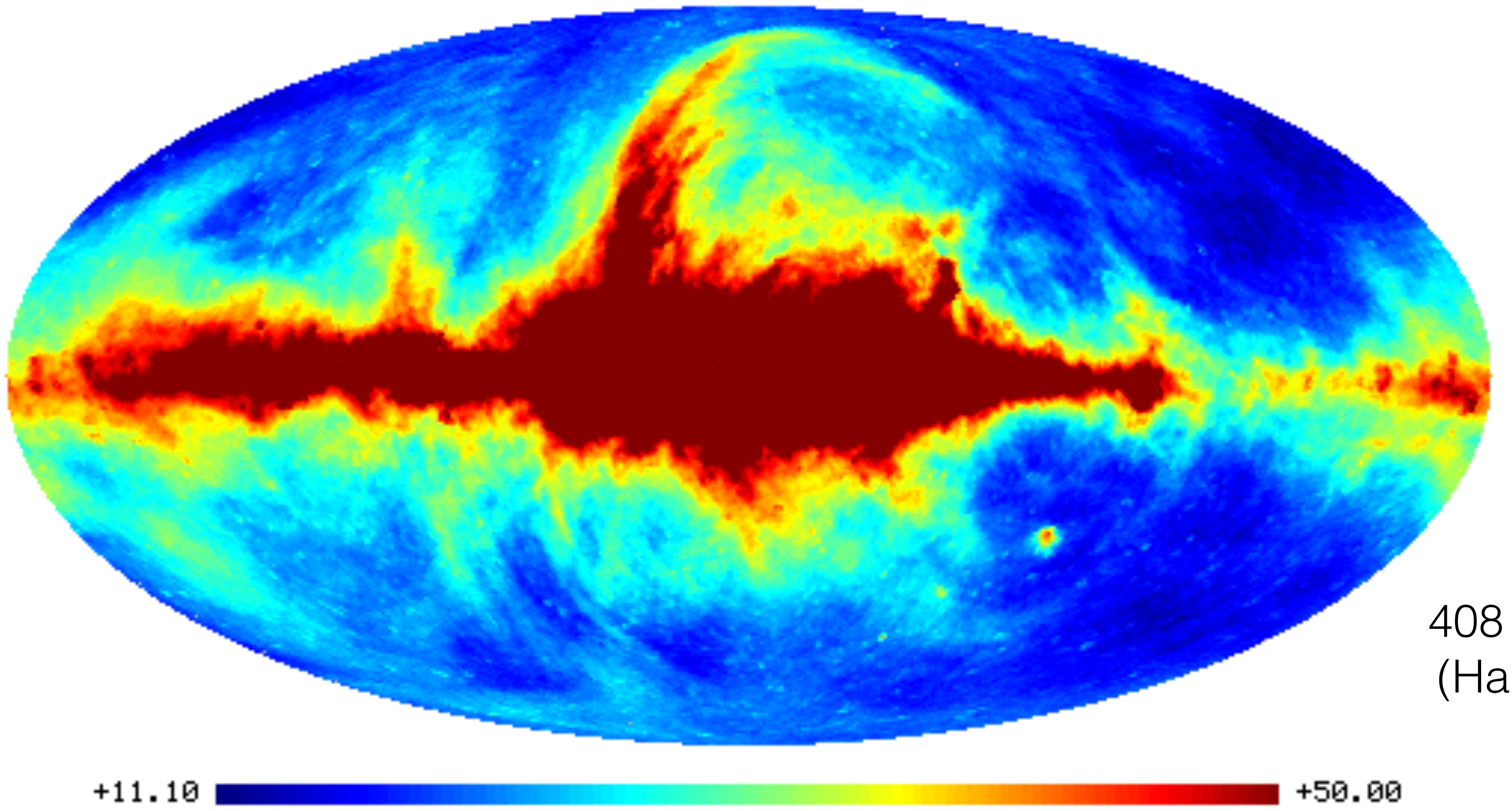
Diffuse Foregrounds at Low Frequencies



408 MHz all-sky map
(Haslam et al. 1982)

Dr Clive Dickinson
Jodrell Bank Centre for Astrophysics (The University of Manchester)

ISSS Summer School, Observing the Universe with the Cosmic Microwave Background, L'Aquila, Italy, 22-26 April 2014

This lecture

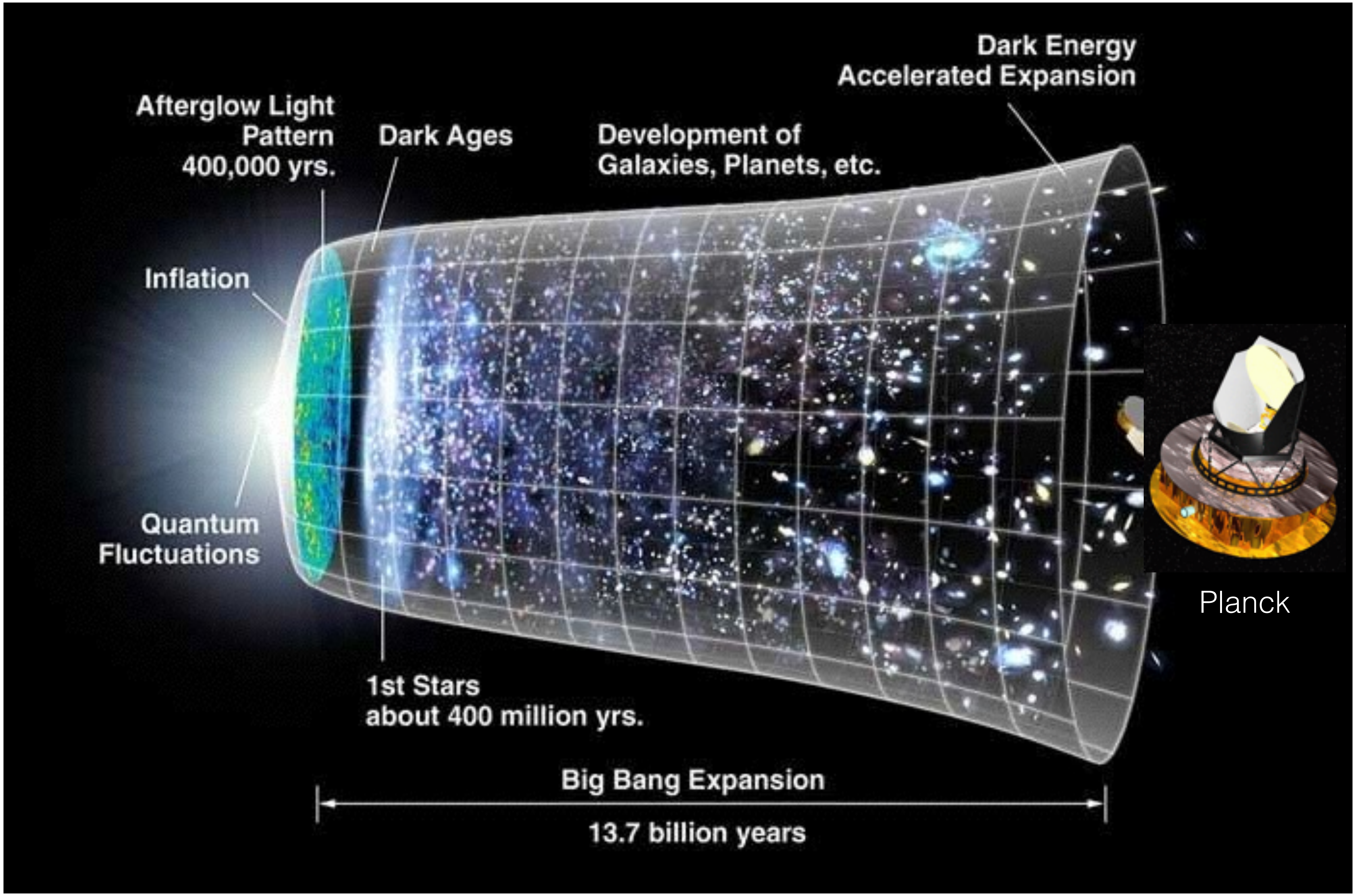
- **1. Introduction to foregrounds**
  - What are they?
  - Diffuse Galactic foregrounds
  - Why study them?
- **2. Physical emission mechanisms at “low” (WMAP/LFI) frequencies**
  - Synchrotron radiation
  - Free-free radiation
  - Electric dipole radiation from spinning dust (AME)
  - Thermal fluctuations from magnetic dust
- **3. Observational properties of diffuse foregrounds**
  - Generic observations of diffuse foregrounds - what do we know?
  - Impact on CMB data
- **4. Summary**

- *N.B. This lecture focuses on intensity (temperature) foregrounds. Polarization will be covered later in the week. High frequency foregrounds, component separation and extragalactic sources covered in other lectures by J. Delabrouille and G. de Zotti.*

1. Introduction

CMB foregrounds

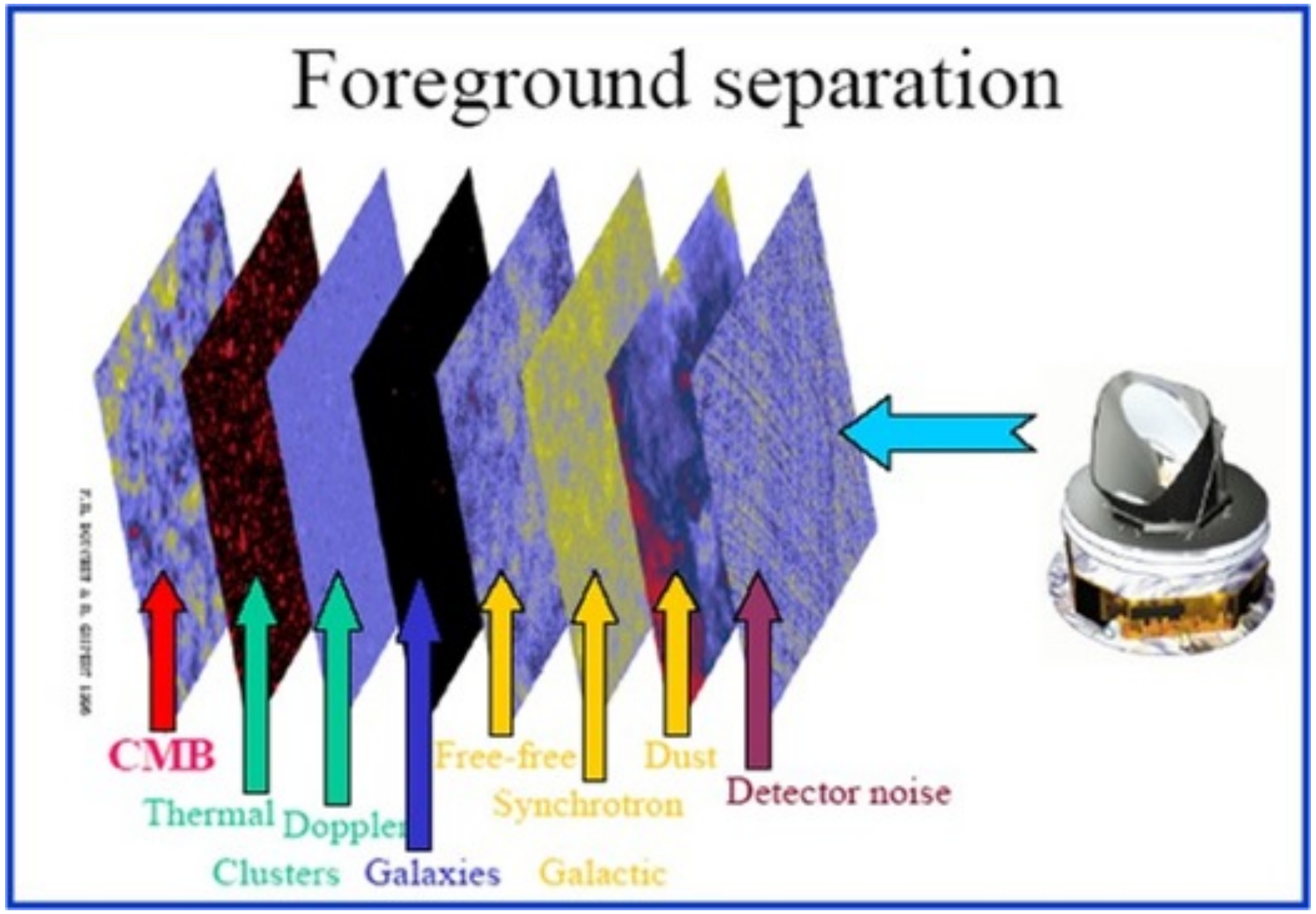
- There are many types (components) of “foregrounds”
  - A foreground is anything between the surface of last scattering (z=1090/380,000 years after the Big Bang) and the instrument detector!



Courtesy NASA/WMAP

Types of CMB foregrounds

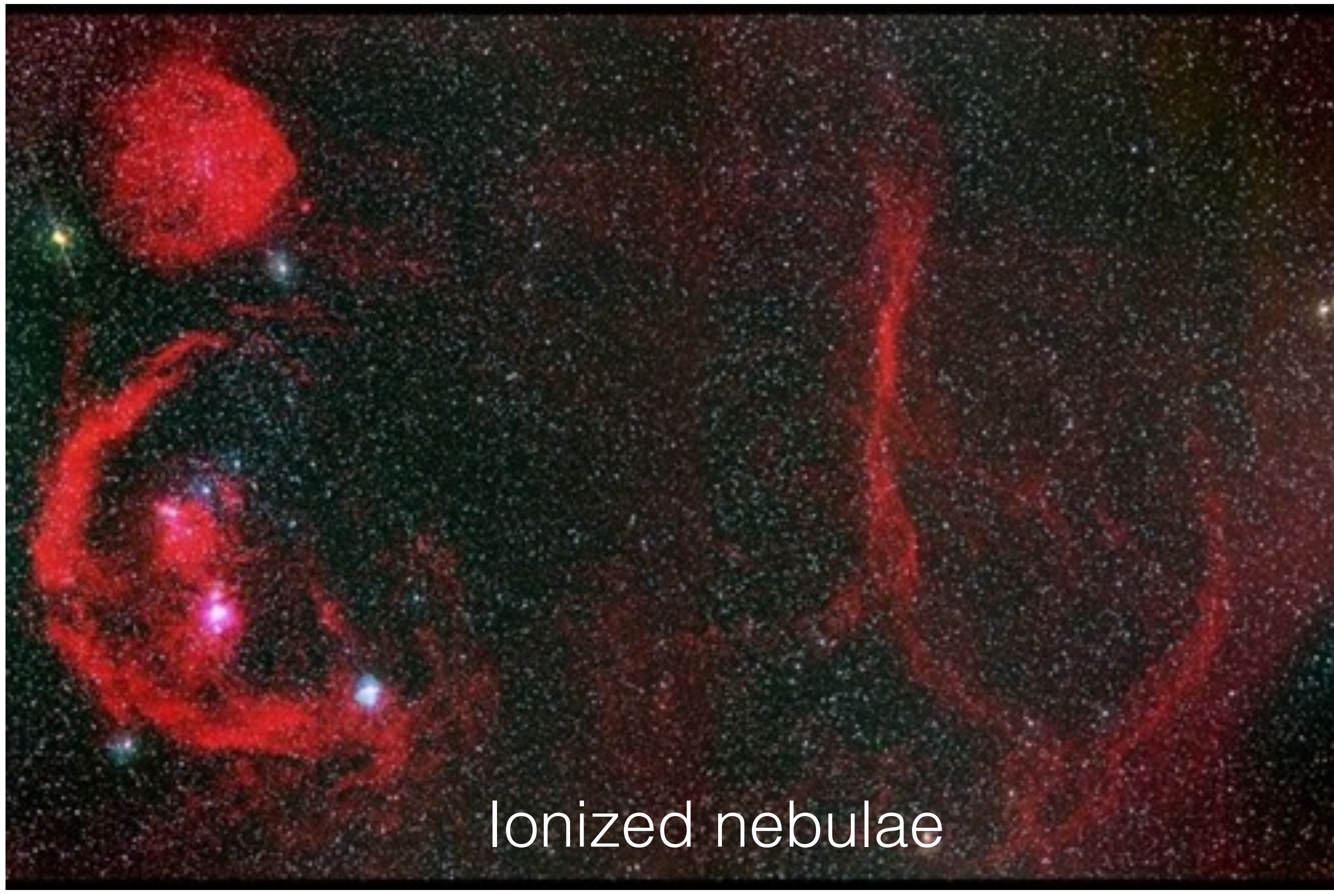
- **Diffuse Galactic foregrounds**
  - **Synchrotron**
  - **Free-free**
  - **Anomalous Microwave Emission (AME)/spinning dust**
  - **Magnetic dust**
  - Thermal dust emission (*see J. Delabrouille talk*)
- Extragalactic sources (*see G. de Zotti talk*)
  - Radio galaxies
  - “Normal” (star-forming) galaxies
- Solar system
  - Sun
  - Moon
  - Zodiacal light
  - Asteroids
- Atmosphere
  - Clouds, H_2O, O_2
- Radio Frequency Interference (RFI)
- Secondary CMB anisotropies
  - Sunyaev-Zeldovich, Sachs-Wolfe effect…



Courtesy of F. Bouchet

Why are they of interest?

- 1. **Understanding them allows us to more effectively remove them from CMB data** (*“component separation” - see talk by J. Delabrouille)*
  - Spatial structure (brightness maps, coherence, statistics)
  - Frequency spectra
  - Polarization
- 2. **They are of astrophysical interest in their own right (see Planck papers!)**
  - Physical emission mechanisms
  - Galactic structure - on the sky, line-of-sight, 3D
  - Physics and evolution of the interstellar medium (ISM)
    - Dust temperatures, electron temperatures, cosmic ray energy spectrum, equilibrium…
  - Star formation
  - Galactic magnetic field



A few definitions

- **Flux density**, S [W m^{-2} Hz^{-1}] related to "**brightness temperature**" T_b via simple equation involving solid angle Ω and λ^2
  - T_b often defined in the Rayleigh-Jeans (R-J) limit i.e. hν << kT (not to be confused with "**thermodynamic temperature**" T_{CMB} which is defined relative to a blackbody at T=2.725 K)
  - *Also see talk by A. Mennella*

$$\boxed{\frac{S}{\Omega} = \left(\frac{2k}{\lambda^2}\right) T_b}$$

- **Optical depth** τ related to T_b via the effective temperature T
  - τ << 1 optically thin (transparent)
  - τ >> 1 optically thick (opaque)

$$\boxed{T_b = T(1 - e^{-\tau})}$$

- **Spectral index** is the slope of the spectrum between two frequencies in log-space
  - e.g. S = A ν^{-3} has a spectral index α=-3

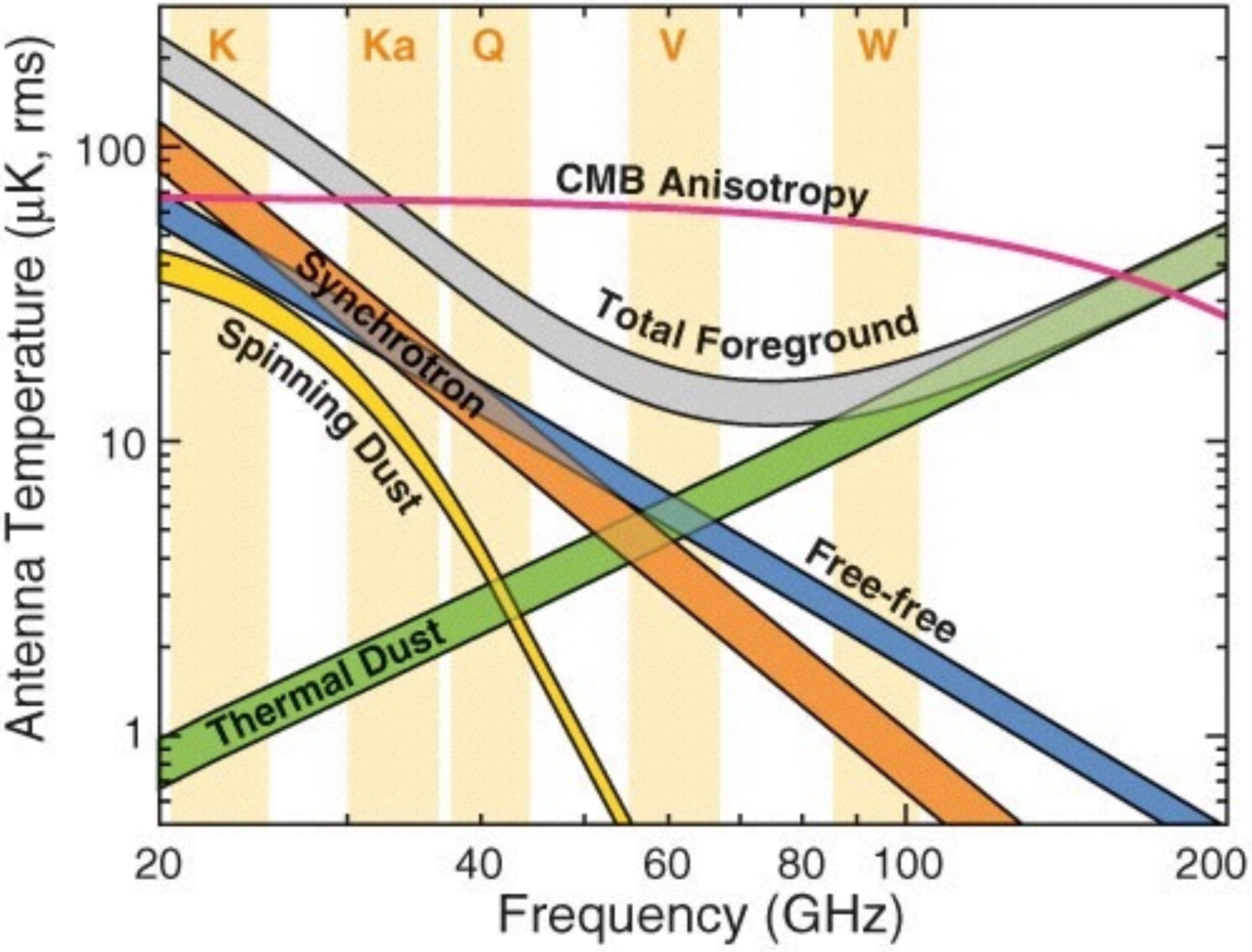
$$\boxed{\alpha = \frac{ln(S_1/S_2)}{ln(\nu_1/\nu_2)}}$$

  - Brightness temperature spectral index is the flux density spectral index - 2

$$\boxed{S \propto \nu^{\alpha} \quad (T \propto \nu^{\beta}\ ;\ \alpha = \beta + 2)}$$

Low frequencies and CMB

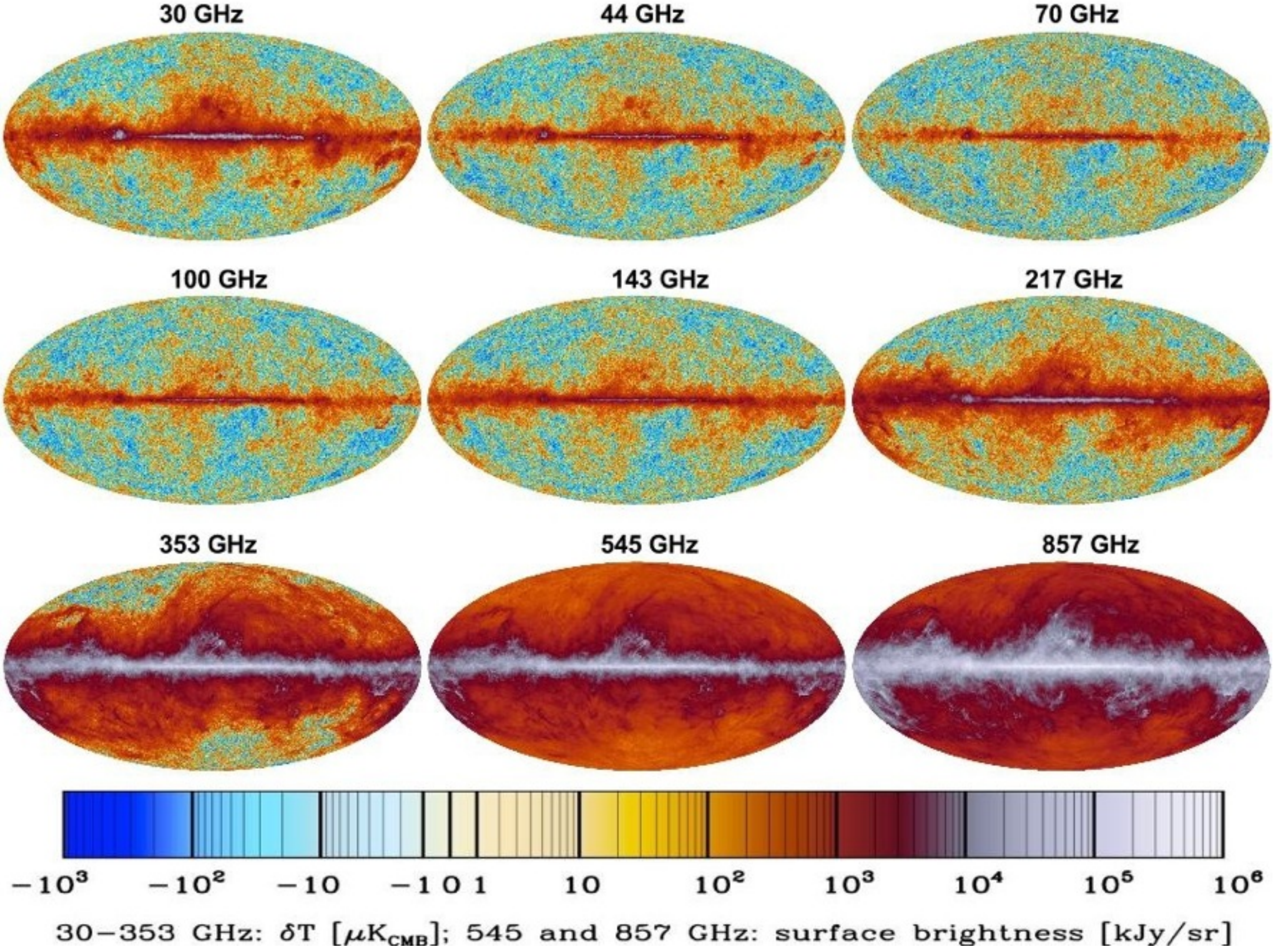
- CMB “window” where CMB is visible through the foregrounds
  - CMB observations typically made at frequencies ~10-300 GHz
- Foreground minimum at ~60-150 GHz (position dependent)
  - $\nu_{\rm min}$ ~80 GHz



$(T_a \propto T_b)$

Bennett et al. (2013)

Planck temperature maps



2. Physical emission mechanisms

Cyclotron vs Synchrotron radiation

- Cyclotron radiation: Electrons in magnetic field move in a spiral around field lines
  - Acceleration of charged particles produces EM radiation (radio waves)
  - **F** = e **v** x **B** = m_e **a**
  - e $v_\perp$ B = m_e r ω^2 = m_e $v_\perp$ ω
  - r is the gyro radius
  - Radiation at gyrofrequency $\omega = eB/m_e$ depends only on B
  - Radiates via usual **Lamor formula**
- **Synchrotron radiation: Relativistic (high energy) electrons accelerated by the Galactic magnetic field**
  - Critical difference is due to relativistic beaming

$$\frac{dE}{dt} = \int_{4\pi} \frac{dE}{dt\,d\Omega}\,d\Omega = \frac{q^2|\mathbf{a}|^2}{16\pi^2\varepsilon_o c^3}\int_{4\pi}\sin^2\theta\,d\Omega$$

$$\int = \frac{8\pi}{3}\text{, so:}$$

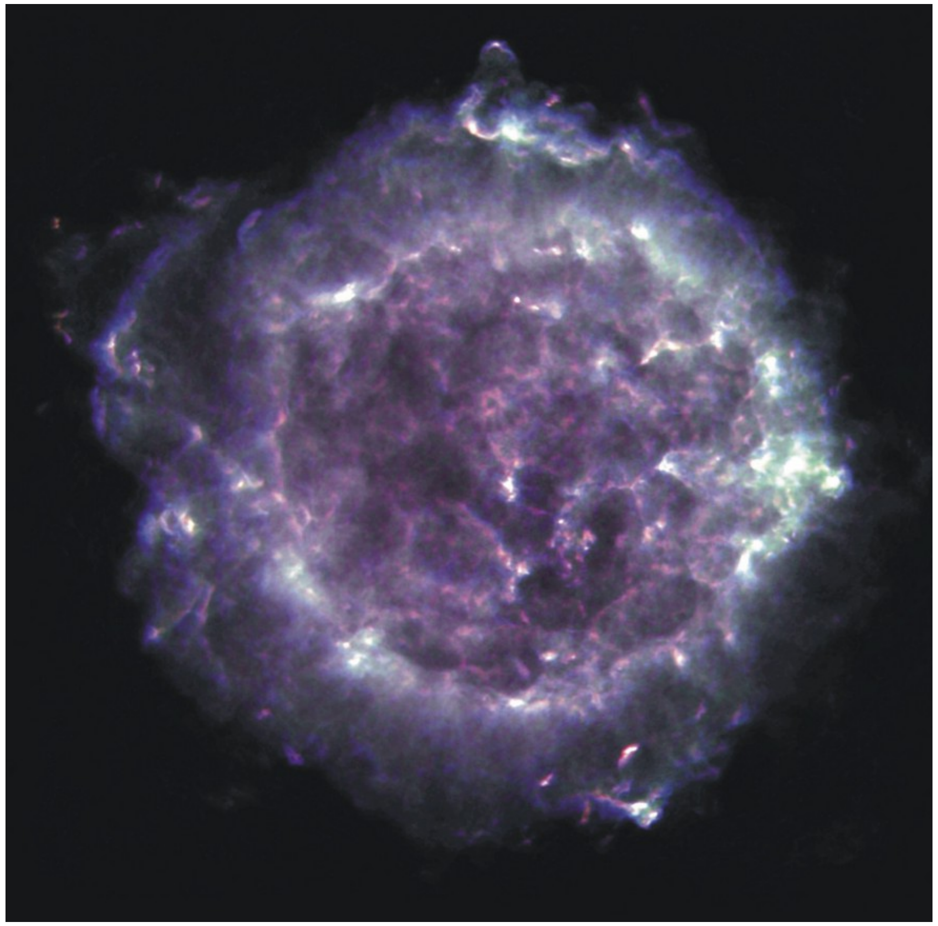
$$\boxed{\frac{dE}{dt} = \frac{q^2|\mathbf{a}|^2}{6\pi\varepsilon_o c^3} = \frac{|\ddot{\mathbf{p}}|^2}{6\pi\varepsilon_o c^3}}$$

See Longair text book for full derivation

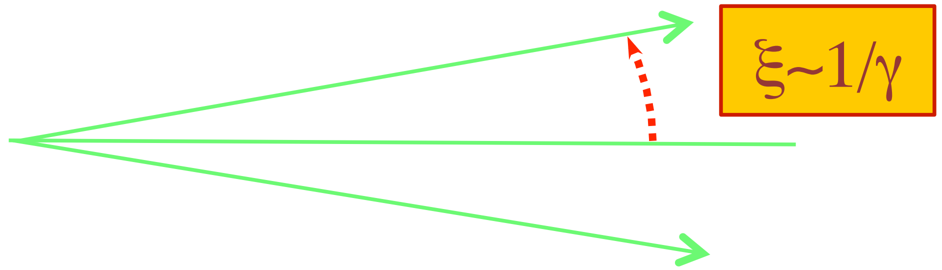
Synchrotron radiation

- Synchrotron radiation important because of cosmic ray population of particles with E >> m c^2
  - Small number but contain significant fraction of total energy!
  - Radiation dominated by electrons
- Relativistic beaming and Doppler boosting
  - We only see particles that are moving along the line-of-sight

$$\tan\theta = \frac{\sin\theta'}{\gamma\left(\cos\theta' + \upsilon/c\right)}$$



Supernova remnant Cassiopeia A
(multiple shocks that accelerate particles to high energies)



Synchrotron radiation

- Radius of curvature of path for electron moving ⊥ to B field lines

$$a_\perp = \frac{eB\upsilon}{\gamma m_\mathrm{e}} = \frac{\upsilon^2}{R_c}$$

$$R_c = \frac{\gamma m_\mathrm{e}\upsilon}{eB}$$

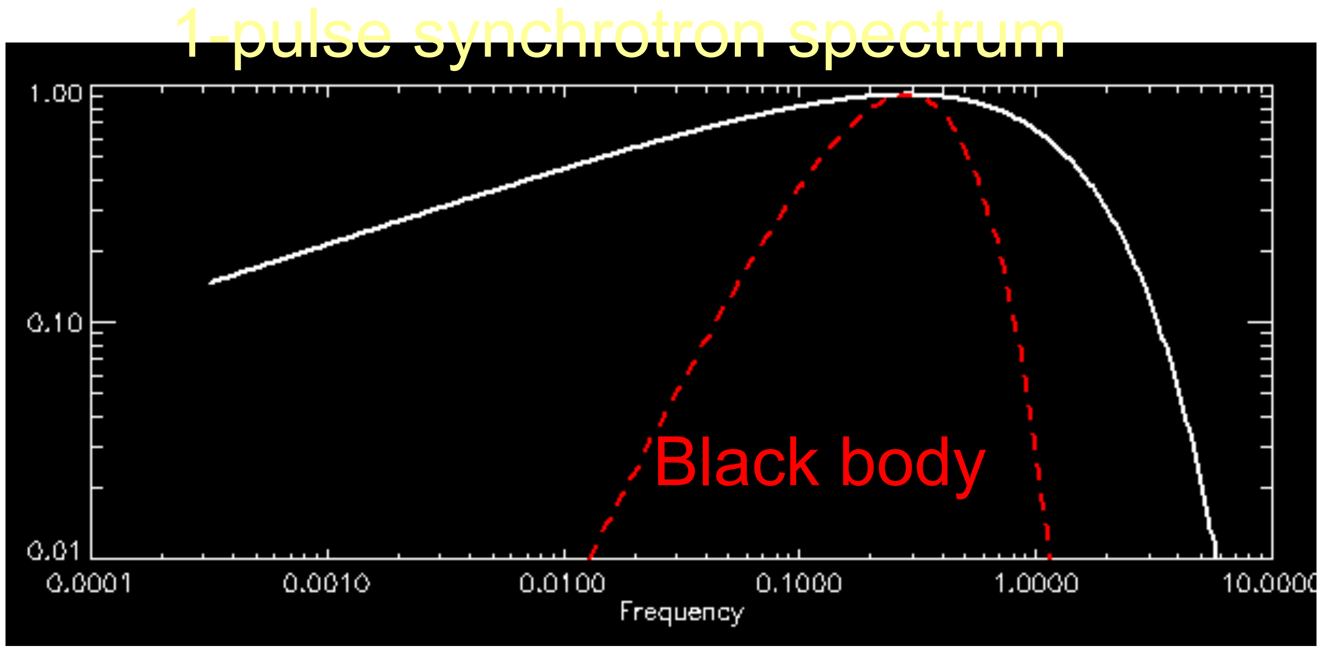
- For electrons at pitch angle α

$$R_c = \frac{\gamma m_\mathrm{e}\upsilon}{eB\sin\alpha}$$

  - ~0.001 pc for GeV electrons!
- Radiation is a series of pulses
- Fourier Transform (FT) pulse to get spectrum of emission from one particle, then integrate over particle distribution to get overall spectrum
- FT gives cut-off frequency

$$\nu_c \sim 1/\tau = \frac{\gamma^2 eB\sin\alpha}{m_\mathrm{e}}$$

  - Peak at ~0.3 ν_C



1 pulse of synchrotron spectrum

Synchrotron radiation

- CR particles typically have a power-law distribution of energies
- No. particles per unit energy

$$N(E)\,dE = N_0 E^{-p}\,dE$$

- Observed synchrotron spectra is superposition of pulses at these different energies

$$j_\nu d\nu = \frac{dE}{dt} N_0 E^{-p}\,dE$$

- Since $\nu_g \propto$ B

$$\boxed{j_\nu \propto N_0 B^{(p+1)/2} \nu^{(1-p)/2}}$$

- **Emitted photon spectrum is also a power-law, but with a different slope**
  - **Spectral index $\boxed{\alpha\text{=(1-p)/2}}$**
  - **p~3.0 over GeV range -> α=-1.0**
  - **Intensity $\propto$ B^2**

$$E = \gamma m_e c^2 = \left(\nu_c / \nu_g\right)^{1/2} m_e c^2$$

$$\Rightarrow dE = \frac{m_e c^2}{2\sqrt{\nu_g \nu}}\,d\nu$$

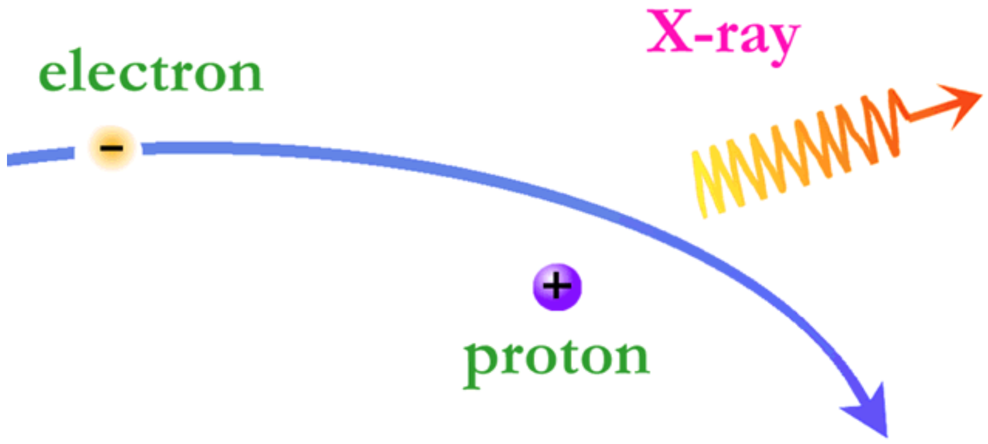
$$j_\nu = \frac{dE}{dt} N_0 E^{-p} \frac{dE}{d\nu}$$

$$= \frac{4}{3}\sigma_T c \frac{B^2}{2\mu_0}\gamma^2 N_0 E^{-p} \frac{m_e c^2}{2\sqrt{\nu_g \nu}}$$

$$= \frac{\sigma_T c}{3\mu_0}\left(m_e c^2\right)^{1-p} N_0 \left(\frac{\nu}{\nu_g}\right)^{1-\frac{p}{2}} B^2 \left(\nu_g \nu\right)^{-\frac{1}{2}}$$

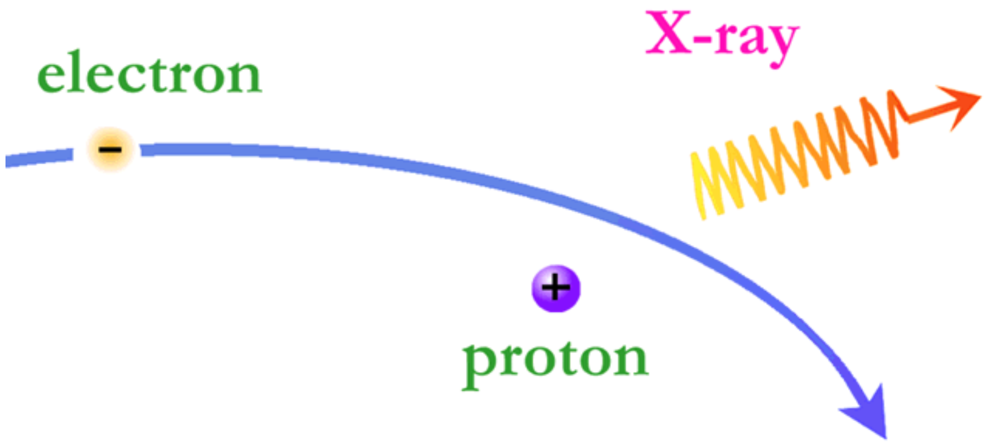
Free-free radiation (1)

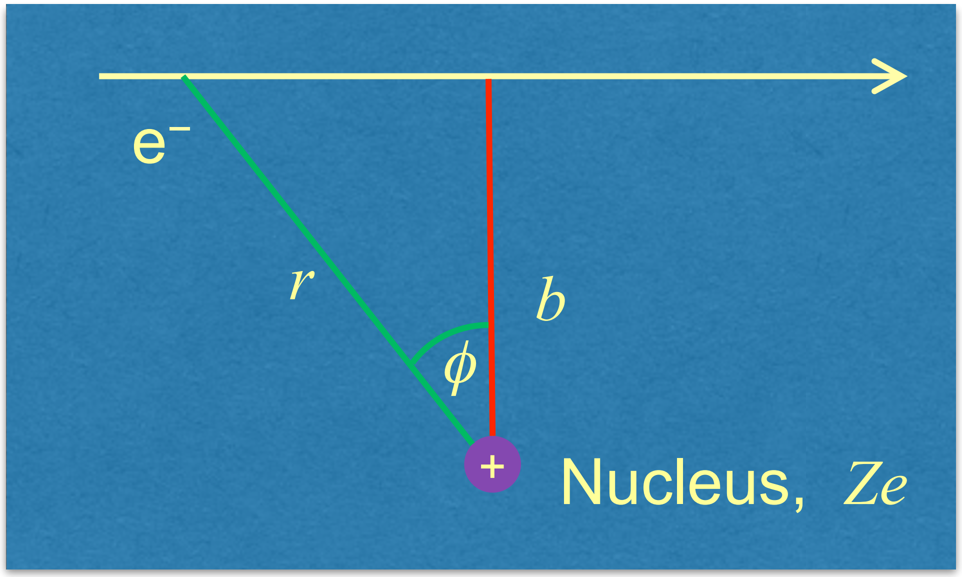
- Free electrons accelerated by ions (proton or alpha-particle)
  - Also known as thermal bremsstrahlung (“braking radiation”)
  - Inevitably produced by hot ionised gas (plasma)
- Can be (mostly) explained by classical electromagnetism
  - Coulomb’s law, Maxwell’s equations…
  - Small quantum mechanical corrections (at high frequencies)



Free-free radiation (2)

See Longair text book for full derivation (or Oster 1960)





- Theory:
  - 1. Work out case of single collision deflection (Lamor formula again!)
  - 2. Integrate effects of deflections by all the nuclei
  - 3. Worry about limits of integration (different for radio vs X-ray)
  - 4. Integrate over a population of electrons with Maxwell-Boltzmann velocity distribution (thermal bremsstrahlung)
    - Gaunt factor g(ν,T) contains all quantum mechanical corrections (~1)

$$j_\nu = 5.4 \times 10^{-52} \frac{Z^2 n_\mathrm{i} n_\mathrm{e}}{\sqrt{T}} g(\nu, T) \exp[-h\nu / k_\mathrm{B} T] \,\mathrm{W\ Hz^{-1} sr^{-1} m^{-3}}$$

- Total energy emitted

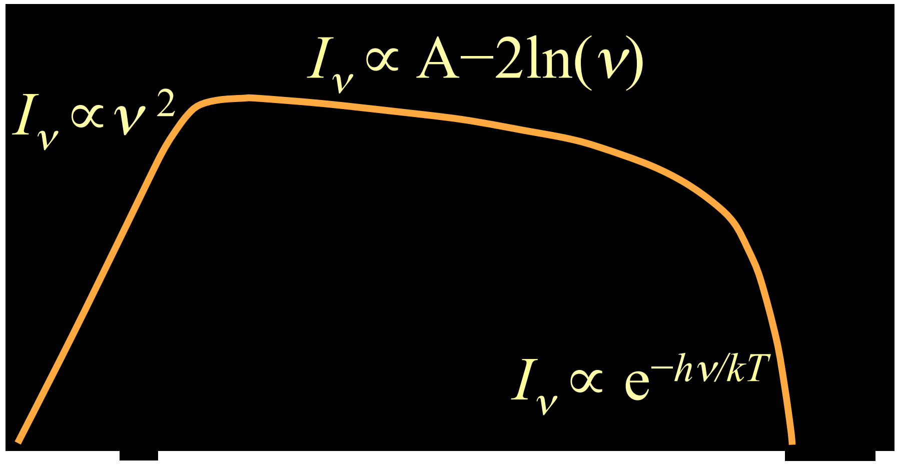
$$\frac{dE}{dt} = 1.435 \times 10^{-40} Z^2 \sqrt{T} n_\mathrm{i} n_\mathrm{e} \bar{g}(T) \,\mathrm{W\ m^{-3}}$$

Free-free radiation (3)

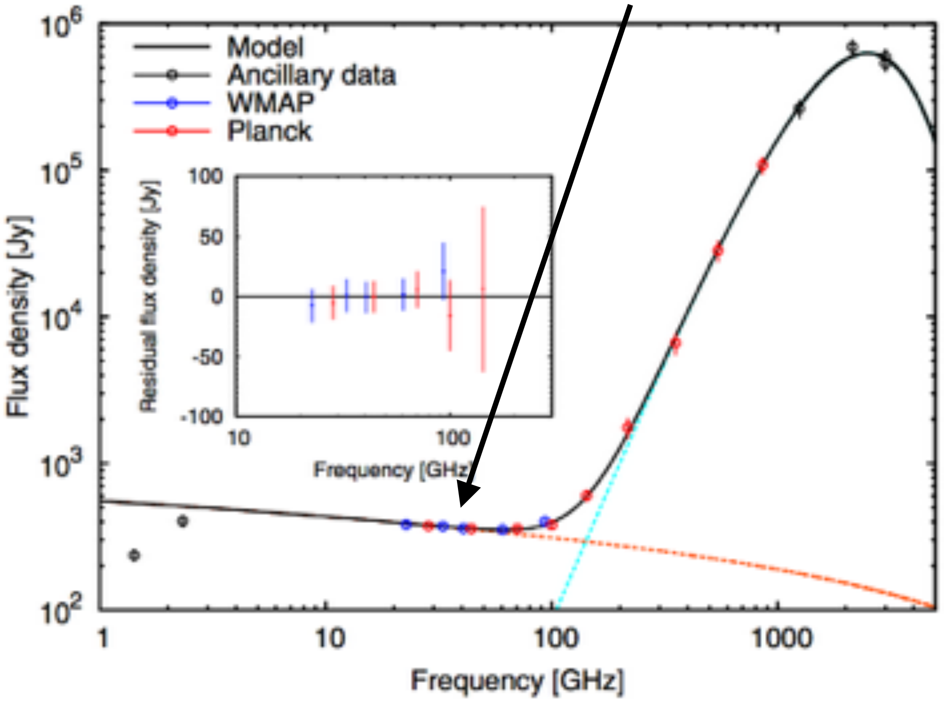
- Free-free emission from thermal plasma must absorb as well as emit, via

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = B_\nu(T)$$

- At low frequencies, τ>1, to give Rayleigh-Jeans spectrum fixed by temperature of the plasma
- At high radio (microwave) frequencies, τ<<1, spectrum is close to α=-0.1 (β=-2.1) and varies little with frequency/temperature
- At VERY high (IR/optical) frequencies, exponential cut-off
- Over relevant range for CMB/foregrounds (few GHz to ~1000 GHz), free-free radiation is a power-law
  - **β=-2.1 (1-10 GHz)**
  - **Steepening slightly to β~-2.13 at 100 GHz.**



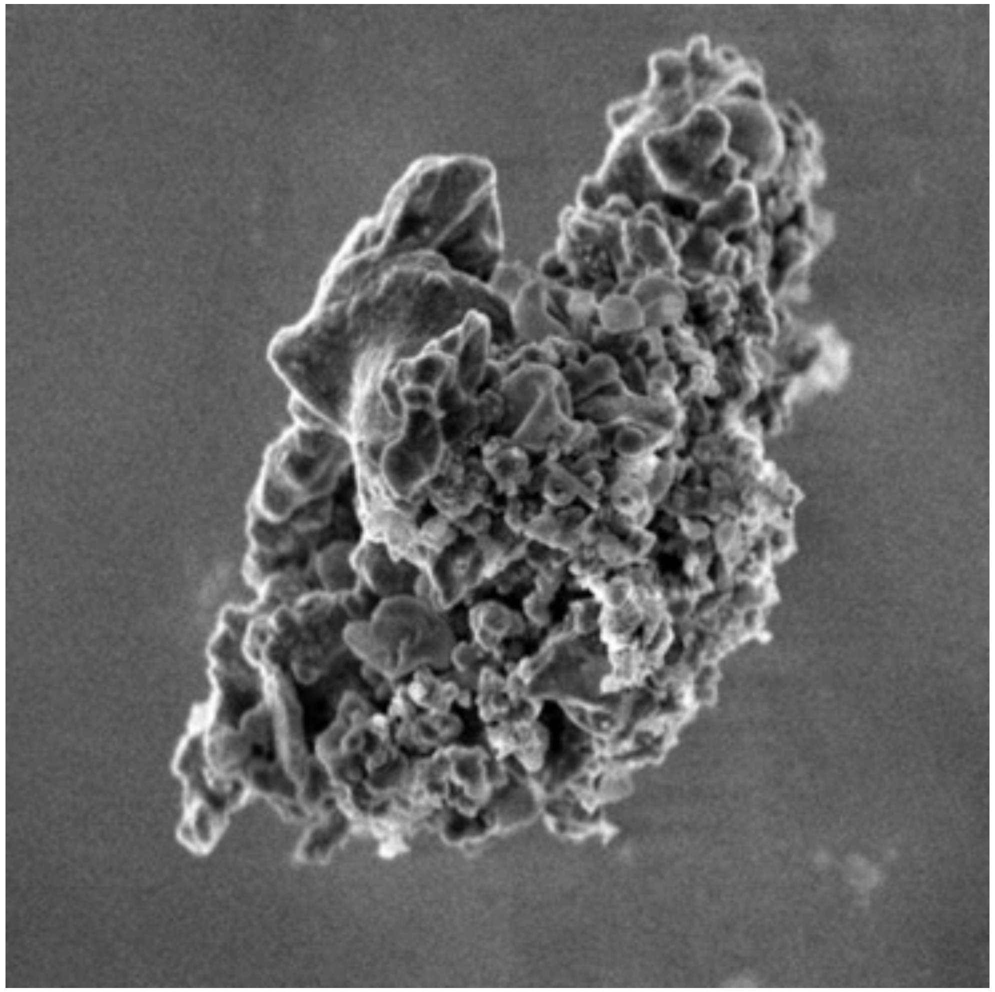
Free-free spectrum α≈-0.1 across CMB frequency band



M42 Orion Nebula spectrum
(Planck Collaboration, 2011, Early Paper XX)

Electric dipole (spinning dust) radiation

- Interstellar dust grains naturally have a residual **electric dipole moment μ**
  - Spherical grains
  - Asymmetric grains
  - Poly-Aromatic Hydrocarbons (PAHs)
- Dust grains and are known to **rotate rapidly** due to interactions in the ISM
  - Rotating electric dipole -> radio waves!
  - First predicted by Erickson (1957)
- Excitation and damping terms
  - Collisions
  - Plasma excitation/drag
  - Emission/absorption of IR/CMB photons
  - H_2 formation & photoelectric ejection
- Other effects to consider
  - Grain wobbling, impulsive talks…



Typical interstellar dust grain - a collection of 100s-1000s of atoms/molecules

$$\boxed{P = \frac{2}{3}\frac{\omega^4 \mu^2 \sin^2\theta}{c^3}}$$

Power radiated by a spinning electric dipole moment

Electric dipole (spinning dust) radiation

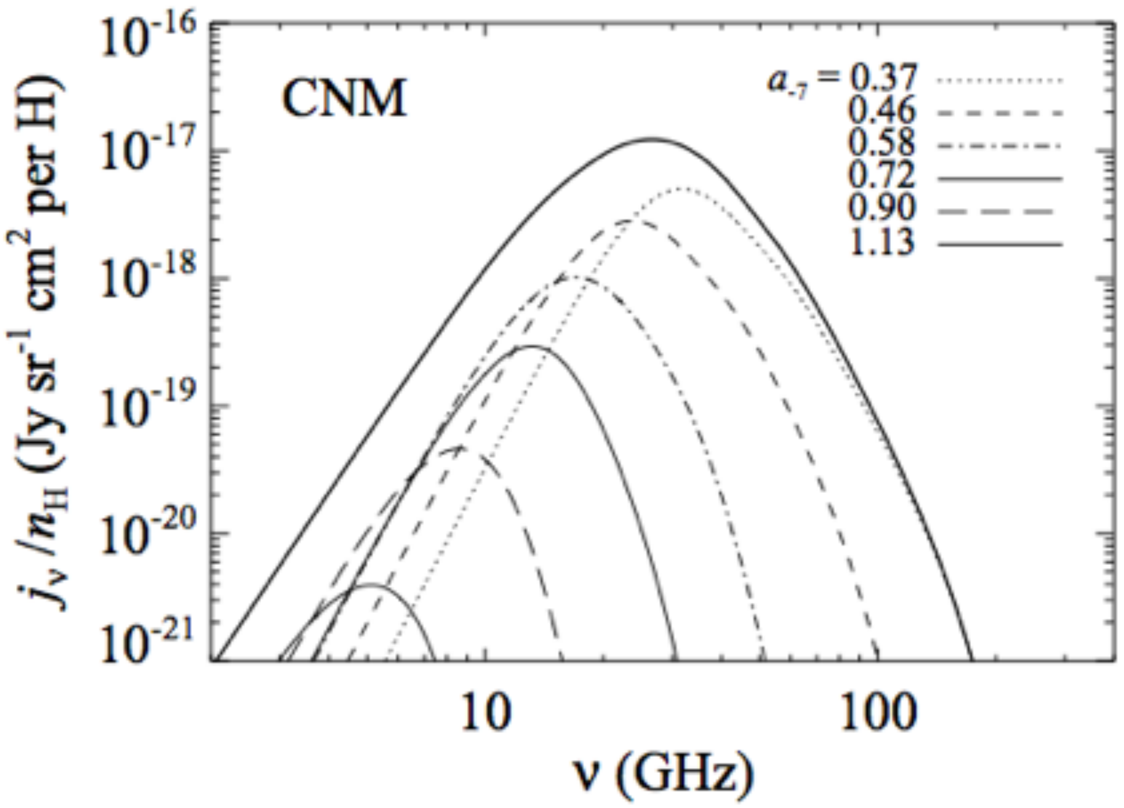
- **Emissivity** of electric dipole radiation per H atom (Jy/s/sr/H atom)
    - Grain size distribution dn/da
    - Electric dipole moments $\boldsymbol{\mu}$ as a function of grain size a
    - Angular velocity distribution function f(ω)

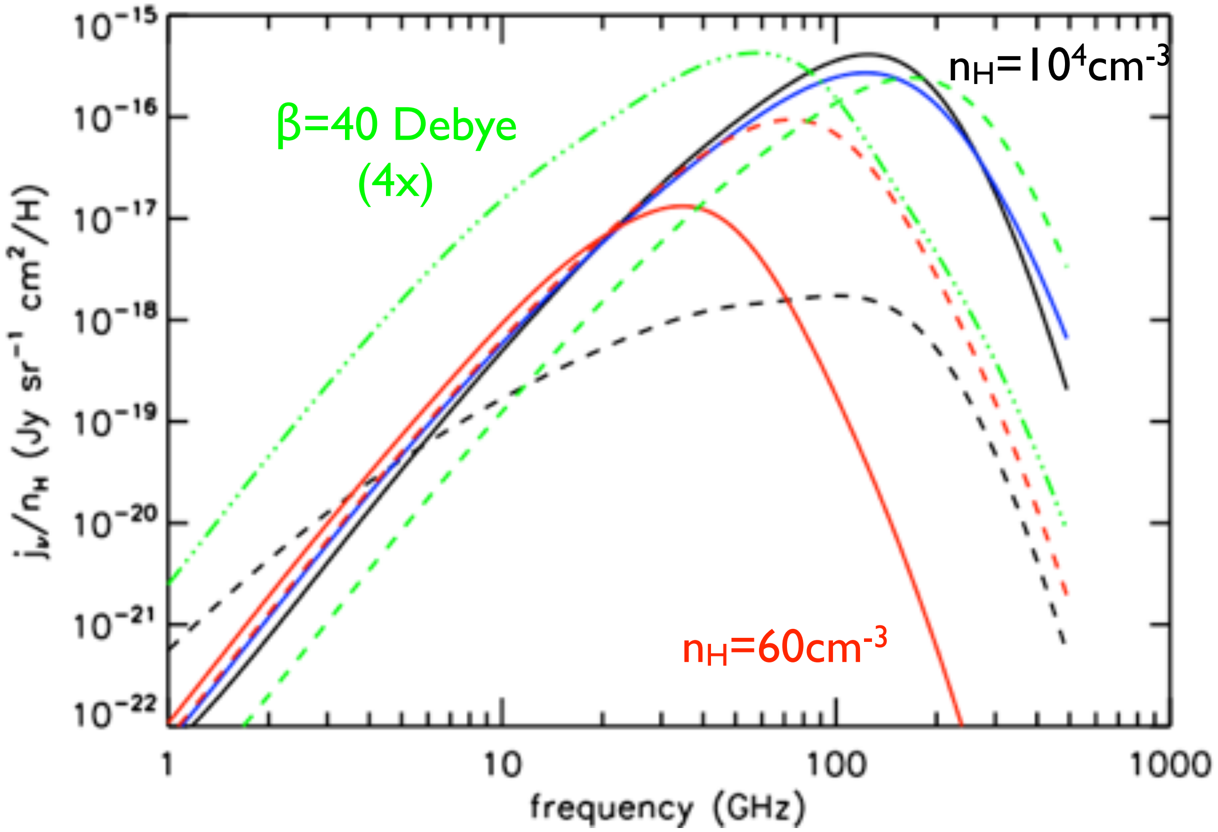
$$\frac{j_\nu}{n_\mathrm{H}} = \frac{1}{4\pi}\int_{a_\mathrm{min}}^{a_\mathrm{max}} \mathrm{d}a\, \frac{1}{n_\mathrm{H}}\frac{\mathrm{d}n_\mathrm{gr}}{\mathrm{d}a} 4\pi\omega^2 f_\mathrm{a}(\omega) 2\pi \frac{2}{3}\frac{\mu_{a\perp}^2\omega^4}{c^3}$$

where $\omega = 2\pi\nu$.

- Very complicated physics!
- But typically gives a peaked spectrum
    - Peak depends on size distribution and rotational velocities
        - Always dominated by smallest grains spinning the fastest
    - ~30 GHz expected for typical parameters

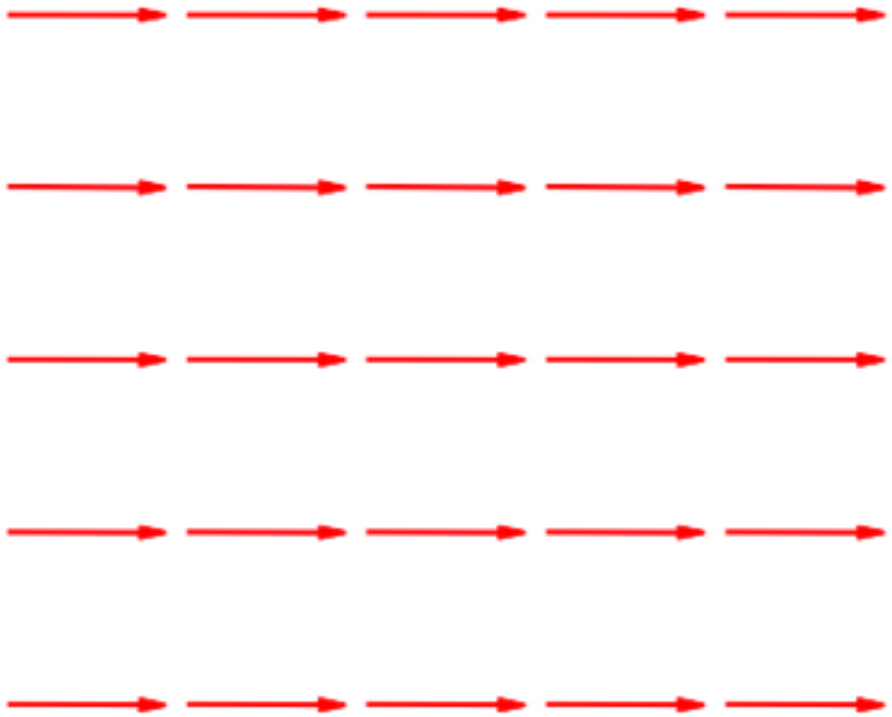
Ali-Hamoud, Hirata, Dickinson (2009)





Thermal fluctuations of magnetic dust

- Much of Fe could be in magnetic material (matallic Fe, magnetite, maghemite etc.)
- Lowest energy state of metallic Fe
  - Spins are parallel (magnetized)
  - Magnetization M is aligned with one of the crystal axes
- Excited state: spins parallel, but oriented away from crystal axis
- Oscillations in magnetization -> magnetic dipole radiation
- Finite temperature -> **thermal magnetic dipole emission**
- Detailed predictions depend very much on form of Fe
  - Draine & Lazarian (1999), Draine & Hensley (2013)



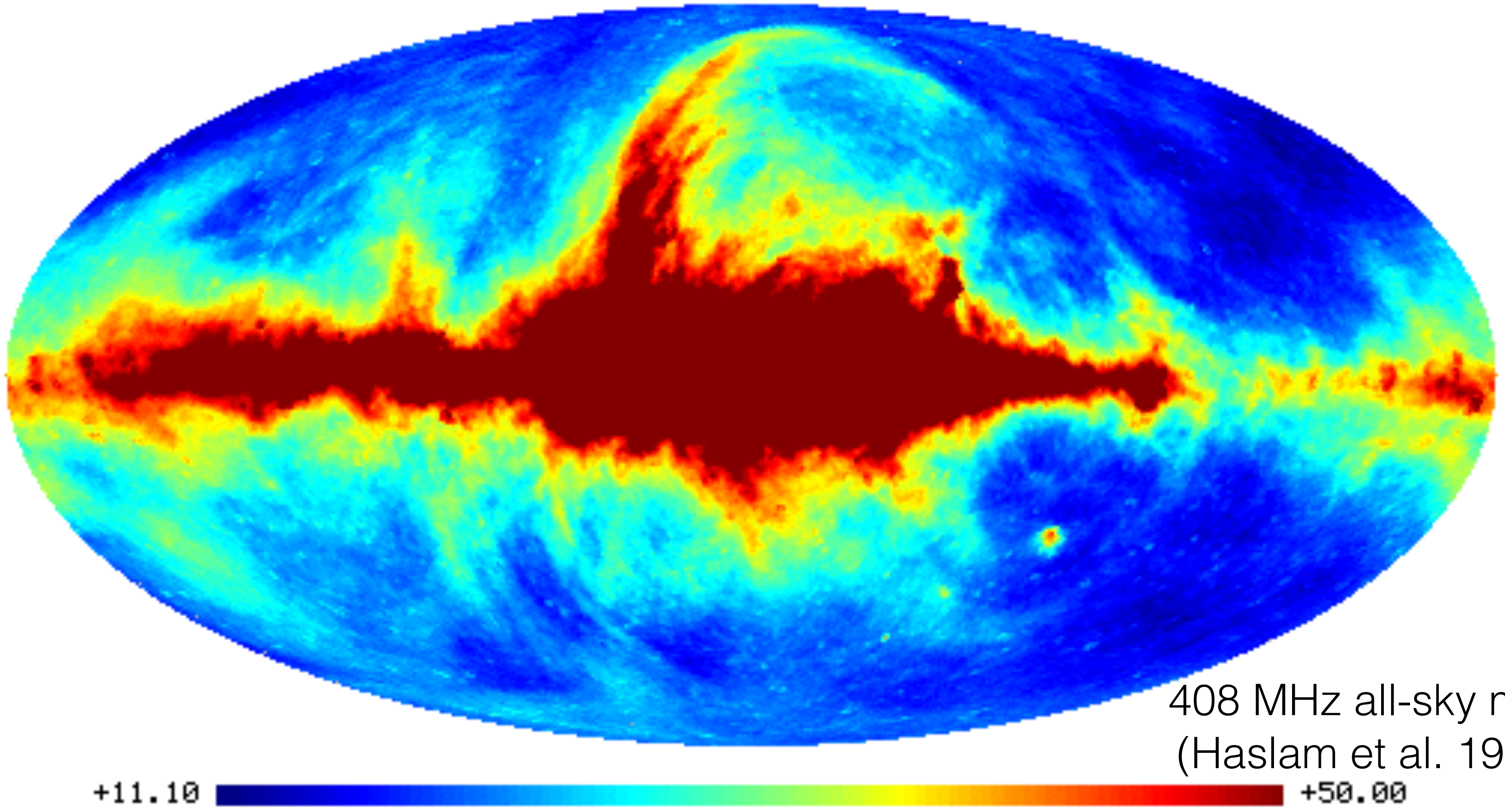
Ferromagnetic lattice with spins aligned
Thermal fluctuations will move them away producing dipole radiation

3. Observational properties of diffuse foregrounds

Synchrotron radiation observations

- Synchrotron radiation dominates at **low frequencies**
  - CR energy spectral index p~3.0 at GeV energies (related to radio emission)
  - Synchrotron spectral index β=-2.7 at ~1 GHz
- -> Observe synchrotron at frequencies of ~1 GHz and below
  - Free-free, CMB, spinning dust etc are weak/negligible
- Relatively weak at frequencies above ~10-20 GHz
  - Except bright supernova remnants and radio galaxies (bright up to 100 GHz and higher)

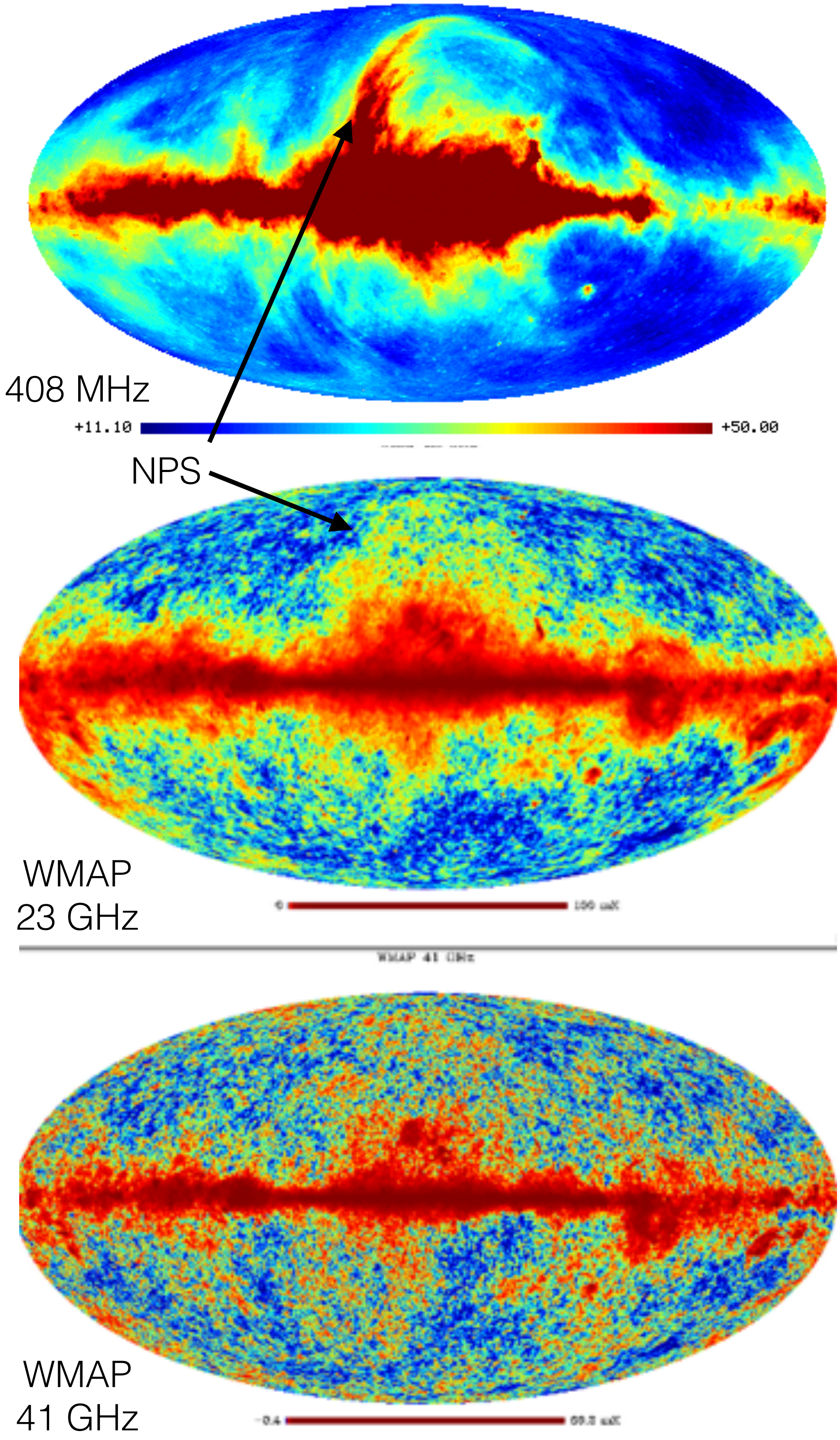
Crab nebula (pulsar wind nebula)



408 MHz all-sky map (Haslam et al. 1982)

Synchrotron at CMB frequencies

- Synchrotron emission expected to fall off quickly with frequency (β=-3.0 above ~10 GHz)
- **Even bright North Polar Spur (NPS) is weak above 40 GHz**
  - **It is still bright in 20-40 GHz range**
- **Most other features are weak above ~30 GHz**
- Flat (hard) spectrum (β=-2.5) synchrotron appears to be minimal for most of the sky
- But requires high sensitivity ~5-15 GHz data to be sure!…



Low frequency foreground surveys

C-Band All-Sky Survey (C-BASS)

5 GHz All-Sky Survey
~6m dishes in California & South Africa
I, Q, U
45 arcmin resolution
~0.1 mK r.m.s.

6.1m C-BASS dish at Owens Valley, California

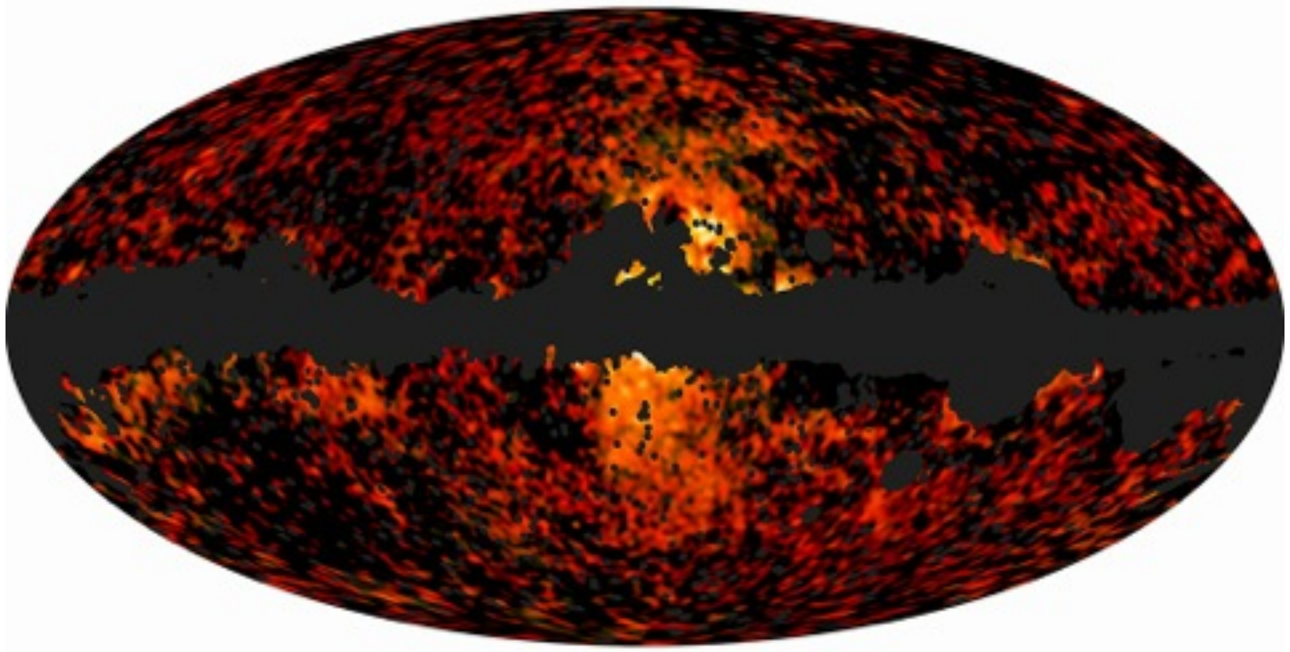
Q-U-I JOInt Tenerife Experiment (QUIJOTE)

11,13,17, 19 GHz Northern Sky Survey
Dedicated telescope, Tenerife
I, Q, U
~1 deg resolution
~few μK r.m.s. (pol channels)

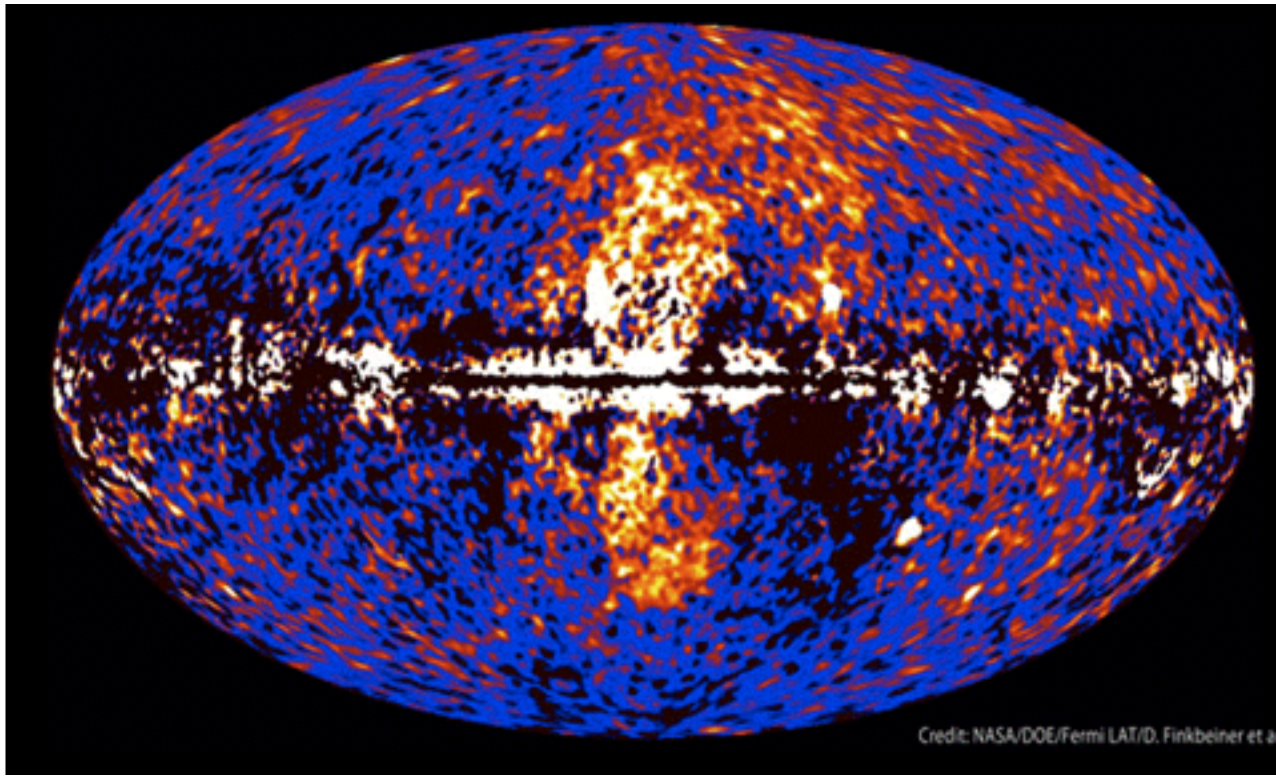
QUIJOTE telescope scanning the sky

WMAP/Planck Galactic “haze”

- Mysterious new component discovered in CMB data (Finkbeiner et al. 2004; Planck Collaboration 2013)
  - Residual “haze” emission around the Galactic centre after subtraction of soft synchrotron, free-free, AME, CMB and thermal dust
- **Spectrum appears to be hard (flatter than normal) β=-2.5**
- Thought to be hard synchrotron from a different population of CR electrons
  - Probably related to the Fermi haze
- Several ideas about origin of these
  - Most plausible is starburst period ~10^5-10^6 years ago providing energy injection ~10^{54+} ergs!



WMAP/Planck Haze (Planck Collaboration 2013)



Credit: NASA/DOE/Fermi LAT/D. Finkbeiner et a

Fermi lobes (Fermi Collaboration)

Free-free radiation

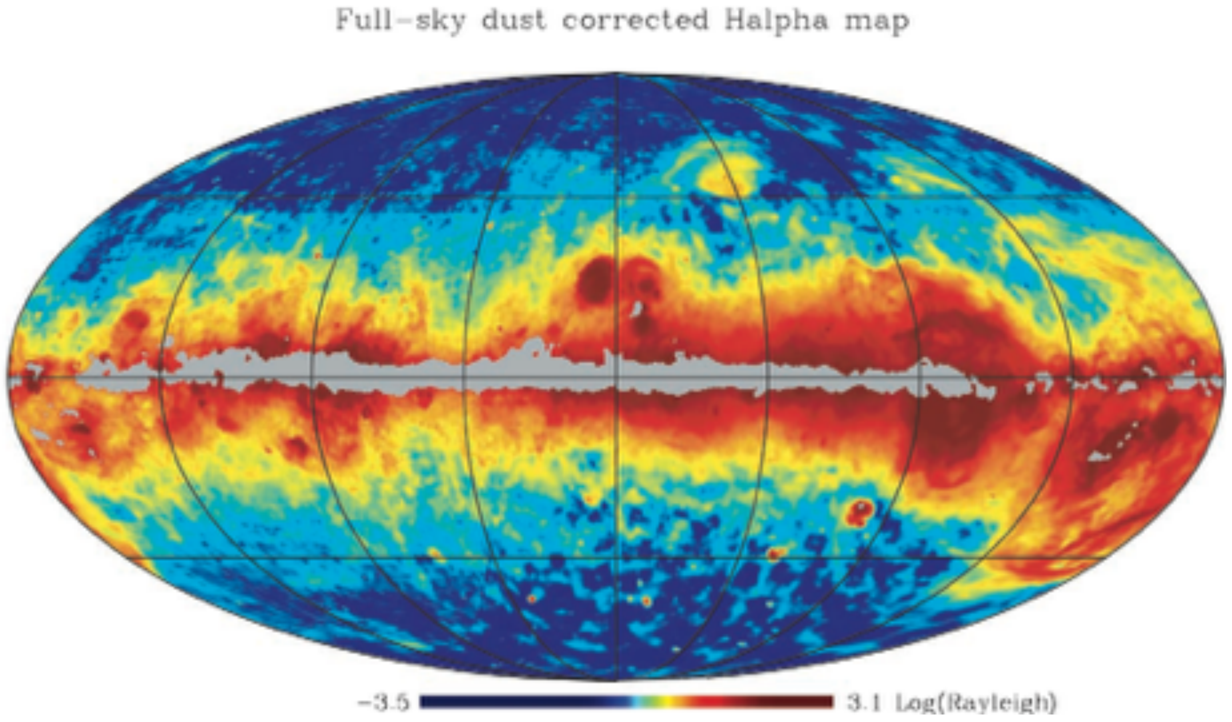
- Free-free also relatively weak above 10 GHz
- Flatter spectrum (β=-2.1) means it could be dominant foreground at frequencies ~30-100 GHz
- Can compare with optical Hα (656.28 nm) line data
    - Balmer recombination transition from n=3 to n=2 level
    - Hα traces Warm Ionized Medium (WIM) with $T_e \sim 10^4$K
- Hα intensity goes as the **Emission Measure** (EM)
    - Exactly the same as free-free emission!
- **Free-free intensity (units of rayleighs, R) can be predicted from optical Hα data!**

$$\boxed{\frac{T_b^{ff}}{I_{H\alpha}} = 8.396 \times 10^3 a \nu_{GHz}^{-2.1} T_4^{0.667} 10^{0.029/T_4} (1 + 0.08)}$$

$$\boxed{I_{H\alpha} \propto EM = \int n_e^2 dl}$$

$$I(H\alpha) \overset{\text{case B}}{=} 9.41 \times 10^{-8} T_4^{-1.017} 10^{-0.029/T_4} (EM)_{cm^{-6}\,pc}$$

$$\boxed{T_b = 8.235 \times 10^{-2} a T_e^{-0.35} \nu_{GHz}^{-2.1} (1 + 0.08)(EM)_{cm^{-6}\,pc}}$$

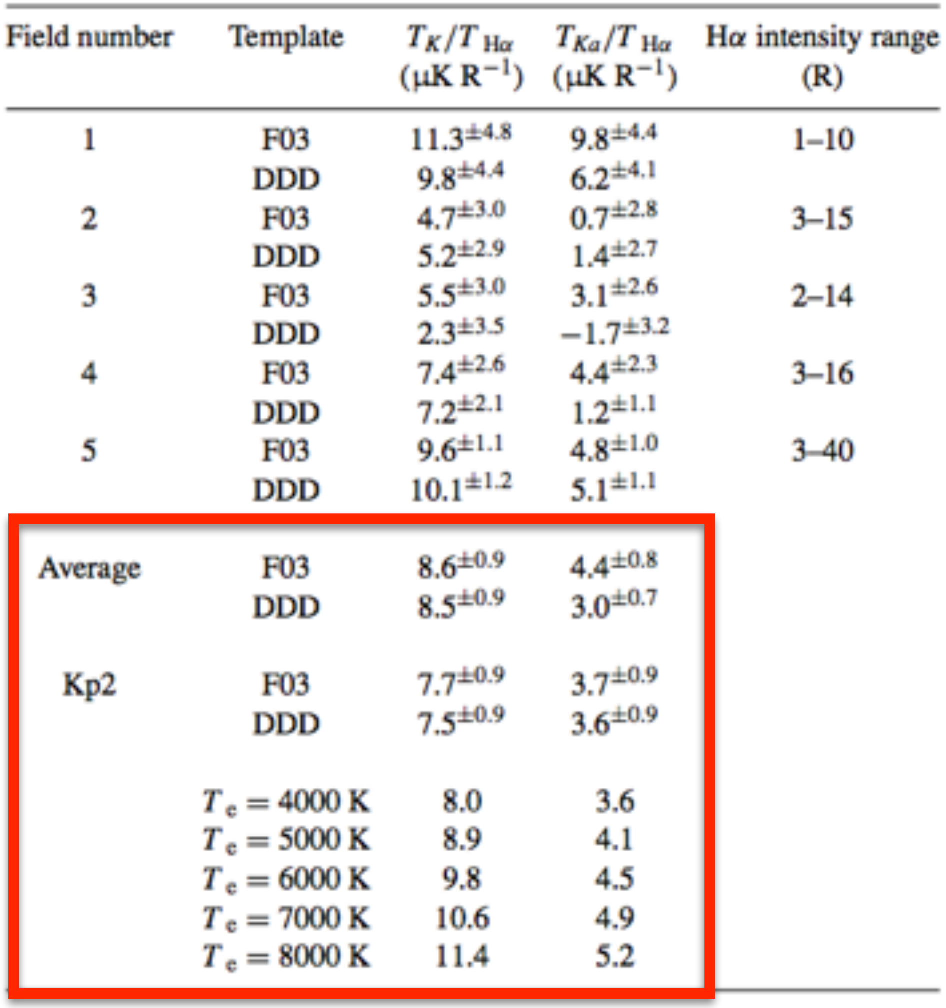


Full-sky Hα map (Dickinson, Davies, Davis 2003)

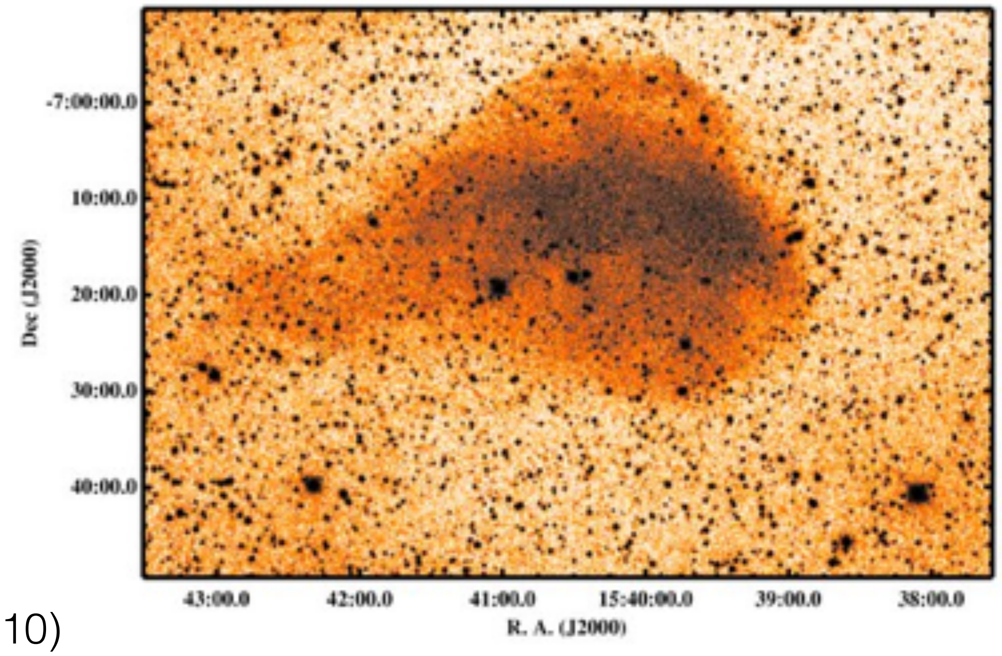
Free-free vs Hα

- Free-free amplitude given by Hα intensity if
  - Electron temperature T_e known
  - No significant absorption by dust
  - In Local Thermodynamic Equilibrium (LTE)
  - No scattered Hα light
- Amplitude of free-free component seems to be low relative to Hα
  - High latitudes should be (mostly) absorption free
  - If T_e ~7000-8000K, expect ~ 11.4 μK/R at 23 GHz (K-band)
  - Best-fit gives T_e ~4000K
- New results (Witt et al. 2010; Brandt & Draine 2013) suggest significant fraction (~1/2!) of **scattered Hα light** which would bring this back to expected range

Davies et al. (2006)



Field number	Template	$T_K/T_{H\alpha}$ (μK R^{-1})	$T_{Ka}/T_{H\alpha}$ (μK R^{-1})	Hα intensity range (R)
1	F03	$11.3^{\pm 4.8}$	$9.8^{\pm 4.4}$	1–10
	DDD	$9.8^{\pm 4.4}$	$6.2^{\pm 4.1}$	
2	F03	$4.7^{\pm 3.0}$	$0.7^{\pm 2.8}$	3–15
	DDD	$5.2^{\pm 2.9}$	$1.4^{\pm 2.7}$	
3	F03	$5.5^{\pm 3.0}$	$3.1^{\pm 2.6}$	2–14
	DDD	$2.3^{\pm 3.5}$	$-1.7^{\pm 3.2}$	
4	F03	$7.4^{\pm 2.6}$	$4.4^{\pm 2.3}$	3–16
	DDD	$7.2^{\pm 2.1}$	$1.2^{\pm 1.1}$	
5	F03	$9.6^{\pm 1.1}$	$4.8^{\pm 1.0}$	3–40
	DDD	$10.1^{\pm 1.2}$	$5.1^{\pm 1.1}$	
Average	F03	$8.6^{\pm 0.9}$	$4.4^{\pm 0.8}$	
	DDD	$8.5^{\pm 0.9}$	$3.0^{\pm 0.7}$	
Kp2	F03	$7.7^{\pm 0.9}$	$3.7^{\pm 0.9}$	
	DDD	$7.5^{\pm 0.9}$	$3.6^{\pm 0.9}$	
	$T_e = 4000$ K	8.0	3.6	
	$T_e = 5000$ K	8.9	4.1	
	$T_e = 6000$ K	9.8	4.5	
	$T_e = 7000$ K	10.6	4.9	
	$T_e = 8000$ K	11.4	5.2	

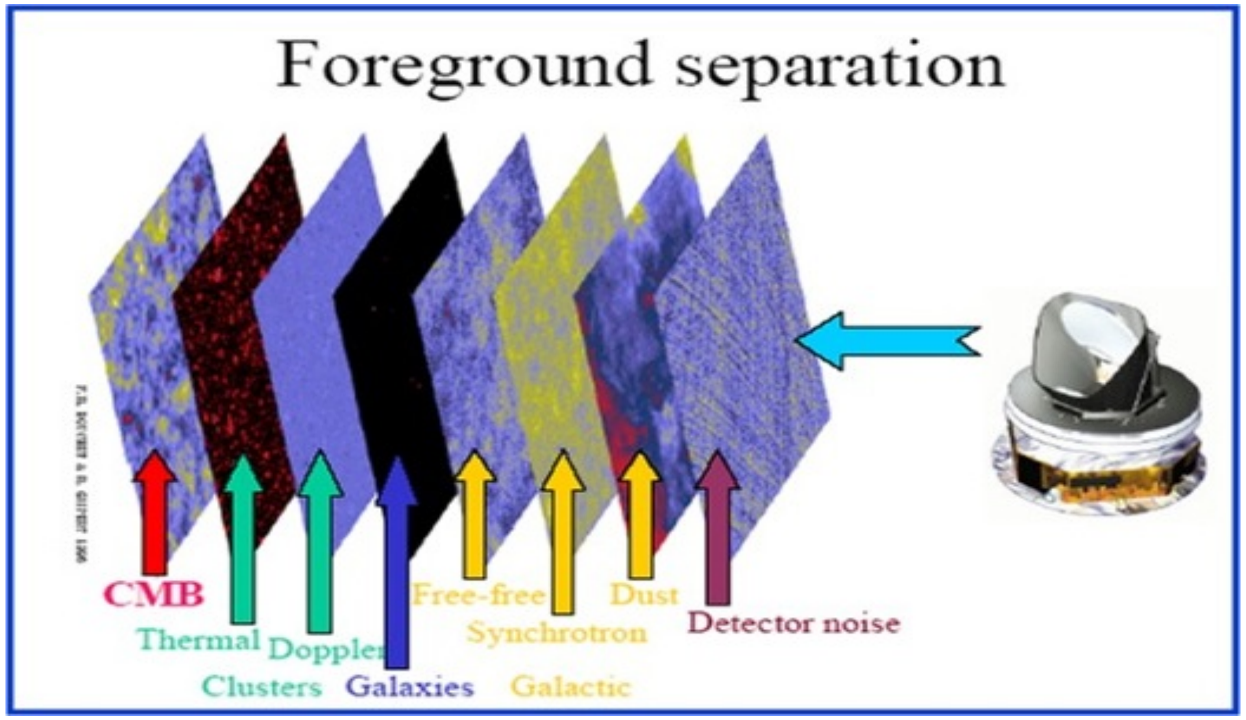


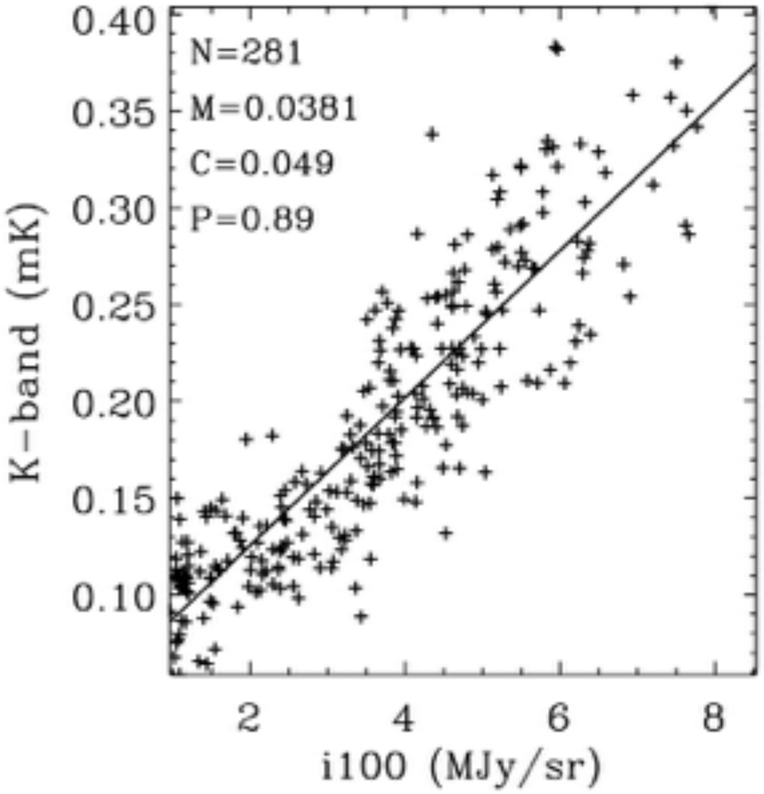
LDN1780 dark cloud emits ~1/2 scattered light (Witt et al. 2010)

Anomalous Microwave Emission (AME)

- Additional foreground detected at frequencies ~10-60 GHz (Kogut et al. 1996; Leitch et al. 1997)
  - Cannot easily be explained by synchrotron/free-free/CMB emission mechanisms
  - Strongly correlated with FIR (~100μm) thermal dust emission
  - Steeply falling spectrum above ~30 GHz
  - Relatively low level of polarisation (few % or less)
- Difficult to separate from CMB and rest of the foregrounds especially at high latitudes
- AME has been detected by many experiments
- Best explanation is **electric dipole radiation from ultra-small rapidly spinning dust grains (Draine & Lazarian 1998)**

Adapted from Leitch et al. (1997)

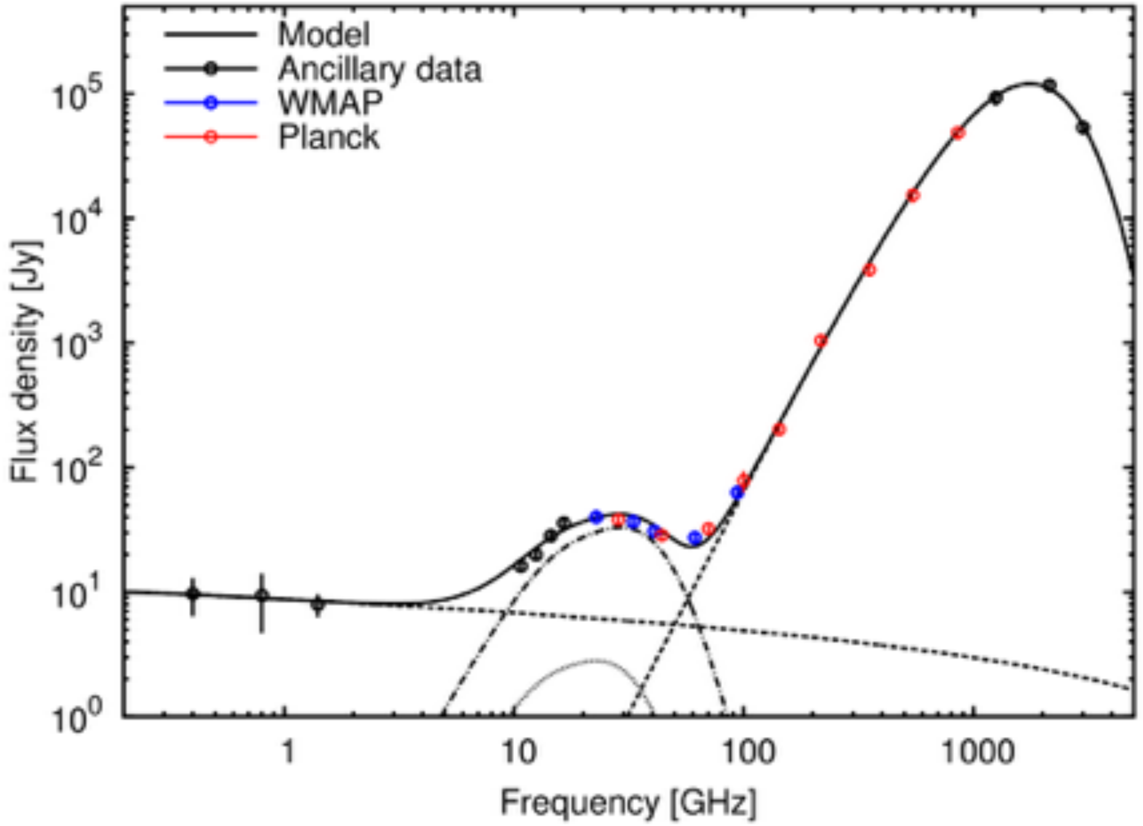


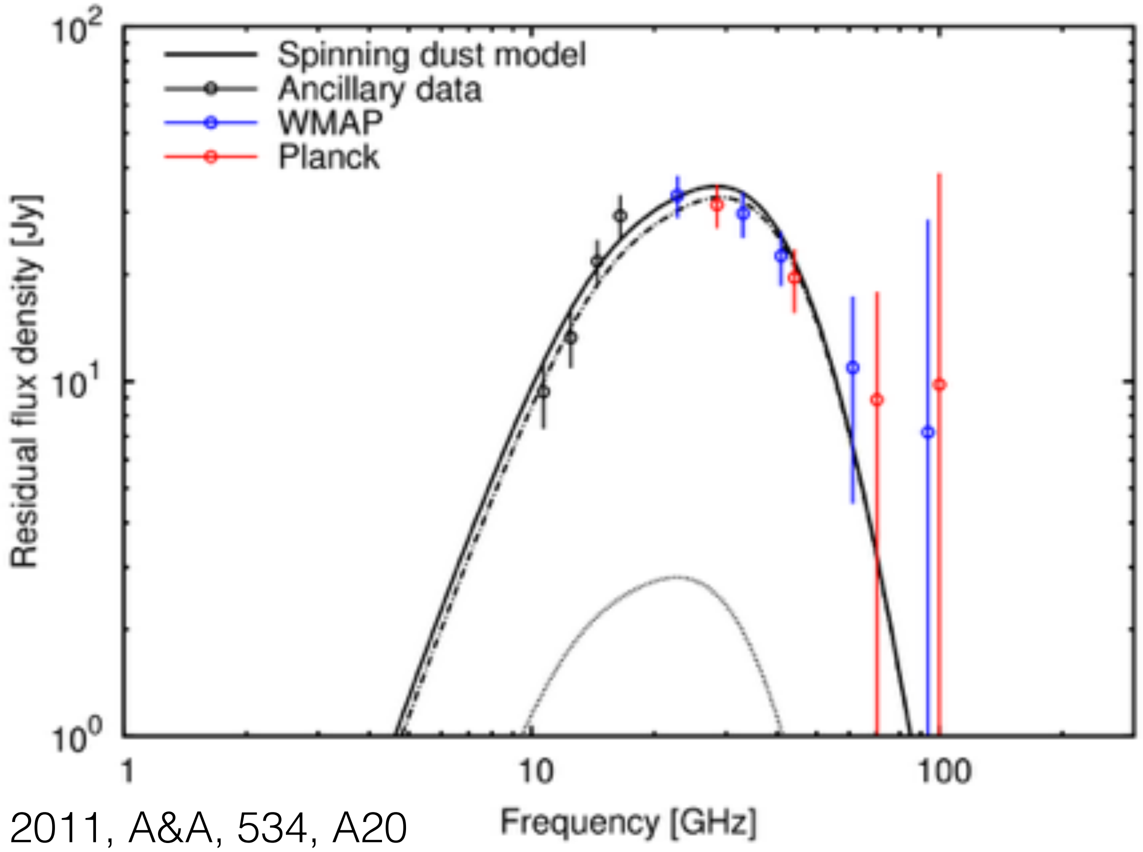


Correlation plot of 23 GHz and 100 μm (Davies et al. 2006)

Spinning dust - observations

- Observations in some regions fit the spinning dust model very well!
- Early Planck paper showed **definitive evidence for spinning dust in 2 molecular clouds**
  - Perseus
  - Rho Ophiuchi
- Strong spinning dust signature
  - **Peaked spectrum around 25-30 GHz**
- **Fitted spinning dust models (grain sizes, density, radiation field etc.) are compatible with expectations!**

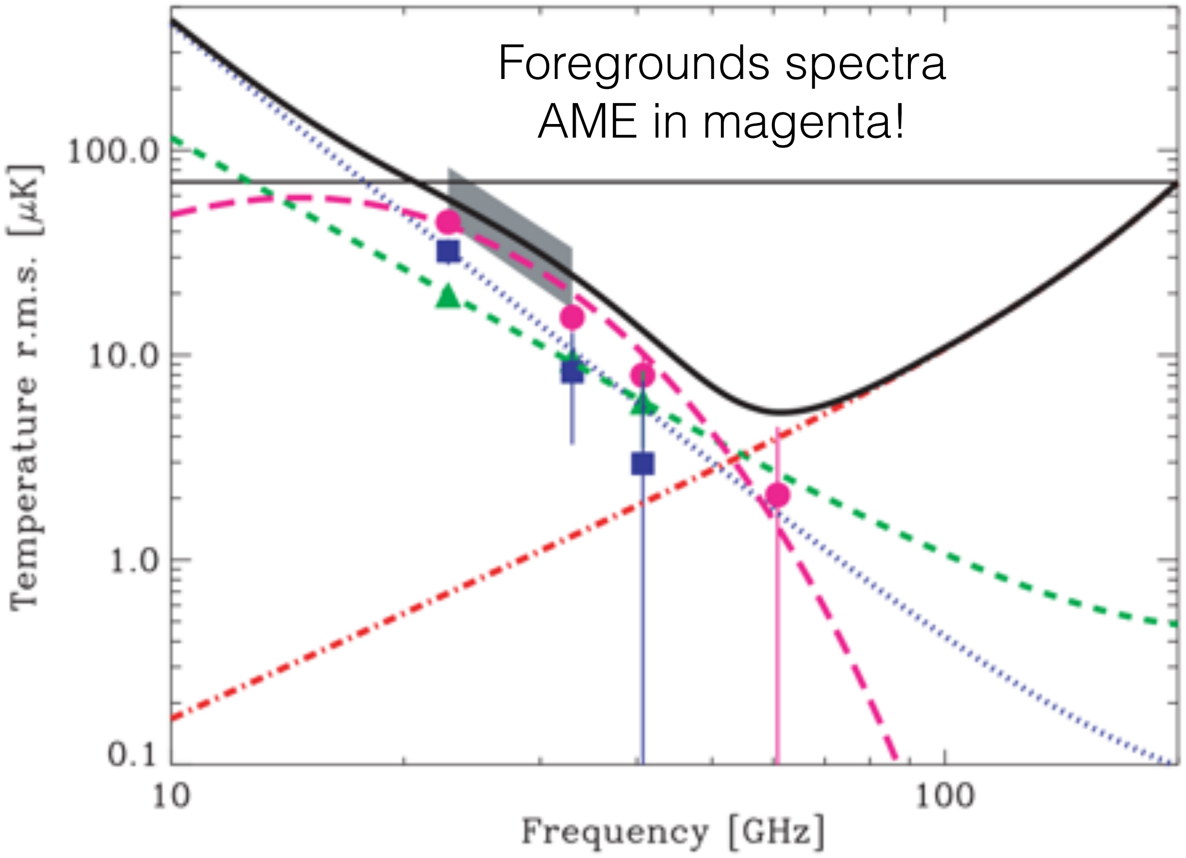




Planck collaboration, Early Paper, 2011, A&A, 534, A20

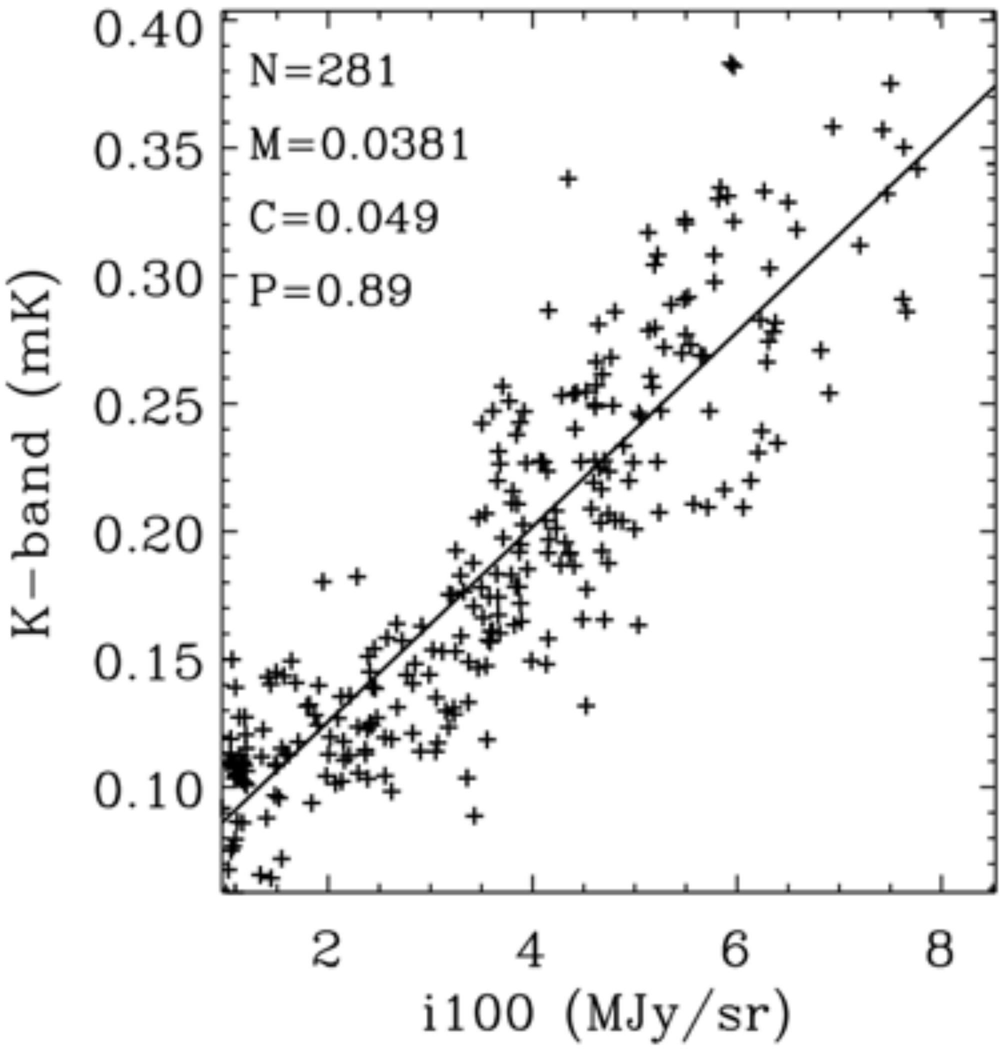
AME at high Galactic latitudes

- Still debate about diffuse AME at high Galactic latitudes
  - Spinning dust?
  - Flat spectrum synchrotron?
  - Magnetic dust?
  - Hot bremsstrahlung
  - Quantum dust
  - ….
- Difficult to separate the (relatively weak) diffuse components and CMB!
- N.B. AME may be the dominant foreground at 20-60 GHz !



Foregrounds spectra
AME in magenta!

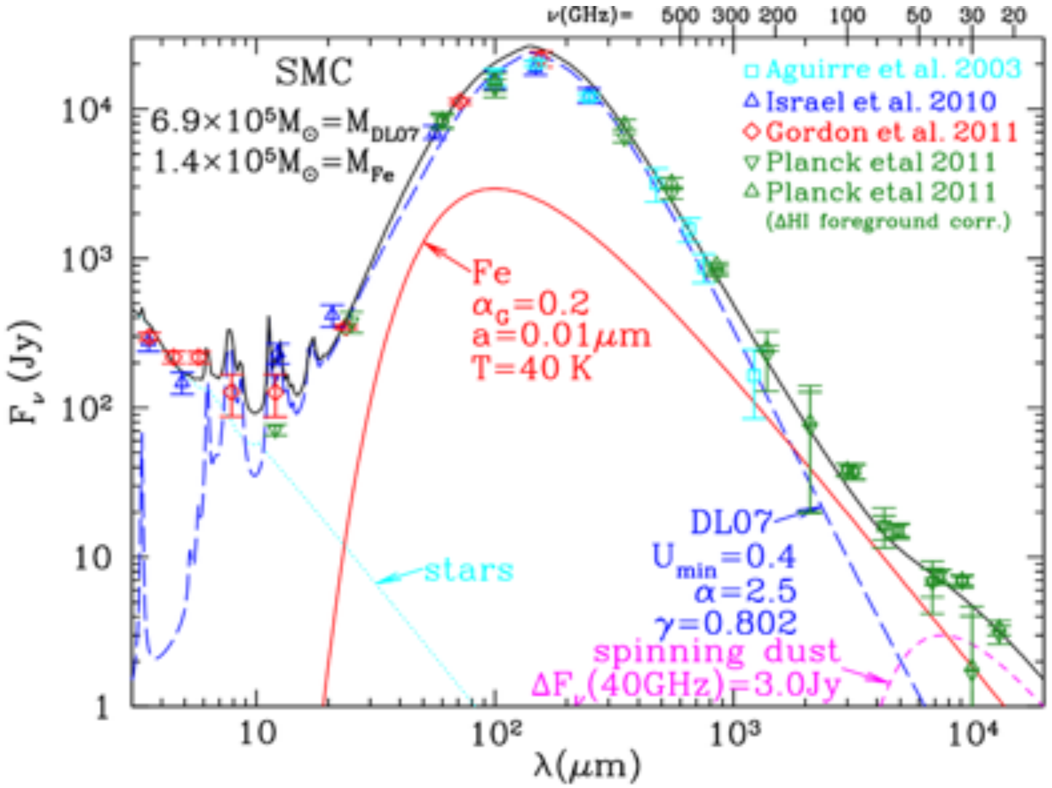
Correlation plot of 23 GHz and 100 μm
(FIR thermal dust) on random bit of sky



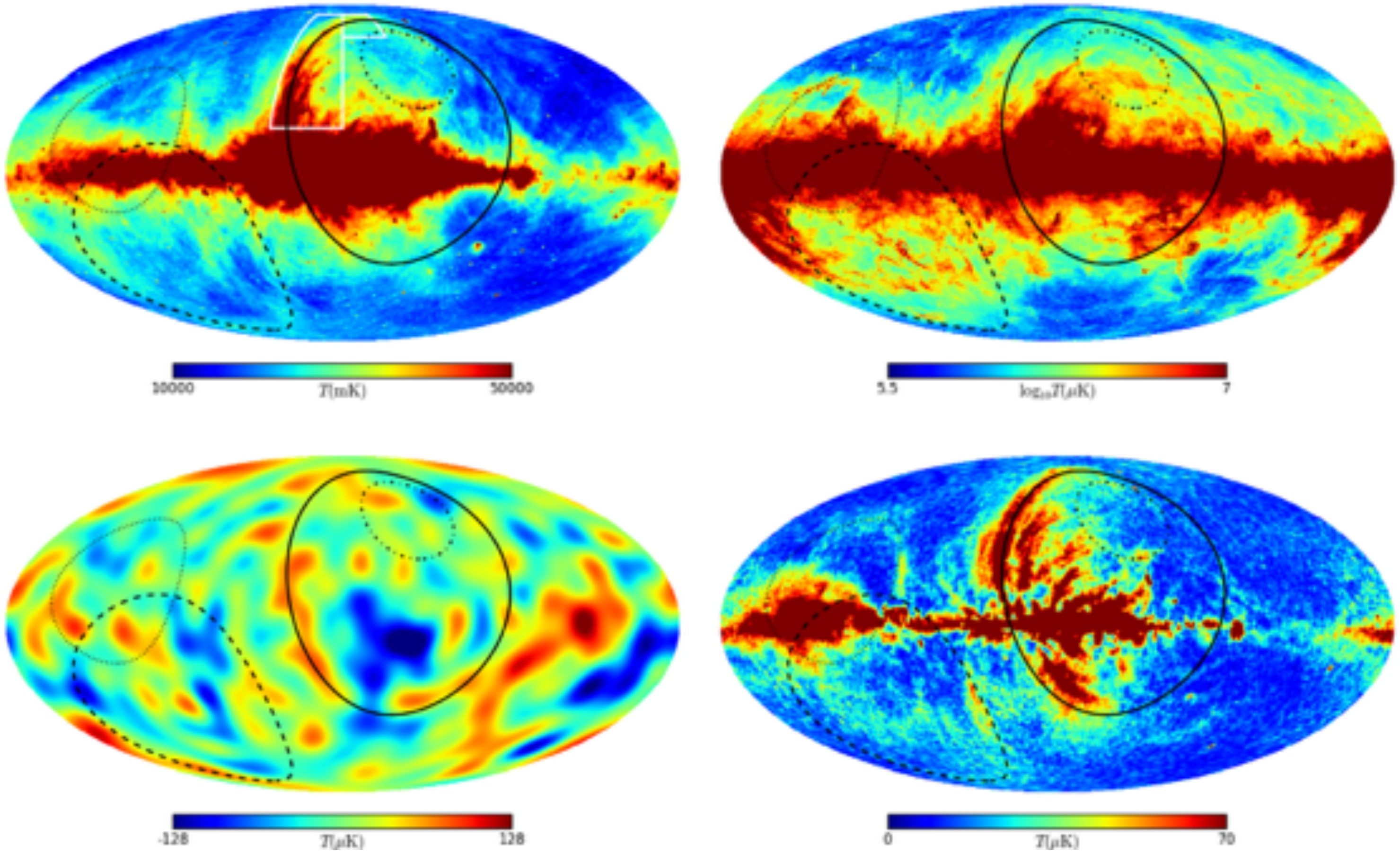
Davies et al. (2006)

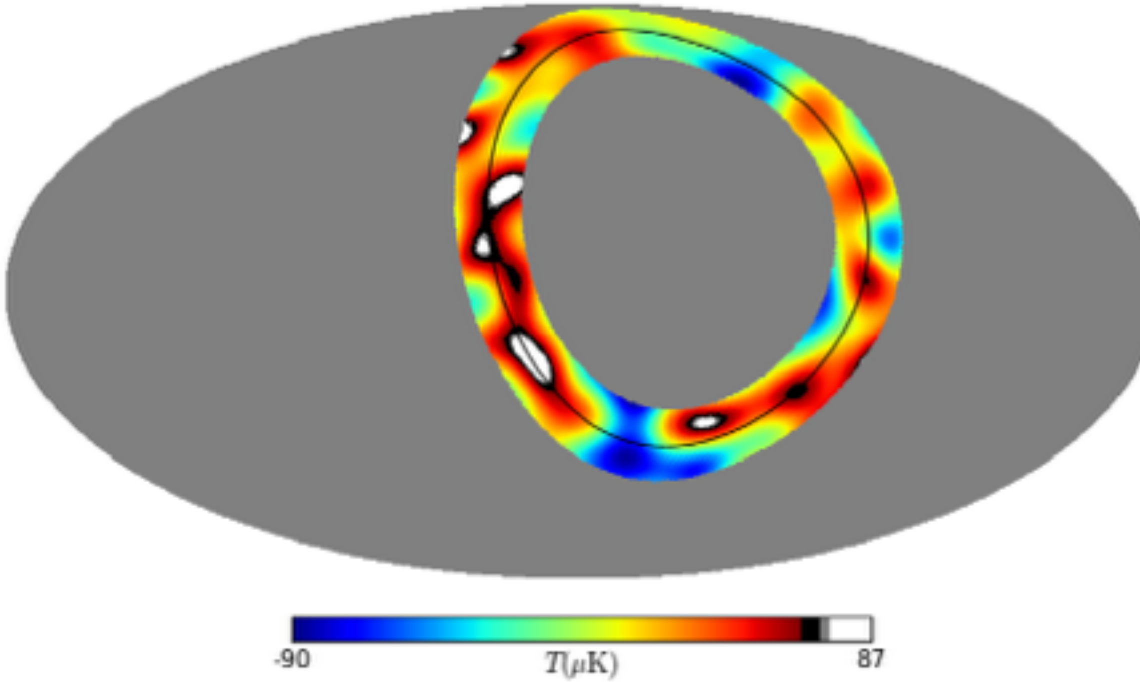
Magnetic dipole radiation

- No hard evidence for magnetic dipole emission
    - BUT, some interesting hints:
- Excess emission at ~30-300 GHz in SMC can fitted with component of magnetic dipole emission
- Flatenning of sub-mm index at 100-353 GHz (Planck Collaboration et al. 2014)
- Excess power associated with loops? (Liu et al. 2014)
- **Requires (Planck) polarisation data to confirm origin**



Draine & Hensley (2013)



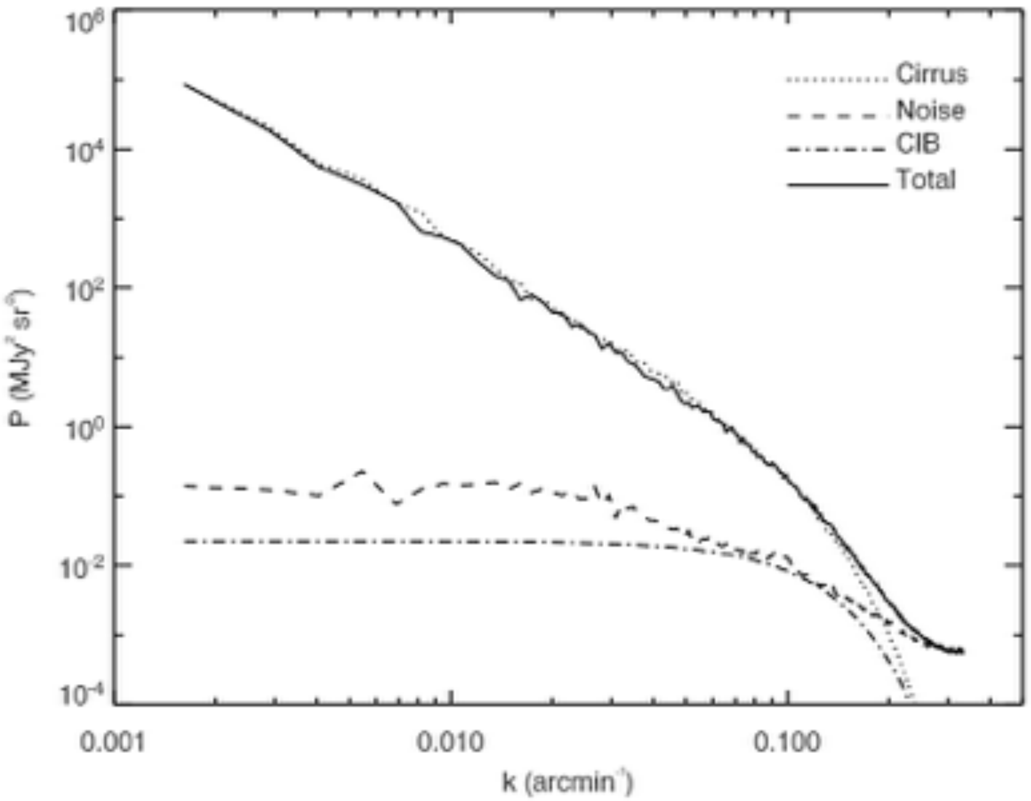


Liu et al. (2014)

Power spectrum of foregrounds

- Diffuse foregrounds are by their nature stronger on larger angular scales (low-l)
- Galactic plane and ~cosec(b) dominates with large-scale features superimposed
    - Loops, filaments etc.
- Small-scales likely driven by turbulence in the ISM
- Diffuse foregrounds power spectra typically power-laws with
    - $C_l \propto -3.0$
    - Power $C_l\, l(l+1) \propto -1.0$
    - Dominant on scales >~1 deg
- Extragalactic sources
    - $C_l \propto 0.0$ (flat, like white noise)
    - Power $\propto +2.0$
    - Dominant on small scales (<1 deg)

Power spectrum of typical IRAS (100um) map
(Miville-Deschenes et al., 2007)



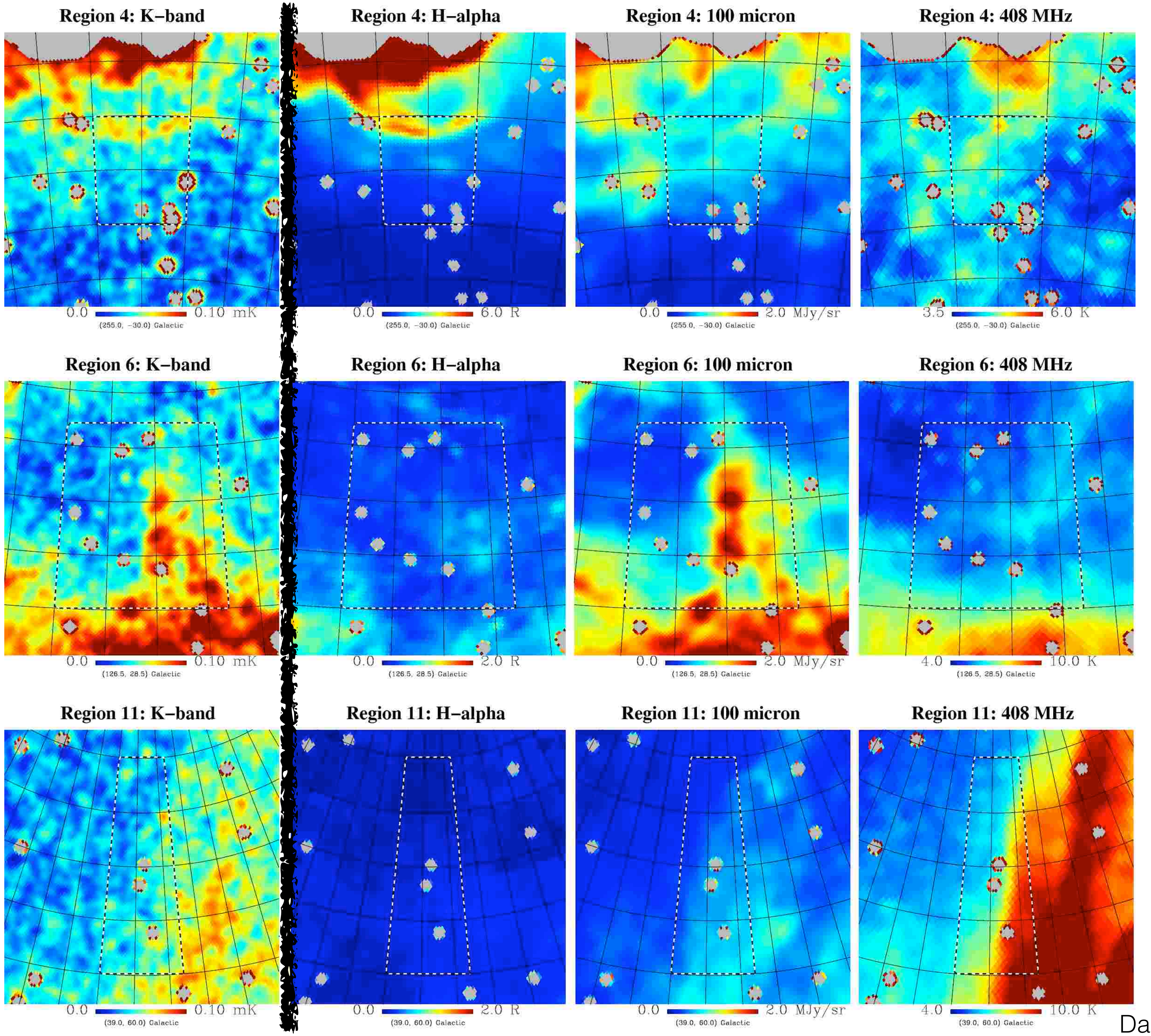
4. Impact on CMB data

Visualisation of diffuse foreground components

Free-free

Dust

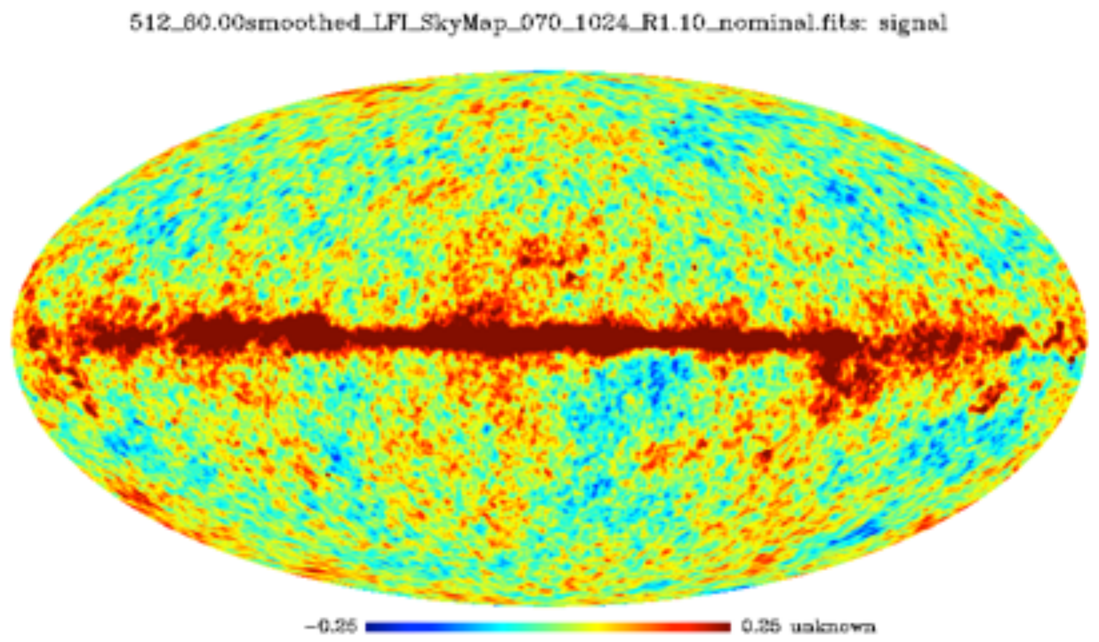
Synchrotron



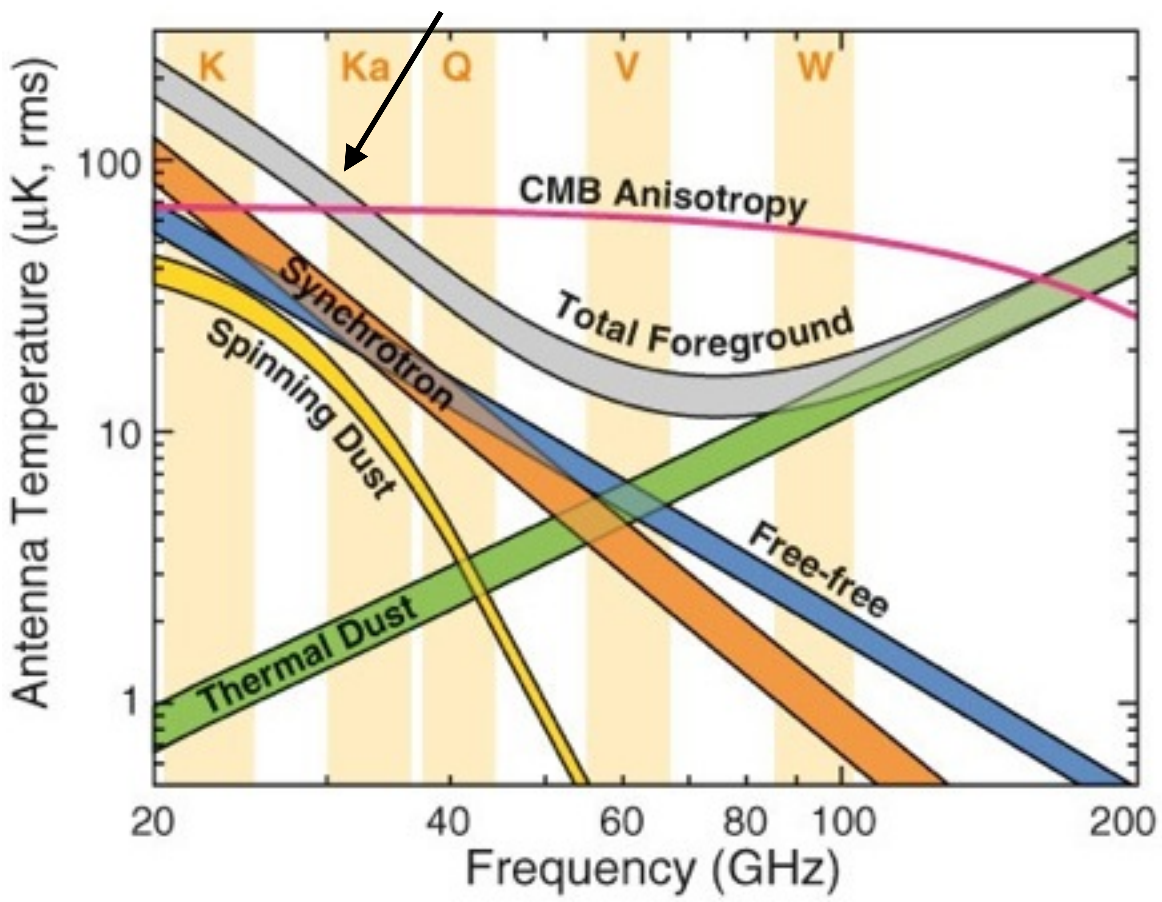
Davies et al. (2006)

Impact on CMB data - not too bad for T

- CMB window allows us to peer through our Universe back to z~1000
  - CMB dominates ~50% of sky at ~ 1deg scales and frequencies ~30-200 GHz
  - CMB visible in raw (calibrated) maps!
- Typically requires removal by a factor of a few in amplitude (~10 in power)
  - Plus some masking of the plane and brightest regions
- **Combined low frequency spectrum is approximately a power-law! (Planck Collaboration 2013, XII)**
  - Makes it easier for component separation algorithms to remove it
- However, separating the individual low frequency components not so easy!



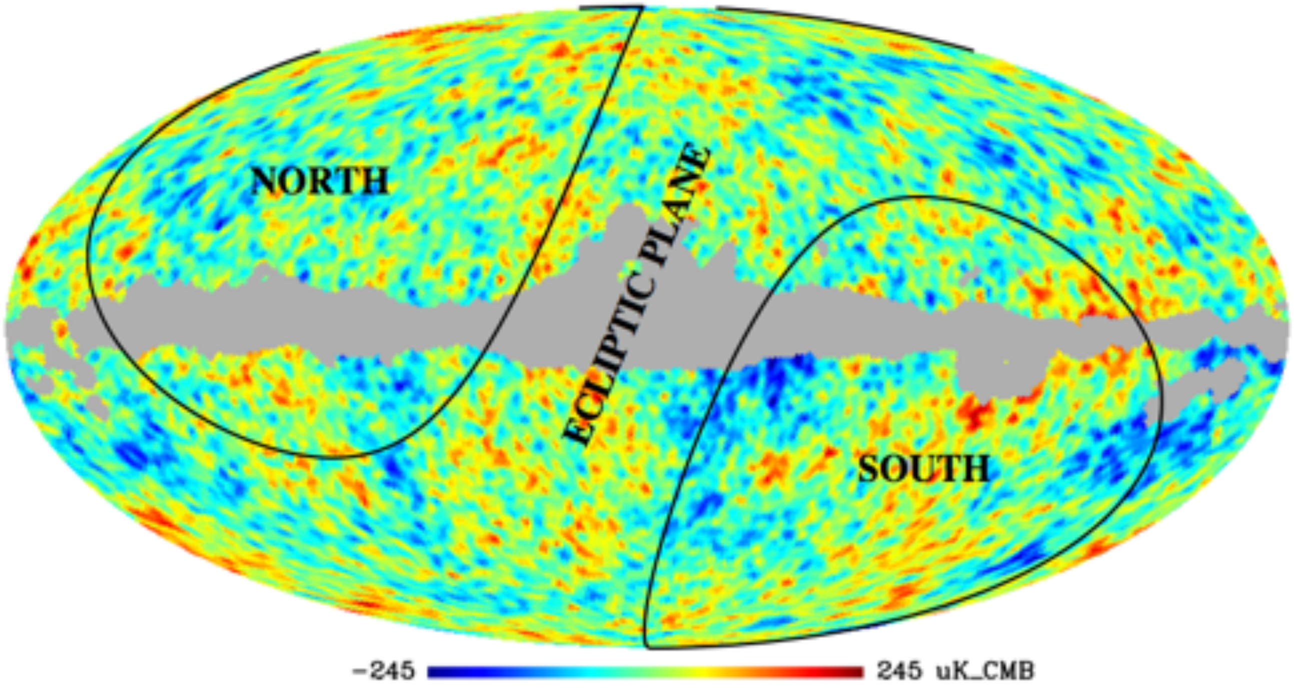
Planck LFI 70 GHz map smoothed to 1 deg

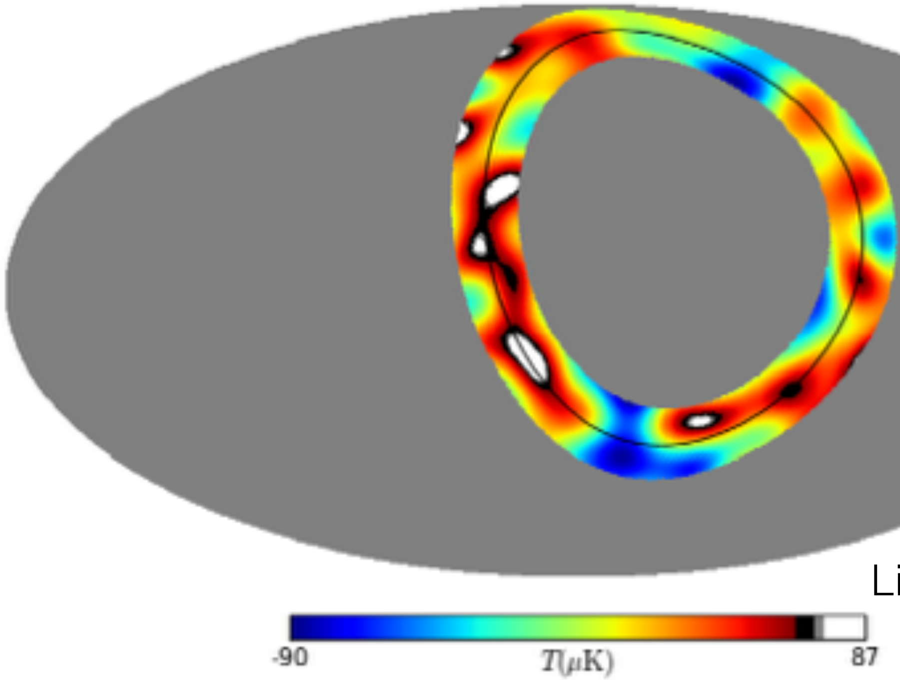


Total low frequency foreground ~power-law!

Anomalies don’t appear to be compromised

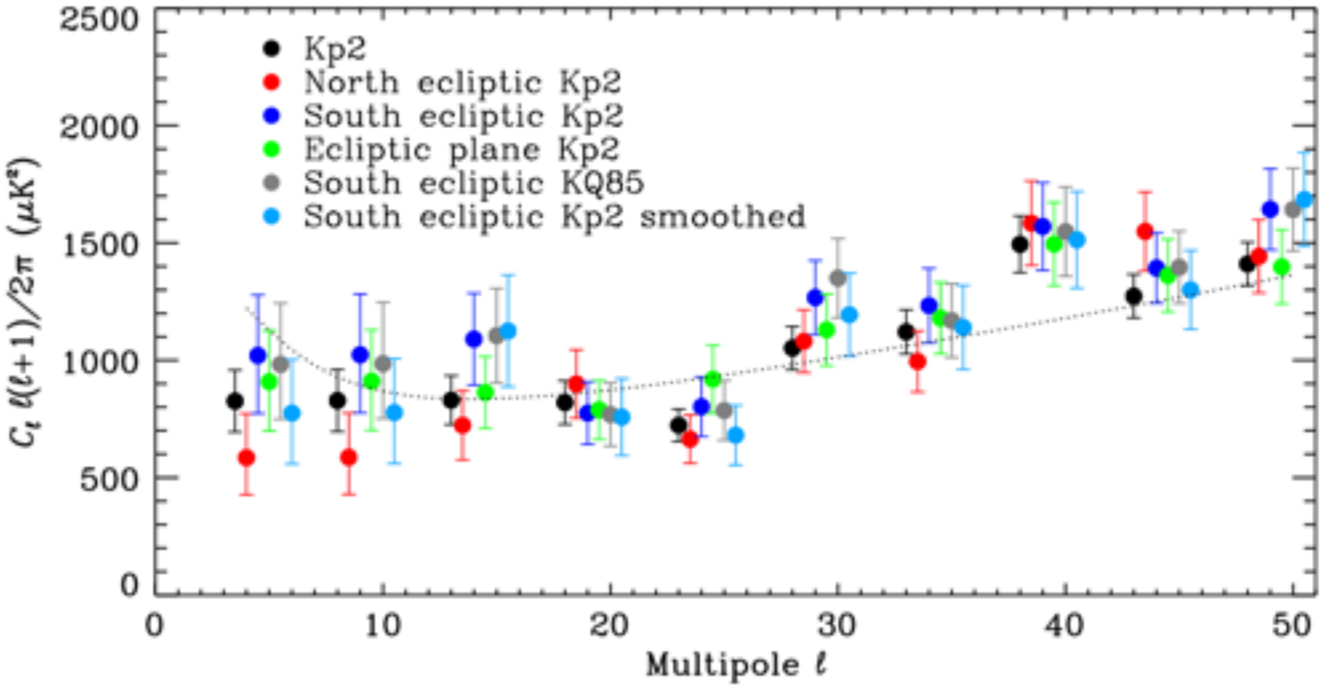
- **Anomalies appear to be robust to foreground analyses**
  - Low quadrupole
  - Glitches in power spectrum
  - N/S asymmetry
  - Quadrupole/octapole alignments
  - Cold spot
- But there have been some claims w.r.t. this, especially along ecliptic plane
  - **We should keep our minds open to new foregrounds!** (e.g. see Liu et al. 2014)





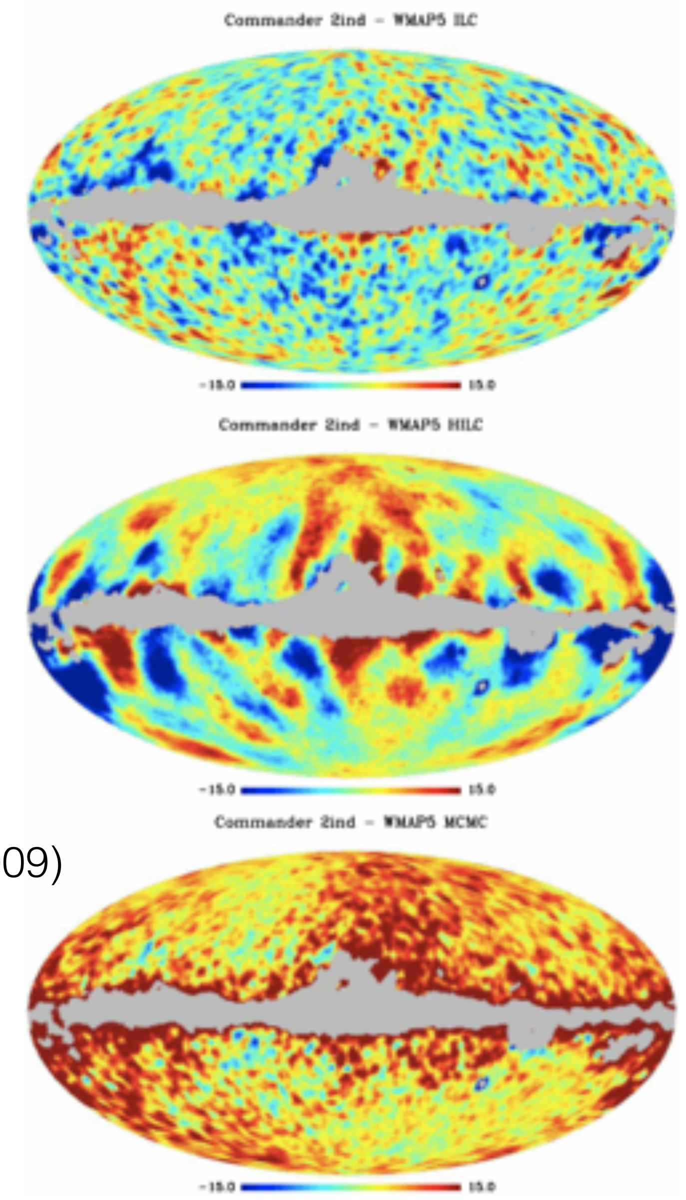
Liu et al. (2014)

Re-analysis of WMAP data using Commander Gibbs sampling component separation algorithm shows robustness w.r.t. foreground model (Dickinson et al. 2009)



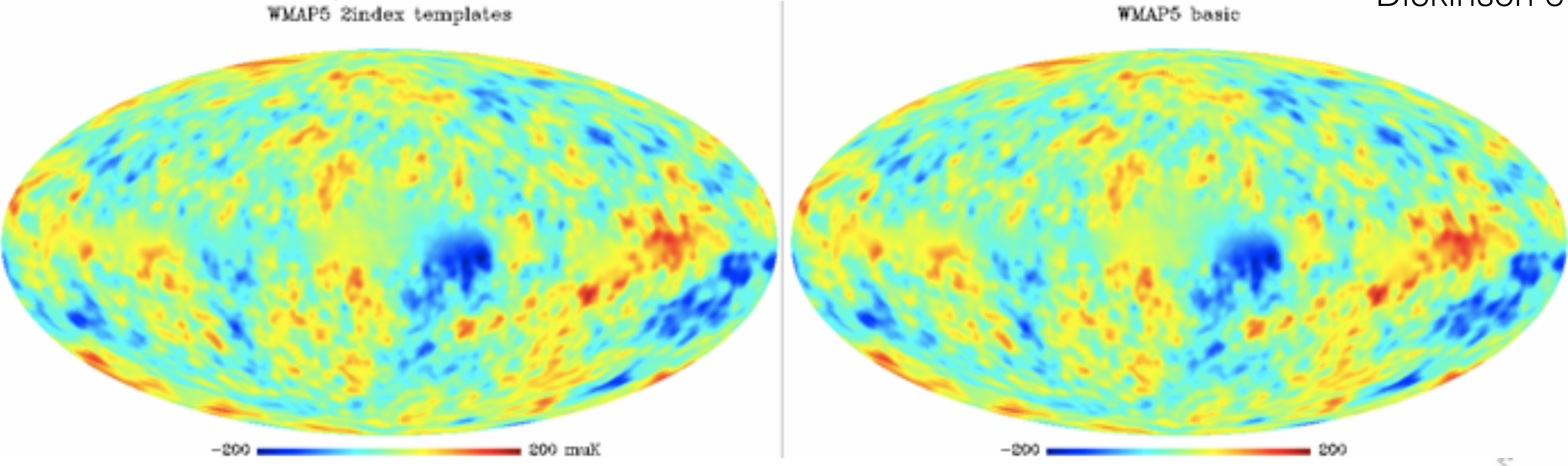
Impact on CMB data - a warning!

- Foregrounds are generally small after component separation
    - But they are there! Typically at ~10 μK level!
    - Usually not an issue, but should be kept in mind and tested for!



Can you see the difference between these 2 maps?

Dickinson et al. (2009)



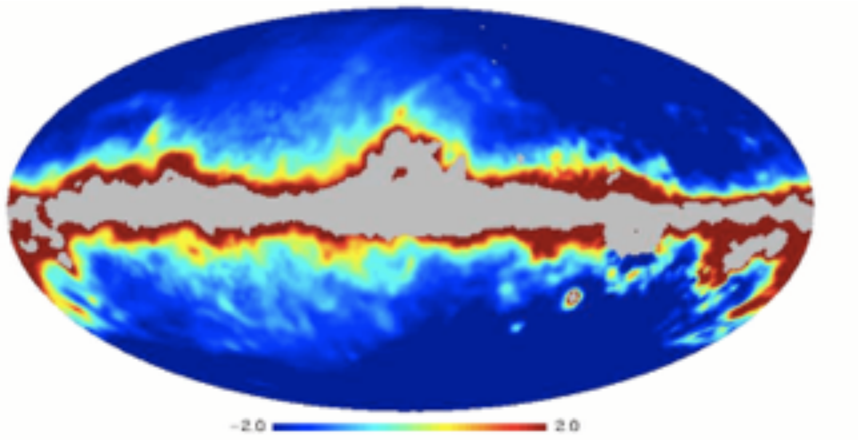
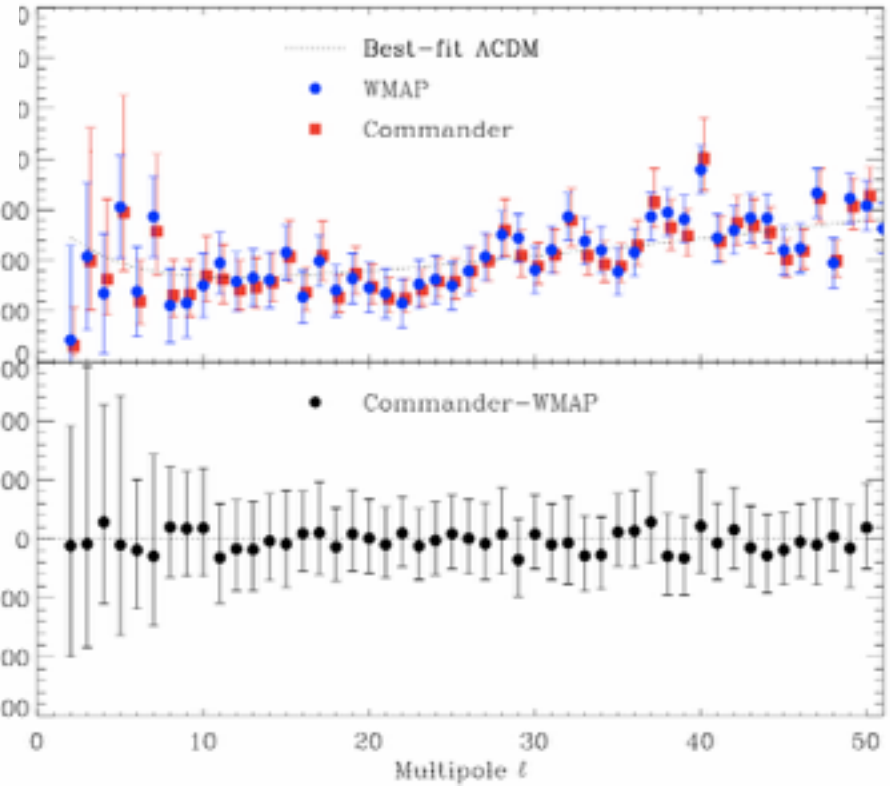
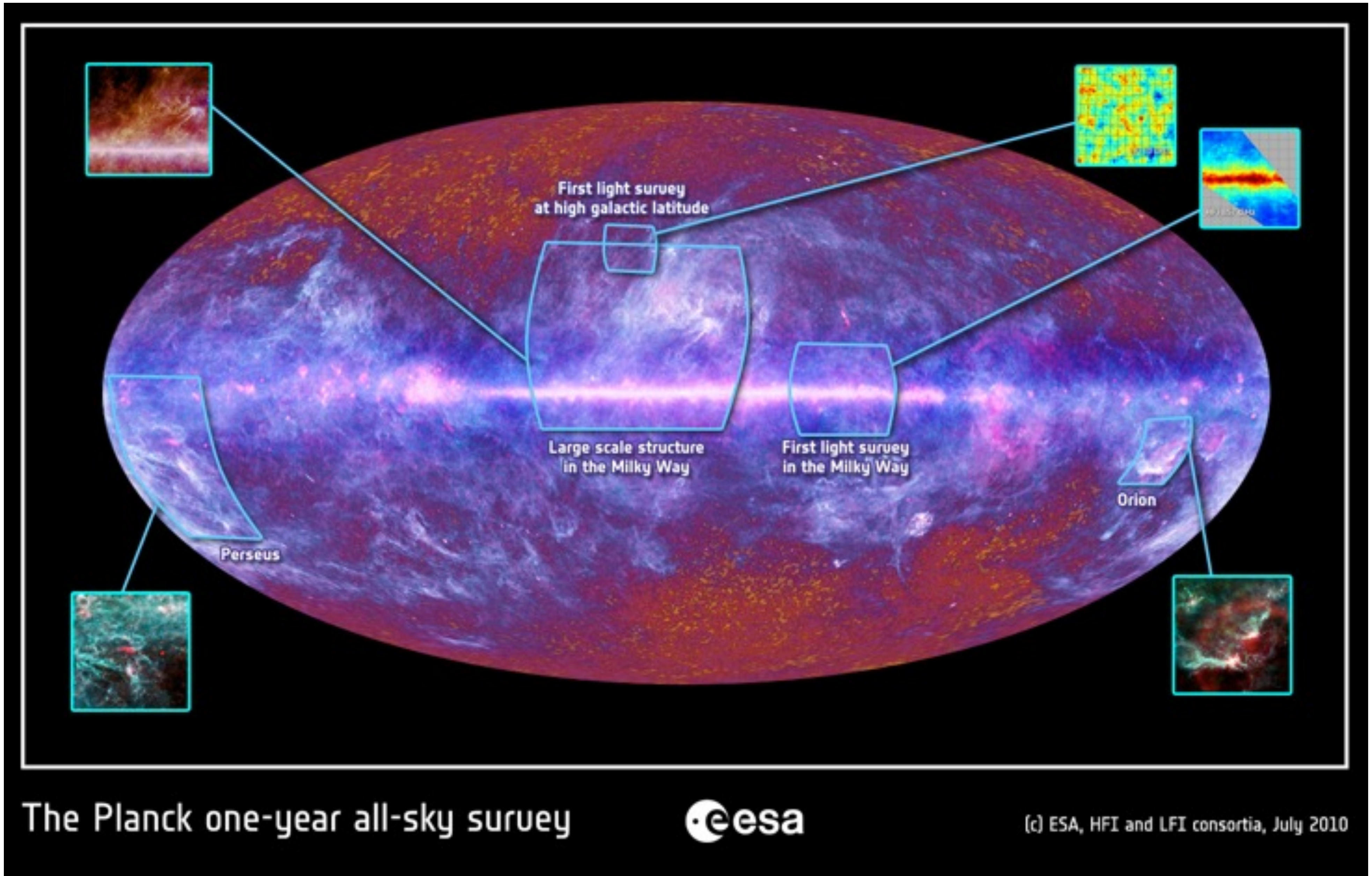


FIG. 9.— Difference map of the mean CMB maps from the basic *WMAP5* model (1 template) and when an additional Hα template is included (2 templates). Units are μK.



Summary

- **Diffuse low frequency foregrounds** consist of several distinct emission processes which dominate on large angular scales
  - **Synchrotron** radiation - steep spectrum (β=-3.0 with variation Δβ~0.2)
  - **Free-free** radiation - flat spectrum (β=-2.1)
  - **AME** - likely due to **spinning dust grains** with peaked spectrum ~30 GHz in flux density
  - Possible contribution from **magnetic dipole radiation**
- Important for achieving the most precise CMB maps and cosmological information
- **Foregrounds are very interesting in their own right!**
  - Learn a lot about physics, structure of our Galaxy/ISM etc! (many of the Planck papers are about foregrounds!)



Galactic foreground templates

- Use spatial “**template**” maps to model each component
  - Synchrotron: Low frequency (408 MHz) map
  - Free-free: Hα data
  - AME: FIR thermal dust emission models (e.g. 100 um)
- Fit for the amplitudes for a given region, taking into account the noise and the CMB (covariance matrix)

$$c_\nu = c_{CMB} + c_N + \alpha_X c_X$$

$$\chi^2 = (c_\nu - \alpha_X c_X)^T M^{-1} (c_\nu - \alpha_X c_X)$$

$$\alpha_X = c_X M^{-1} c_\nu / c_X M^{-1} c_X$$