Acceleration of Radiation Belt Electrons by VLF Chorus and Magnetosonic Waves

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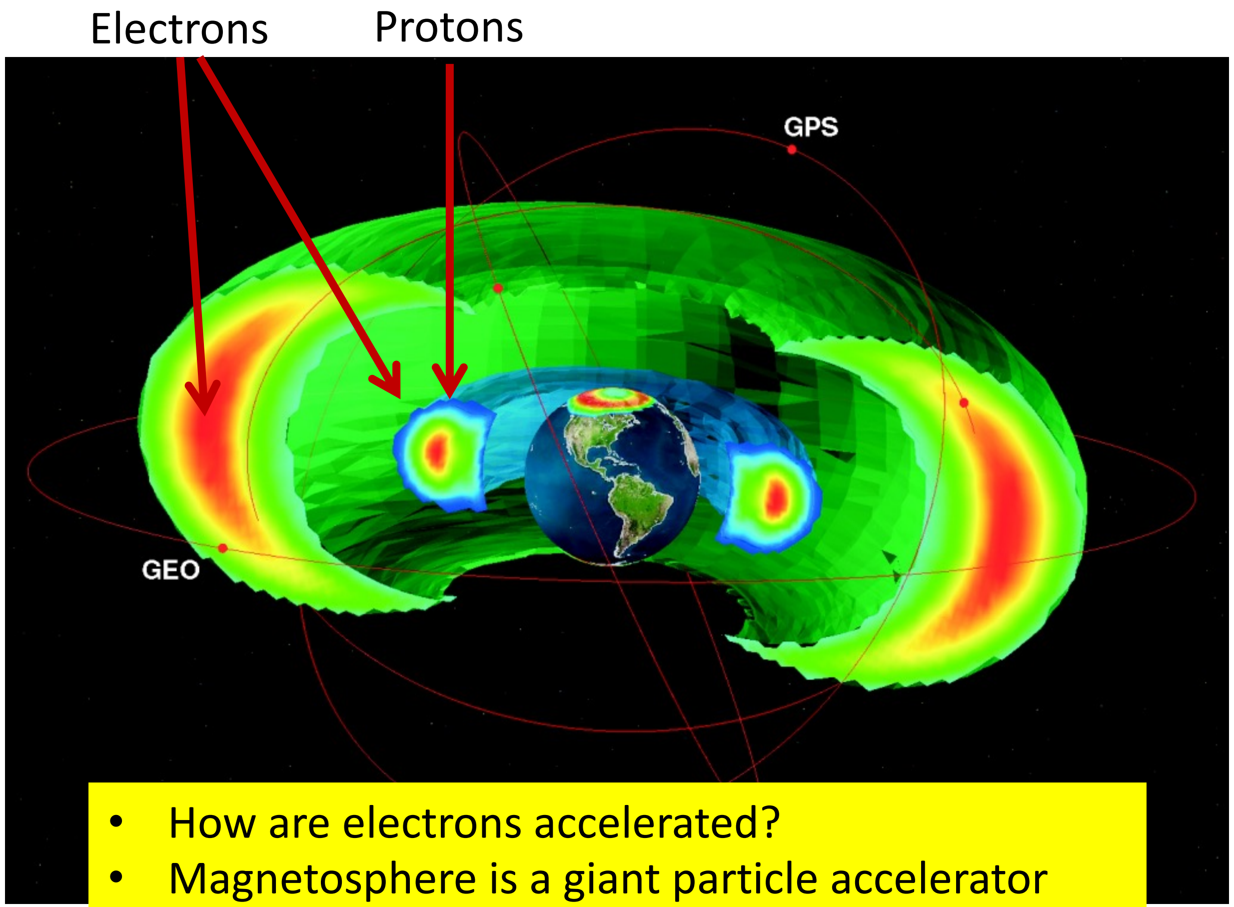
Int. School of Space Science, University of L'Aquila, Italy, 27th September, 2022

Earth’s Radiation Belts

- One proton belt ~ 100s MeV
- Two electron belts
  - Energies up to 10 MeV
  - Peaks near 1.6 and 4.5 Re
- Hazardous for spacecraft and humans



- How are electrons accelerated?
- Magnetosphere is a giant particle accelerator

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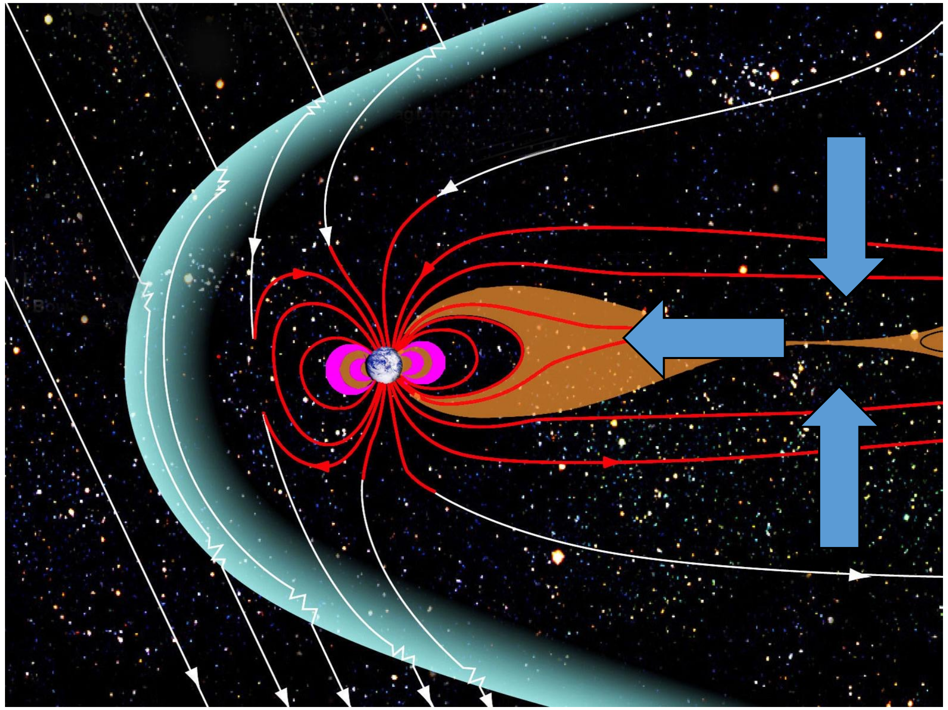
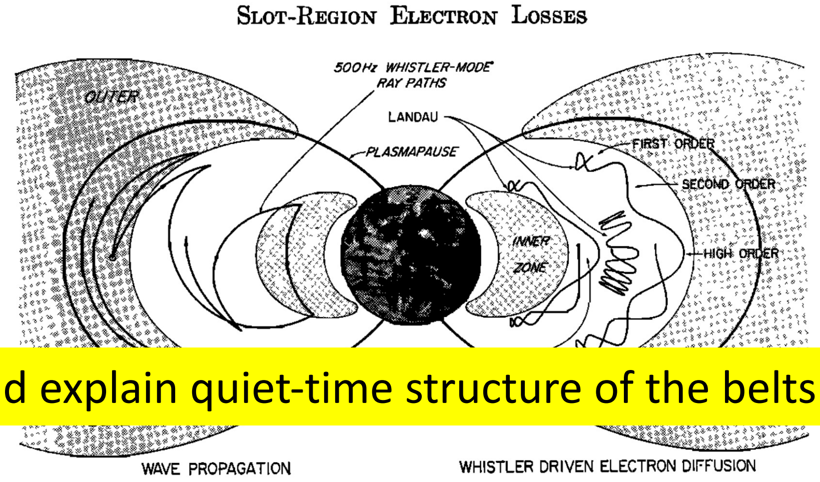
Electron Acceleration – 1960s, 70s, 80s, early 90s





- Could explain quiet-time structure of the belts

1. Low energy electrons from the solar wind
2. Transport towards the planet by radial diffusion, E fields and substorms
3. Electrons drift around the planet
4. Magnetic field fluctuations cause inward diffusion
5. Energy gain by conservation of first invariant

$$\mu = M = \frac{p^2 \sin^2 \alpha}{2 m_0 B}$$

6. Wave-particle interactions + collisions cause losses and inner boundary

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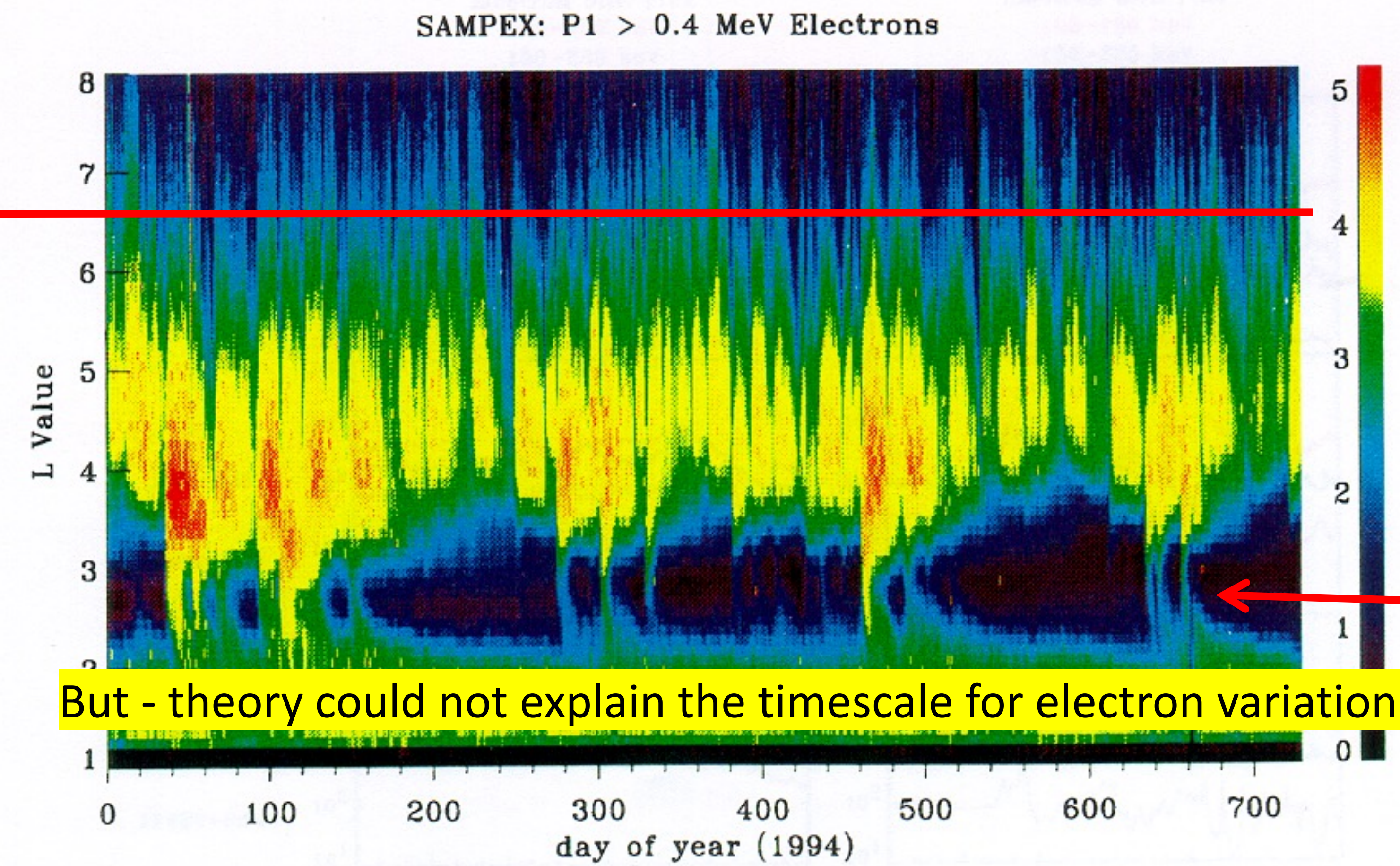
1990s – Time Variability

Geostationary orbit



Highly variable

Injection + decay

But - theory could not explain the timescale for electron variations

Li et al., (1997)

1998 - Two New Theories

Wave Acceleration

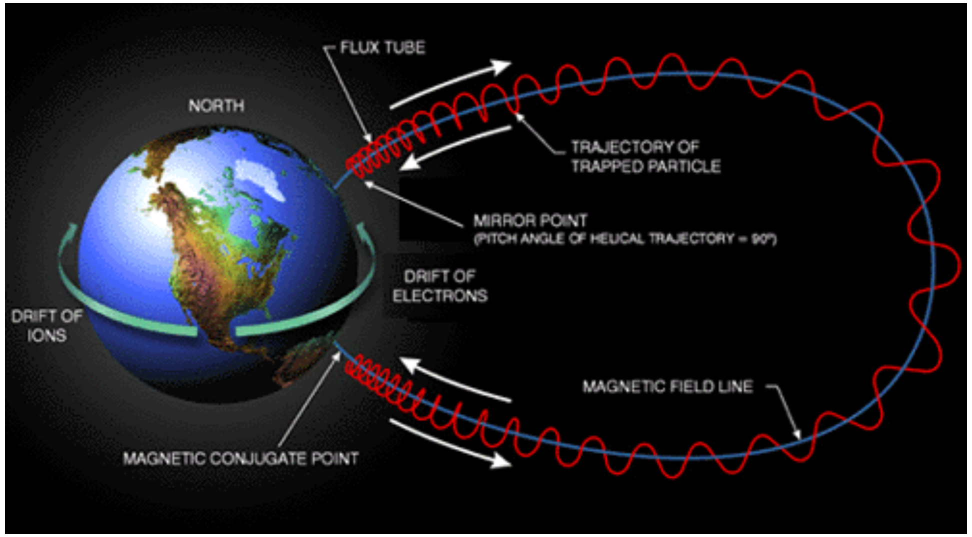
- Solar wind drives substorms and convection which transport low energy electrons toward the Earth
- Electron distribution becomes unstable and excites plasma instabilities – waves - inside geostationary orbit
- Waves (~ few kHz) cause electron precipitation and acceleration to energies of several MeV
- "Local acceleration"

Enhanced Radial Diffusion

- Solar wind flow past the magnetosphere drives Kelvin Helmholtz instabilities – waves at ULF frequencies (mHz)
- ULF waves propagate towards the Earth and drive field line resonances - increase magnetic field fluctuations
- Radial diffusion much faster, and energies reach several MeV

Both theories transfer energy from the solar wind into electrons, but via different multi-stage processes

Adiabatic Invariants

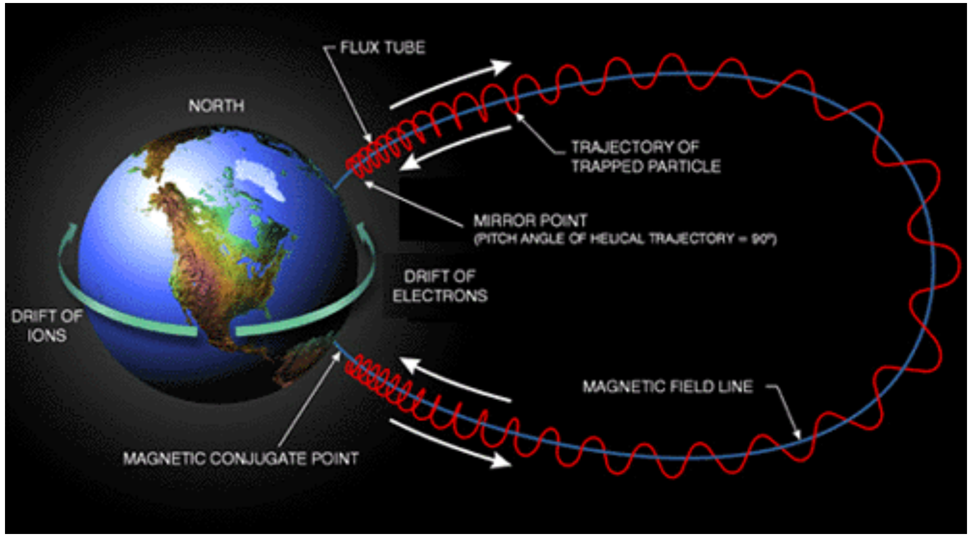


- Particles trapped in the geomagnetic field have 3 types of cyclic motion:
  - Cyclotron about the magnetic field
  - Bounce between northern and southern hemisphere
  - Drift around the Earth
- Associated with each cyclic motion is an approximate conservation law - called the adiabatic invariants
- Adiabatic invariants are conserved if the system changes very slowly
- They are violated if there are variations comparable to the drift, bounce or cyclotron frequency

1st Adiabatic Invariant

$$\mu = M = \frac{p^2 \sin^2 \alpha}{2 m_0 B}$$

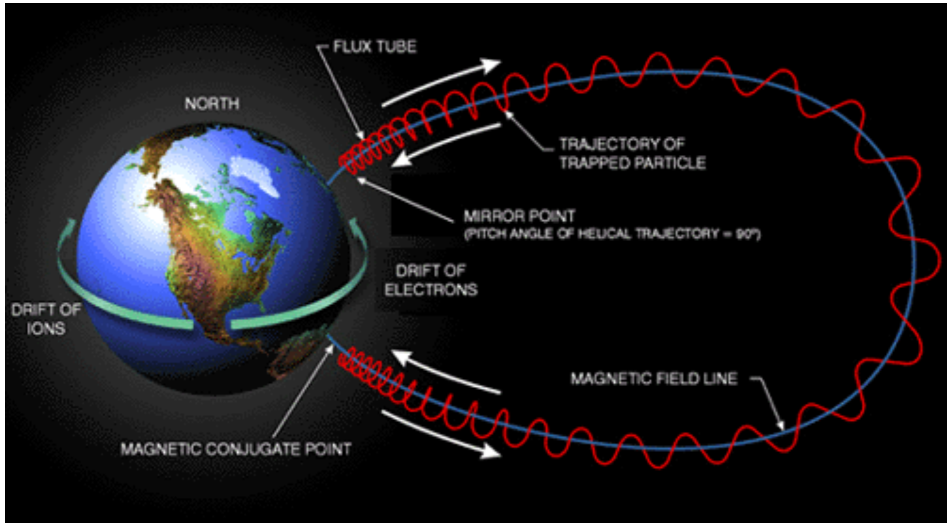
- Associated with cyclotron motion around the field B
- Timescale is less than a millisecond for electrons (frequency of a few kHz)
- p is momentum, α = pitch angle
- Pitch angle α of the particle changes as it moves along the field line
- Conservation of *Mu* allows us to relate the pitch angle at higher latitudes to that at the equator
- We define the loss cone as the field at 100 km (B100) and put $\alpha = 90^o$, so $\alpha_{lc} = sin^{-1}\sqrt{\frac{Beq}{B100}}$



- Electrons with $\alpha < \alpha_{lc}$ are lost to the atmosphere
- Electrons with $\alpha > \alpha_{lc}$ remain trapped

2^{nd} Adiabatic Invariant

$$\mathbf{J}_2 = 2\oint_{m1}^{m2} p_{\|}\, dl$$



- Associated with the bounce motion between the mirror points in the northern and southern hemisphere
- Timescale typically a few tenths of a second for MeV electrons (frequency of a few Hz)
- For $\alpha = 90^o$ J_2 tends to 0
- As the mirror points are set by the field, very approximately it can be thought of as the length of the field line between the mirror points
- Note – if particles are transported towards the Earth and J is conserved then $p_{\|}$ increases

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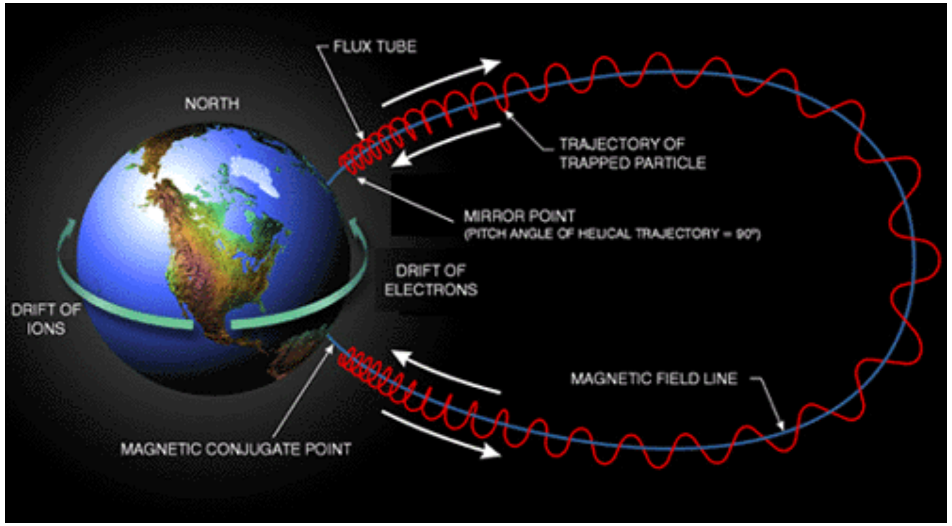
3rd Adiabatic Invariant

$$\mathbf{J}_3 = q \int \mathbf{B}.d\mathbf{s} = q\Phi$$

- Associated with the drift period around the Earth
- Timescale typically 10-15 mins for MeV electrons (mHz)
- Magnetic flux enclosed by the drift orbit is conserved
- If the Earth's magnetic field changes very slowly, then all 3 invariants are conserved and no net acceleration or loss
- Acceleration/loss requires breaking 1 or more invariant
- Breaking invariants involves E, B fields at frequencies comparable to drift, bounce and cyclotron frequencies



- Period of adiabatic invariants are widely separated
- E.G. Field fluctuations (E or B) on timescale of minutes can break the 3rd invariant, but still conserve the first and second invariant

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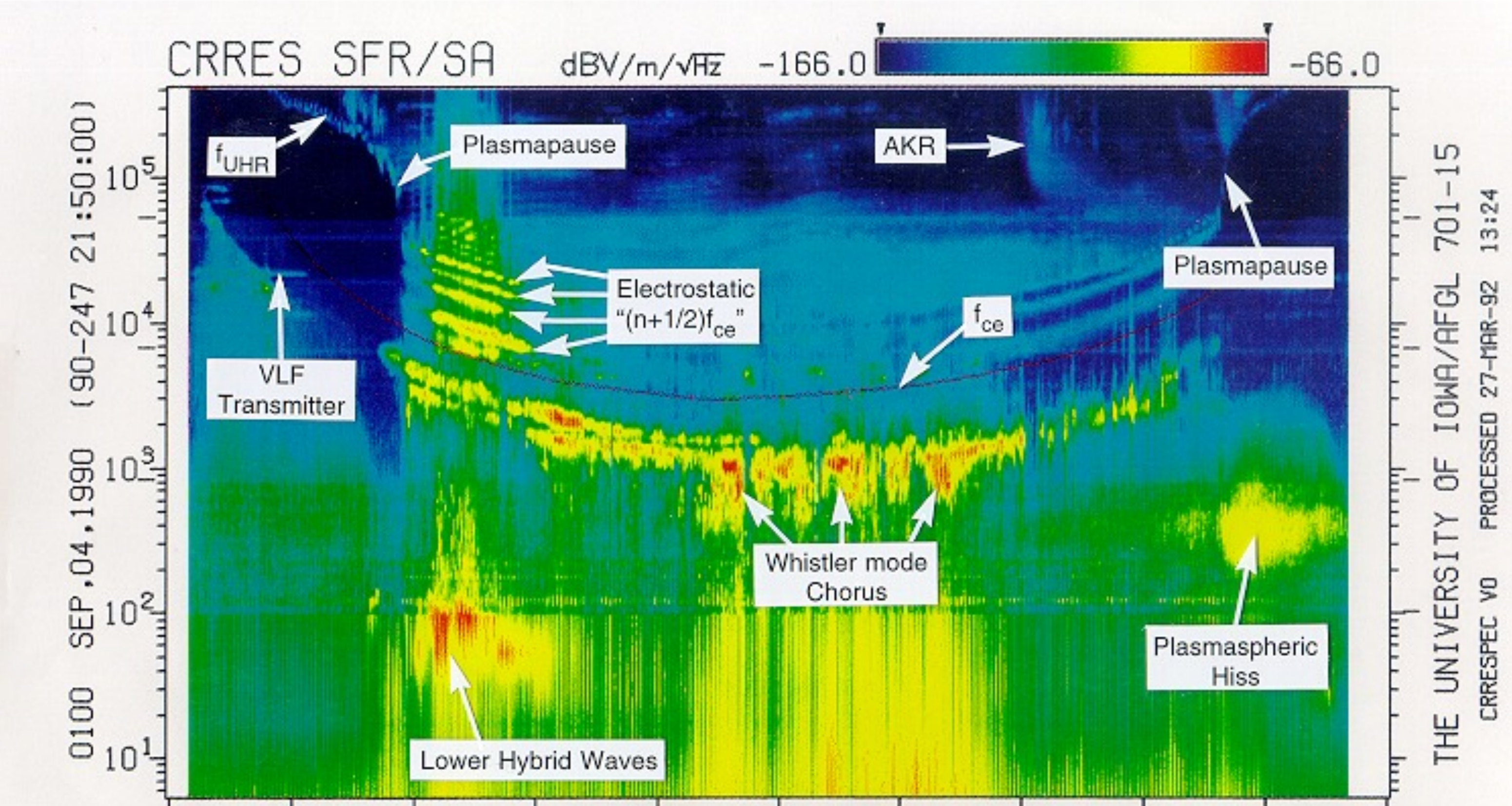
Breaking the 1st Invariant - Plasma Waves



UT	21:50		23:50		01:50		03:50		05:50		07:50
R	1.12	2.83	4.47	5.50	6.07	6.27	6.10	5.54	4.55	2.96	1.07
MLAT	4.19°	-9.11°	-2.44°	1.69°	4.95°	7.80°	10.36°	12.51°	13.69°	11.14°	-21.07°
MLT	16:34	02:36	04:19	05:13	05:54	06:31	07:09	07:51	08:48	10:29	19:24
L	1.17	2.93	4.59	5.63	6.22	6.46	6.37	5.88	4.87	3.08	1.21

Dispersion Relation for a Hot Plasma – Kinetic Theory

- The dispersion relation relates the wave frequency ω to the wavevector k in a hot plasma. We solve the equation:

$$D(\mathbf{k},\omega) = An^4 + Bn^2 + C = 0$$

where

$$A = \left[\varepsilon_{11}\left(\frac{k_\perp}{k}\right)^2 + 2\varepsilon_{13}\frac{k_\perp k_\parallel}{k^2} + \varepsilon_{33}\left(\frac{k_\parallel}{k}\right)^2\right]$$

$$B = -\left[\left(\varepsilon_{12}\frac{k_\perp}{k} - \varepsilon_{23}\frac{k_\parallel}{k}\right)^2 + (\varepsilon_{11}\varepsilon_{33} - \varepsilon_{13}^2) + A\varepsilon_{22}\right]$$

$$C = \left[(\varepsilon_{11}\varepsilon_{33} - \varepsilon_{13}^2)\varepsilon_{22} + (\varepsilon_{12}\varepsilon_{33} + 2\varepsilon_{13}\varepsilon_{23})\varepsilon_{12} + \varepsilon_{11}\varepsilon_{23}^2\right]$$

Let's extract some physical understanding

Resonance condition

$$\varepsilon(\mathbf{k},\omega) = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\mathbf{I} + \sum_\sigma \frac{\omega_{p\sigma}^2}{\omega^2}\sum_{n=-\infty}^{\infty}\int d\mathbf{v}\left(\frac{n\Omega_\sigma}{v_\perp}\frac{\partial f_\sigma}{\partial v_\perp} + k_\parallel\frac{\partial f_\sigma}{\partial v_\parallel}\right)\frac{\mathbf{\Pi}_\sigma(v_\perp, v_\parallel; n)}{(\omega - n\Omega_\sigma - k_\parallel v_\parallel)} \qquad (131)$$

where

$$\mathbf{\Pi}_\sigma(v_\perp, v_\parallel; n) = \begin{bmatrix} \frac{n^2\Omega_\sigma^2}{k_\perp^2}J_n^2 & iv_\perp\frac{n\Omega_\sigma}{k_\perp}J_nJ_n' & v_\parallel\frac{n\Omega_\sigma}{k_\perp}J_n^2 \\ -iv_\perp\frac{n\Omega_\sigma}{k_\perp}J_nJ_n' & v_\perp^2(J_n')^2 & -iv_\parallel v_\perp J_nJ_n' \\ v_\parallel\frac{n\Omega_\sigma}{k_\perp}J_n^2 & iv_\parallel v_\perp J_nJ_n' & v_\parallel^2J_n^2 \end{bmatrix} \qquad (132)$$

and

$$\int d\mathbf{v} = 2\pi\int_0^\infty v_\perp dv_\perp \int_{-\infty}^{\infty} dv_\parallel \qquad (133)$$

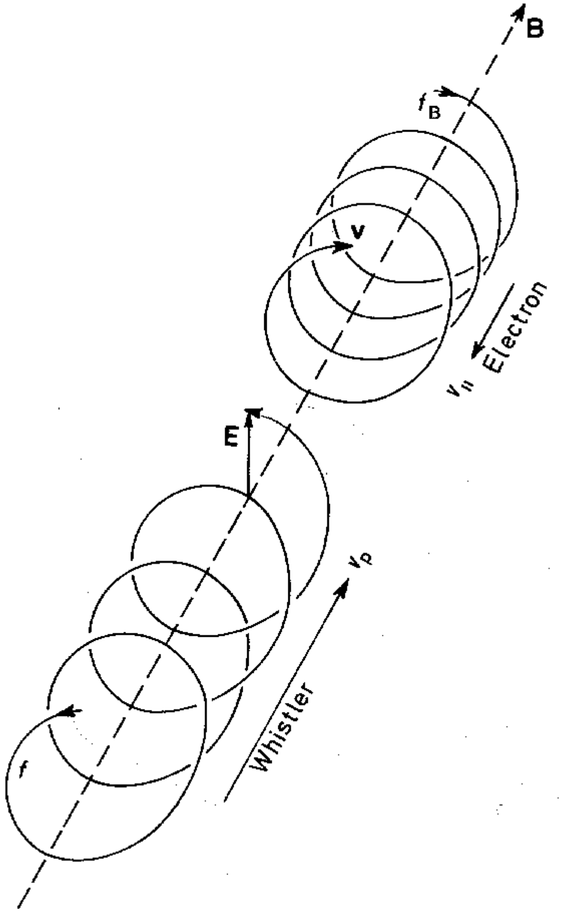
Doppler Shifted Cyclotron Resonance

$$\omega - k_{\|} v_{\|} - n\Omega_{\sigma}/\gamma = 0$$

- For resonance the wave frequency is Doppler shifted up to the electron cyclotron frequency
- For parallel propagation whistler mode waves are right hand circularly polarised and n = -1
- At resonance - wave electric field rotates in unison with electrons
- Re-Write:

$$v_{\|} = \frac{\omega}{k_{\|}} \left(1 + \frac{n|\Omega_e|}{\gamma \omega} \right)$$

- Waves and electrons must travel in opposite directions



- Resonance = efficient exchange of energy and momentum

Resonant Ellipse

- The resonance condition is an ellipse

$$\frac{v_\perp^2}{a^2} + \frac{(v_\parallel - d)^2}{b^2} = 1$$

- where

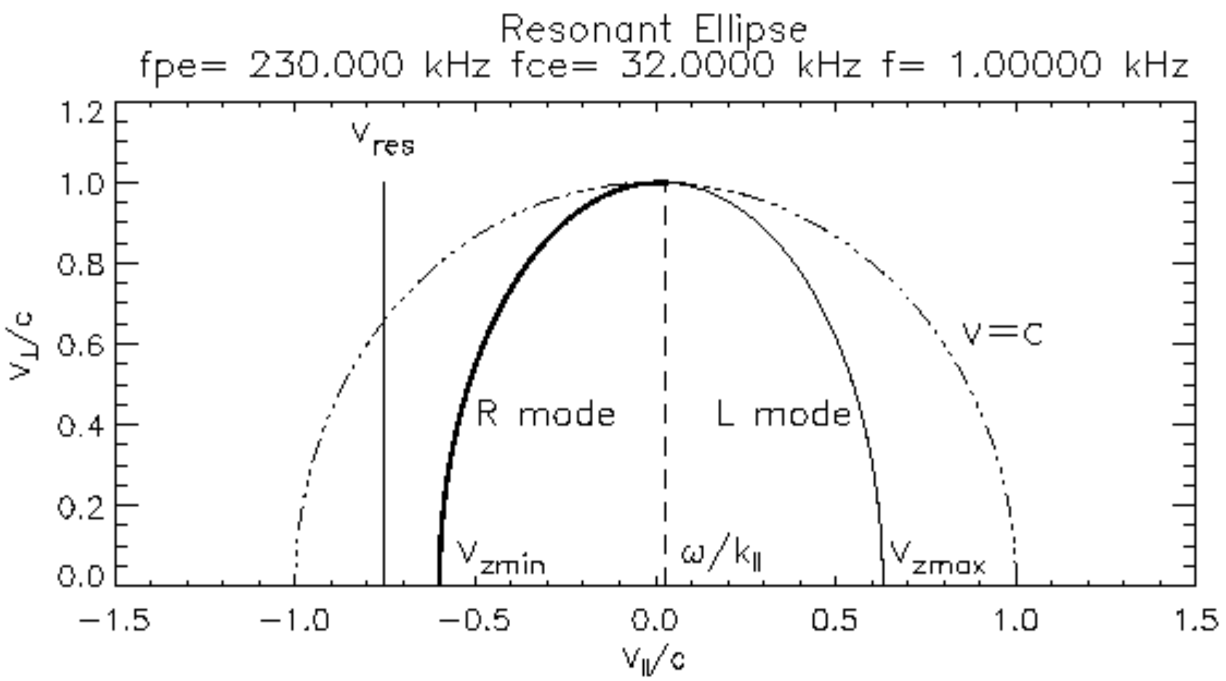
$$a^2 = c^2\left[1 - \frac{(\omega/\Omega_q)^2}{(n^2+h^2)}\right]$$

$$b^2 = \frac{n^2 a^2}{(n^2+h^2)}$$

$$h = \frac{ck_\parallel}{\Omega_q}$$

$$d = \frac{ch}{(n^2+h^2)}\frac{\omega}{\Omega_q}$$

- For n = 0, we have Landau resonance and the ellipse becomes a line with $v_\parallel = \omega/k_\parallel$
- For al other n the ellipse touches the circle at v = c
- The minimum resonant energy (Eres) is where the ellipse crosses the $v_\parallel$ axis



- To solve – we require the plasma frequency, cyclotron frequency, wave frequency, propagation angle and we must solve the plasma dispersion relation
- The resonant ellipse shows us the energy and pitch angle of the electrons that 1 monochromatic wave will interact with

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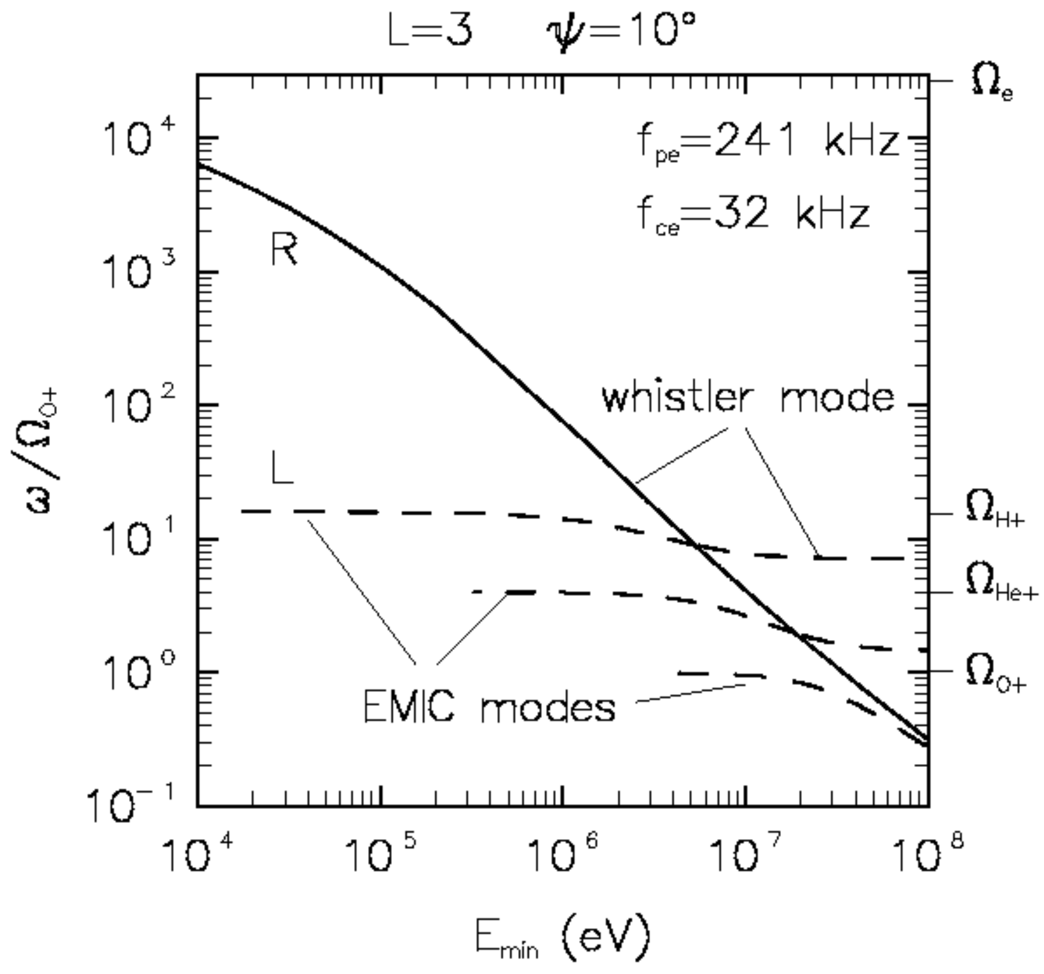
Resonant Energies



- Horne and Thorne (1998)
- To accelerate electrons waves must be able to resonate with 0.1-few MeV electrons
- Found 5 wave modes
  - Whistler mode
  - Magnetosonic
  - Z mode
  - RXZ
  - LO
- Whistler mode is a prime candidate for acceleration (and loss)
- Electromagnetic ion cyclotron waves (EMIC) contribute to loss

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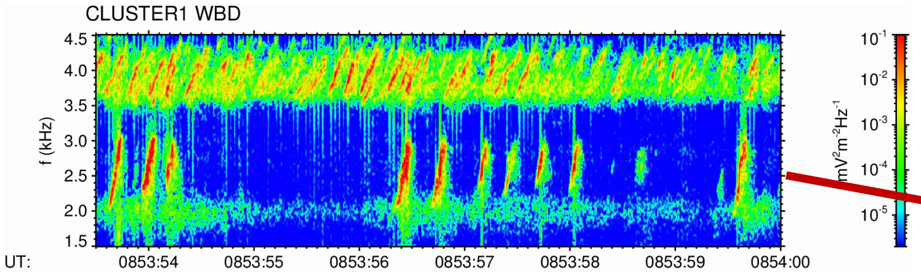
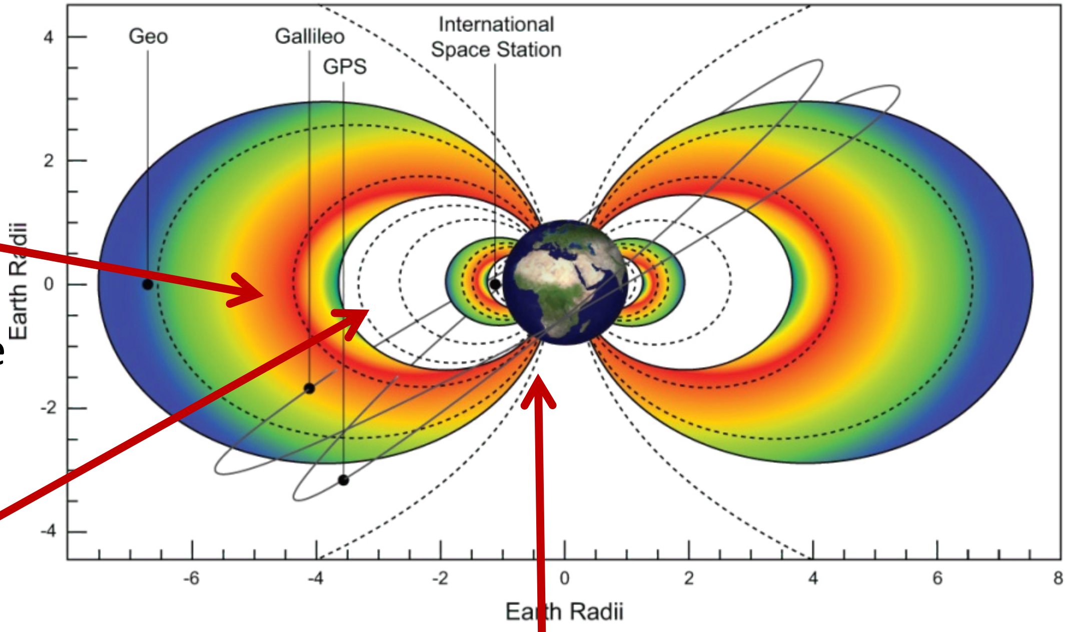
Chorus and Hiss Waves

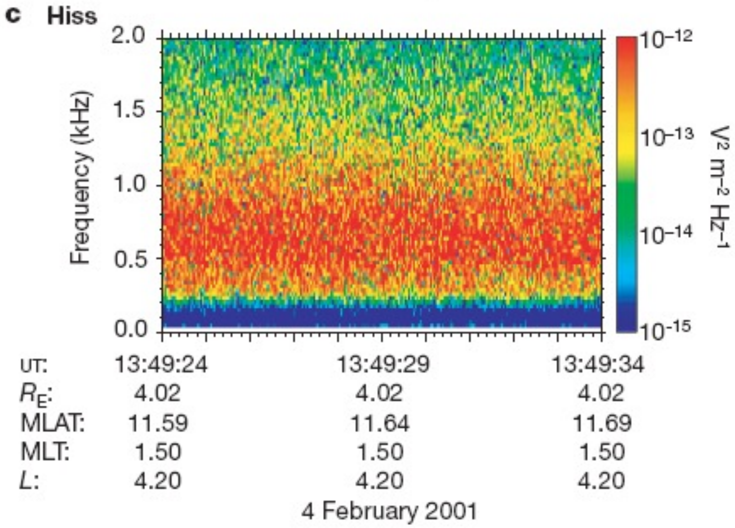
Satellite observations



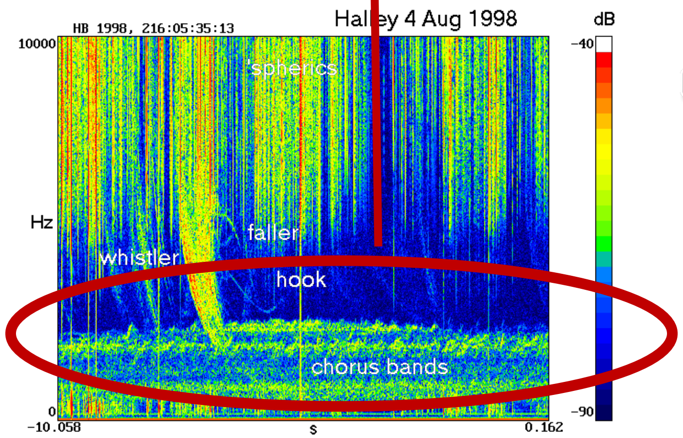


- Chorus plasma waves observed outside the plasmapause
- Most intense waves

- Hiss Waves observed inside Plasmapause



Antarctic observations



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Energy Transfer Between Waves and Electrons

- Consider an electron in a magnetic field B_0 and a right hand circularly polarized whistler mode wave. The force on the electron is

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times(\mathbf{B_0} + \mathbf{B}))$$

- The change in energy is

$$\begin{aligned} dE &= \mathbf{F}.\mathbf{ds} \\ &= \mathbf{F}.\mathbf{v}dt \\ &= q(\mathbf{E} + \mathbf{v}\times(\mathbf{B_0} + \mathbf{B})).\mathbf{v}dt \\ &= q\mathbf{E}.\mathbf{v}dt \end{aligned}$$

- A large change in energy requires the dot product to last a long time. This is achieved during resonance.

- To calculate the change in energy, transform to a frame of reference moving with the wave along the background field at phase velocity $v_{ph} < c$

$$\begin{aligned} \mathbf{F} &= q(\mathbf{E} + \mathbf{v}\times(\mathbf{B_0} + \mathbf{B})) \\ \mathbf{F} &= q(\mathbf{E} + \mathbf{v_{ph}}\times(\mathbf{B_0} + \mathbf{B}) + (\mathbf{v} - \mathbf{v_{ph}})\times(\mathbf{B_0} + \mathbf{B})) \\ &= q((\mathbf{v} - \mathbf{v_{ph}})\times(\mathbf{B_0} + \mathbf{B})) \end{aligned}$$

- Where we have assumed plane waves $\mathbf{B} = \mathbf{k} \times \mathbf{E}/\omega$
- And $|v_{ph}| = |\omega/k_{||}|$
- In the wave frame the force is orthogonal to the particle displacement and so energy is conserved
- The particle velocity and pitch angle can change, but energy must be conserved in the wave frame. Hence electrons must move along lines of constant energy given by

$$v_{\perp}'^2 + v_{||}'^2 = v_0'^2$$

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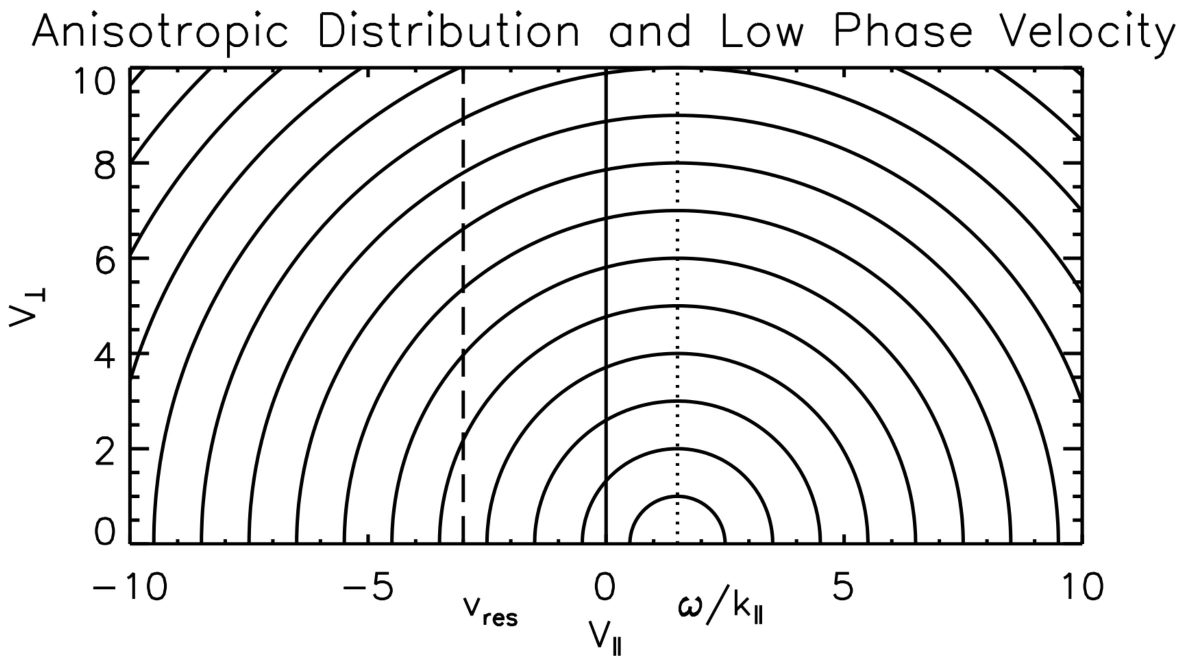
Single Wave Characteristics

- Transform back to the rest frame

$$v_\perp^2 + \left(v_\parallel - \frac{\omega}{k_\parallel}\right)^2 = v_0^2$$

- In the rest frame the electrons move along surfaces that are circles centred at $v_\parallel = \omega/k_\parallel$ and the energy can change
- These surfaces are known as single wave characteristics



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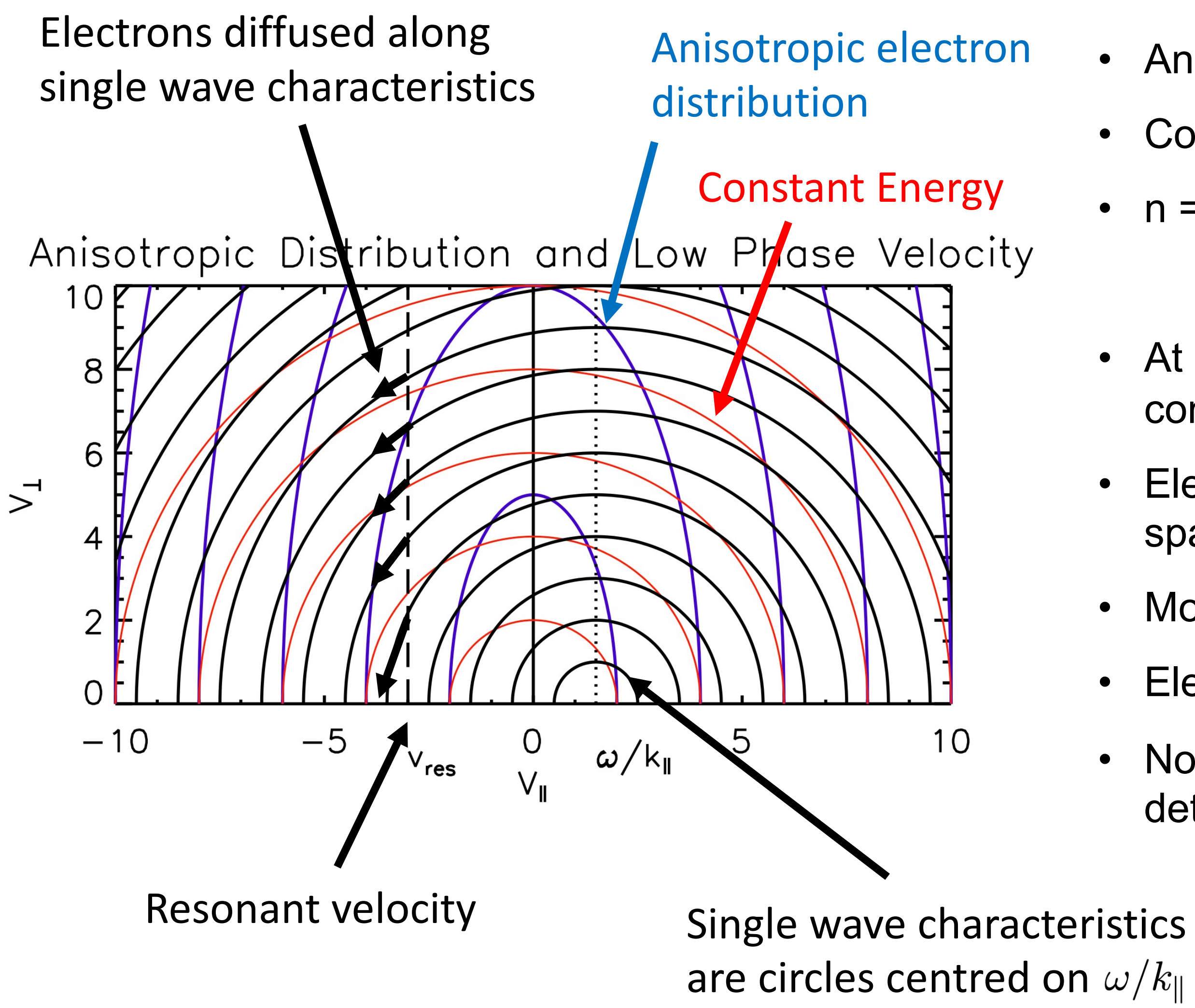
Single Wave Characteristics - Low Phase Velocity

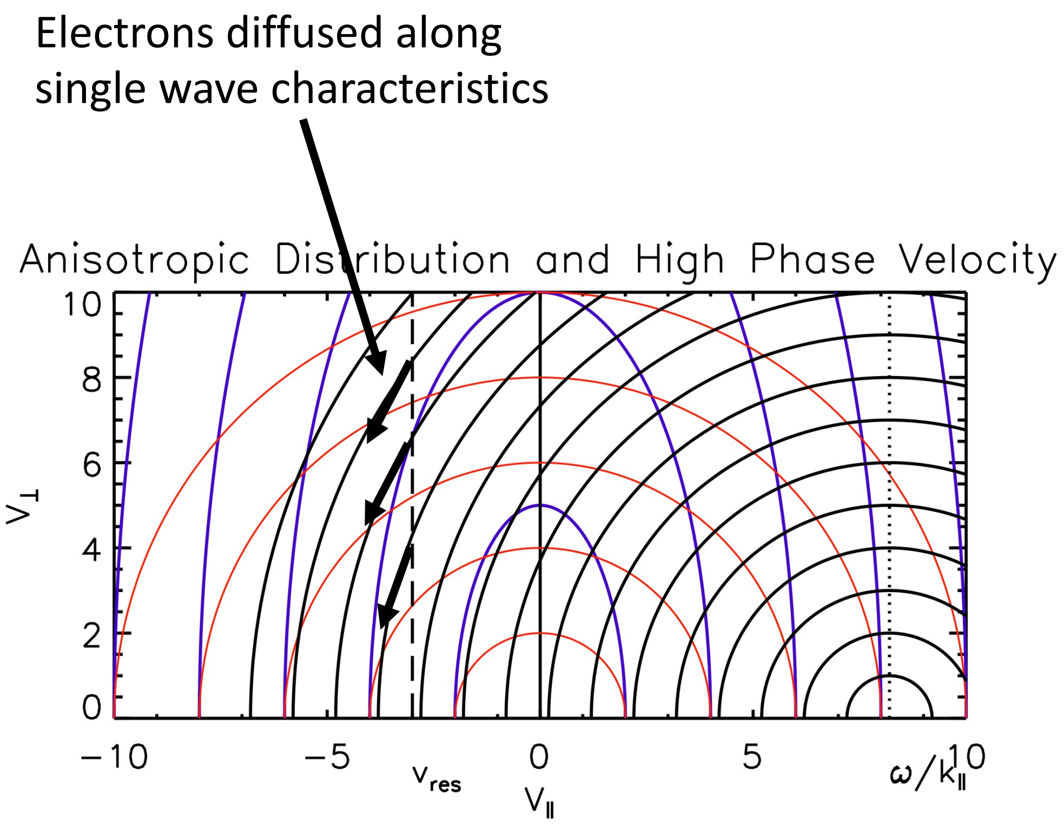


- Anisotropic distribution f(vp,vz) where Tp > Tz
- Consider a whistler mode wave propagating along B_0
- n = -1.

$$v_{\parallel} = \frac{\omega}{k_{\parallel}} \left(1 + \frac{n|\Omega_e|}{\gamma\omega} \right)$$

- At the resonant velocity $v_{//}$ = v_res electrons are confined to move along single wave characteristics
- Electrons move towards regions of lower phase space density or f(vp,vz)
- Move anti-clockwise - to smaller pitch angles
- Electrons lose energy - waves gain energy
- Note – we need to solve the full dispersion relation to determine net wave growth or decay

Single Wave Characteristics - High Phase Velocity



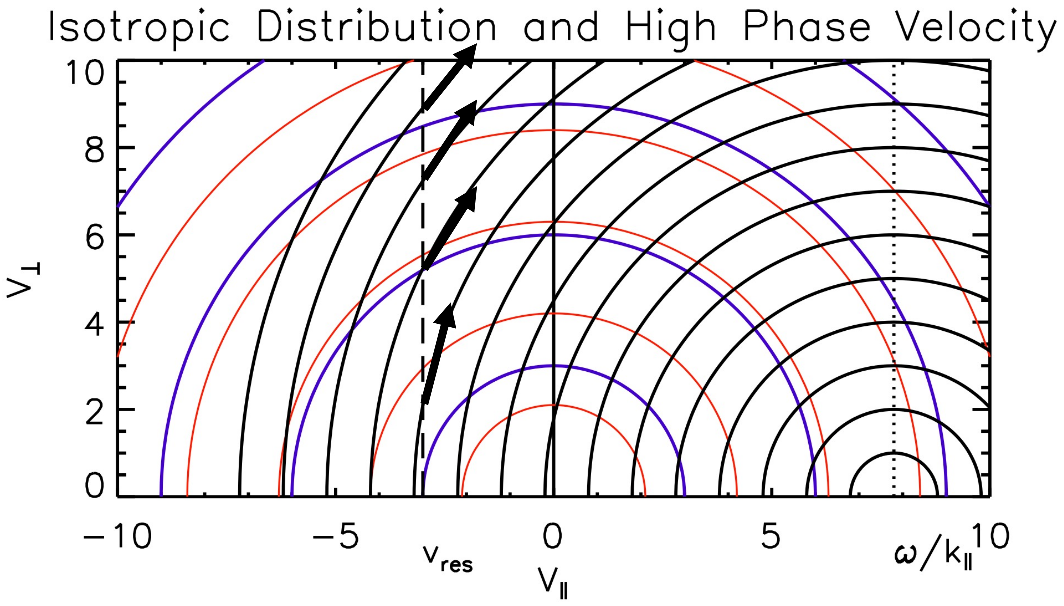
- Anisotropic distribution f(vp,vz) where Tp > Tz
- Consider a whistler mode wave propagating along B_0
- n = -1

$$v_\parallel = \frac{\omega}{k_\parallel}\left(1 + \frac{n|\Omega_e|}{\gamma\omega}\right)$$

- Electrons move anti-clockwise to smaller pitch angles and lower phase space density
- Electrons lose much more energy – stronger wave growth

Single Wave Characteristics - High Phase Velocity

Electrons diffused along
single wave characteristics



- Isotropic distribution f(vp,vz) where Tp = Tz
- Consider a whistler mode wave propagating along B_0
- n = -1

$$v_\parallel = \frac{\omega}{k_\parallel}\left(1 + \frac{n|\Omega_e|}{\gamma\omega}\right)$$

- Electrons move along single wave characteristics towards lower phase space density - to larger pitch angles and higher energy
- Electrons gain energy and waves become damped
- Electrons are accelerated

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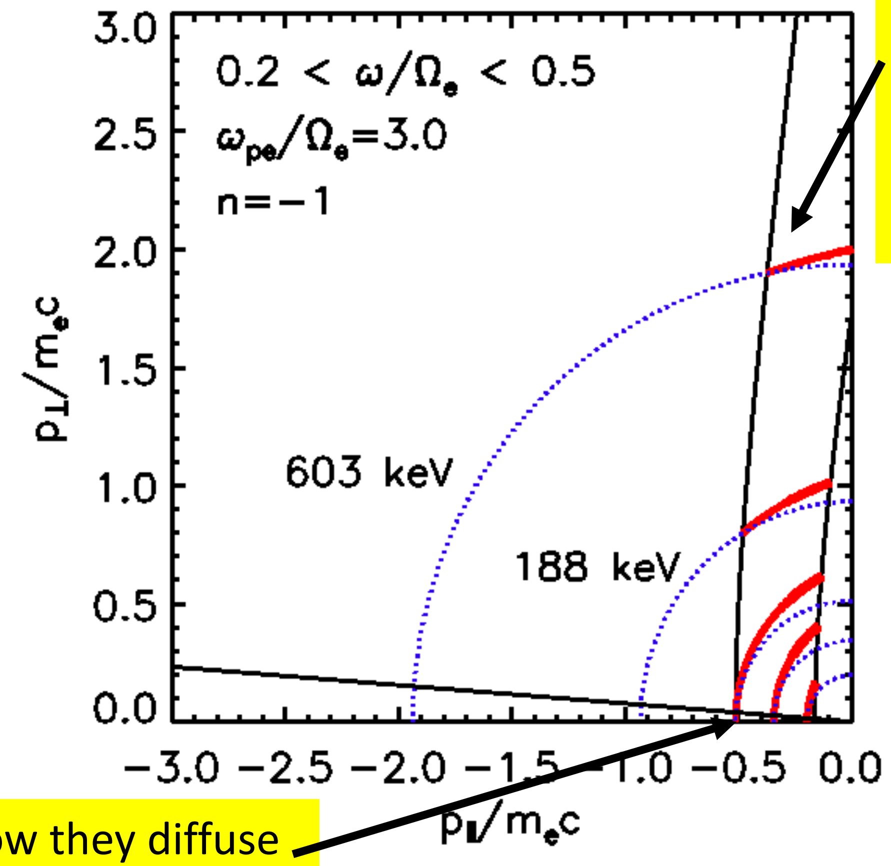
Electron Acceleration by Whistler Mode Chorus Waves

Horne and Thorne, (1998, 2003, 2005a,b)

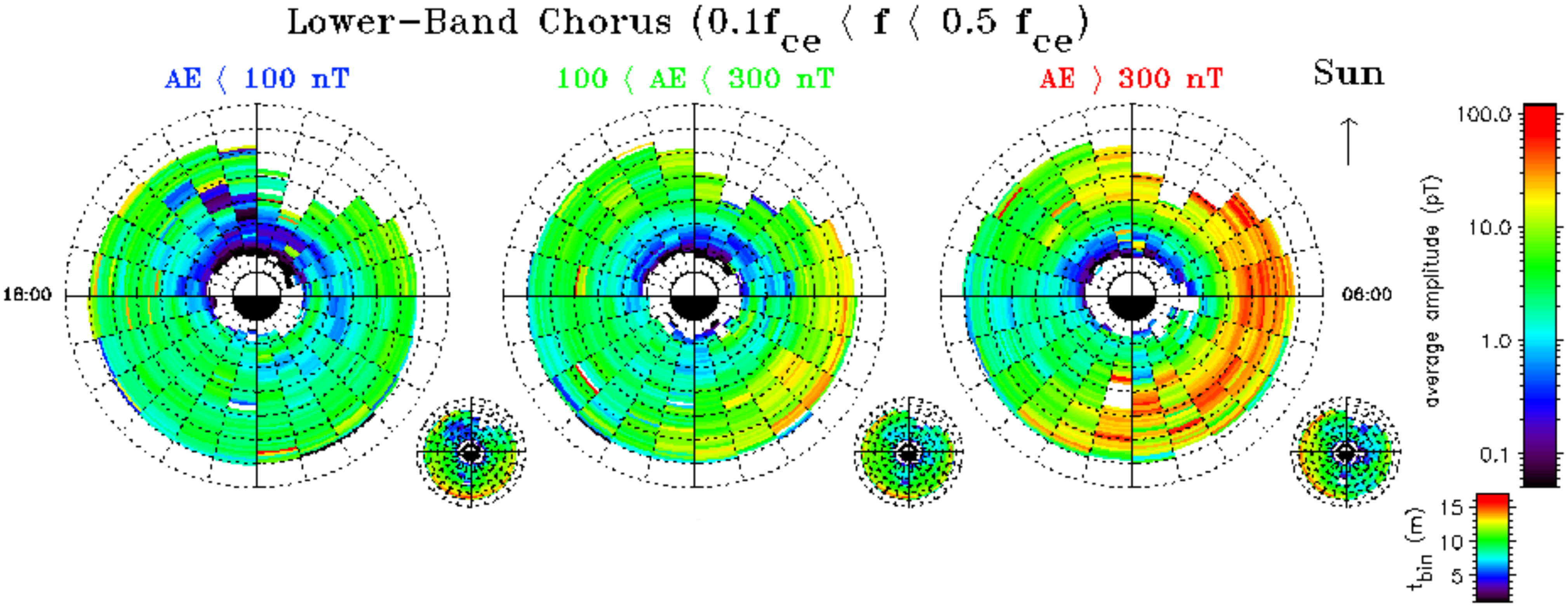
- If we have a broad band of waves then resonances overlap – treat as diffusion problem
- Quasilinear diffusion
  - Small angle scattering by each wave
  - Waves uncorrelated
  - Large enough bandwidth that there is no particle trapping by the waves
  - Diffusion is proportional to wave power
- The quasilinear approach is an approximation which ignores trapping effects



Waves diffuse electrons to higher energies which remain trapped in the radiation belts

As waves grow they diffuse electrons into loss cone at small pitch angles

Chorus Wave Intensity



Meredith et al. [2002]

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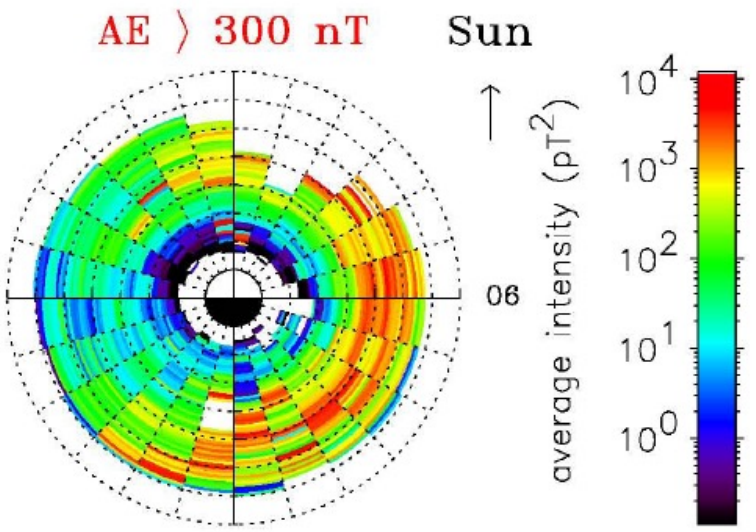
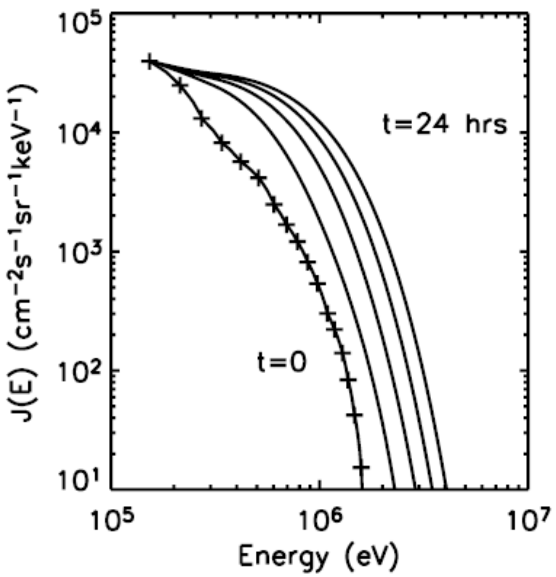
Cyclotron Resonant Electron Acceleration

- Meredith et al. *JGR* (2002)



- Whistler mode waves excited by ~1 - 50 keV electrons
- Diffusion of electrons to higher energy is effective in low density regions

- Horne et al., *JGR (2005), Nature* (2005)



- Solve Fokker-Planck equation
- Electron acceleration to several MeV
- Timescale ~ 1 day – typical of observations

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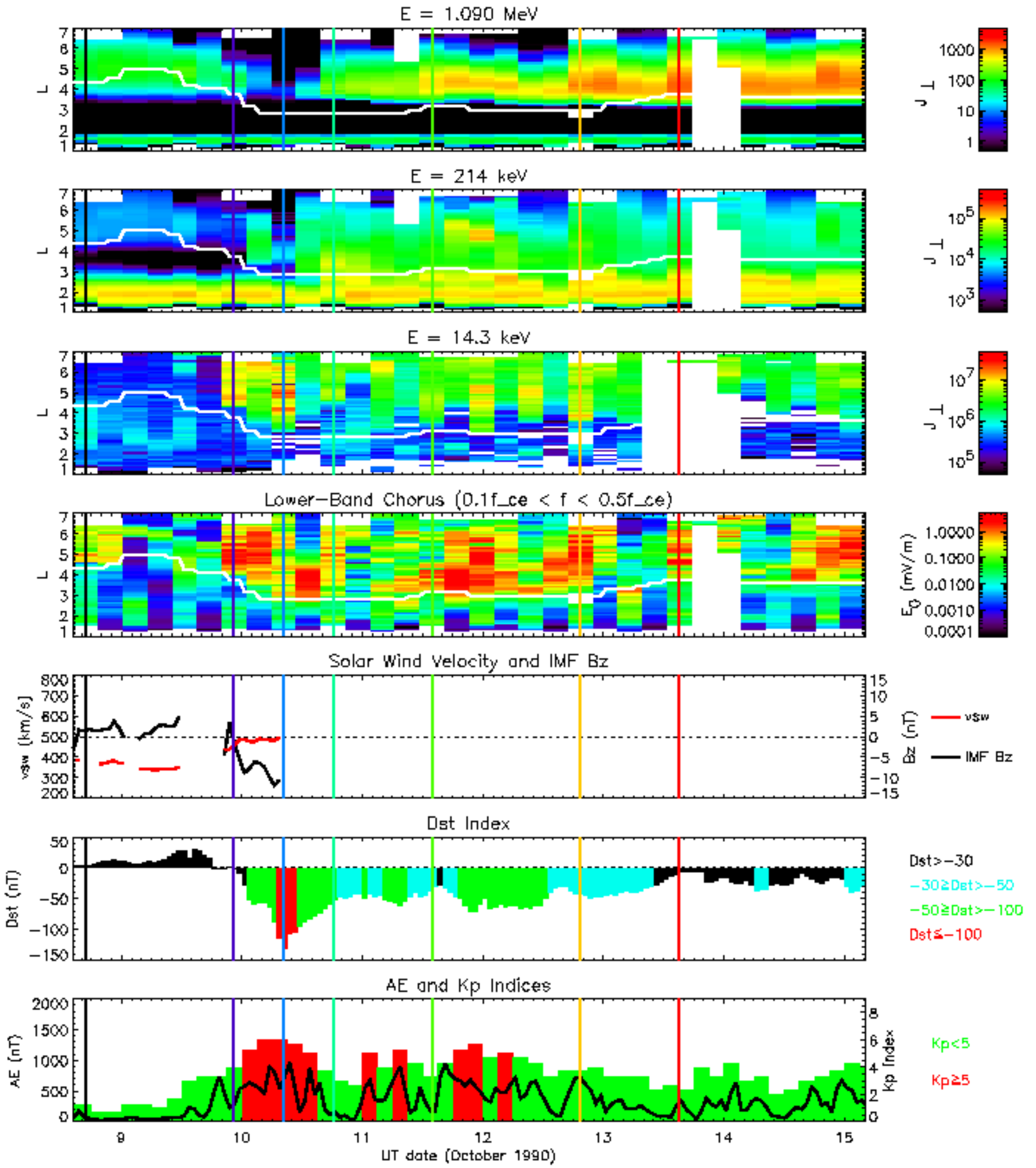
October 1990 Storm



E=1.09 MeV

214 keV

Flux drop-out during storm ~MeV

Injection of 14 keV electrons excite chorus waves (source electrons)

14.3 keV

Increase in seed electrons hundreds keV

Lower band Chorus

Waves accelerate electrons to MeV

Flux increases above pre-storm level

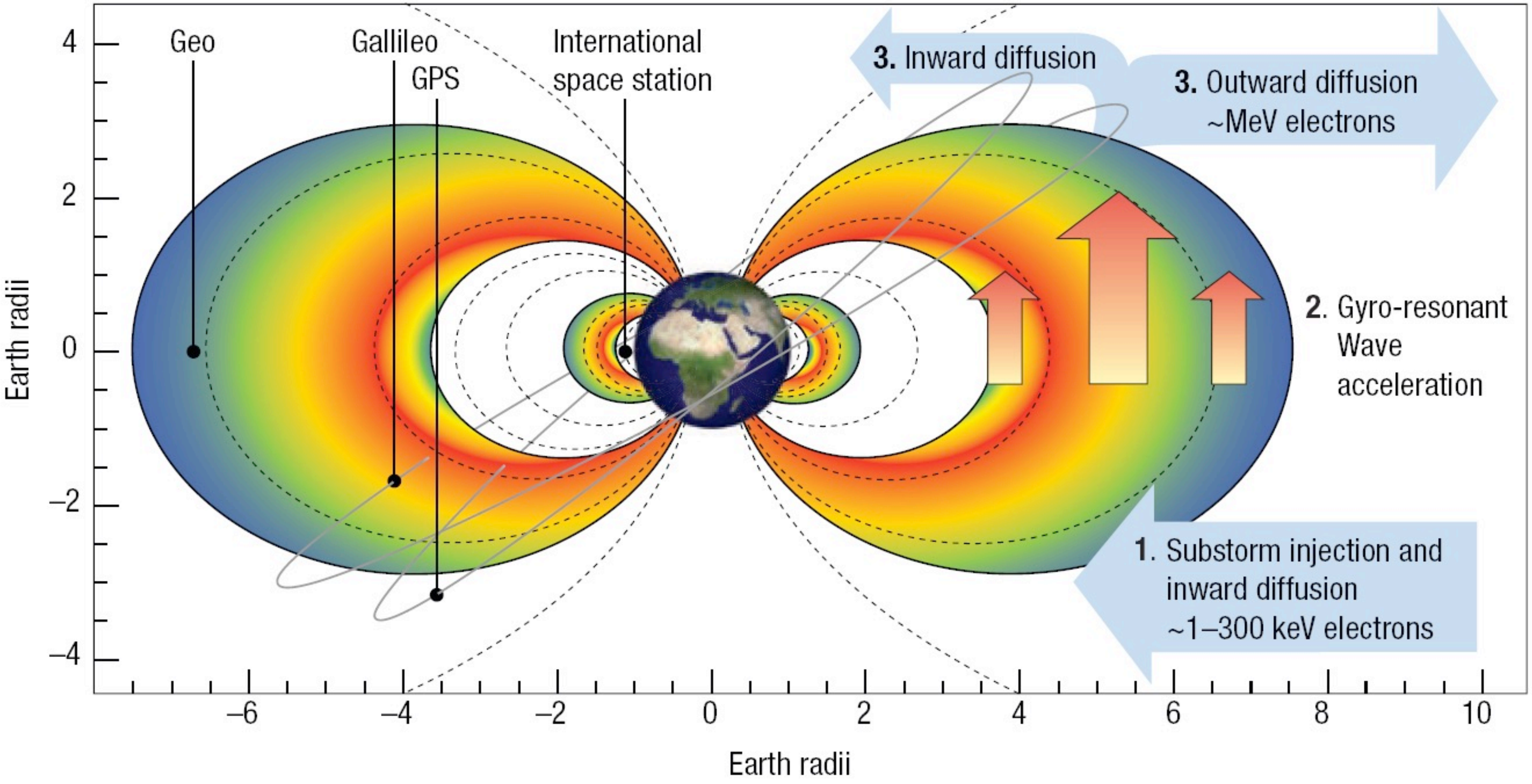
V Solar wind & Bz

Dst

AE & Kp

Meredith et al., (2002)

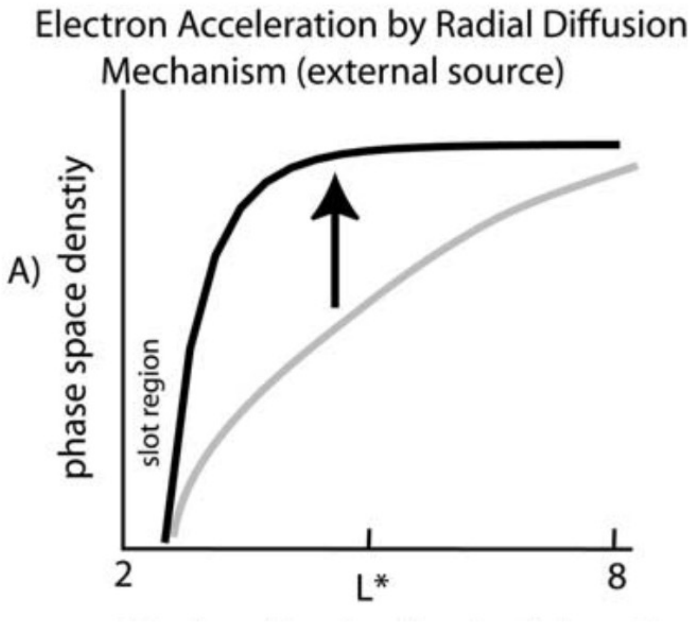
Electron acceleration in the outer radiation belt

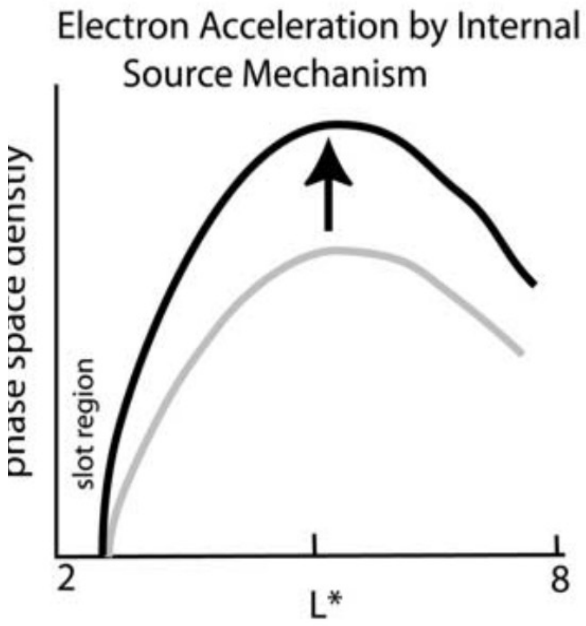


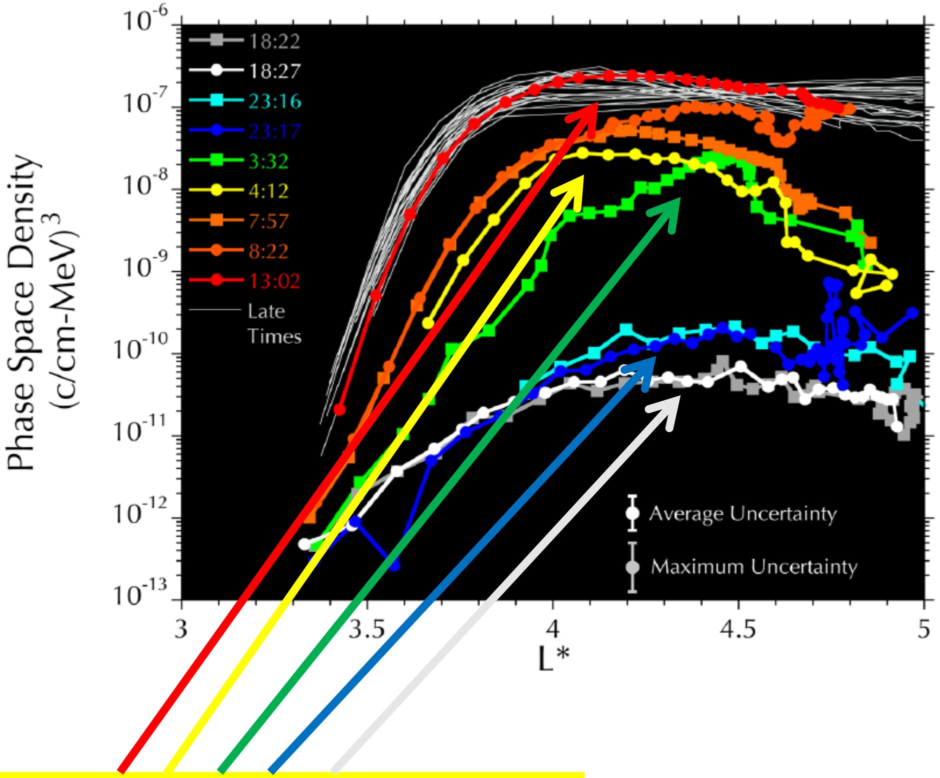
Horne, *Nature Physics* [2007]

Evidence for Local Acceleration

- Reeves et al., *Science*, (2013)

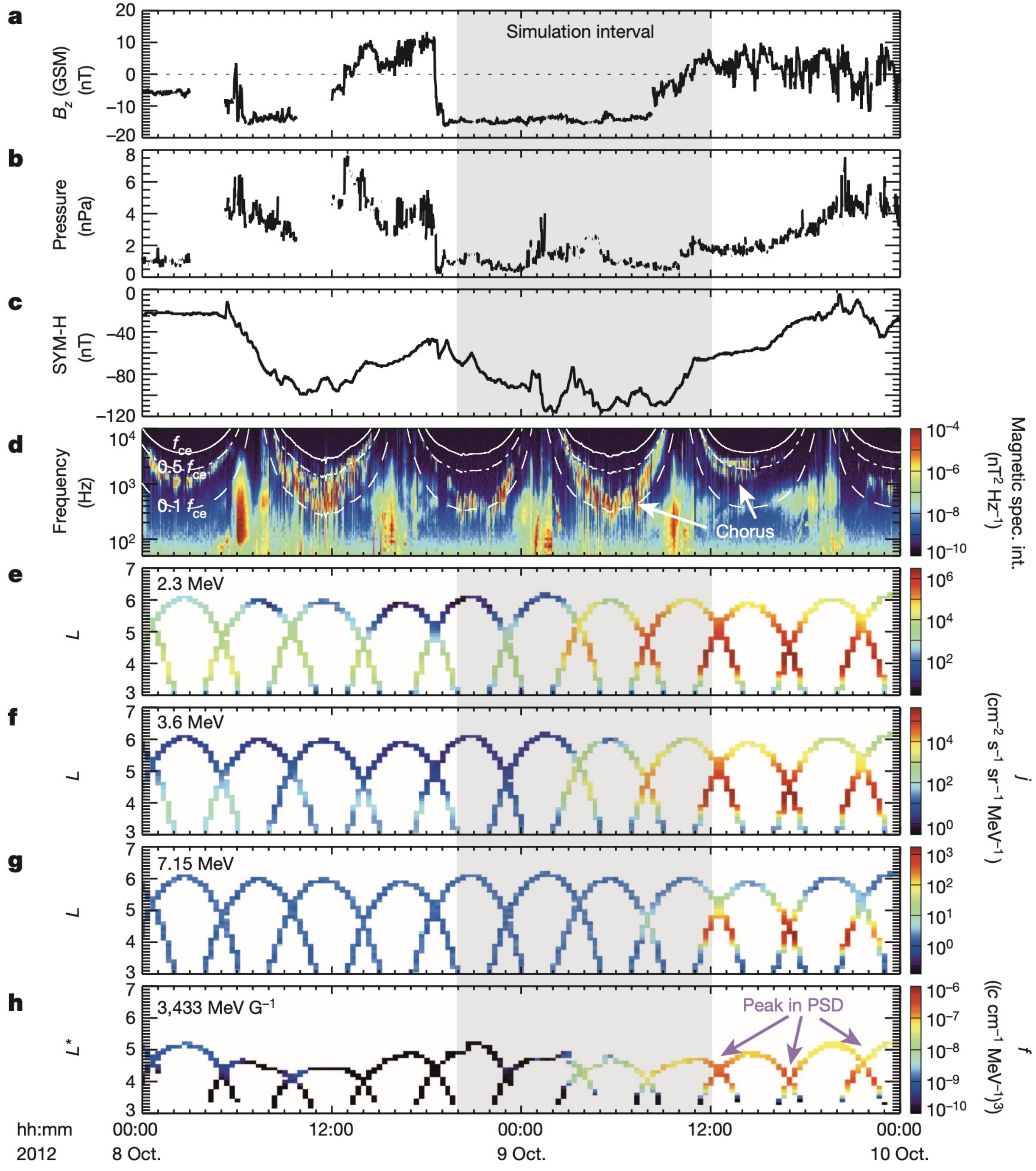


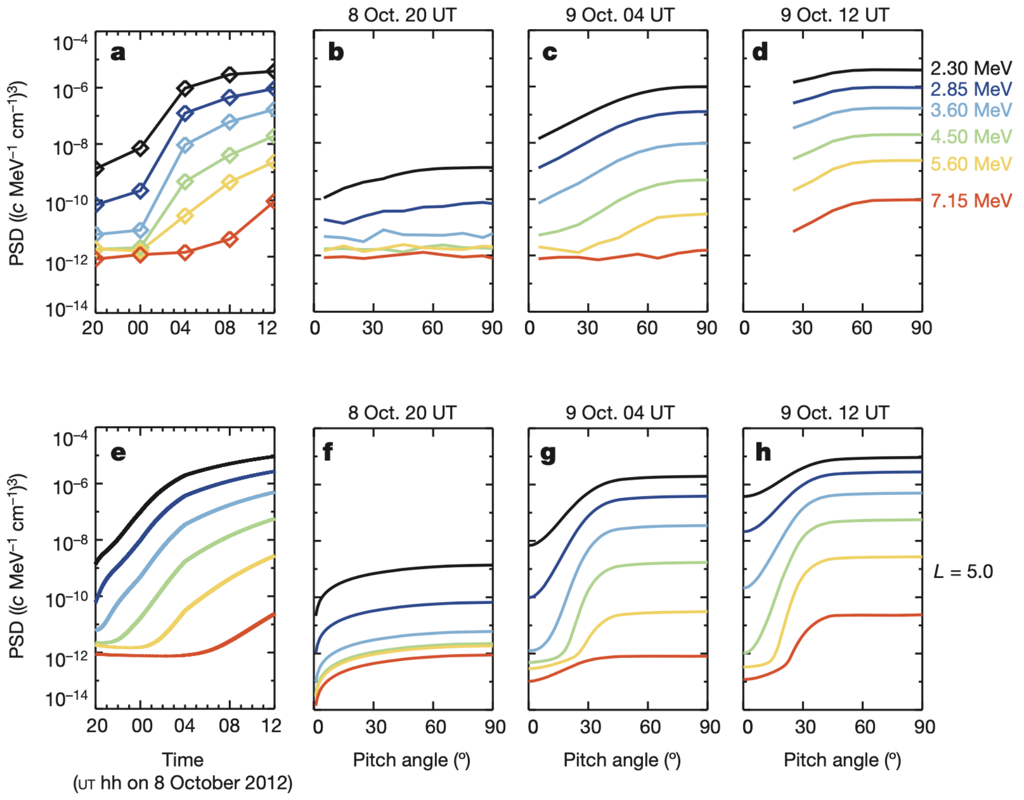




- Data shows the peaks grows with time
- Radial diffusion cannot produce a growing peak
- Evidence for local acceleration by wave-particle interactions

Evidence for Electron Acceleration by Chorus Waves





Thorne et al. *Nature* (2013)

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Fokker-Planck Equation

Glauert et al., JGR, (2014a,b)

Solve the Fokker-Planck Equation

Model includes:

- Wave-particle interactions
- Radial transport
- Loss to the atmosphere
- Loss to the magnetopause

Pitch angle diffusion

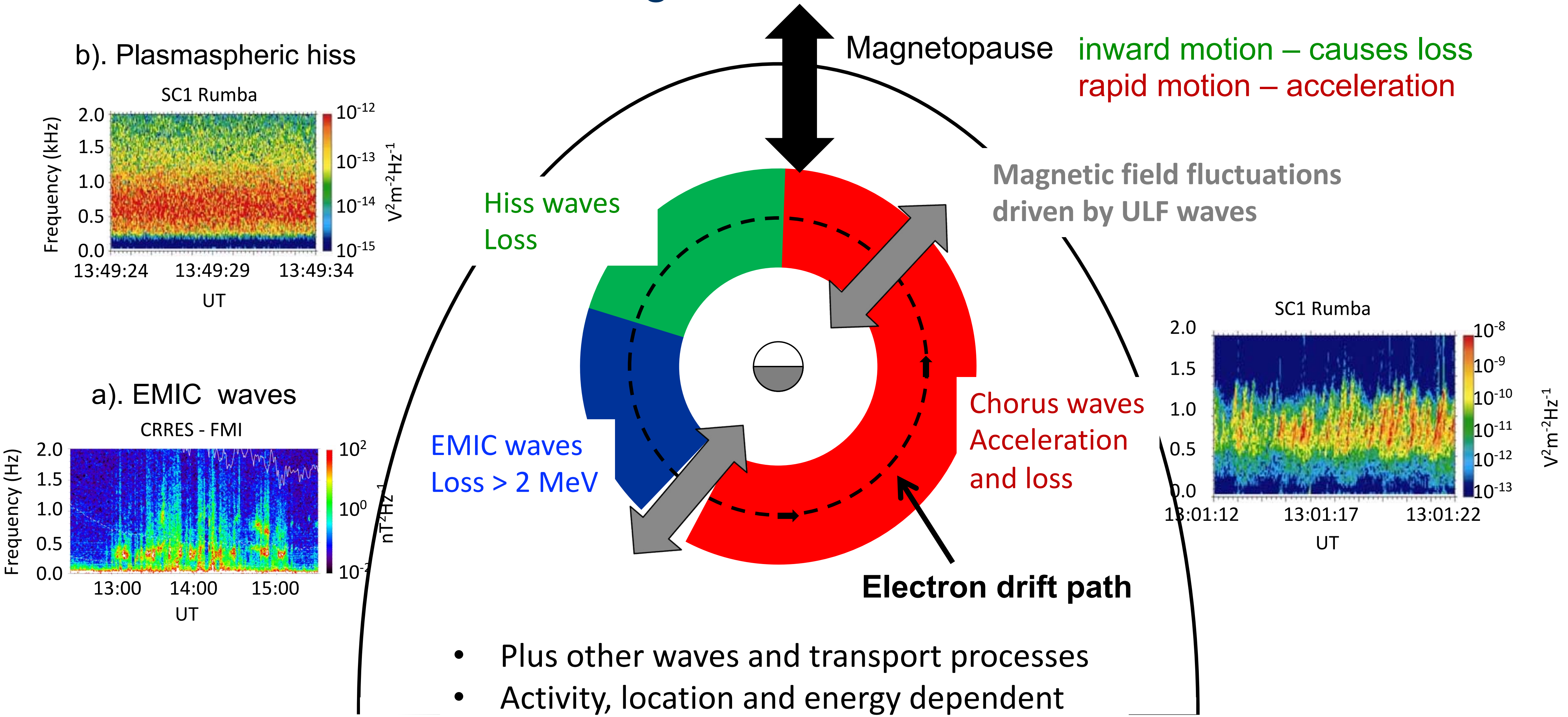
Energy diffusion

$$\frac{\partial f}{\partial t} = \frac{1}{g(\alpha)} \frac{\partial}{\partial \alpha}\bigg|_{E,L} g(\alpha) \left(D_{\alpha\alpha} \frac{\partial f}{\partial \alpha}\bigg|_{E,L} + D_{\alpha E} \frac{\partial f}{\partial E}\bigg|_{\alpha,L} \right) + \frac{1}{A(E)} \frac{\partial}{\partial E}\bigg|_{\alpha,L} A(E) \left(D_{EE} \frac{\partial f}{\partial E}\bigg|_{\alpha,L} + D_{\alpha E} \frac{\partial f}{\partial \alpha}\bigg|_{E,L} \right) + L^2 \frac{\partial}{\partial L}\bigg|_{\mu,J} \left(\frac{1}{L^2} D_{LL} \frac{\partial f}{\partial L}\bigg|_{\mu,J} \right) - \frac{f}{\tau}$$

Radial diffusion

Loss to atmosphere
Loss to the magnetopause

Modelling the Radiation Belts



- Plus other waves and transport processes
- Activity, location and energy dependent
- Nonlinear, timescale – milliseconds to days

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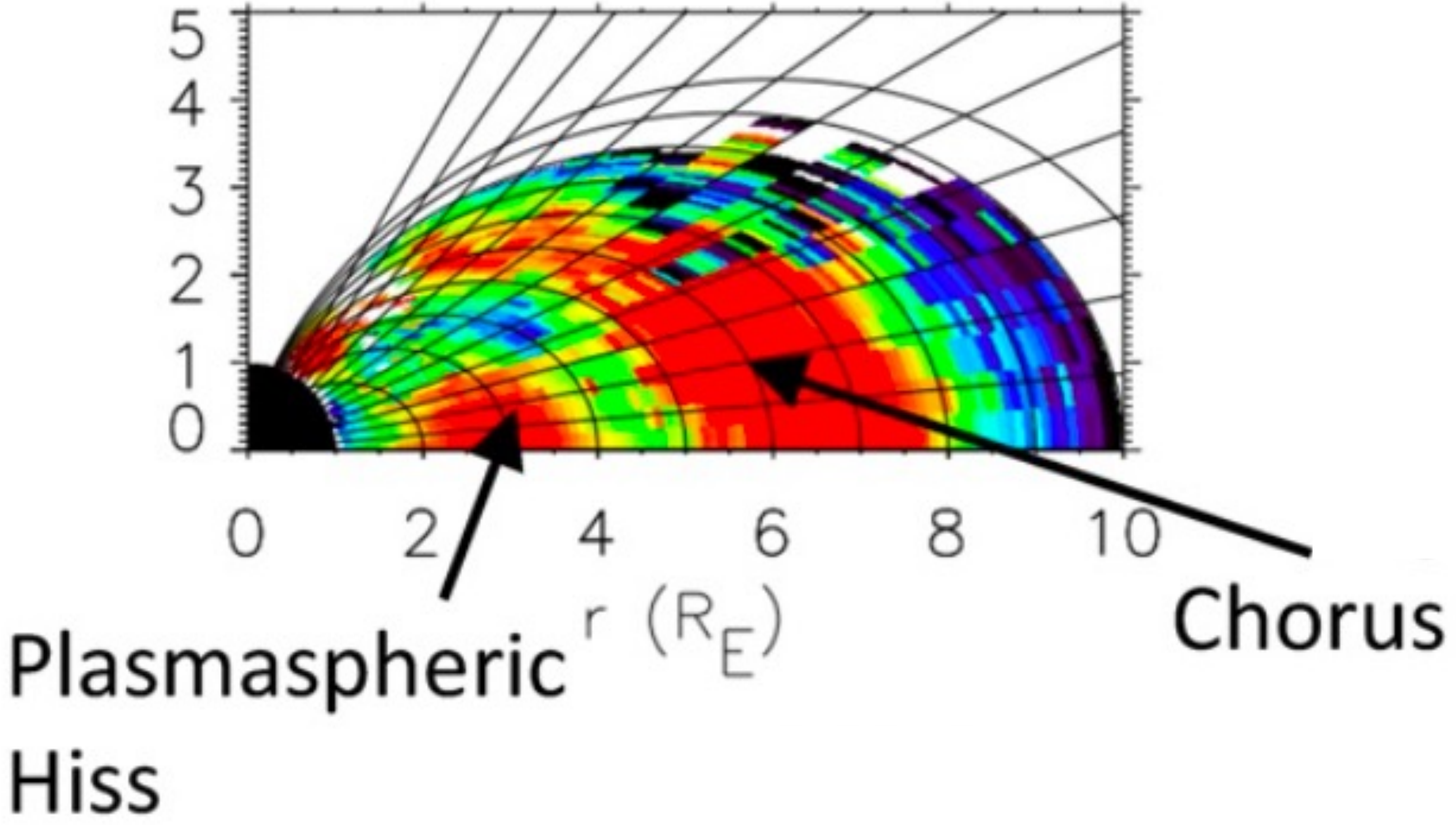
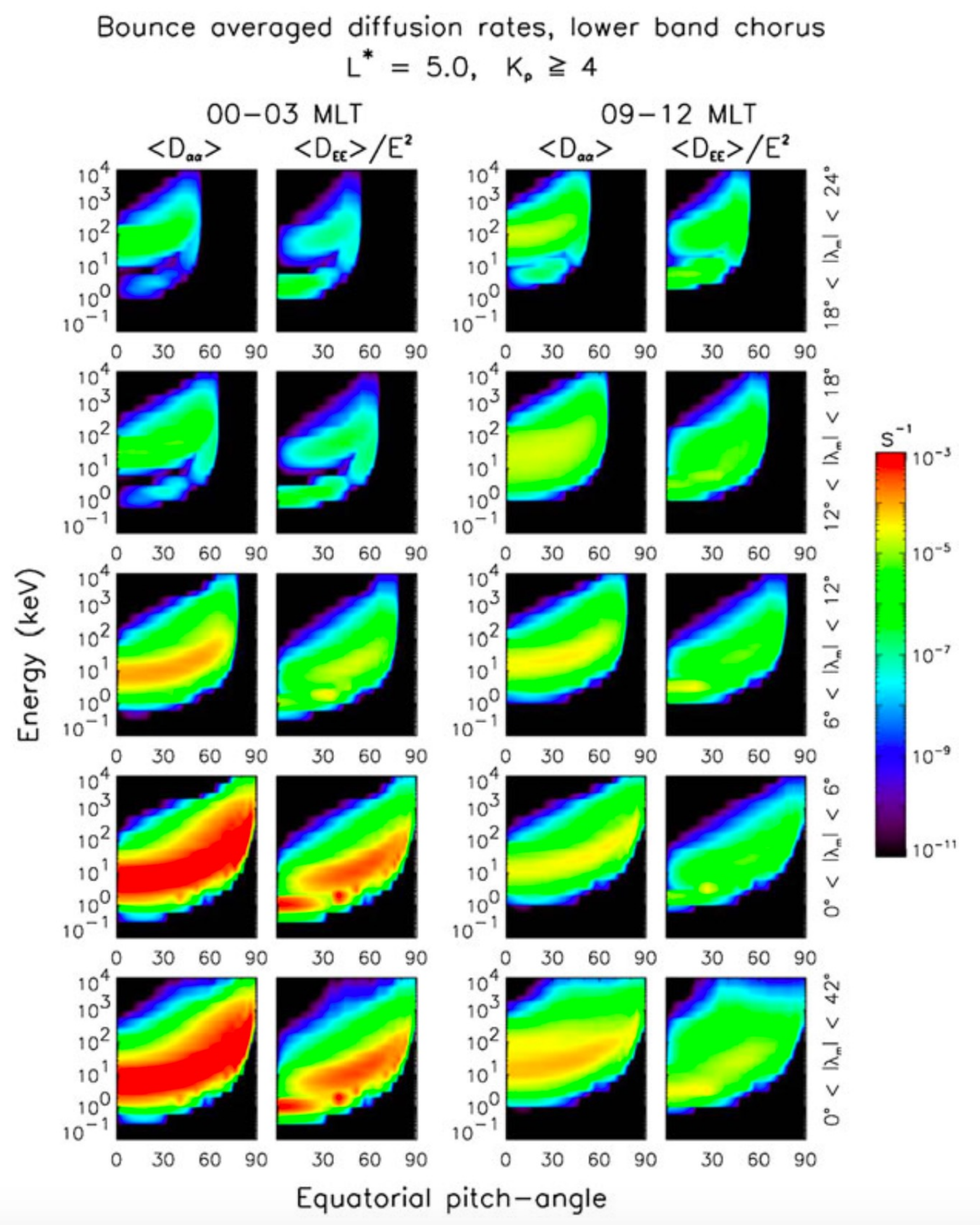
Latitude Distribution of the Waves

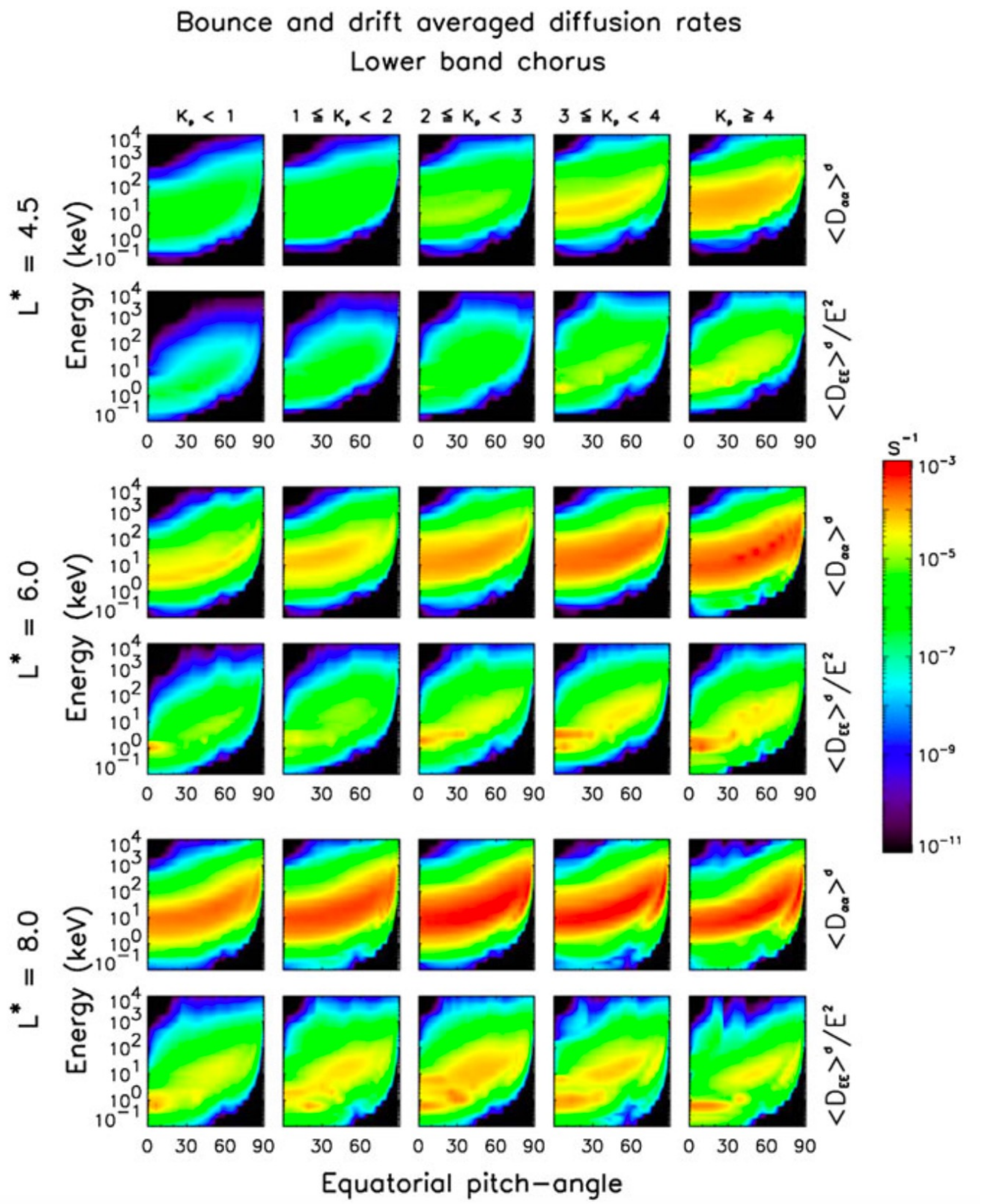
- Merredith et al., (2021)



Chorus Diffusion Rates



- Diffusion is stronger for waves near the equator as waves are intense and wave normal angle is small
- Waves at higher latitude diffuse electrons with smaller pitch angles – so acceleration at large pitch angles must occur near the equator
- Bounce averaging - include diffusion due to all latitudes where waves exist
  - Need latitude distribution of waves
- Electron diffusion varies with Magnetic Local Time of the waves
- Horne et al., *JGR* (2013)

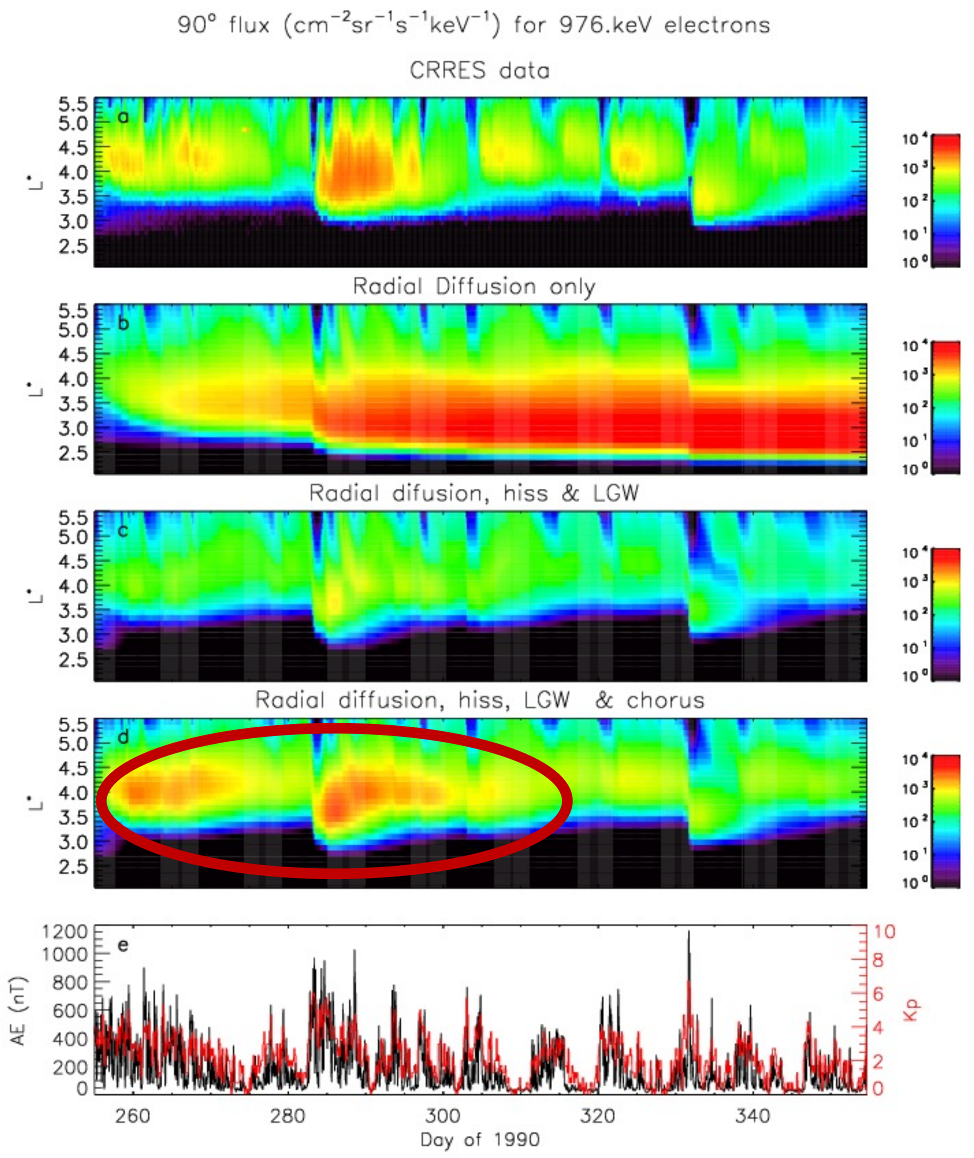


Bounce and Drift Averaged Diffusion Rates

- Chorus diffusion rates increase with magnetic activity as measured by Kp index
- At low energies 1 – 20 keV
- Pitch angle diffusion into the loss cone corresponds to electron loss and wave growth
- PA diffusion does not extend to large pitch angles
- At high energies ~ > MeV
- Pitch angle diffusion into the loss cone is much smaller and so energy diffusion to higher energies can take place and electrons can remain trapped – increase in trapped flux.

Global Simulations

Glauert et al., *JGR*, (2014)



Satellite data

Radial diffusion

Radial diffusion and hiss waves

Radial diffusion, hiss and chorus waves

Geomagnetic activity

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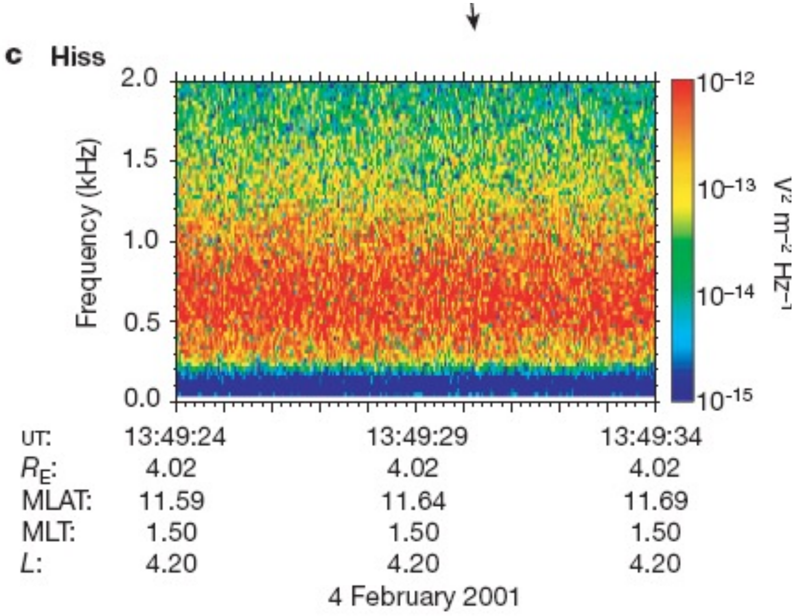
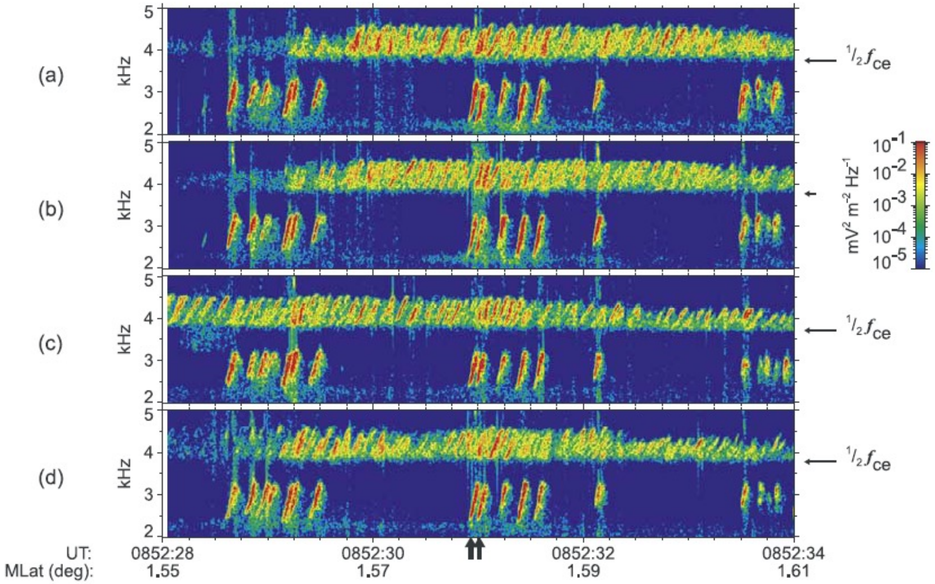
Whistler Mode Hiss and Chorus Waves





- Hiss
- Band of noise – no structure

- Chorus
- Nonlinear, discrete rising tones

- We have used quasilinear theory which allows us to simulate 30 years of radiation belt variability
- Chorus is highly nonlinear – electron trapping and other effects on timescales of milliseconds
- How good is the approximation? Could precipitation and acceleration be higher/lower?

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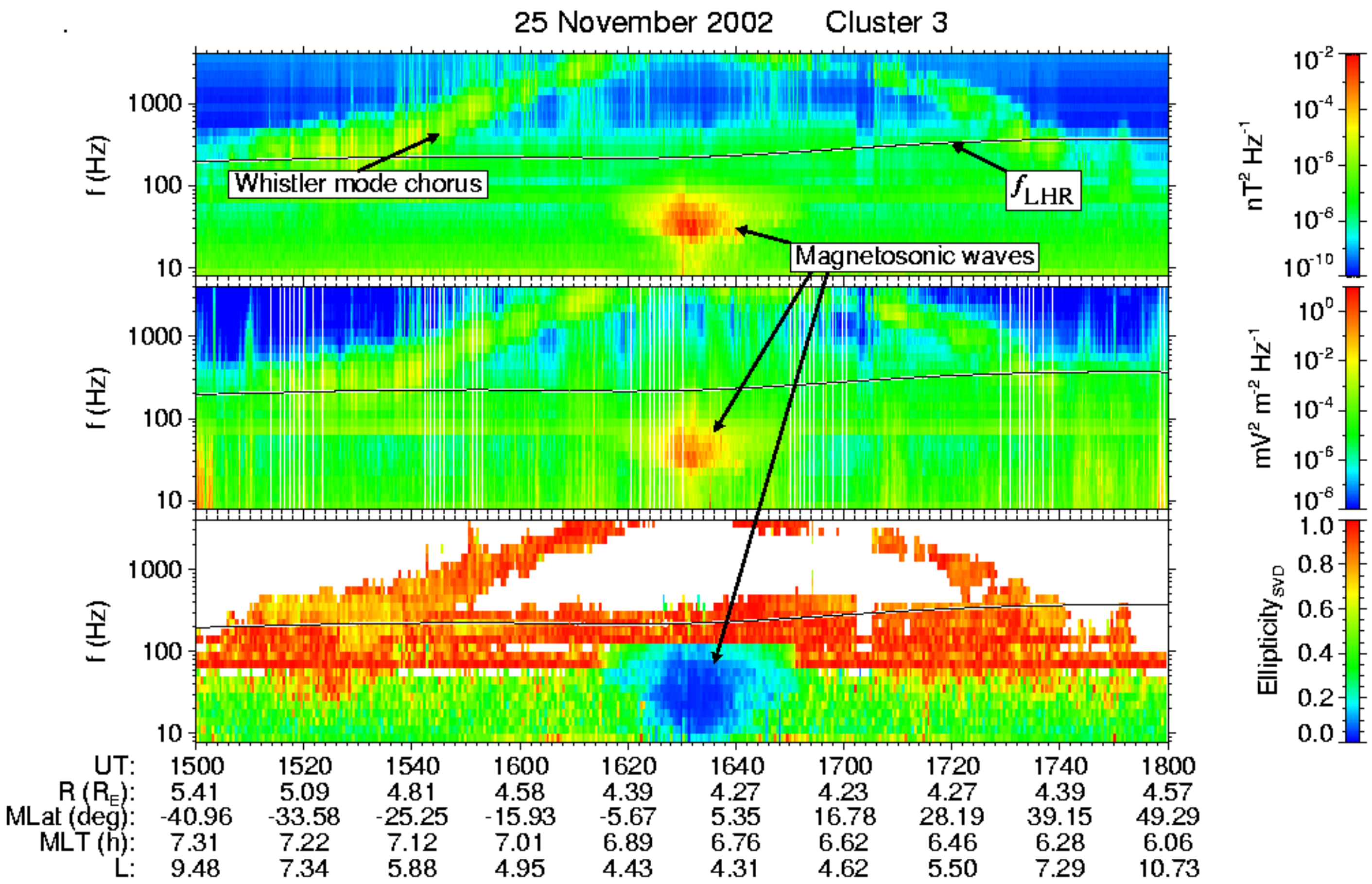
Magnetosonic Waves



- Magnetosonic waves propagate across Bo, fcH < f < fLHR
- Intense
- Generated by proton ring distributions [e.g., Boardsen et al. 1992]

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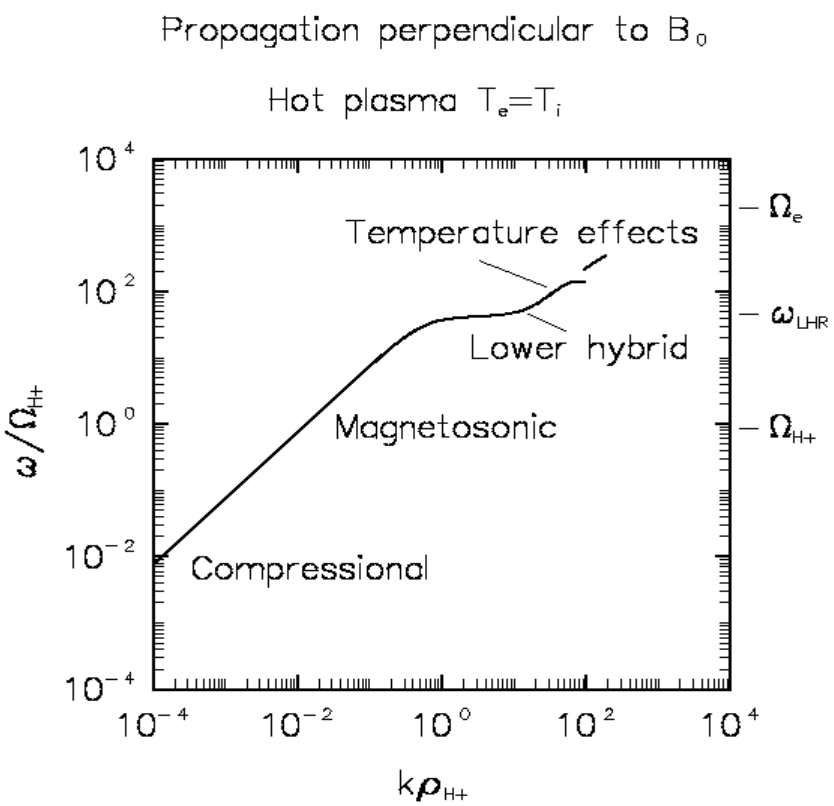
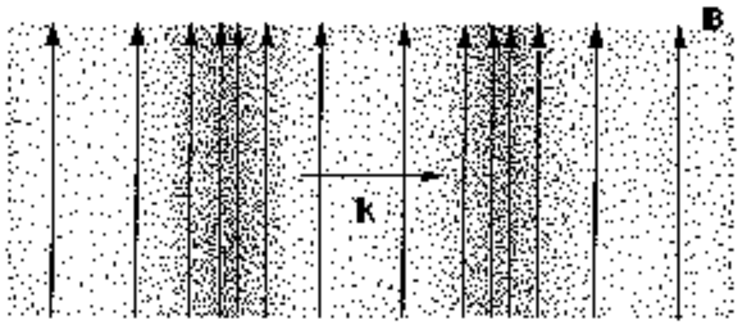
Low Frequency Propagation Perpendicular to B





- Fast compressional magnetosonic wave
  - B field and plasma compressions
- Bw is along Bo, and Ew is perpendicular to Bo and k

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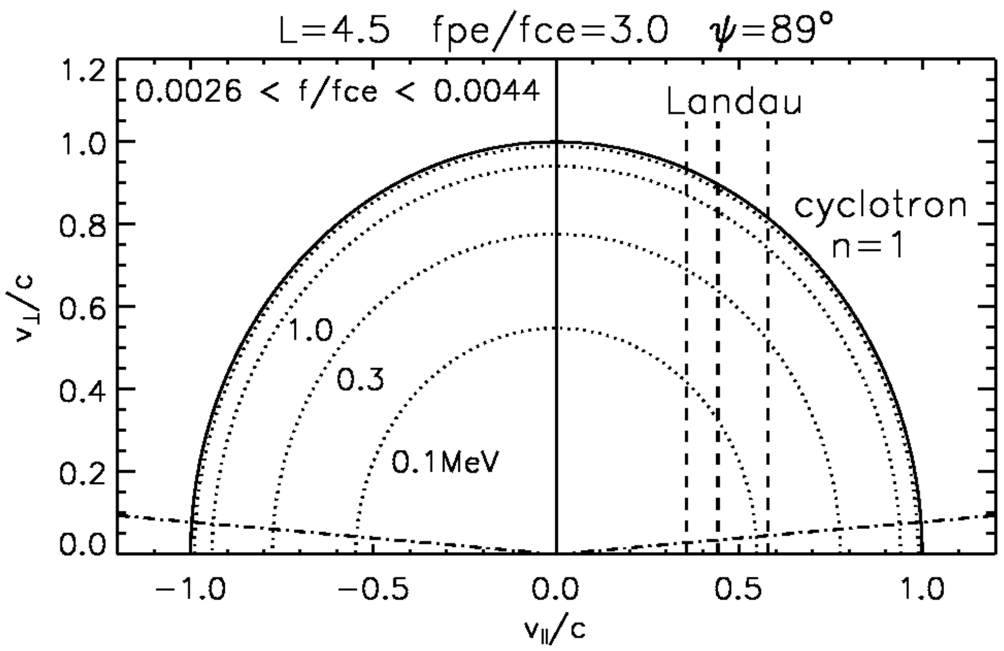
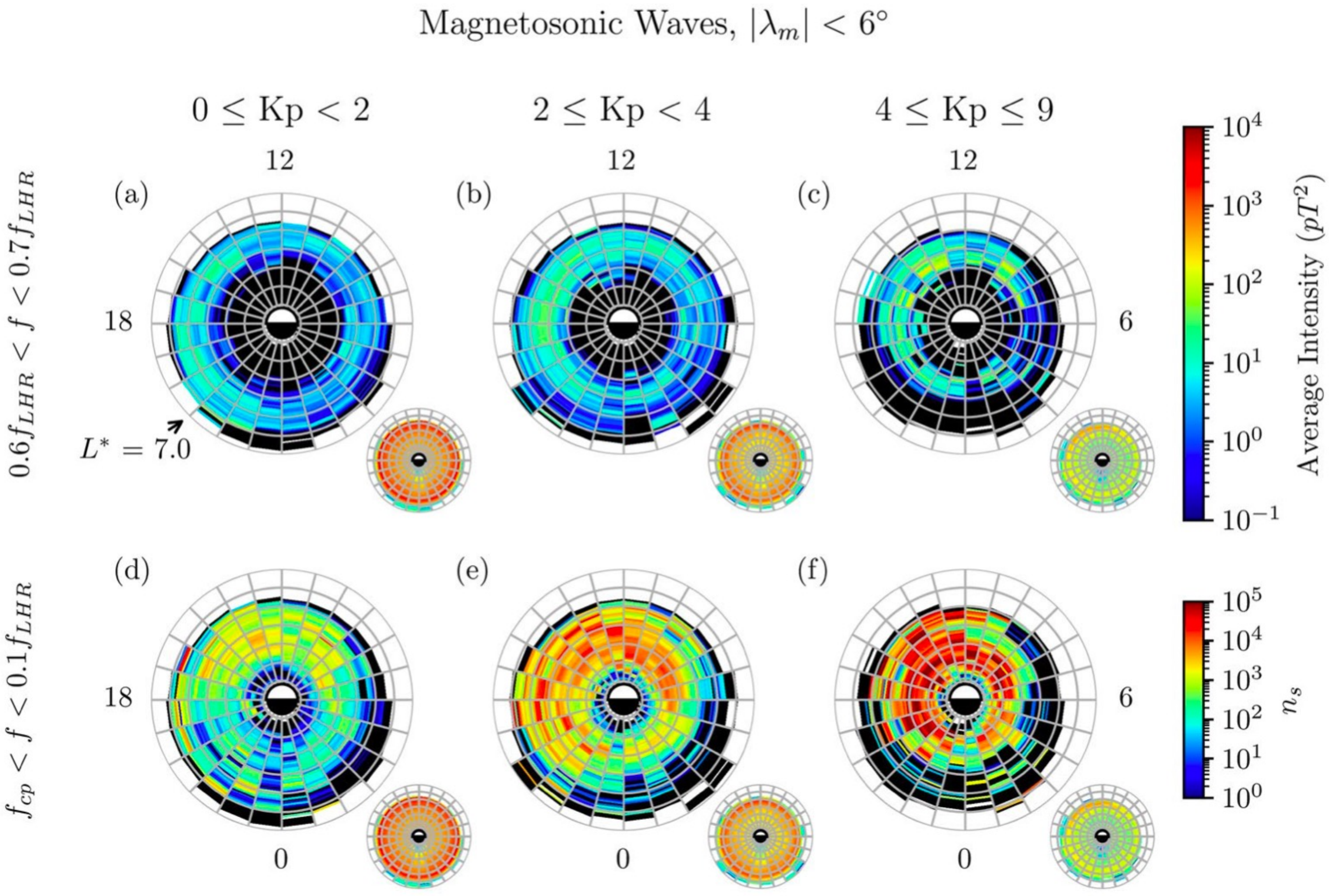
Magnetosonic Waves - Resonant Diffusion



$$\omega - n\Omega_\sigma/\gamma - k_\parallel v_\parallel = 0$$

- Solve with dispersion relation
  - Not field-aligned !
- Cyclotron resonance >3 MeV
  - unlikely to contribute
- Landau resonance possible
  - Energy diffusion
- Higher energies at larger pitch angles
- For a band of waves with spread of directions
  - Landau resonance extended over pitch angles

Magnetosonic Waves



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Ion Ring Distributions – Generate Magnetosonic Waves

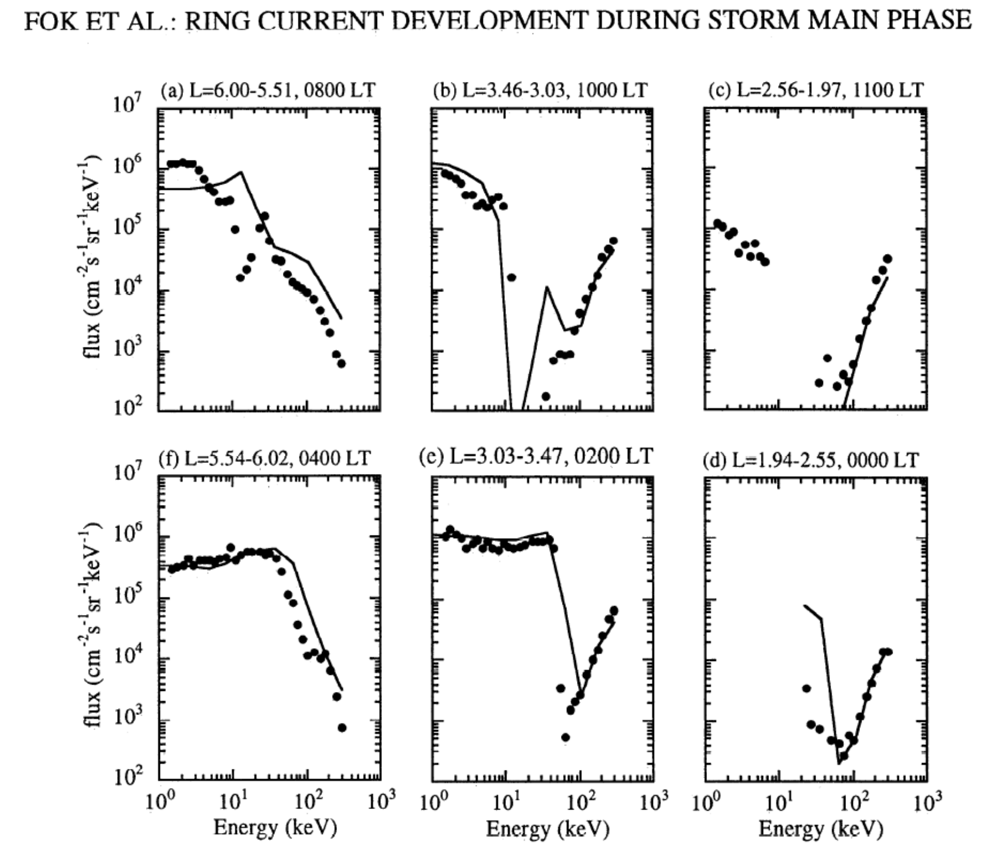

Figure 3. The comparison of calculated average H^+ fluxes (curves) with Active Magnetospheric Particle Tracer Explorers (AMPTE)/CCE measurements (circles) during Orbit 2 at selected locations.

Fok et al. JGR, [1996]

- Ion ring distributions form during magnetic storms
- Energy dependent drift
  - Slow drift - loss
- Injection into existing population
- Waves couple ring current with electron radiation belts

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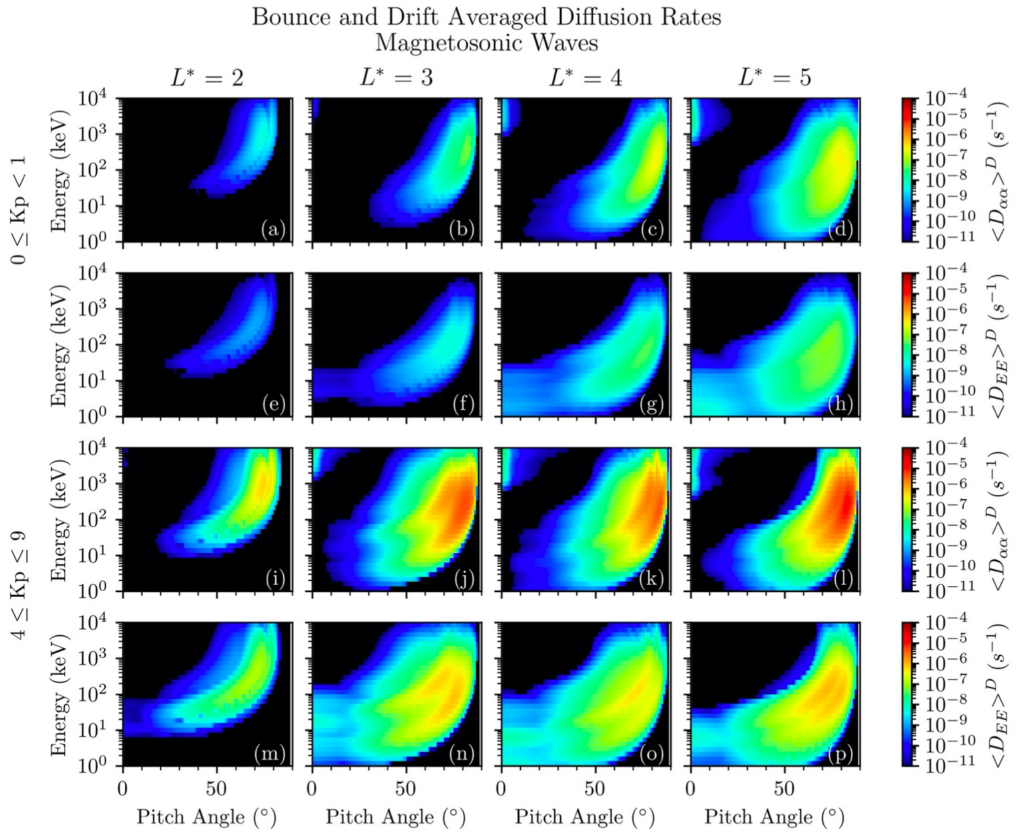
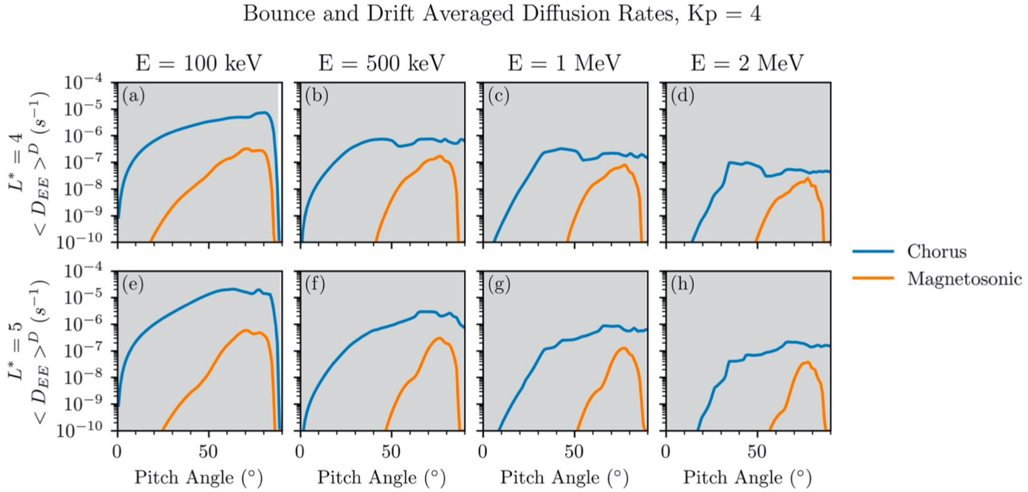
Diffusion Rates



At L=4, Chorus dominates
Wong et al. JGR (2022)



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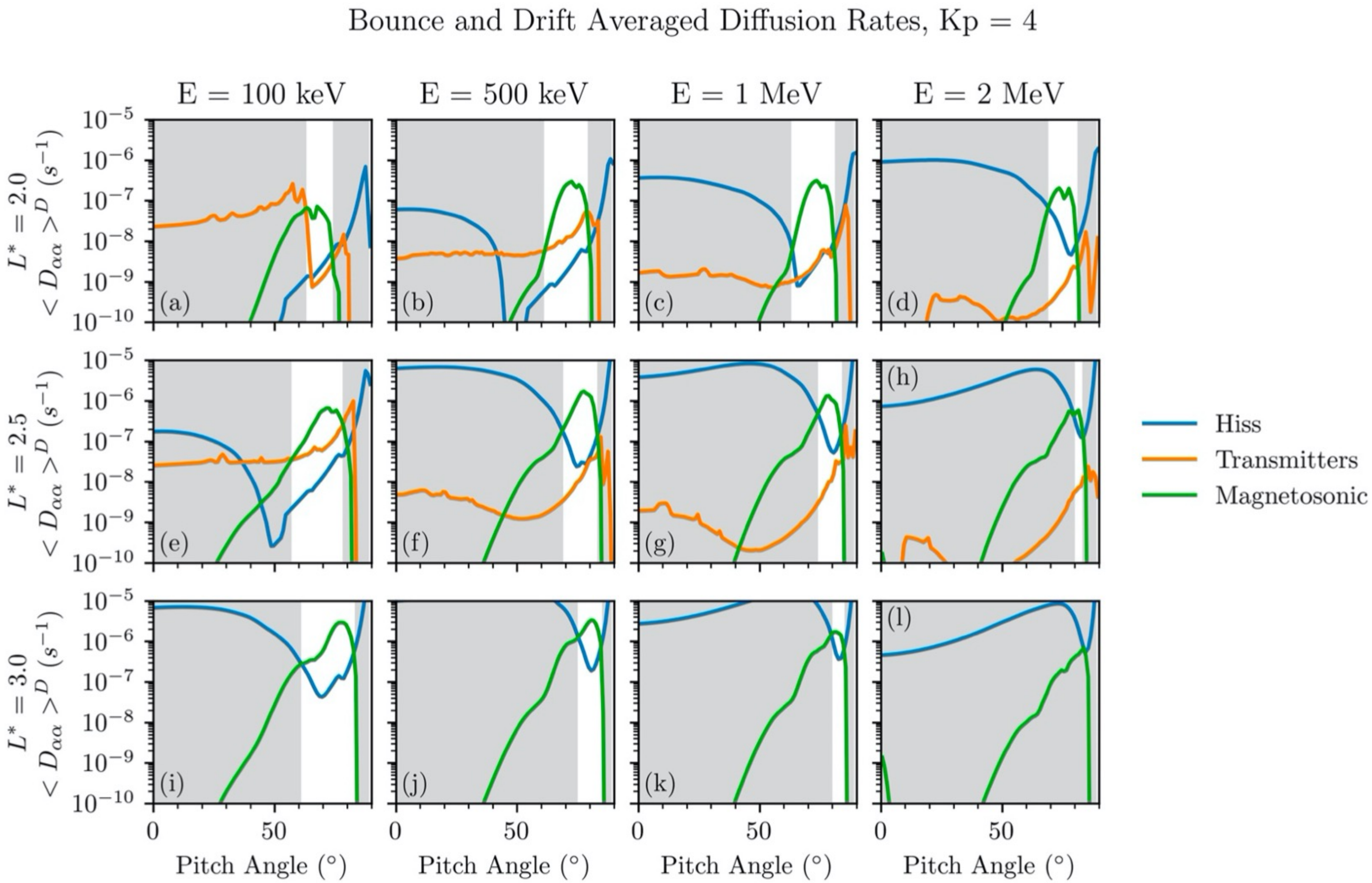
Magnetosonic Waves



- Magnetosonic waves increase electron loss inside the plasmasphere
- They help reduce the “bottleneck” near 70-80 degrees so all electrons can be removed

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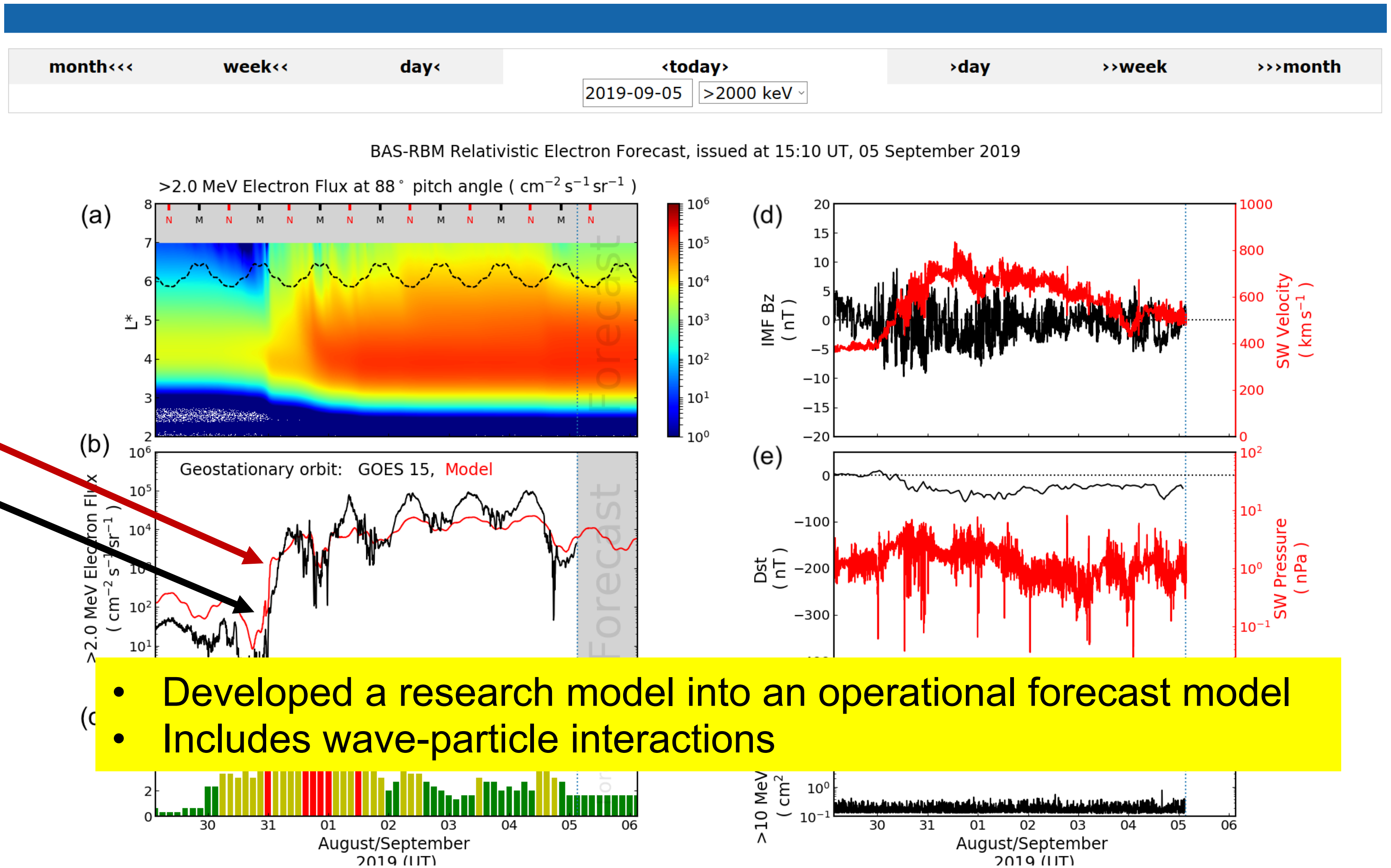
Satellite Risk Prediction and Radiation Forecast System (SaRIF)

Horne et al. *Space Weather*, (2021); Glauert et al. *Space Weather* (2021)

Model

Data



- Developed a research model into an operational forecast model
- Includes wave-particle interactions

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