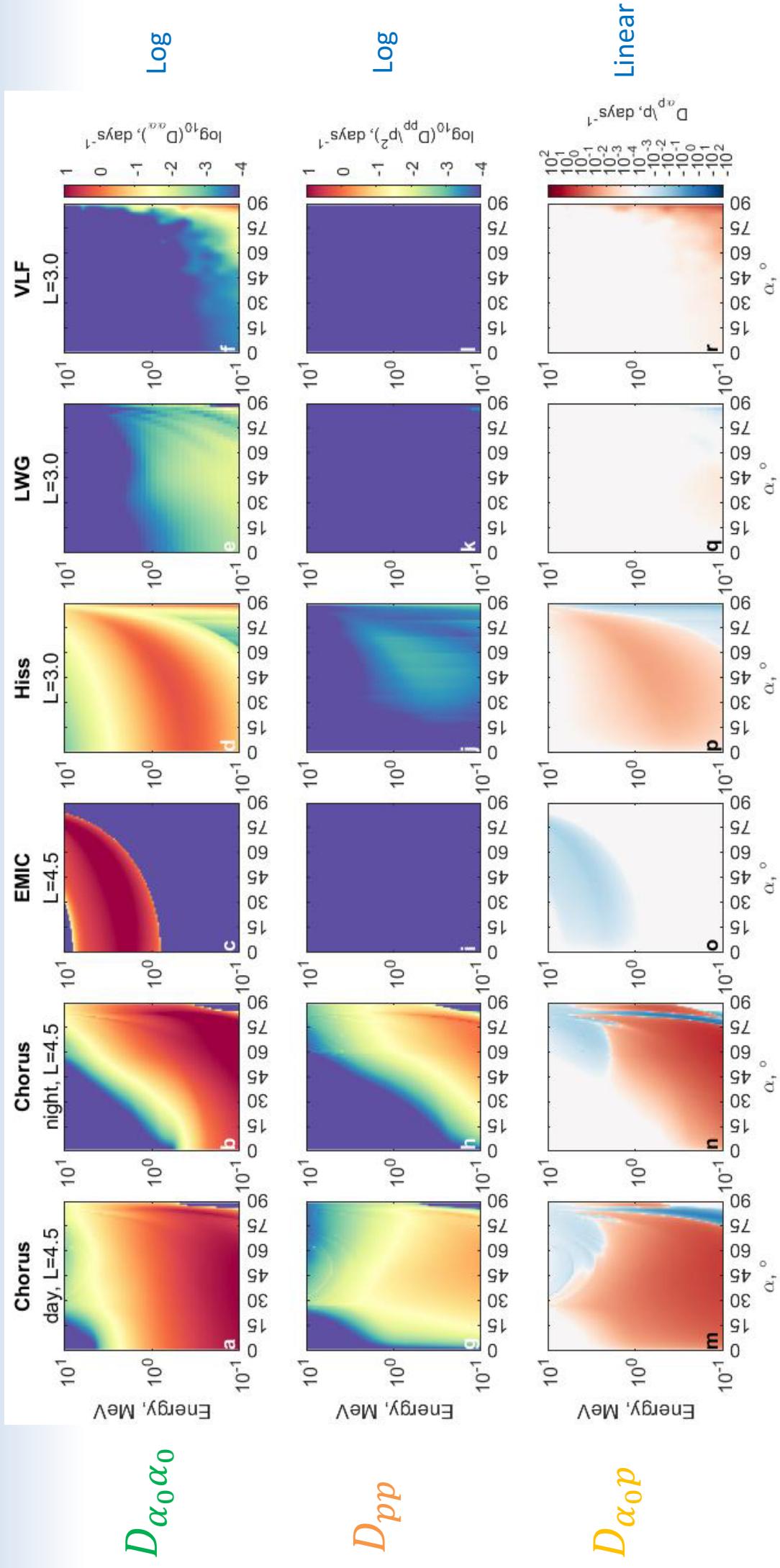
$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial L^*} \Big|_{\mu_J} \frac{1}{L^{*2}} D_{L^* L^*} \frac{\partial f}{\partial L^*} \Big|_{\mu_J} + \frac{1}{p^2} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} p^2 \left( D_{pp} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} f + D_{p\alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} f \right) - \frac{f}{\tau}$$

$$+ \frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} f + D_{\alpha_0 p} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} f \right)$$

## Radial diffusion coefficients ( $D_{LL}$ )

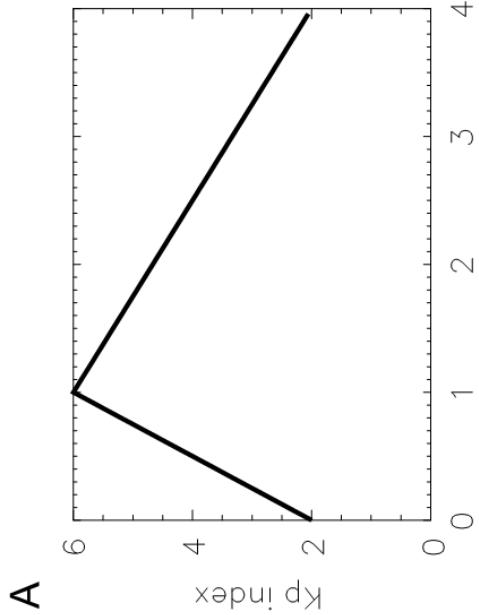


# Local diffusion coefficients



# **1D SIMULATIONS**

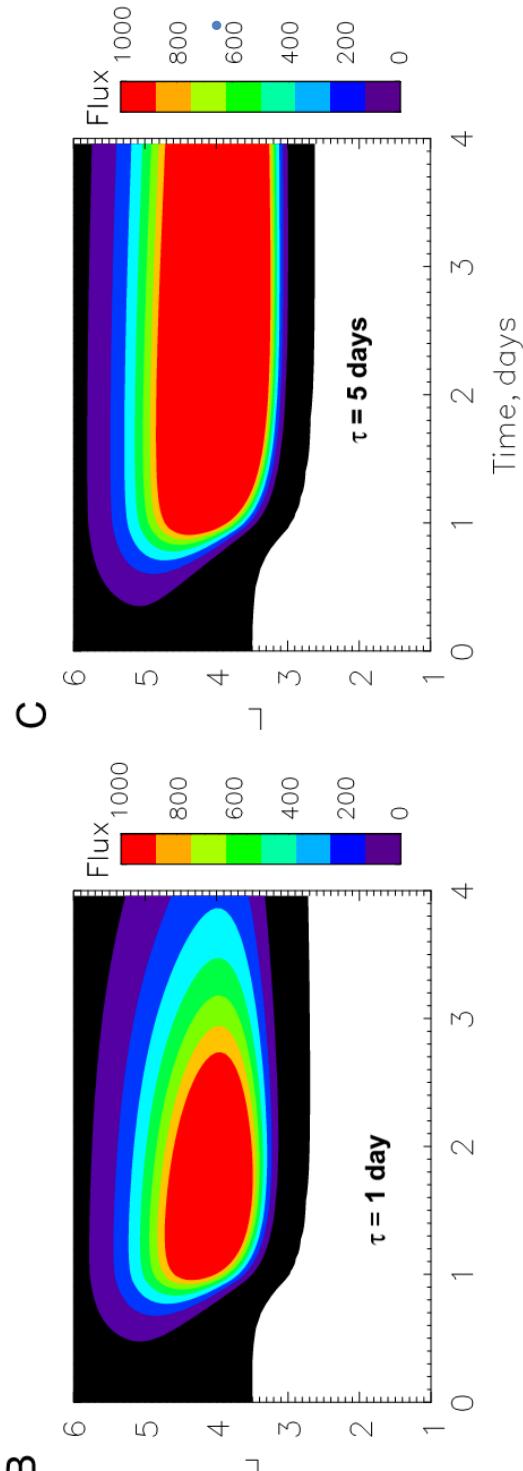
# 1D radial diffusion and the loss term (lifetime)



**Time-dependent code**  
**Radial diffusion of 1 MeV electrons**

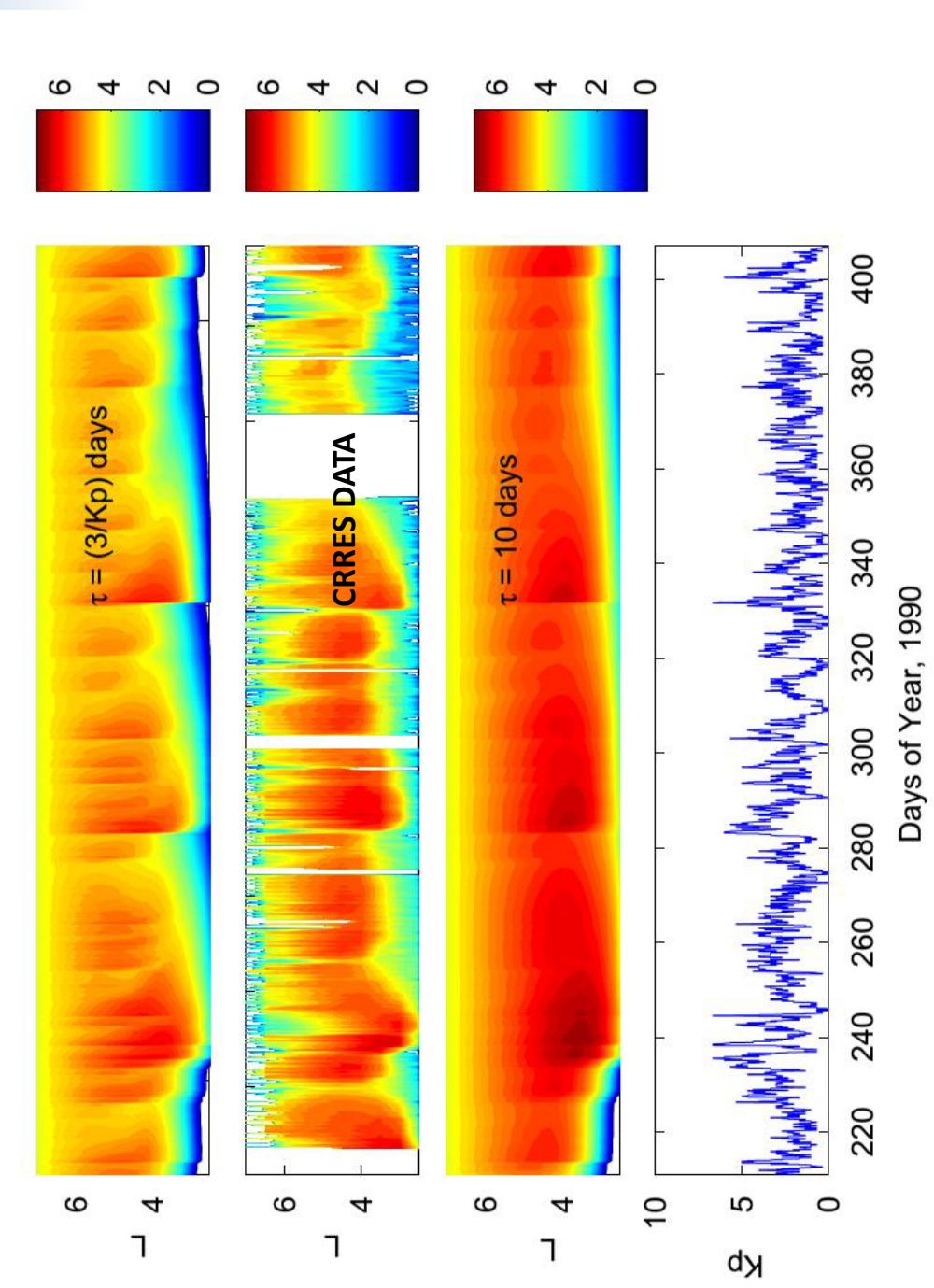
$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial L} L^{*-2} D_{L^* L^*} \frac{\partial f}{\partial L^*} - \frac{f}{\tau}$$

- Leading mechanisms for acceleration to relativistic energies include radial diffusion driven by ULF waves.
- Under steady-state conditions the equilibrium structure of the radiation belts is governed by the balance between local acceleration, inward diffusion, and losses due to pitch-angle scattering. The steady-state solution can be used to set up the initial conditions for the modeling.
- **The equilibrium is never attained during the storm, thus the dynamics of electron fluxes during storms must be studied using a time-dependent code.**



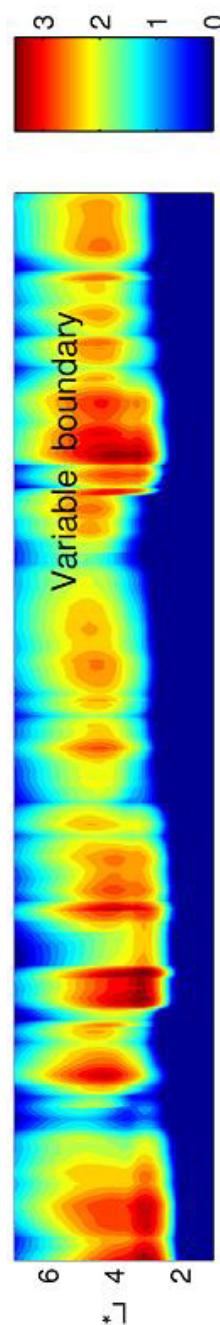
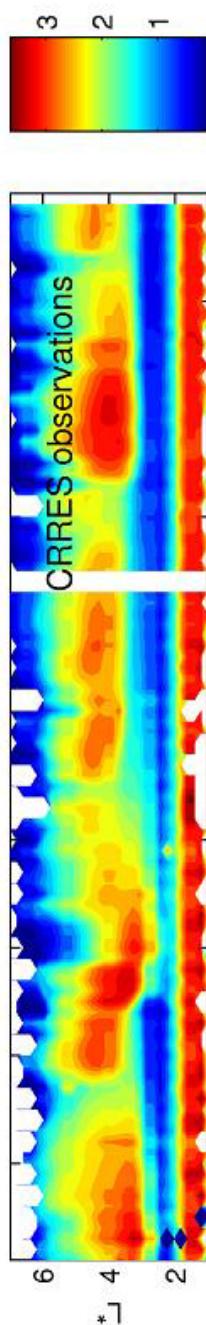
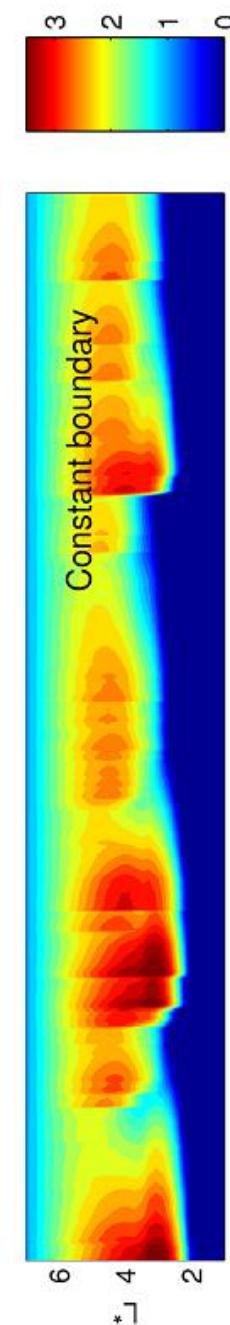
*Adopted from Shprits and Thorne, 2004*

## 1D radial diffusion and empirical lifetime



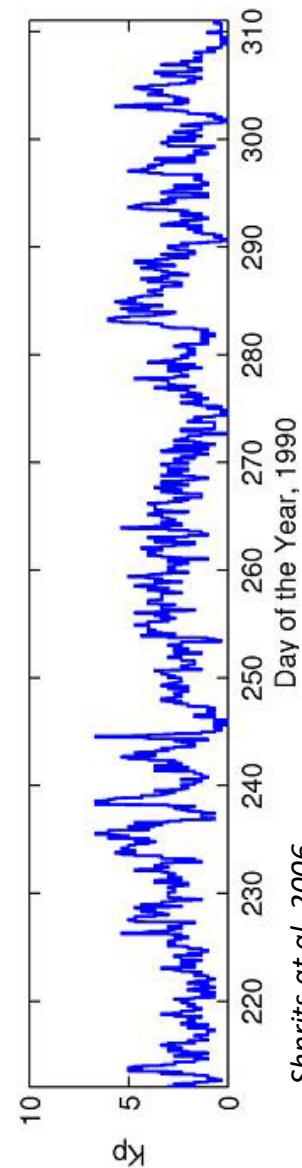
- 500 CRRES orbits starting from 30 July 1990
- Radial diffusion coefficient ( $D_{LL}$ ) from Brautigam and Albert, 2000
- Simulations with empirical lifetimes parameterized as a function of  $K_p$ .
- The radial diffusion model with simplified data-derived lifetimes is capable of predicting the radial extent of high energy fluxes and predicts MeV fluxes within one order of magnitude accuracy for most of the time of the simulation and most L values.

# 1D radial diffusion and boundary conditions (loss to magnetopause)



$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial L} L^{*-2} D_{L^* L^*} \frac{\partial f}{\partial L^*} - \frac{f}{\tau}$$

- 19 July - 6 November 1990
- Simulations with both variable and constant outer boundary conditions.
- The results of the simulations indicate that radial diffusion can effectively redistribute outer radiation belt fluxes and smooth PSD gradients, which are produced by losses to magnetopause or by local acceleration and loss.



$$f(L^* = 6) = f_0$$

$$f(L^* = 6) = f_0 \cdot b f(t)$$

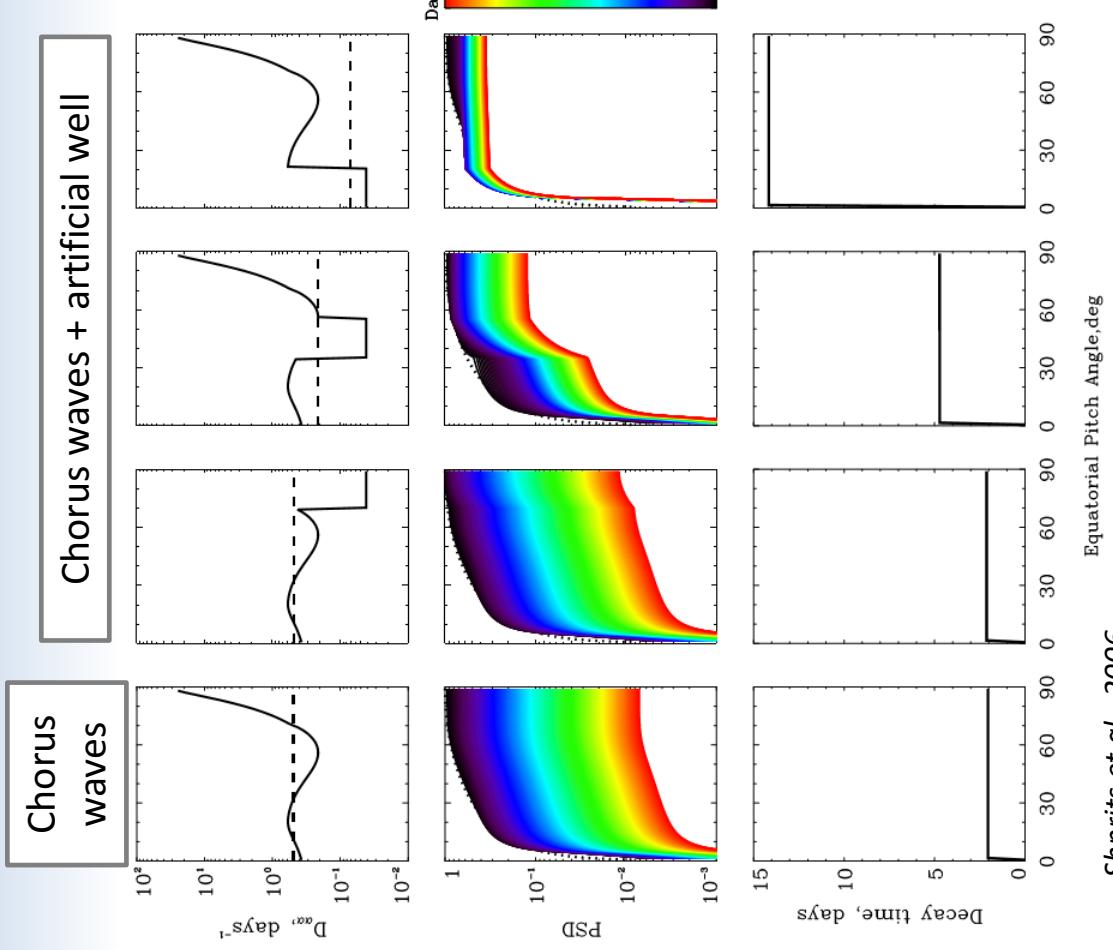
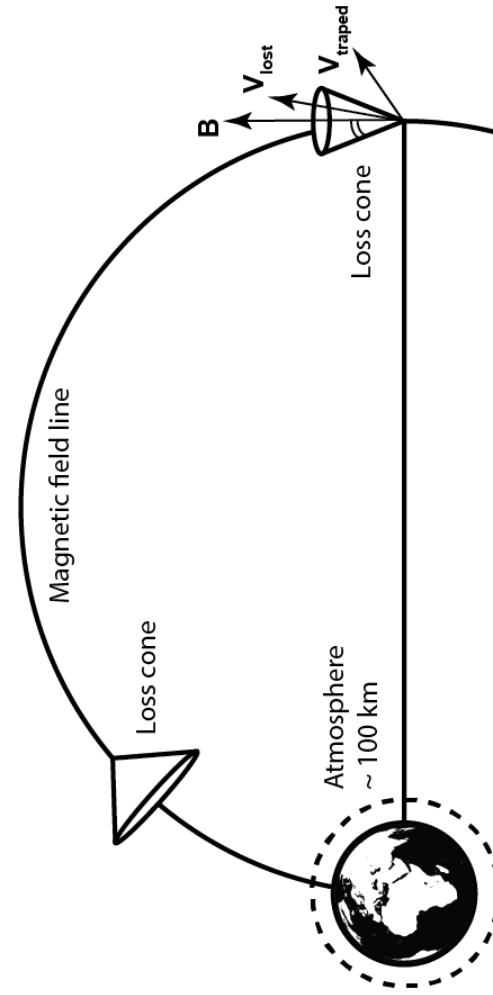
**Time-dependent  
boundary conditions**

# 1D pitch-angle diffusion and controlling effect of scattering rates

$$\frac{\partial f}{\partial t} = \frac{1}{T(y)y} \frac{\partial}{\partial y} \left. T(y)y D_{yy} \frac{\partial}{\partial y} \right|_{p,L} f - \frac{f}{\tau_{lc}},$$

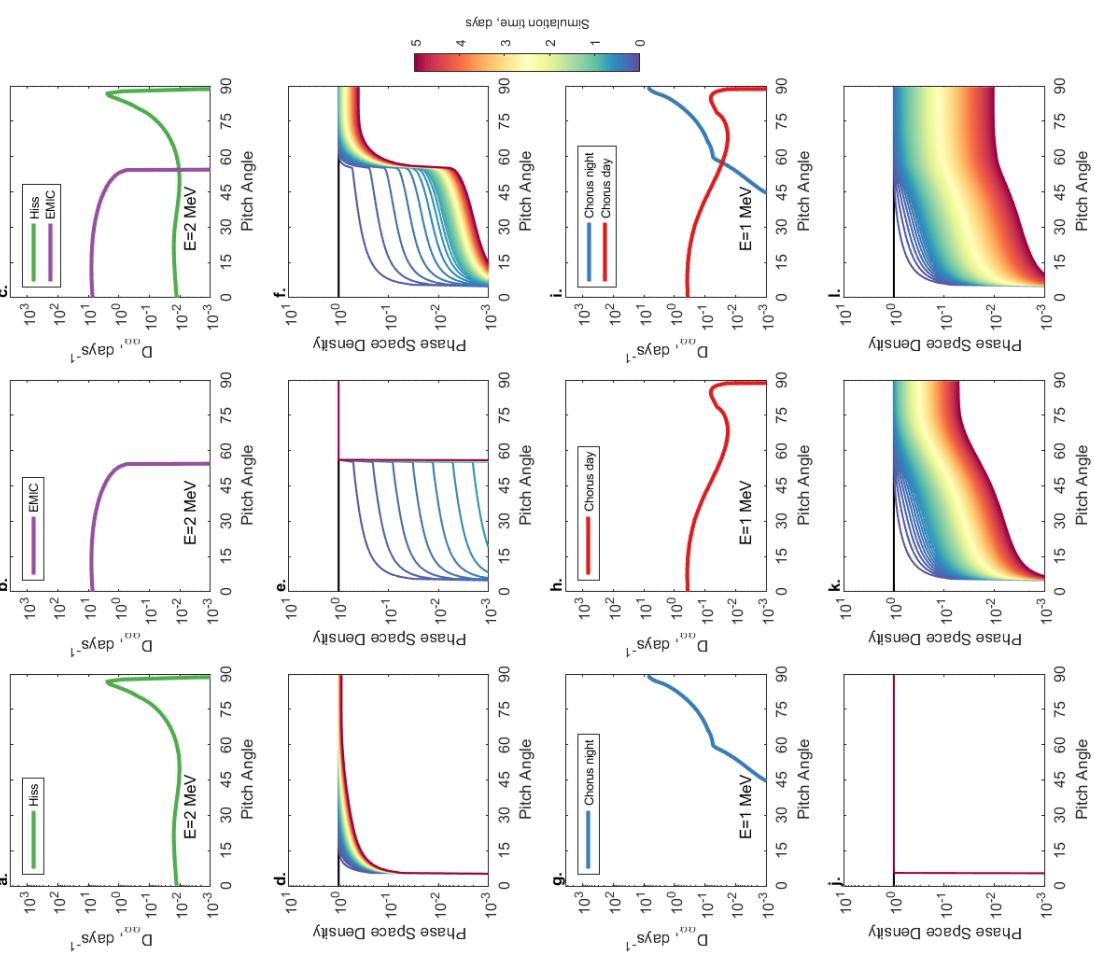
$$y = \sin(\alpha_0)$$

If the pitch angle distribution reaches an equilibrium shape the decay of the PSD can be described by a single lifetime parameter, independent of the pitch angle of particles and **lifetimes are primarily controlled by the diffusion rates near the edge of the loss cone.**



Shprits et al., 2006

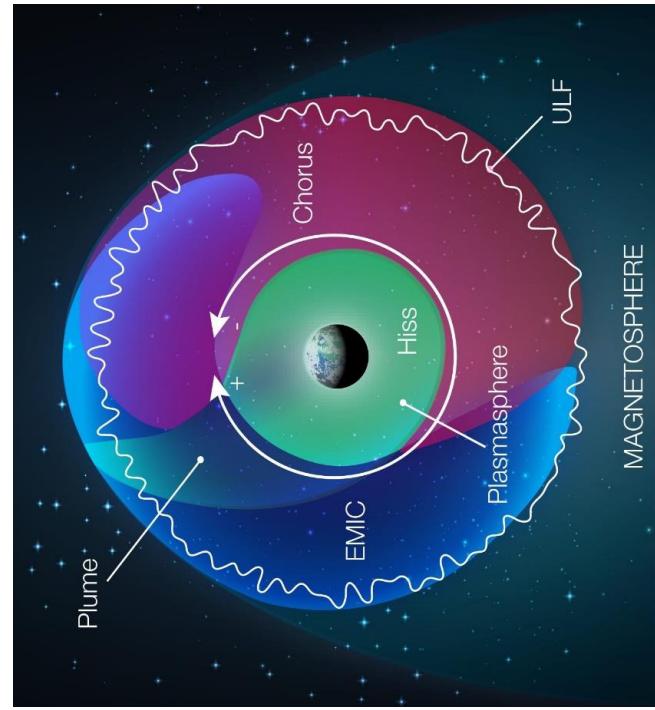
# 1D pitch-angle diffusion and local waves at different MLT



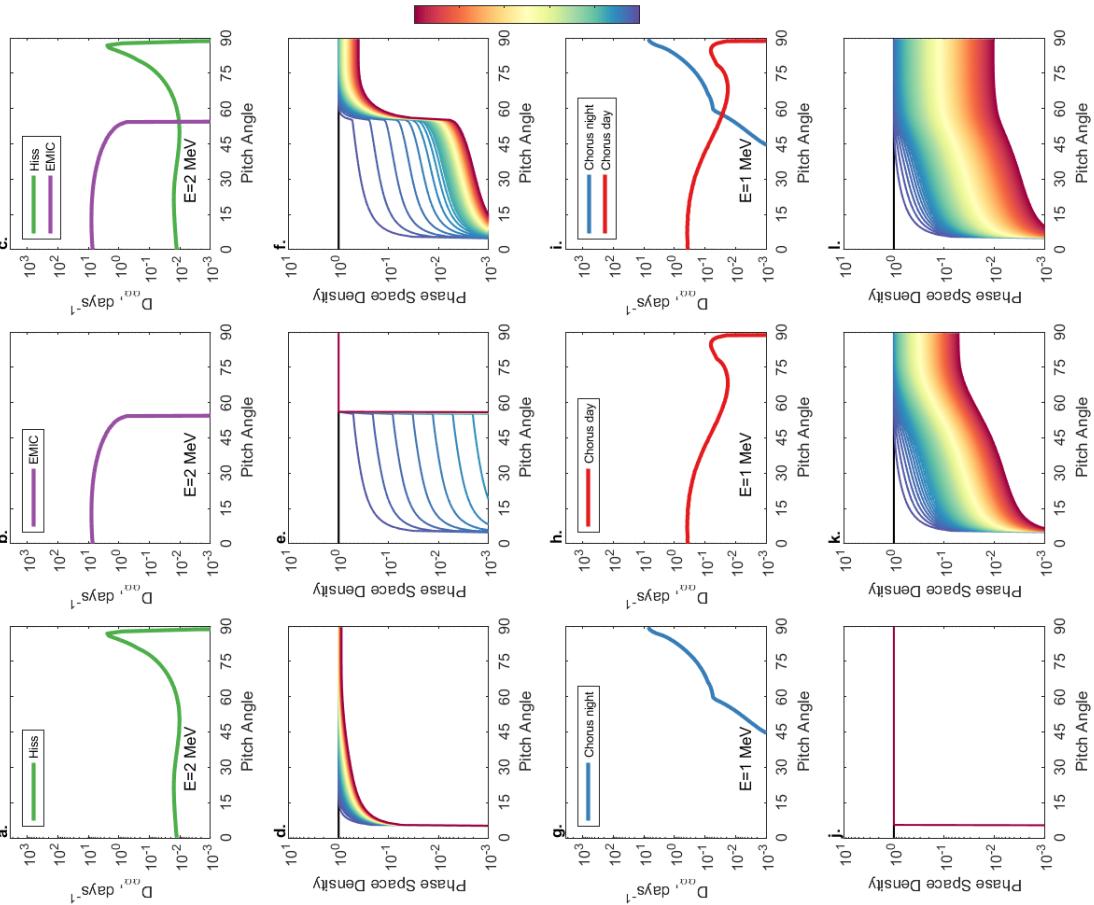
$$\frac{\partial f}{\partial t} = \frac{1}{T(y)y} \frac{\partial}{\partial y} \Big|_{p,L} T(y)y D_{yy} \frac{\partial}{\partial y} \Big|_{p,L} f - \frac{f}{\tau_{lc}},$$

$$y = \sin(\alpha_0)$$

- Electrons drift around the Earth and interact with different waves along their drift
- Hiss and chorus waves assist EMIC waves in scattering the core population



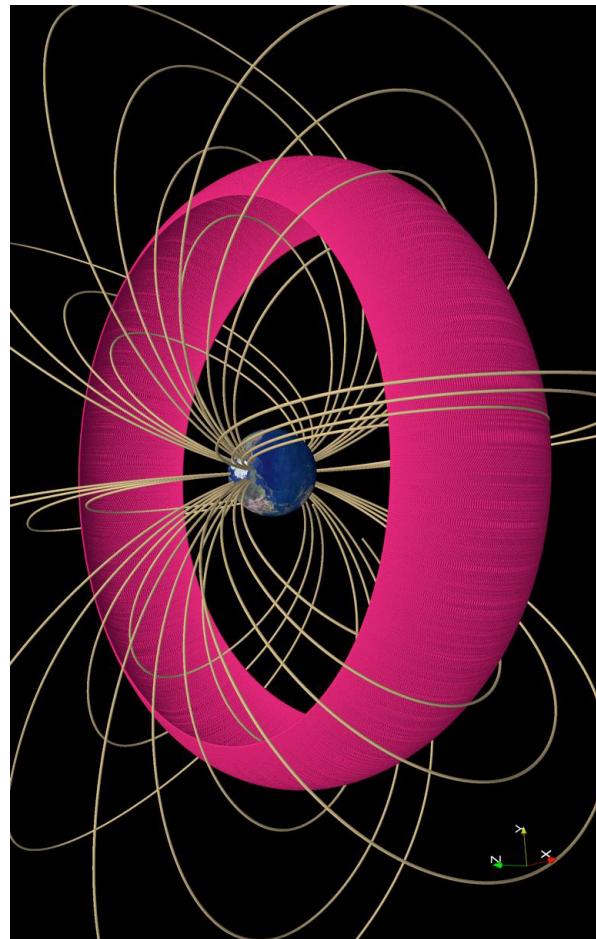
# 1D pitch-angle diffusion and local waves at different MLT



$$\frac{\partial f}{\partial t} = \frac{1}{T(y)y} \frac{\partial}{\partial y} \Big|_{p,L} T(y)y D_{yy} \frac{\partial}{\partial y} \Big|_{p,L} f - \frac{f}{\tau_{lc}},$$

$$y = \sin(\alpha_0)$$

- Electrons drift around the Earth and interact with different waves along their drift
- Hiss and chorus waves assist EMIC waves in scattering the core population



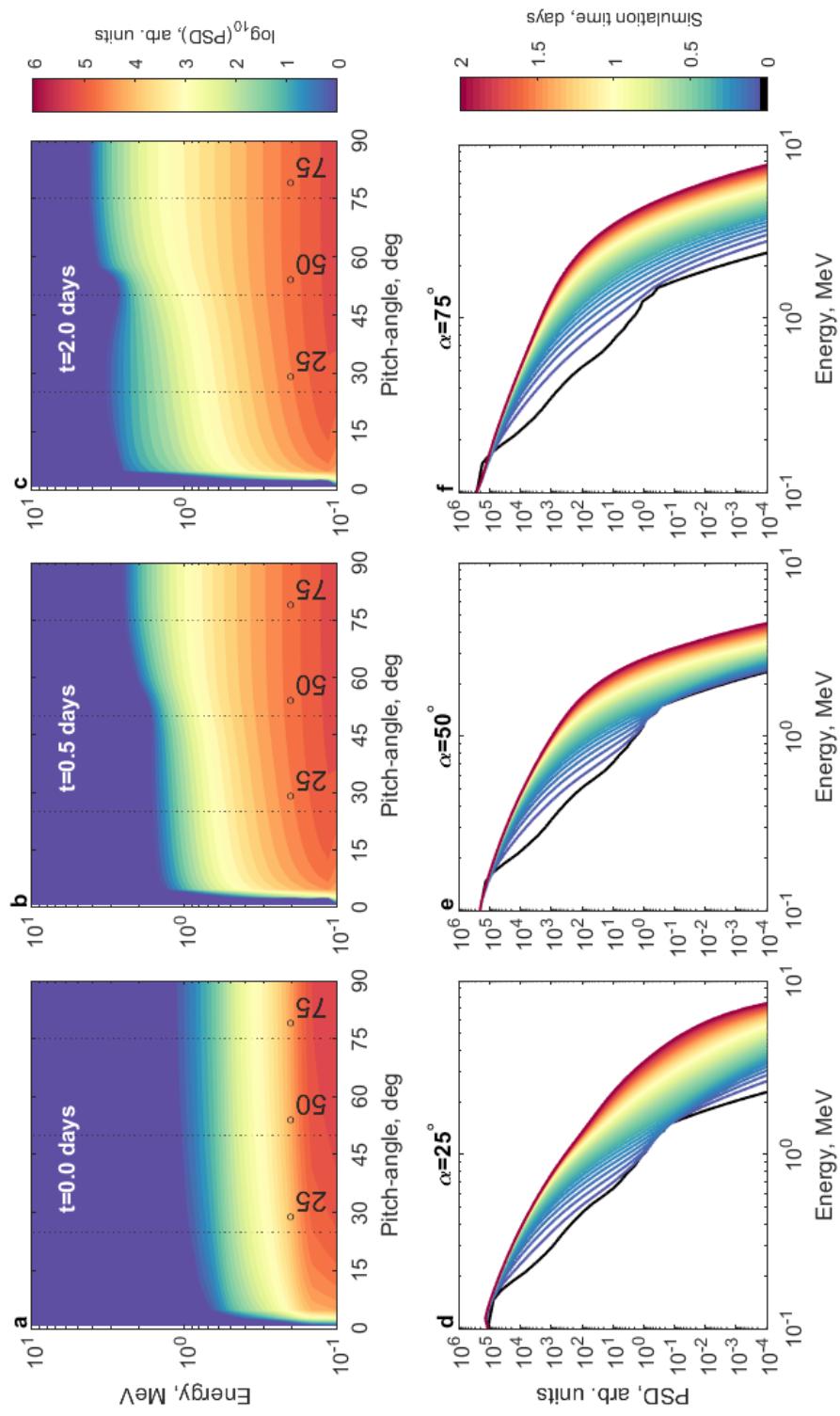
# **2D SIMULATIONS**

## 2D simulation of local acceleration

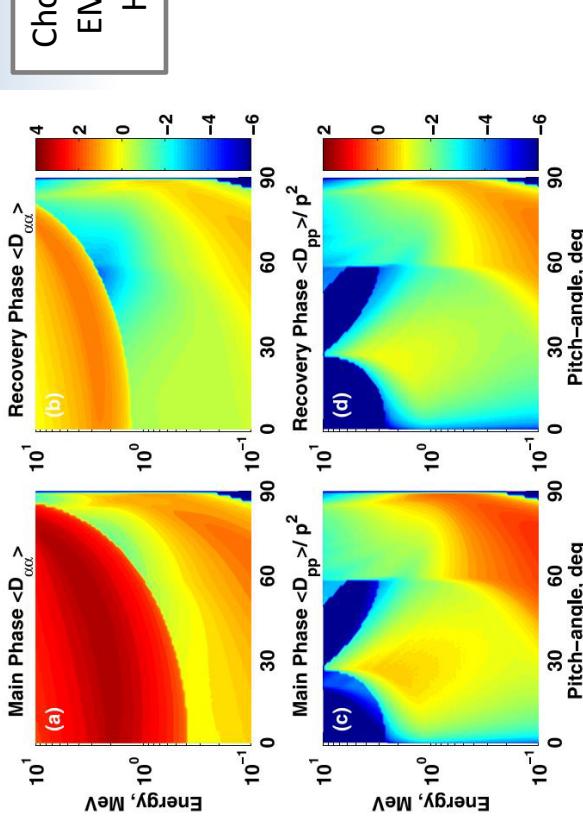
- The wave-particle interactions with whistler mode chorus waves result in a net acceleration

$$\frac{\partial f}{\partial t} = \frac{1}{T(y)y} \frac{\partial}{\partial y} \left|_{p,L} T(y) y D_{yy} \frac{\partial}{\partial y} \right|_{p,L} f + \frac{1}{p^2} \frac{\partial}{\partial p} \left|_{\alpha_0,L} p^2 D_{pp} \frac{\partial}{\partial p} \right|_{\alpha_0,L} f - \frac{f}{\tau_{lc}},$$

$$y = \sin(\alpha)$$



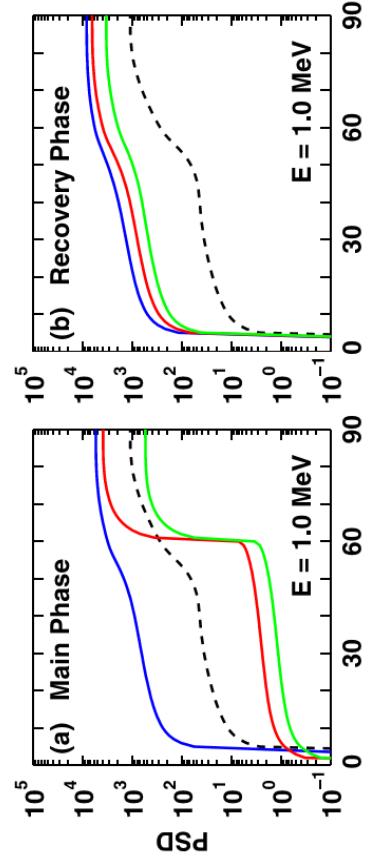
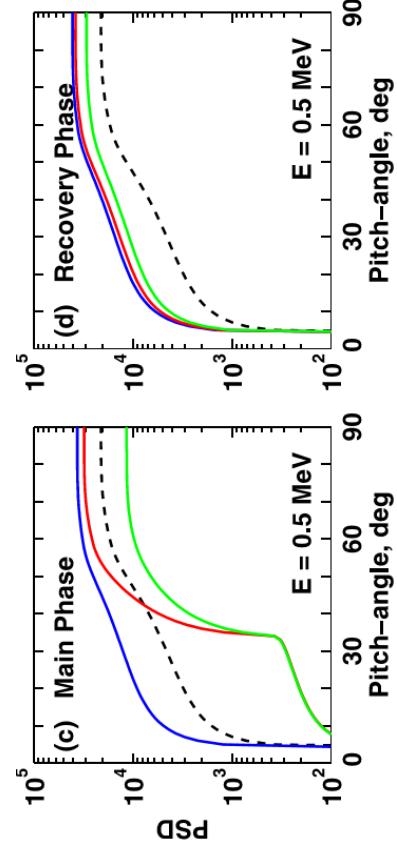
## 2D simulation of the electron dynamics during storms



$$\frac{\partial f}{\partial t} = \frac{1}{T(y)y} \frac{\partial}{\partial y} \left|_{p,L} T(y)y D_{yy} \frac{\partial}{\partial y} \right|_{p,L} f + \frac{1}{p^2} \frac{\partial}{\partial p} \left|_{\alpha_0,L} p^2 D_{pp} \frac{\partial}{\partial p} \right|_{\alpha_0,L} f - \frac{f}{\tau_{lc}},$$

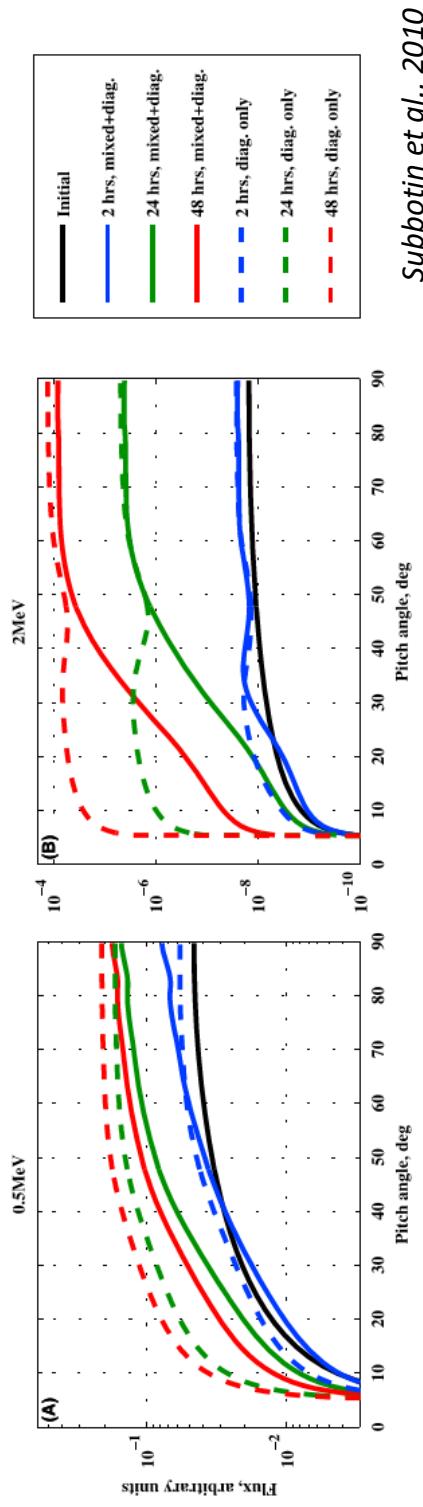
$y = \sin(\alpha)$

- The wave-particle interactions with whistler mode chorus waves result in a net acceleration of relativistic electrons, while EMIC waves, which provide very fast scattering near the edge of the loss cone, may be a dominant loss mechanism during the main phase of a storm.



## 2D simulation with mixed terms

$$\frac{\partial f}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} |_{\alpha_0,L} p^2 \left( D_{pp} \frac{\partial}{\partial p} |_{\alpha_0,L} f + D_{p\alpha_0} \frac{\partial}{\partial \alpha_0} |_{p,L} f \right) + \\ + \frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} |_{p,L} T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} |_{p,L} f + D_{\alpha_0 p} \frac{\partial}{\partial p} |_{\alpha_0,L} f \right) - \tau_{lc}$$



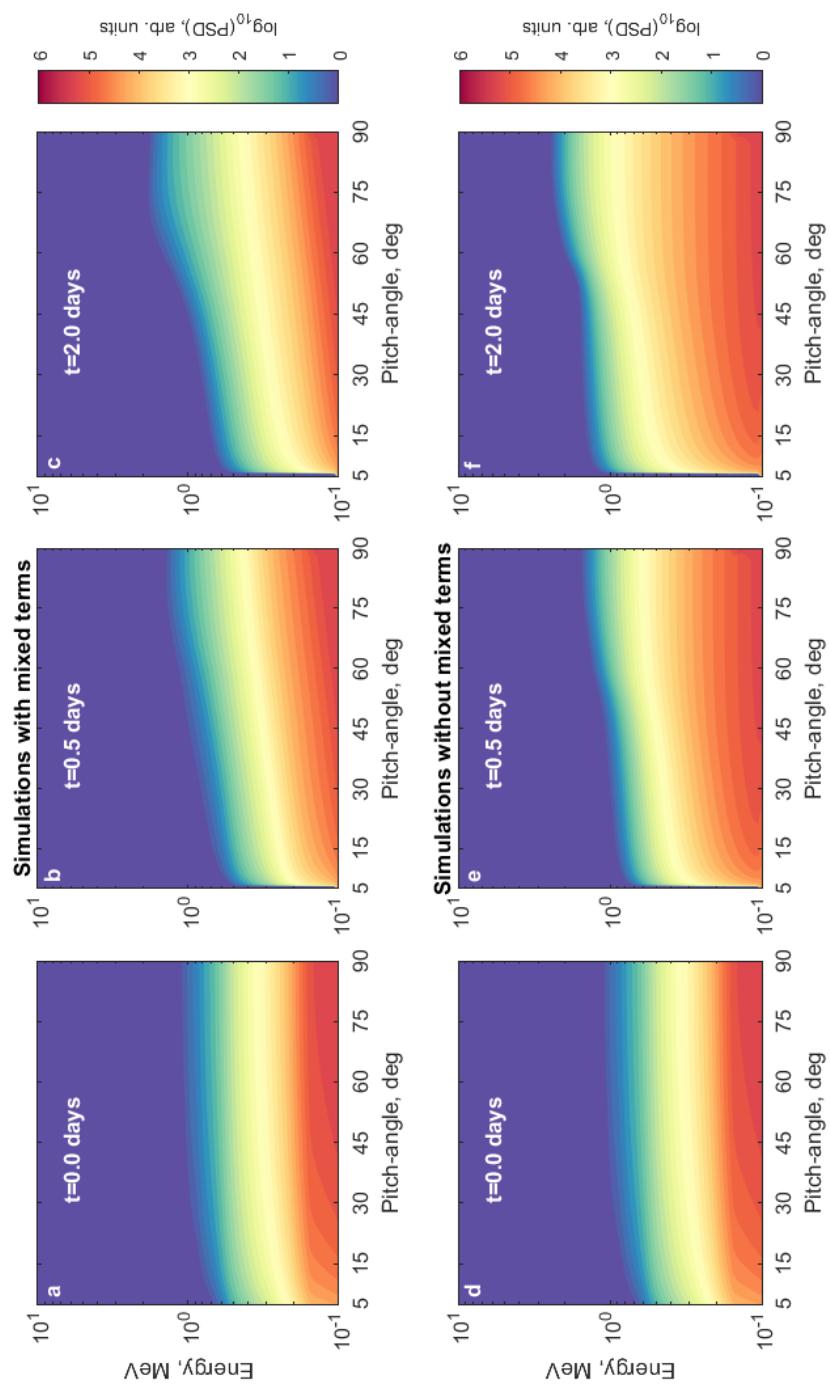
*Subbotin et al., 2010*

- The mixed diffusion terms act to **decrease the enhancement of relativistic electron flux by chorus waves**
- The effect of the mixed diffusion terms is most significant at small pitch angles

*Albert and Young, 2005*

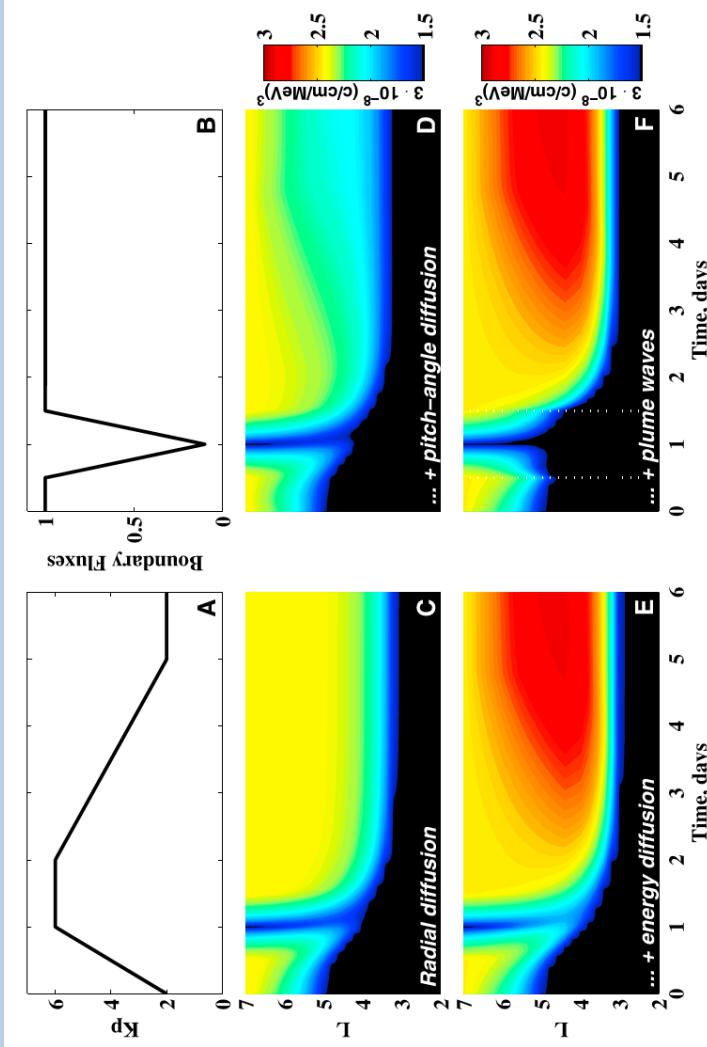
## 2D simulation with mixed terms

$$\begin{aligned} \frac{\partial f}{\partial t} = & \frac{1}{p^2} \frac{\partial}{\partial p} |_{\alpha_0,L} p^2 \left( D_{pp} \frac{\partial}{\partial p} |_{\alpha_0,L} f + D_{p\alpha_0} \frac{\partial}{\partial \alpha_0} |_{p,L} f \right) + \\ & + \frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} |_{p,L} T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} |_{p,L} f + D_{\alpha_0 p} \frac{\partial}{\partial p} |_{\alpha_0,L} f \right) - \tau_{lc} \end{aligned}$$



# **3D SIMULATIONS**

# 3D simulations of an Idealized Storm

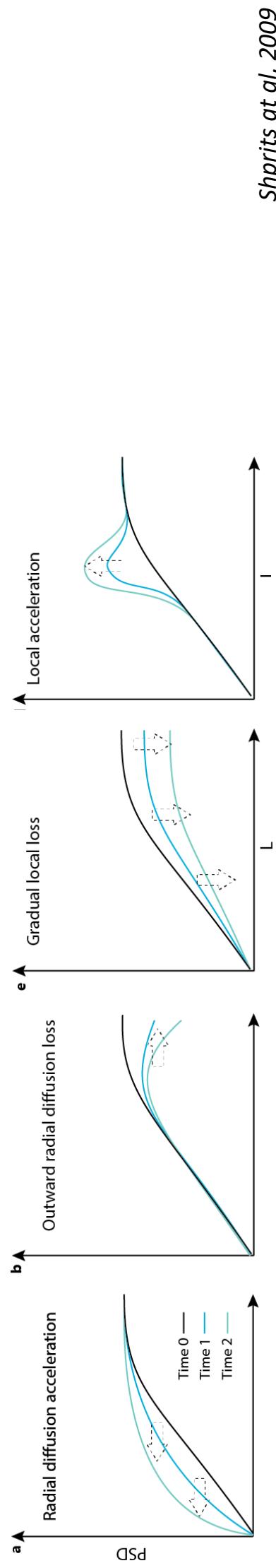


$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial L^*} \left|_{\mu_J} \right. \frac{1}{L^{*2}} D_{L^* L^*} \frac{\partial f}{\partial L^*} \left|_{\mu_J} \right. +$$

$$\frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} \left|_{p,L} \right. T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} \left|_{p,L} \right. f \right) +$$

$$\frac{1}{p^2} \frac{\partial}{\partial p} \left|_{\alpha_0,L} \right. p^2 \left( D_{pp} \frac{\partial}{\partial p} \left|_{\alpha_0,L} \right. f \right) - \frac{f}{\tau}$$

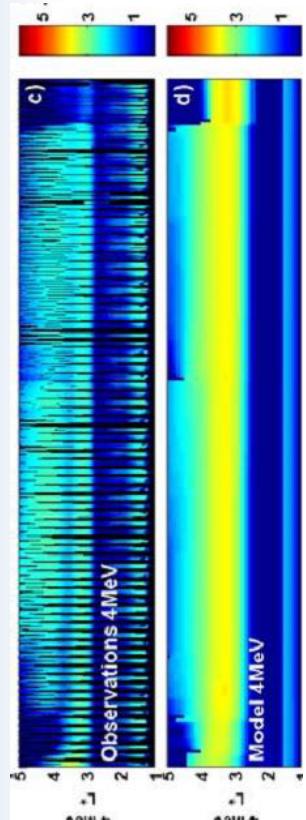
- Radial diffusion can transport electrons inward and accelerate them. Outward – loss to the magnetopause.
- Pitch-angle diffusion produces loss to the atmosphere
- Energy diffusion accelerates particles and creates peaks in phase space density.



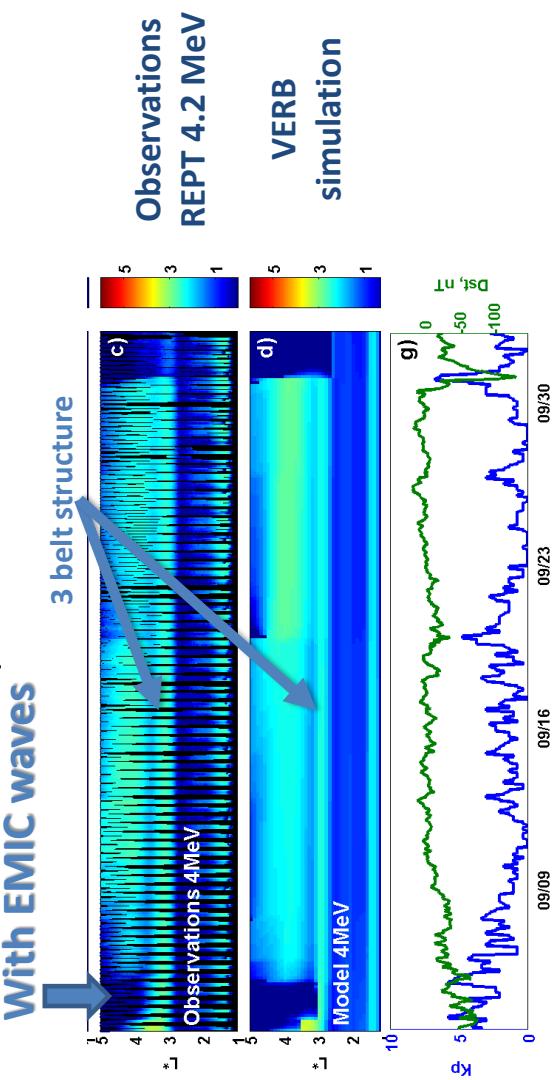
Shprits et al, 2009

# Modeling of the 3<sup>rd</sup> radiation belt

## Without EMIC waves



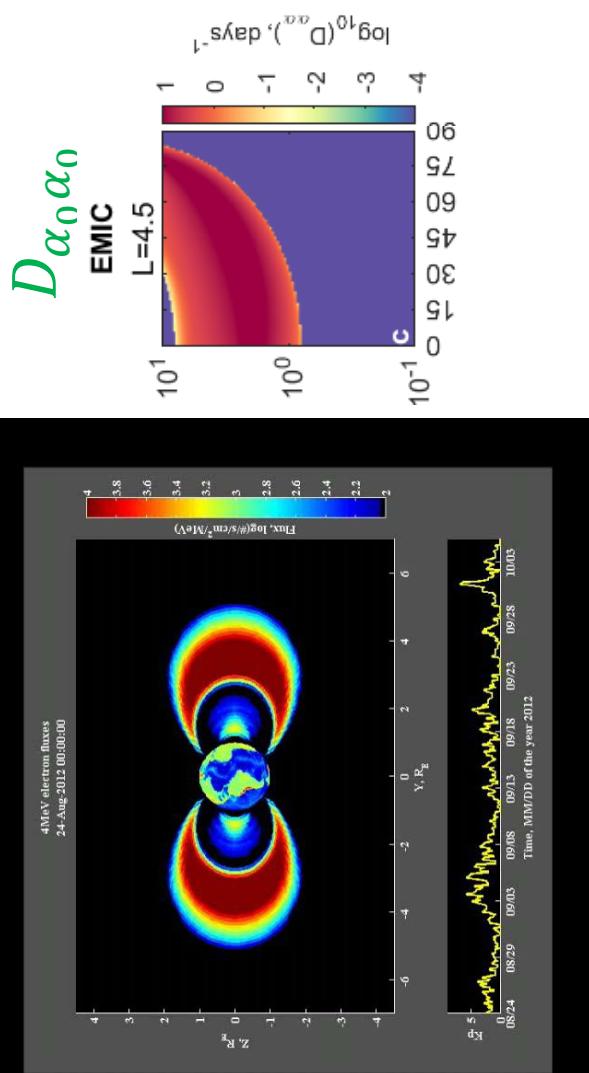
## With EMIC waves



$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial L^*} \left|_{\mu_J} \right. \frac{1}{L^{*2}} D_{L^* L^*} \frac{\partial f}{\partial L^*} \left. \right|_{\mu_J} +$$

$$\frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} \left|_{p,L} \right. T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} \left|_{p,L} \right. f + D_{\alpha_0 p} \frac{\partial}{\partial p} \left|_{\alpha_0, L} \right. f \right) +$$

$$\frac{1}{p^2} \frac{\partial}{\partial p} \left|_{\alpha_0, L} \right. p^2 \left( D_{pp} \frac{\partial}{\partial p} \left|_{\alpha_0, L} \right. f + D_{p \alpha_0} \frac{\partial}{\partial \alpha_0} \left|_{p,L} \right. f \right) - \frac{f}{\tau}$$



The VERB code simulation with EMIC waves reproduces the third belt structure during September 2012.

Community Coordinated Modeling Center

**NASA CCMC**

# VERB code at CCMC

<https://ccmc.gsfc.nasa.gov/models/VERB-3D~2.5>

Welcome to the new CCMC website!

Please note that some pages may have moved during the migration. If you experience any issues with the new website, please reach out to [esic-ccmc-support@lists.hq.nasa.gov](mailto:esic-ccmc-support@lists.hq.nasa.gov).

VERB-3D

Version 2.5

Runs-on-Request

The Veritable Electron Radiation Belt code (VERB) was developed by the Space Environment Modeling Group SEMG (<http://ibm.nps.edu/>) at the University of California, Los Angeles. The model has been described in Shprits et al., 2009, and Subbotin et al., 2010. It solves the Fokker-Planck equation for electron PSD Schulz and Lancerotti, 1974; Shprits et al., 2009b; Subbotin and Shprits, 2009]. The equation is solved using a finite differences approach and an implicit numerical scheme. The stability of such scheme is independent of the used time step. Following the approach used for the solution without mixed diffusion terms, described by Subbotin and Shprits[2009], the equation (1) is split into radial diffusion and local (energy, pitch angle, mixed) diffusion. The further separation of energy and pitch angle diffusion is impossible due to the existence of the mixed diffusion terms. Therefore, the implicit solution requires inversion of a model matrix of the  $\times$ -D operator on each time step. Inversion of such a big matrix is a quite time consuming computational operation, which is not required for the solution of the Fokker-Planck equation in the explicit formulation [Press et al., 1992]. However, the implicit scheme allows to use a longer model time step, while the time step in explicit scheme is limited by the Courant-Friedrichs-Lowy stability condition [Courant et al., 1928; Press et al., 1992], and the overall computational wall clock time with implicit scheme is lower.



- Creating a simulation request:
  - Credentials
  - Name of the simulation
  - Simulation period

\*\*You must agree to the CCMC DATA Collection Consent Agreement in order to submit a run\*\*

Do you give your consent?  YES

First (Given) Name:  (required, alphanumeric characters and '-' sign allowed)

Last (Family) Name:  (required, alphanumeric characters and '-' sign allowed)

E-mail:  (required, alphanumeric characters, '.', ',', '@', ':' signs allowed)

Run Number:  1 Select a different Run Number if you have already requested any Inner Magnetosphere run today (unless you want to overwrite a run request)

Title/Introduction  (required (no punctuation or special characters allowed))

Key words  (required (no punctuation or special characters allowed))

Inner Magnetosphere (IM) model:  
 VERB model  
 VEEB model

Run type and time interval:

Start date (on or after 2012/01/01 00:00):  2017/01/01 (YYYY/MM/DD) Start time:  00:00 (HH:MM)

End date (on or after 2012/01/01 00:00):  2017/01/07 (YYYY/MM/DD) End time:  00:00 (HH:MM)

Event run  
 Next (2 of 4: Options)



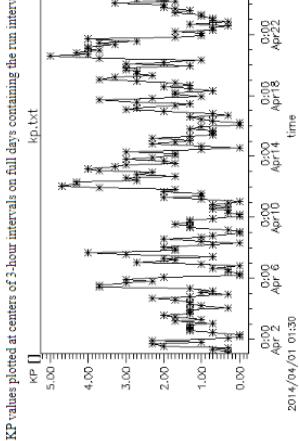
The VERB code is written in C++, and was designed for use on a single-CPU computer and conforms to the C++ 2011 standard. The cross-platform code supports the compilation on various systems (Linux, Windows, Mac OS). Please see the link to the model description for more details

# Setup simulation parameters

## Geomagnetic index data availability:

Definite or provisional K<sub>p</sub> data available  
**K<sub>p</sub> input in VERB:**

- Use observed K<sub>p</sub>



Constant value:   Uploaded time series

Choose File  No file chosen

The file size should not exceed 1MB/byte. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Boundary flux data:

• Use prescribed flux scaling (BF) at outer boundary (L=7)  
• Constant value  (must be > 0)

Uploaded time series

Choose File  No file chosen

The file size should not exceed 1MB/byte. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format  
 Use GOES flux data at outer boundary ([experimental](#))

Model grid (L, Pitch Angle, Energy)

- Low resolution grid (31x40x39). Grid will not allow to use mixed diffusion terms
- High resolution grid (31x101x101), use this grid to include mixed diffusion terms  
Mixed terms for local diffusion will be turned off with grids smaller than 100 in pitch angle or energy

**Lower-band and Upper-band Chorus Wave Diffusion**

Wang et al.(2019), Wang and Shprits (2019)

**plasmaspheric Hiss Wave Diffusion**

Spasovitch et al. (2015), Oliora et al. (2016)

**Plasmapause Model**

Cane and Anderson (1992)

## Setup Kp-index:

- **Default K<sub>p</sub> from the World Data Center for Geomagnetism, Kyoto.**
- Fixed K<sub>p</sub> value, e.g., K<sub>p</sub> = 1
- K<sub>p</sub> form the data file

## Setup L upper boundary or boundary flux (bf) scaling:

- **Default Fixed Dirichlet boundary condition (e.g., =1)**
- Bf scaling from data files
- Bf scaling from observations (experimental feature)
  - GOES 13
  - GOES 15

## Setup grid size

- **Low resolution (no mixed terms)**
- **High resolution (with mixed terms)**
- **Radial diffusion, chorus and hiss waves are scaled by K<sub>p</sub>.**

## Setup simulation parameters

- Geomagnetic index data availability:**

  - Definitive or provisic Boundary flux data:
    - KP input in VI
    - Use obse
    - KP values ph
    - KF
    - 5.0
    - Range: 0.8 - 2 MeV; default: 1 MeV. The target energy is used to interpolate between the two energy channels to obtain the time-dependent scaling factor.
    - The energy spectrum used at the boundary is fixed at this time.
    - GOES satellites
    - These GOES 13 spacecraft are available:
      - ✓ goes13
  - Target energy for boundary flux scaling
    - 1.0
    - Range: 0.8 - 2 MeV; default: 1 MeV. The target energy is used to interpolate between the two energy channels to obtain the time-dependent scaling factor.
    - Use GOES 13 data [experimental]

**Corrected Fluxes from GOES-13**

**Boundary flux**

  - Use presc
  - Constraint
  - Uploaded
  - Class
  - The fil
  - DOS V

**Model grid ( $L, \rho$ )**

  - Low resolutio
  - High resolutio
  - Mixed terms for

**Lower-band and Plasmapause Mo**

Wang et al (2011)  
Spasolevich et al  
Carpenter and A

**Plasmapause H**

Wang et al (2011)  
Spasolevich et al  
Carpenter and A

**Setup Kp-index:**

  - **Default Kp from the World Data Center for Geomagnetism, Kyoto.**
    - Fixed Kp value, e.g., Kp = 1
    - Kp form the data file

**Setup L upper boundary or boundary flux (bf) scaling:**

  - **Default Fixed Dirichlet boundary condition (e.g., =1)**
    - Bf scaling from data files
    - Bf scaling from observations (experimental feature)
    - GOES 13
    - GOES 15

**Setup grid size**

  - Low resolution (no mixed terms)
  - High resolution (with mixed terms)

**Radial diffusion, chorus and hiss waves are scaled by Kp.**

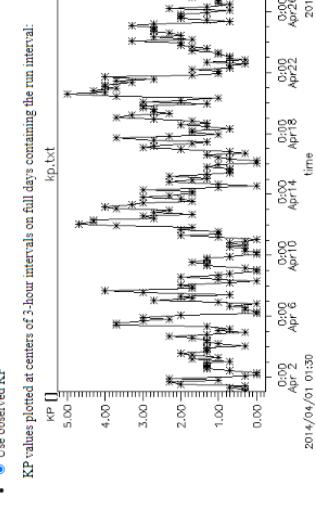
# Setup simulation parameters

## Geomagnetic index data availability:

Definite or provisional K<sub>p</sub> data available

### K<sub>p</sub> input in VERB:

- Use observed K<sub>p</sub>



Constant value:

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

Choose File  No file chosen

The file size should not exceed 1MB/ye. The full path to the input file at your local disk should not contain any blank spaces or quotation marks. On

DOS/Windows systems the recommended extension for the input file name is ".txt". Please read the detailed instructions on VERB input file format

## Setup K<sub>p</sub>-index:

- **Default K<sub>p</sub> from the World Data Center for Geomagnetism, Kyoto.**
- **Fixed K<sub>p</sub> value, e.g., K<sub>p</sub> = 1**
- **K<sub>p</sub> form the data file**

## Setup L upper boundary or boundary flux (bf) scaling:

- **Default Fixed Dirichlet boundary condition (e.g., =1)**
- **Bf scaling from data files**
- **Bf scaling from observations (experimental feature)**

- **GOES 13**
- **GOES 15**

## Setup grid size

- **Low resolution (no mixed terms)**
- **High resolution (with mixed terms)**
- **Radial diffusion, chorus and hiss waves are scaled by K<sub>p</sub>.**

## Model grid (L, Pitch Angle, Energy)

- Low resolution grid (31x40x39). Grid will not allow to use mixed diffusion terms
- High resolution grid (31x101x101), use this grid to include mixed diffusion terms Mixed terms for local diffusion will be turned off with grids smaller than 100 in pitch angle or energy

## Lower-band and Upper-band Chorus Wave Diffusion

Wang et al.(2019), Wang and Shprits (2019)

## plasmaspheric Hiss Wave Diffusion

Spasovitch et al. (2015), Oliora et al. (2016)

## Plasmapause Model

Cane and Anderson (1992)



Next (3 of 4: Process inputs)

# Review input and submit

## Confirmation page

**Welcome to the new CCMC website!**

Please note that some pages may have moved during the migration. If you experience any issues with the new website, please reach out to [ccmc-support@lists.lnl.nasa.gov](mailto:ccmc-support@lists.lnl.nasa.gov).

**User Message**

Summary of Submission  
Title: Introduction April 2014 with data b4f  
Key Word: ccmc-meeting  
Node 1 Type: IM  
Node 1: VERB  
Model Version: 2.5  
Start Time: 2014/04/01 00:00  
End Time: 2014/05/01 00:00  
Duration of the run: 720 hours  
KP input option:  
observed Kp  
BF (boundary scaling) input option:  
BF at 1 Mev from GOES satellites: ('goes13', 'goes15')  
target energy: 1 (1 Mev is the default for constant or user-defined scaling)  
Grid:  
Output Frequency: 7200 seconds  
Special Request: 0  
Justification:

**The Registration Number For This Run Is: Alexander\_Drozdov\_060322\_IM\_1**

Email has been sent. A copy of the email notification about this submission should arrive in your email inbox shortly.  
Please have the registration number when making inquiries about the run.

[Return to Top](#)

**Corrected Fluxes from GOES-15**

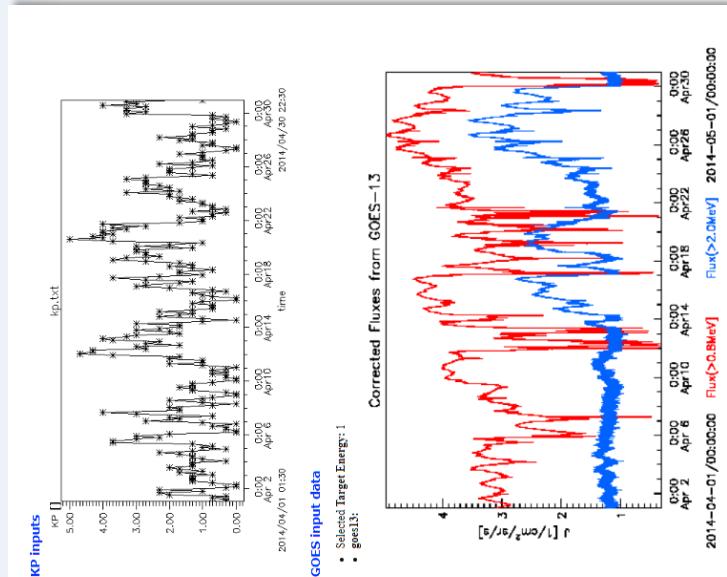
**Model output frequency:** 1 hours

**Options:**  
remove event  
model: VERB

**Special Request:**  
ONLY if you require a customized simulation setup not provided by the standard submission options. Unlike the standard runs that are automatically processed, special requests are reviewed and manually modified by the CCMC staff. Please note that special requests are reviewed and manually modified by the CCMC staff and are executed only if resources are available.

**Execution:**  
Please do not include special characters including the following: ? | / < > =

**Submit (4 of 4 Finalize)** **Reset Form**

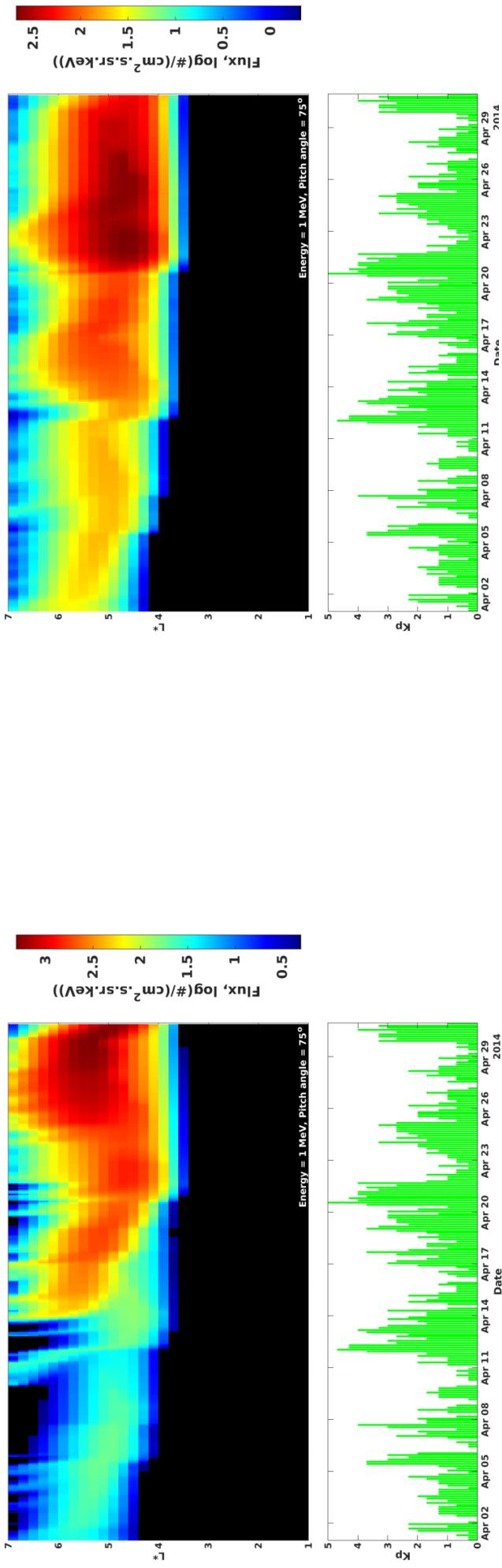


# VERB code simulation results at CCMC

## Flux 1 MeV eq. pitch angle 75°

- Default K<sub>p</sub> from the World Data Center from Geomagnetism, Kyoto.
  - Bf scaling from uploaded file
    - Bf reconstructed by neural network
- GOES 13 and 15
  - Bf scaling from observations (GOES 13 and 15)

- Landis, D. A., Saikin, A. A., Zhelavskaya, I., Drozdov, A. Y., Aseev, N., Shprits, Y. Y., et al. (2022). Title: NARX neural network derivations of the outer boundary radiation belt electron flux. Space Weather. <https://doi.org/10.1029/2021sw002774>



# Find VERB code results at CCMC

<https://ccmc.gsfc.nasa.gov/results>

INNER MAGNETOSPHERE SIMULATION RESULTS

Perform advanced search or simple search (options below) of our full archive.

• View ALL Inner Magnetosphere Runs on Request

• View Runs for the following Model:

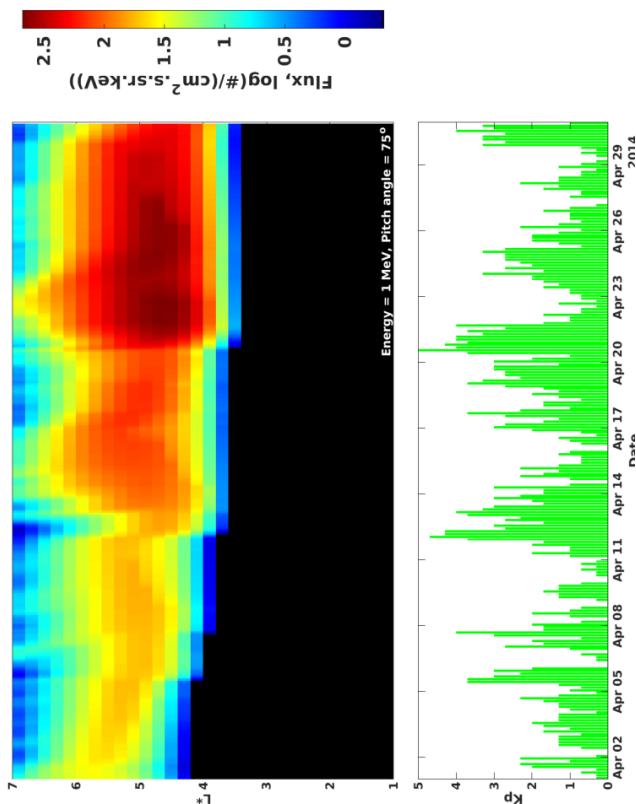
- CMMI
- RCM
- Fygaromenko Magnetic Field
- VERB
- Plasmasphere

**VIEW RUNS**

Check Run Status:  
Enter Run Registration Number: \_\_\_\_\_

CHECK STATUS

1. Find results of any VERB code simulations  
-Bf reconstructed by neural network



# VERB code at CCMC (Unfair competition)

<https://ccmc.gsfc.nasa.gov/models/VERB-3D~2.5>

Welcome to the new CCMC website!

Please note that some pages may have moved during the migration. If you experience any issues with the new website, please reach out to [esic-ccmc-support@lists.hq.nasa.gov](mailto:esic-ccmc-support@lists.hq.nasa.gov).

Community Coordinated Modeling Center

Search

FAQ | Contact

About  Models  Simulation Services  Validation  Community Support  Space Weather  Tools

VERB-3D

Version 2.5

Runs-on-Request

The Veratile Electron Radiation Belt code (VERB) was developed by the Space Environment Modeling Group SEMG (<http://ibmmp.egs.ucla.edu/>) at the University of California, Los Angeles. The model has been described in Shprits et al., 2009, and Subbotin et al., 2010. It solves the Fokker-Planck equation for electron PSD Schulz and Lanzetti, 1974; Shprits et al., 2009b; Subbotin and Shprits, 2009]. The equation is solved using a finite differences approach and an implicit numerical scheme. The stability of such scheme is independent of the used time step. Following the approach used for the solution without mixed diffusion terms, described by Subbotin and Shprits[2009], the equation (1) is split into radial diffusion and local (energy, pitch angle, mixed) diffusion. The

Last Updated: 06/02/2022

Sections in this page

Inputs

Outputs

Figures

Domains

Space Weather Impacts

Publications

Code

Relevant Links

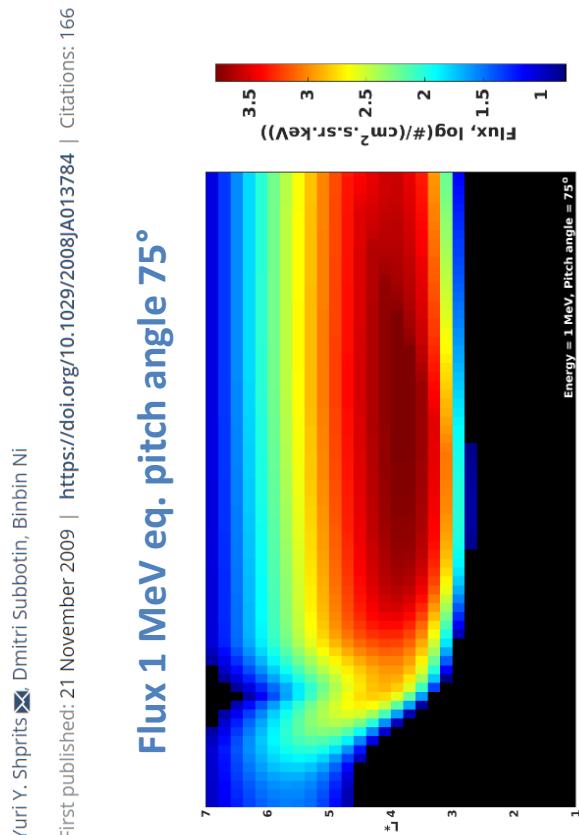
Contacts

Home Model Catalog

1. Find results of any VERB code simulations
  2. Reproduce results from Shprits et al., 2009
- JOURNAL OF GEOPHYSICAL RESEARCH
- Space Physics**
- AN AGU JOURNAL

Magnetospheric Physics | Free Access

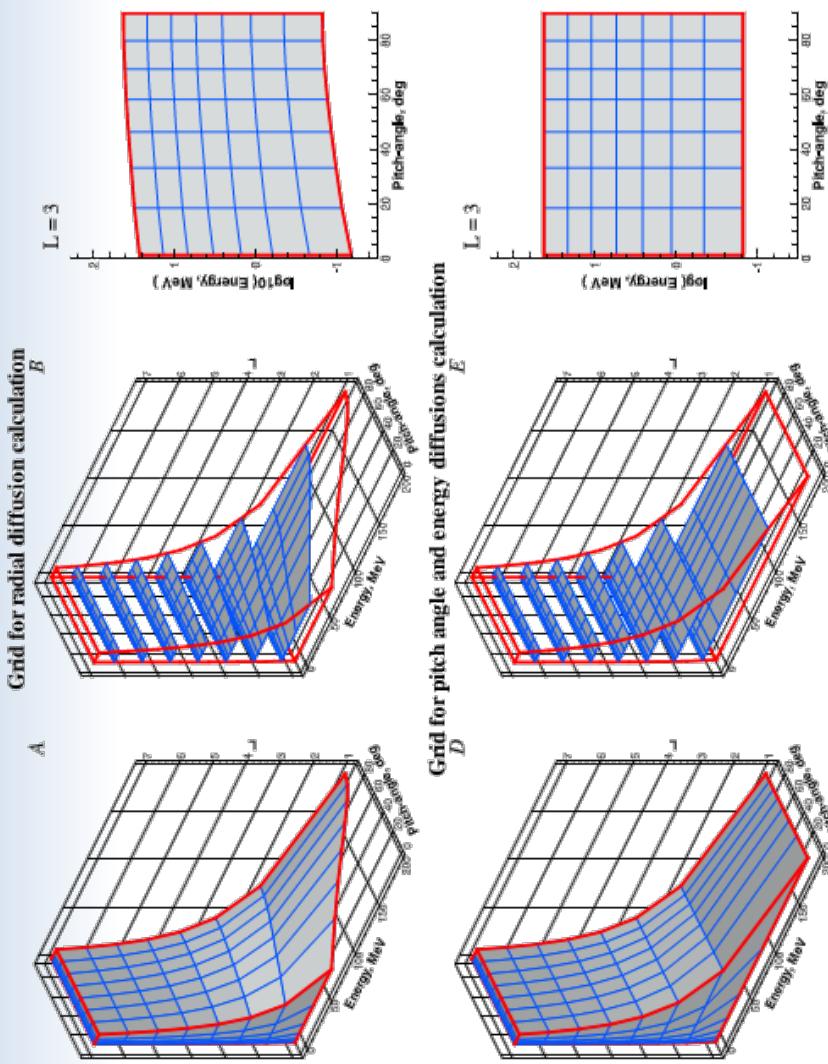
Evolution of electron fluxes in the outer radiation belt computed with the VERB code



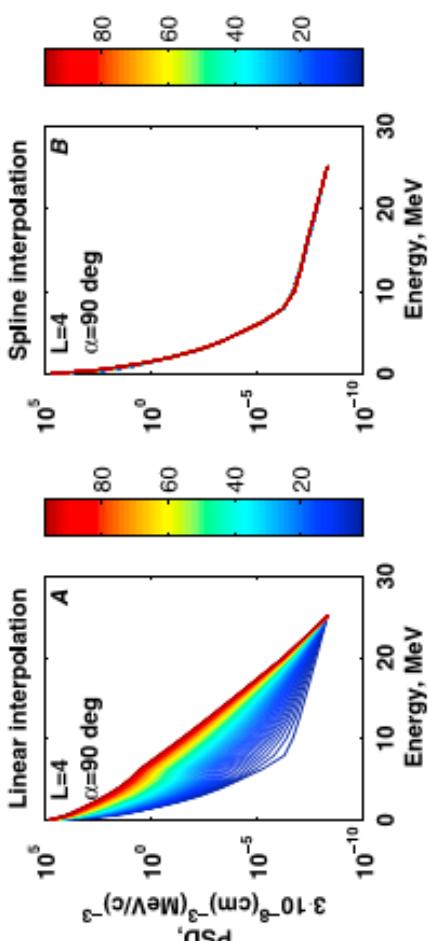
**adrozdov@ucla.edu**

**LvK GRID**

# Three-dimensional grids in the VERB code



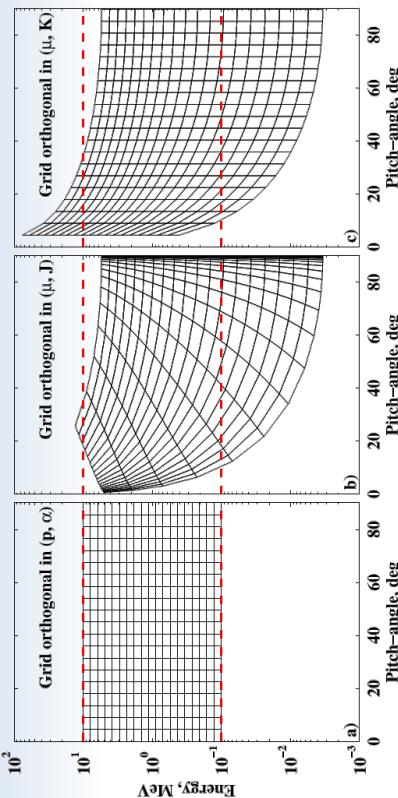
- The radial diffusion term should be solved on the  $(\mu, J, L)$
- The pitch angle and energy operators are written on the  $(p, \alpha_0, L)$  grid. We can set the **boundary conditions**.
- Spline interpolation is used.



$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial L^*} \Big|_{\mu_J} \frac{1}{L^{*2}} D_{L^* L^*} \frac{\partial f}{\partial L^*} + \frac{1}{p^2} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} p^2 \left( D_{p p} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} f + D_{p \alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} f \right) - \frac{f}{\tau}$$

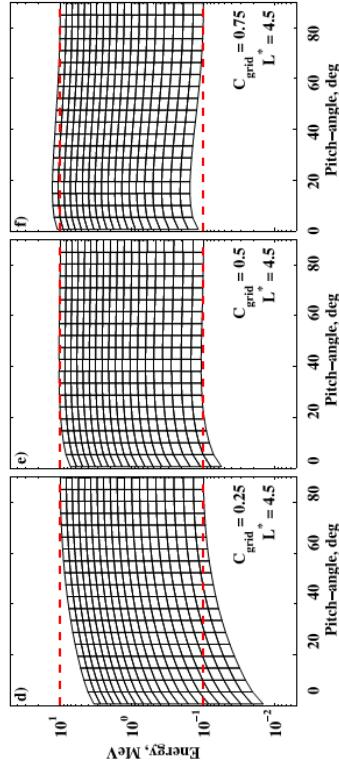
$$+ \frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} \Big|_{p, L^*} f + D_{\alpha_0 p} \frac{\partial}{\partial p} \Big|_{\alpha_0, L^*} f \right) - \frac{f}{\tau}$$

# Fokker-Planck equation in the VERB code using the new invariant grid ( $L^*, V, K$ )



$$V_{C_{grid}} \equiv \mu \cdot (K + C_{grid})^2$$

Grid orthogonal in  $(V_{C_{grid}}, K)$



- The  $(L^*, V, K)$  grid can be used for the computation of radial and local energy and pitch angle diffusion.
- Since the lines of constant  $V_{0.5}$  are almost parallel to the lines of constant energy, while  $K$  grid lines are parallel to the lines of constant pitch angle, the same boundary conditions can be imposed on the  $(V, K, L^*)$  grid as on the  $(\rho, \alpha_0, L^*)$  grid

$$\frac{\partial f}{\partial t} = L^{*2} \frac{\partial}{\partial L^*} \left|_{\mu_J} \right. \frac{1}{L^{*2}} D_{L^* L^*} \frac{\partial f}{\partial L^*} \left. \right|_{\mu_J} + \frac{1}{p^2} \frac{\partial}{\partial p} \left|_{\alpha_0 L^*} \right. p^2 \left( D_{pp} \frac{\partial}{\partial p} \left|_{\alpha_0 L^*} \right. f + D_{\alpha_0 p} \frac{\partial}{\partial \alpha_0} \left|_{p L^*} \right. f \right)$$

$$+ \frac{1}{T(\alpha_0) \sin(2\alpha_0)} \frac{\partial}{\partial \alpha_0} \left|_{p L^*} \right. T(\alpha_0) \sin(2\alpha_0) \left( D_{\alpha_0 \alpha_0} \frac{\partial}{\partial \alpha_0} \left|_{p L^*} \right. f + D_{\alpha_0 p} \frac{\partial}{\partial p} \left|_{\alpha_0 L^*} \right. f \right) - \frac{f}{\tau}$$



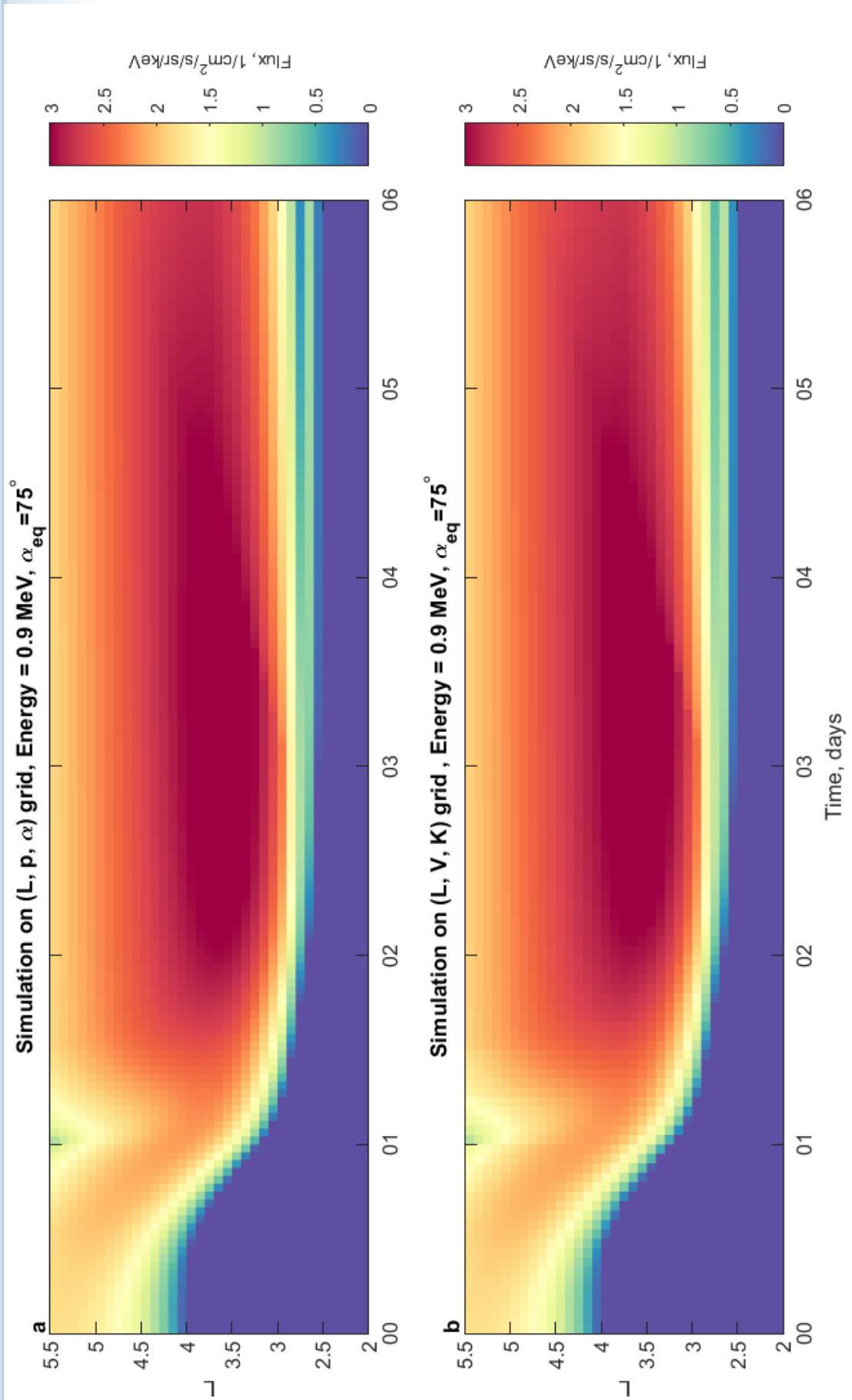
$$\frac{\partial f}{\partial t} = \frac{1}{G} \frac{\partial}{\partial L^*} G D_{L^* L^*} \frac{\partial f}{\partial L^*}$$

$$+ \frac{1}{G} \frac{\partial}{\partial V} G \left( D_{VV} \frac{\partial f}{\partial V} + D_{VK} \frac{\partial f}{\partial K} \right)$$

$$+ \frac{1}{G} \frac{\partial}{\partial K} G \left( D_{KK} \frac{\partial f}{\partial K} + D_{VK} \frac{\partial f}{\partial V} \right) - \frac{f}{\tau}$$

- Radial diffusion
- Local diffusion (hiss, chorus, EMIC)
- Loss term

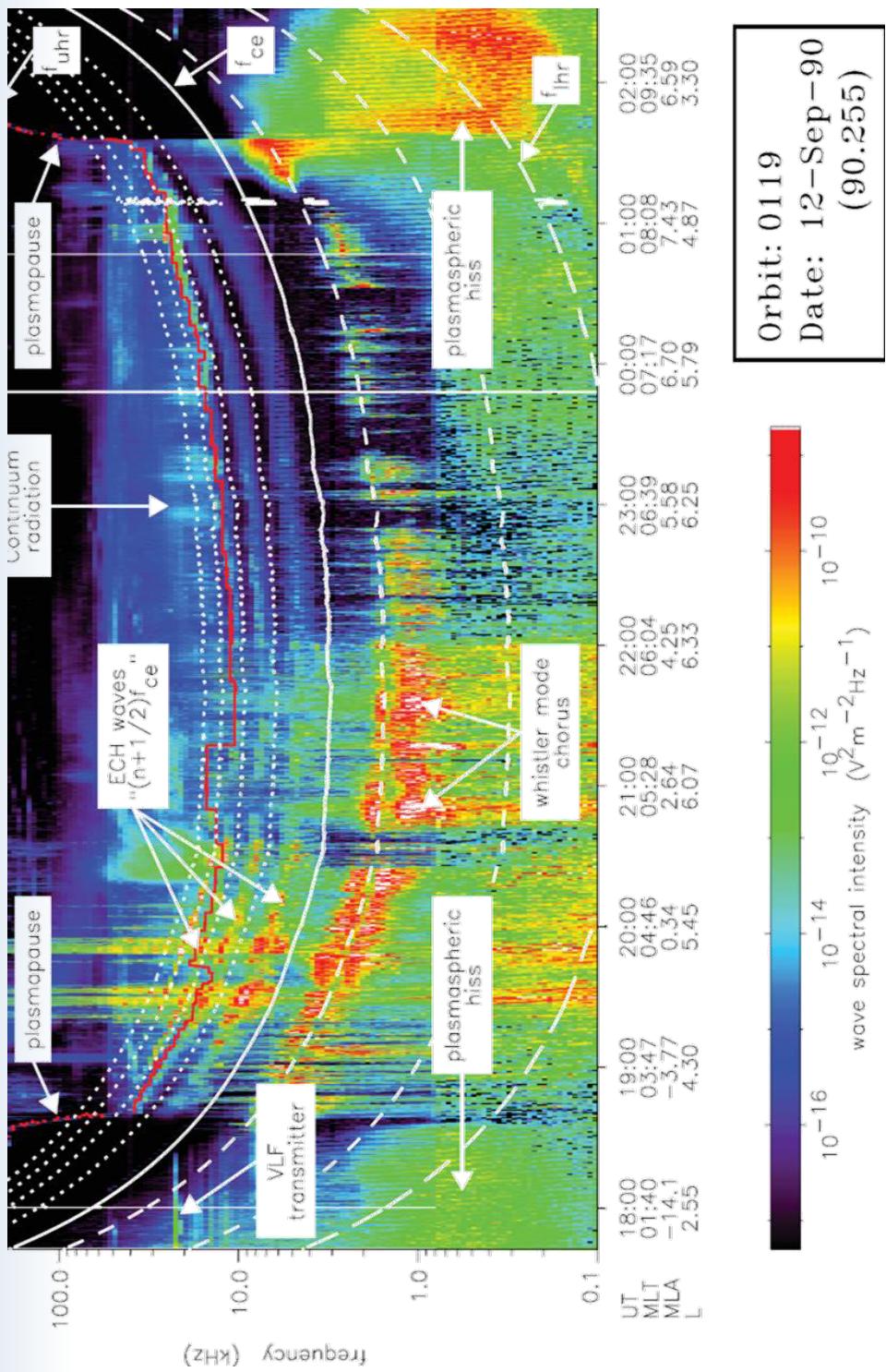
## Simulations on different grids



# **MORE PRACTICAL RESULTS**

The VERB code

# Waves inside the magnetosphere



## Comparison between observations and model

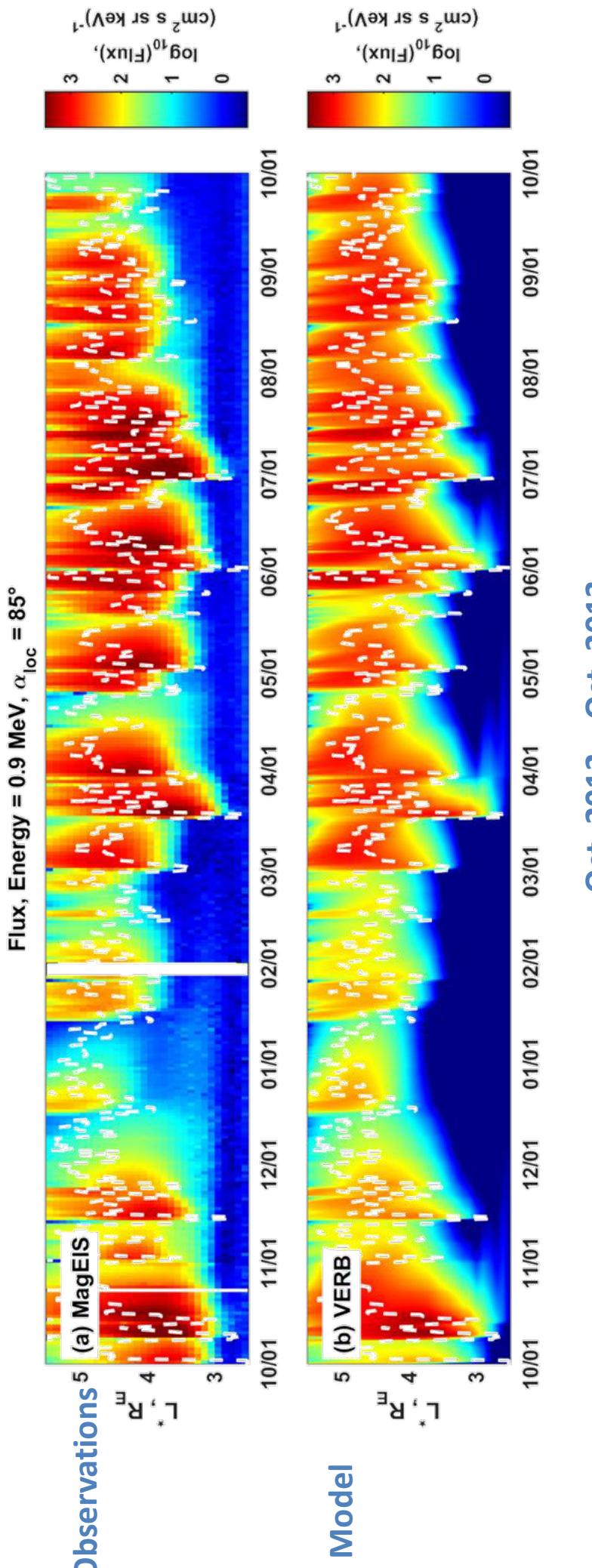
Observations

vs

Model

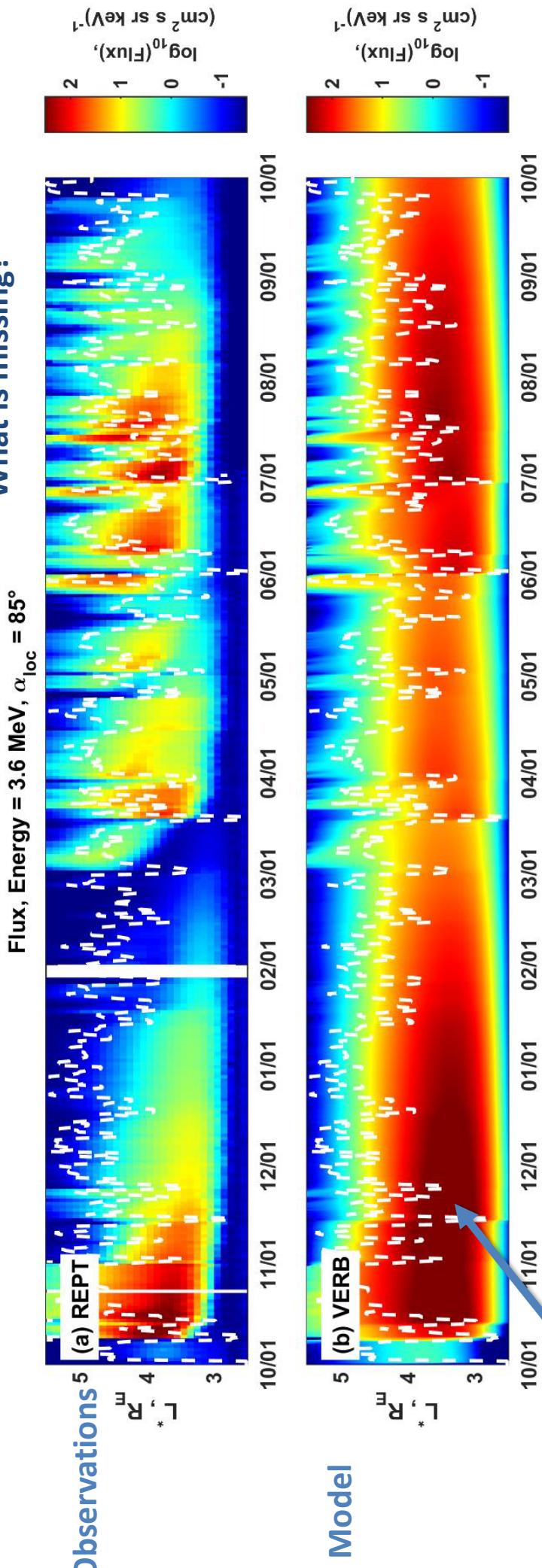
# Comparison between observations and model

We reproduce the dynamics of relativistic electrons ( $\sim 1$  MeV)



# Comparison between observations and model

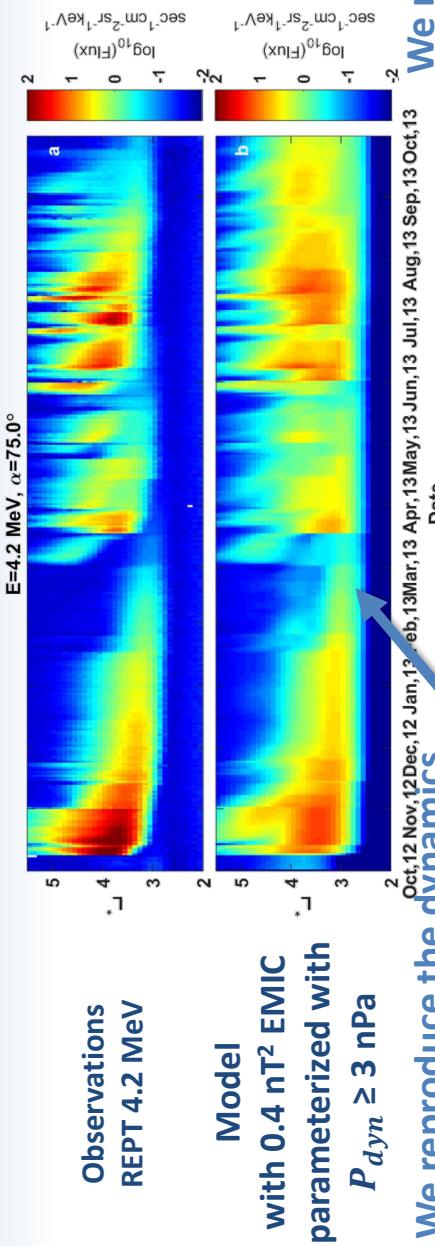
We reproduce the dynamics of relativistic electrons ( $\sim 1$  MeV),  
but not for the ultra-relativistic energies ( $> \sim 2$  MeV)  
What is missing?



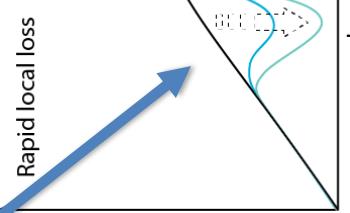
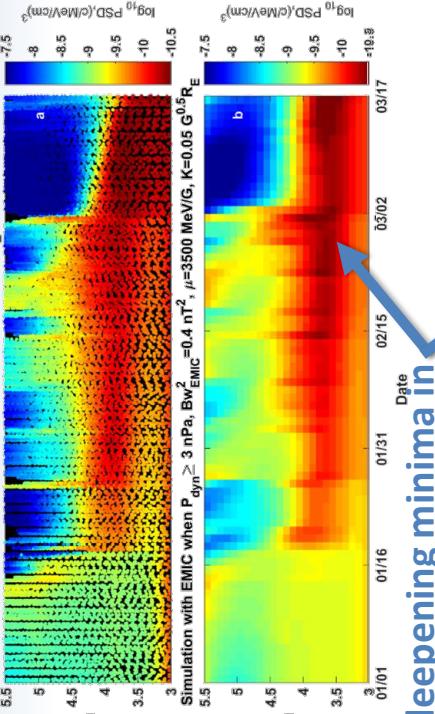
# Some VERB code simulation results

Drozgov et al. 2017

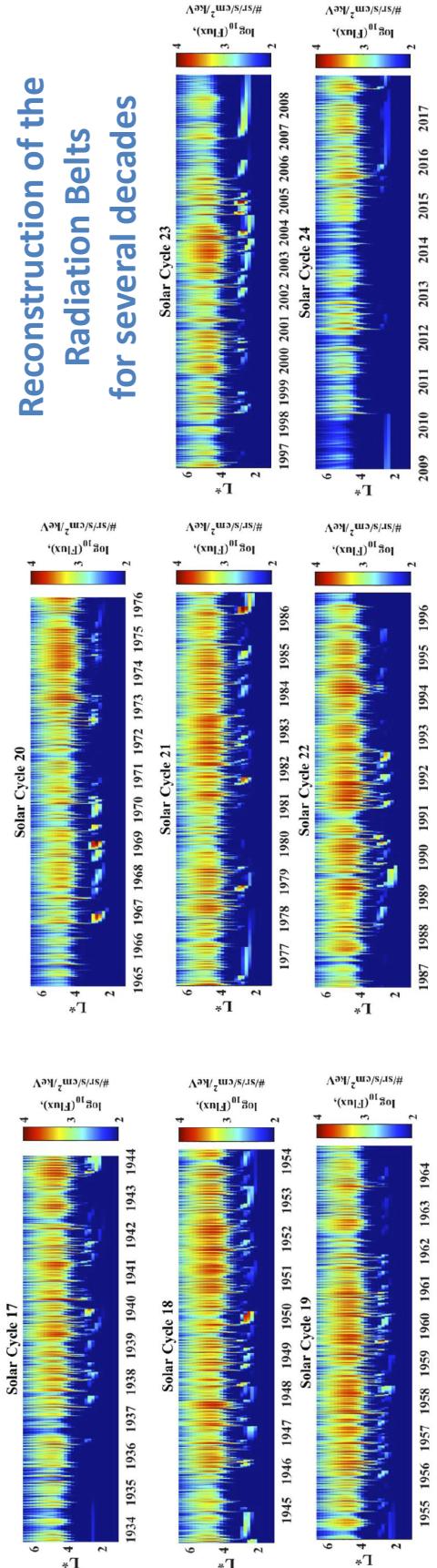
## Oct. 2012 – Oct. 2013



## 17 January 2013 storm



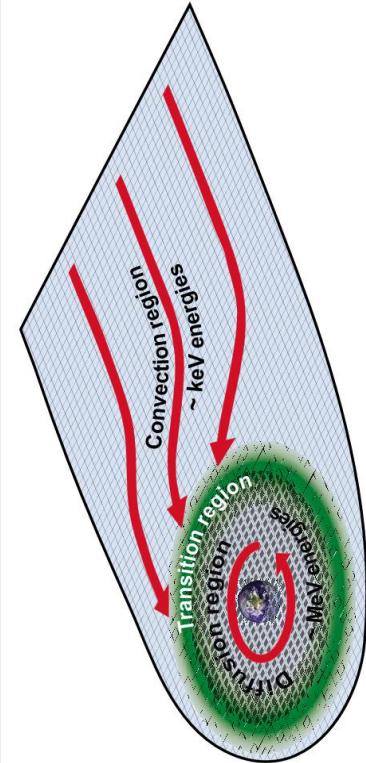
**Reconstruction of the Radiation Belts for several decades**



Saikin et al. 2021

# **4D SIMULATIONS**

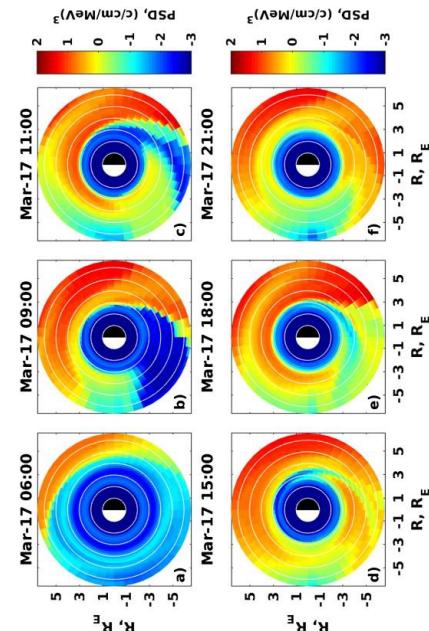
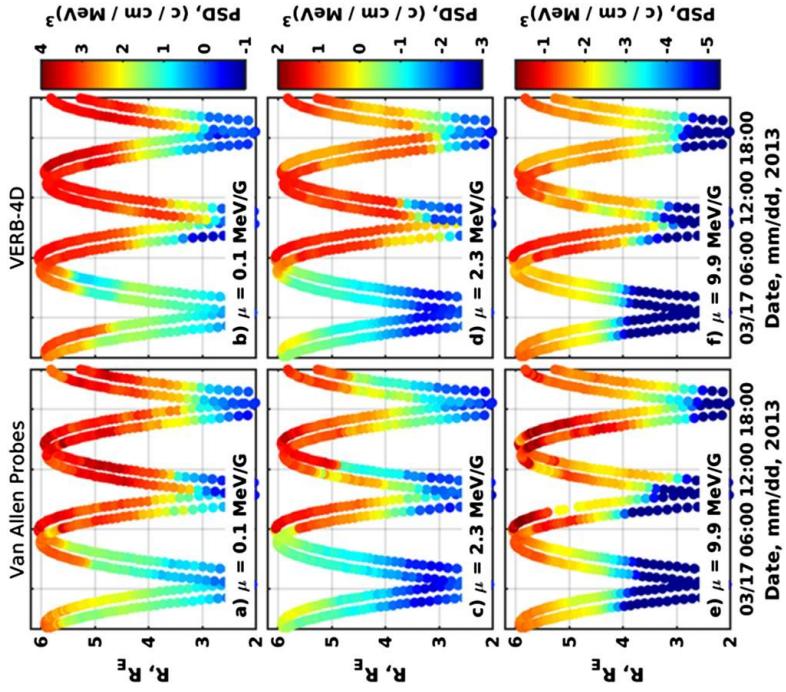
# 4D simulations



Equation of the 4D simulation in the VERB code

$$\frac{\partial f}{\partial t} = -\langle v_\phi \rangle \frac{\partial f}{\partial \phi} - \langle v_R \rangle \frac{\partial f}{\partial R} + \frac{1}{G} \frac{\partial}{\partial L^*} G D_{L^* L^*} \frac{\partial f}{\partial L^*} + \frac{1}{G} \frac{\partial}{\partial V} G \left( D_{VV} \frac{\partial f}{\partial V} + D_{VK} \frac{\partial f}{\partial K} \right) + \frac{1}{G} \frac{\partial}{\partial K} G \left( D_{KK} \frac{\partial f}{\partial K} + D_{VK} \frac{\partial f}{\partial V} \right) - \frac{f}{\tau}$$

The storm time dynamics of the low energy electrons is reproduced relatively well by the VERB-4D code.



The IRBEM library can be used to calculate to calculate  $E \times B$  and gradient-curvature **drift velocities** (with electric field model, e.g. Weimer (2005)).

$$V_F = \frac{E \times B_0}{B_0^2}$$

$$V_{GC} = \frac{\sqrt{8m_0\mu}}{q\tau_B B_0^2} \nabla_0 K \times B_0$$



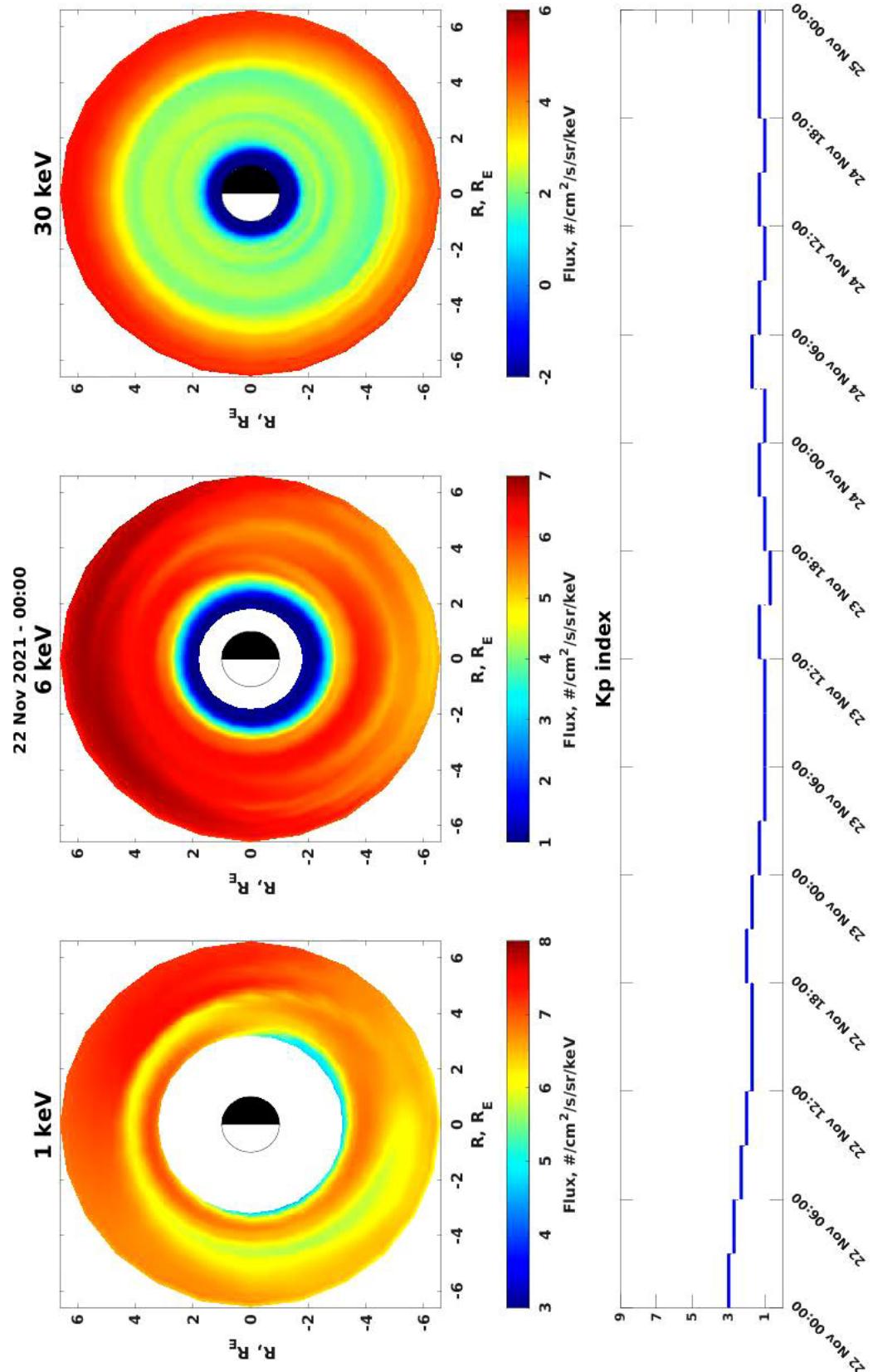
**IRBEM library**



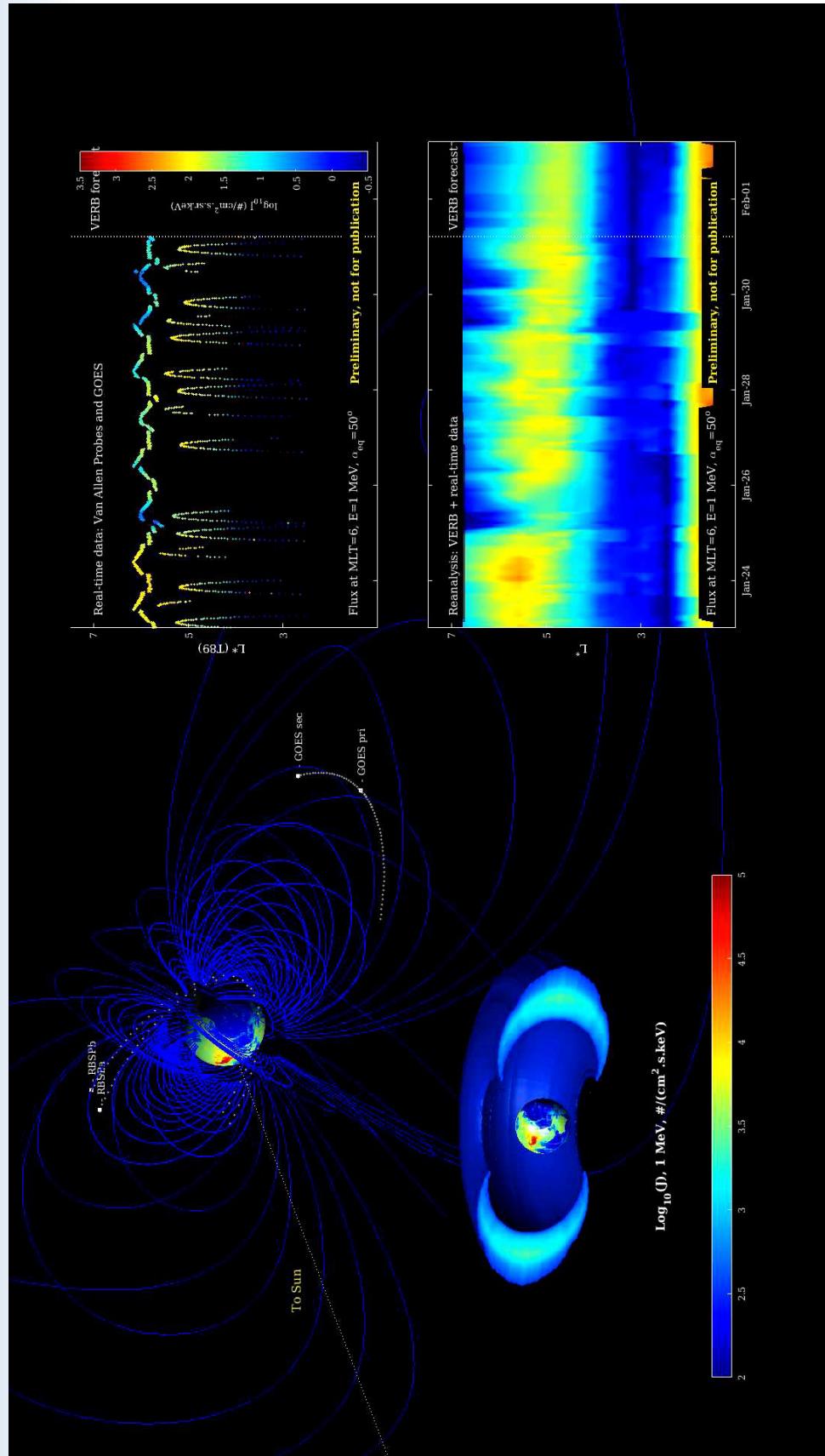
Aseev et al., 2019

Date, mm/dd, 2013  
03/17 06:00 12:00 18:00  
03/17 06:00 12:00 18:00  
Date, mm/dd, 2013  
03/17 06:00 12:00 18:00  
03/17 06:00 12:00 18:00  
Date, mm/dd, 2013

# Electron ring current dynamics



# Radiation belts



**THANK YOU**