NeQuick model and data assimilation

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NeQuick

• The NeQuick is an ionospheric electron density model developed at the T/ICT4D Laboratory of The Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy, and at the Institute for Geophysics, Astrophysics and Meteorology (IGAM) of the University of Graz, Austria.

• It is a quick-run empirical model particularly designed for trans-ionospheric propagation applications, conceived to reproduce the median behavior of the ionosphere ("climate").

• http://t-ict4d.ictp.it/nequick2
NeQuick 2

• The model profile formulation includes 6 semi-Epstein layers with modeled thickness parameters and is based on anchor points defined by foE, foF1, foF2 and M(3000)F2 values.

• These values can be modeled (e.g. ITU-R coefficients for foF2, M(3000)F2) or experimentally derived.

• NeQuick inputs are: position, time and solar flux; the output is the electron concentration at the given location and time.

• NeQuick package includes routines to evaluate the electron density along any “ground-to-satellite” ray-path and the corresponding Total Electron Content (TEC) by numerical integration.

Epstein function

\[ N(h; h_{\text{max}}, N_{\text{max}}, B) = \frac{4N_{\text{max}}}{(1 + \exp\left(\frac{h - h_{\text{max}}}{B}\right))^2} \exp\left(\frac{h - h_{\text{max}}}{B}\right) \]
NeQuick 2 formulation

The model is represented by a sum of Epstein functions for the E, F1 and F2 layers:

\[ N_{\text{bot}}(h) = N_{E}(h) + N_{F1}(h) + N_{F2}(h) \]

where

\[ N_{E}(h) = \frac{4Nm^*E}{\left(1 + \exp\left(\frac{h-hmE}{BE}\xi(h)\right)\right)^2} \exp\left(\frac{h-hmE}{BE}\xi(h)\right) \]

\[ N_{F1}(h) = \frac{4Nm^*F1}{\left(1 + \exp\left(\frac{h-hmF1}{B1}\xi(h)\right)\right)^2} \exp\left(\frac{h-hmF1}{B1}\xi(h)\right) \]

\[ N_{F2}(h) = \frac{4NmF2}{\left(1 + \exp\left(\frac{h-hmF2}{B2}\right)\right)^2} \exp\left(\frac{h-hmF2}{B2}\right) \]
NeQuick 2 formulation

with

\[ Nm^*E = NmE - N_{F1}(hmE) - N_{F2}(hmE) \]
\[ Nm^*F1 = NmF1 - N_E(hmF1) - N_{F2}(hmF1) \]

and

\[ \xi(h) = \exp \left( \frac{10}{1 + 1|h - hmF2|} \right) \]

is a function that ensures a “fade out” of the E and F1 layers in the vicinity of the F2 layer peak in order to avoid secondary maxima around hmF2.
NeQuick 2 formulation

The model topside is represented by a semi-Epstein layer with a height-dependent thickness parameter $H$:

$$N (h) = \frac{4NmF^2}{(1 + \exp(z))^2} \exp(z)$$

with

$$z = \frac{h - hmF^2}{H}$$

$$H = H_0 \left[1 + \frac{rg (h - hmF^2)}{rH_0 + g (h - hmF^2)} \right]$$
NeQuick 2 formulation

Peak heights

\[ hmE = 120 \]
\[ hmF1 = \frac{hmE + hmF2}{2} \]
\[ hmF2 = \frac{1490MF}{M + \Delta M} - 176 \]

with

\[ \Delta M = \begin{cases} 
0.253/(foF2/foE) \\
-1.215 - 0.012, \\
-0.012 & \text{if } foE = 0,
\end{cases} \]
\[ MF = M\sqrt{\frac{0.0196M^2 + 1}{1.2967M^2 - 1}} \]
\[ M = M(3000)F2. \]
NeQuick 2 formulation

Thickness parameters

\[ BE_{bot} = 5 \]
\[ BE_{top} = \max (0.5 \, (hmF1 - hmE) , 7) \]
\[ B1_{bot} = 0.5 \, (hmF1 - hmE) \]
\[ B1_{top} = 0.3 \, (hmF2 - hmF1) \]
\[ B2_{bot} = \frac{0.385 NmF2}{(dN/dh)_{\text{max}}} \]
\[ H = k B2_{bot} \left[ 1 + \frac{rg \, (h - hmF2)}{rk B2_{bot} + g \, (h - hmF2)} \right] \]
NeQuick 2 formulation

where

$$\ln \left( \left( \frac{dN}{dh} \right)_{max} \right) = -3.467 + 1.714 \ln (f_0 F^2) + 2.02 \ln (M(3000) F^2)$$

and

$$k = 3.22 - 0.0538 f_0 F^2 - 0.00664 hm F^2 + 0.113 \frac{hm F^2}{B_{2,bot}} + 0.00257 R_{12}$$
NeQuick 2 formulation

Critical frequencies and propagation factor

\[(foE)^2 = \left(a_e \sqrt{F107}\right)^2 \left(\cos \chi_{eff}\right)^{0.6} + 0.49\]

\[foF1 = \begin{cases} 
1.4 \ foE & \text{if} \ foE \geq 2 \\
0 & \text{if} \ foE < 2 \\
0.85 \ 1.4 \ foE & \text{if} \ 1.4 \ foE > 0.85 \ foF2 
\end{cases}\]

\[foF2\] modeled in terms of ITU – R coefficients

\[M = M(3000)F2.\] modeled in terms of ITU – R coefficients
Empirical models like NeQuick have been developed as climatological models, able to reproduce the typical median condition of the ionosphere.

For research purposes and practical applications, in order to pass from “climate” to “weather”, there is a need to have models able to reproduce the current conditions of the ionosphere.

Considering that there is an increasing availability of experimental data even in real time (ground and space-based GNSS, ionosondes), several assimilation schemes have been developed. They are of different complexity and rely on different kinds of data.
Data Assimilation

• “Data assimilation is an analysis technique in which the observed information is accumulated into the model state by taking advantage of consistency constraints with laws of time evolution and physical properties” (Bouttier, and Courtier, 1999).

• "Data assimilation is fundamentally a model specification and prediction technique that uses data to improve the fidelity of the model" (Bust and Mitchell, 2008).

• The ionospheric data assimilation schemes are based on different mathematical techniques; the most widely used are:
  • variational techniques (3D-VAR and 4D-VAR)
  • Kalman filters (extended Kalman filter (EKF), Ensemble Kalman filter (EnKF))
Assimilation schemes (example)

- Utah State University (USU) Global Assimilation of Ionospheric Measurements (GAIM) [Schunk et al., 2004] or the Jet Propulsion Laboratory (JPL)/University of Southern California (USC) Global Assimilative Ionospheric Model (GAIM) [Wang et al., 2004], or [Schunk et al., 2014], for example, are based on assimilation of data originating from different sources and imply the use of first principle models.
Assimilation schemes (example)

• The Electron Density Assimilative Model (EDAM) [Angling and Khattatov, 2006; Angling, M. J., and N. K. Jackson-Booth, 2011] provides a mean to assimilate ionospheric measurements into a background ionospheric model.

• Assimilated data are: ground-based and space-based GPS-derived TEC, ionosondes-derived parameters
  • Currently IRI is used as a background model (electron density only)

• Extended, localised Gauss Markov Kalman Filter
  • BLUE + time evolution of the differences between the measurements and the background ionosphere
  • Model variances are propagated
  • Covariance are estimated as required
Assimilation schemes (example)

- The Multi Instrument Data Analysis System (MIDAS) [Mitchell C. N. and Spencer P. S. 2003] is a tomographic approach where TEC data are inverted to evaluate the distribution and time evolution of electron concentration.

- Orthonormal basis functions and SVD are used to solve the inverse problem.

Assimilation schemes (example)

- TOMographic IONosphere model (TOMION), [Hernández-Pajares, M. et al., 1999] generates Global Ionospheric Maps (GIMs) of vertical TEC and includes an interpolation module using Kriging technique [Orús et al., 2005]. The ionosphere is represented by two or more layers of voxels and in each voxel the electron density is assumed to be constant. No background model is used.
Assimilation schemes (example)

• IRI Real Time Assimilative Model (IRTAM) [Galkin, I. A., et al. 2012], has been developed to assimilate Global Ionosphere Radio Observatory (GIRO) data (foF2, hmF2) in order to “update” the IRI electron density distribution, while preserving the IRI’s typical ionospheric feature representations.

• The technique calculates the corrected coefficients for the spherical/diurnal expansion used by the CCIR-67/URSI-88 model to specify the global foF2 maps, and similarly the maps for all other IRI profile parameters.
Assimilation schemes (example)

• A similar approach has been used by Brunini et al., [2013] in order to update the ITU-R database using radio occultation (COSMIC) electron density profiles.

• For this purpose the La Plata Ionospheric Model (LPIM) (after linearisation) is adjusted by Least Squares to every RO profile available for the time period of interest.

Global representation of the NmF2 estimated value within the 18–20 UT interval
a) NmF2 for the 2007 September equinox c) NmF2 for the 2011 December solstice
Assimilation schemes (example)

- The Ionospheric Data Assimilation Three-Dimensional (IDA3D), [Bust et al., 2004] uses a three-dimensional variational data assimilation technique (3DVAR).

- It is capable of incorporating ground based and space based GPS-TEC measurements and electron density measurements from radars and satellites.

- The background specification is based upon empirical ionospheric models, but IDA3D is capable of using any global ionospheric specification as a background. IDA3D produces a spatial analysis of the electron density distribution at a specified time. A time series of these specifications can be created using past specifications to determine the background for the current analysis.
Assimilation schemes (example)

TRAIN – A realtime ionospheric DA model

- TRAIN is based on the Malvern Mathematical Data Assimilation Model (MMaDAM)
  - Generalised data assimilation system
  - Hybrid approach of ensemble Kalman filter (EnKF) and particle filter (PFs)
- Background model
  - Ensemble of NeQuick models
- Observation operator
  - Direct integration through each ensemble member
- Forward operator
  - Collapse ensemble to the mean
  - ICTP method for NeQuick forecast

TRAIN electron density grid for 1200 UT, 4/6/2017 assimilating TEC data from ground based GPS receivers, a shipborne GPS receiver and a space based dual frequency altimeter.
Use of effective parameters

Effective parameters

• One of the first effective parameter that has been proposed is the “effective sunspot number” (SSNe). This parameter valid for a set of foF2 observations has been defined as the SSN value that, when used as input to the URSI foF2 model, gives a weighted zero-mean difference between the observed and the modeled foF2 values.

• T-index
  http://www.sws.bom.gov.au/Educational/5/2/1
  The T index can be best regarded as an "equivalent sunspot number" - the sunspot number which would best match the observations made by ionosondes.
Effective parameters

- IRI IG 12
  http://gauss2.gge.unb.ca/grads/attila/papers/52am/ion52am.pdf
  An effective sunspot number (inferred IG12 index) is used which is defined as the product of the IG12 index and the scaling factor.

- Klobuchar-Style Ionospheric Coefficients
  http://aiuws.unibe.ch/ionosphere/#cgim
  Klobuchar-style alpha and beta coefficients best fitting VTEC data.

Effective parameters can be defined using diverse kind of data/models.
NeQuick features relevant to implement adaptation techniques based on the use of effective parameters.

• For a given epoch & ray-path the model TEC is a monotonic function of the solar activity index, that can be regarded as an “effective ionization level” parameter.

∀ link, minimize TEC mismodeling → effective F10.7 (grid)

Use NeQuick to reconstruct the 3D electron density of the ionosphere and the other relevant parameters where needed
vTEC map

grid points:
lat. = -90°, 90° step 2.5°
lon. = -180°, 180° step 5°
Reconstructed foF2 map

grid points:
lat.=-90°, 90° step 2.5°
lon.=-180°, 180° step 5°
NeQuick2: validation results (example: HSA)

foF2 error statistics

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<td>99%</td>
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Nava et al. (2011)
STEC data ingestion; error statistics (000405)

Nava et al., 2006
foF2 data ingestion; error statistics, Sep 2011

Reconstructed foF2

Elvidge et al. (2014)

"Weather"
Adapting NeQuick model to experimental slant TEC and foF2 data at a given location

(Use of slab thickness to constrain the NeQuick profile shape parameter)
At a given epoch:

1. **One GPS receiver**
   - n experimental slant TEC (n satellites)

2. **One ionosonde**
   - One experimental foF2
     - Using ITU-R coeff., minimize foF2 mismodeling as a function of (formally) F10.7
     - Az_foF2 (effective parameter related to foF2)

3. **One experimental hmF2**
   - Using Dudeney formula & ITU-R coeff., minimize hmF2 mismodeling as a function of (formally) F10.7
   - Az_hmF2 (effective parameter related to hmF2)

4. Using NeQuick with Az_foF2 and Az_hmF2 (to locally constrain model peak values), minimize RMS of TEC mismodelings as a function of the model thickness parameter B2bot

5. Obtain the correction factor for B2bot for the area of interest, at the given epoch

6. Use NeQuick to retrieve (locally) the 3D electron density of the ionosphere
Remarks

• The use of two effective parameters has been considered in order to use the ITUR coefficients to estimate foF2 and hmF2 in a region surrounding the ground station.

• In this way the peak parameter values can be estimated for a slant TEC computation.

Use JRO profiles to simulate the process of adapting NeQuick to GPS derived TEC and ionosonde peak parameters data.

TEC and peak parameters are known from the profile.

After model adaptation it is possible to compare profiles in order to evaluate the adaptation technique effectiveness.
Adaptation method validation

Jicamarca Radio Observatory (JRO) location
Adaptation method validation

Model: NeQuick
NeQuick adaptation to foF2 & hmF2

Ionosonde-derived Ne profiles are obtained with AUTOSCALA

AUTomatic Scaling of Polar Ionograms and Cooperative Ionospheric Observations (AUSPICIO) Project

Pietrella et al., 2017
NeQuick adaptation to foF2 & hmF2

Pietrella et al., 2017
NeQuick adaptation to foF2, hmF2 (and Tau$_{top}$)

-Tau$_{top}$ >

- Tau$_{top}$ =

- Tau$_{top}$ <

**COSMIC**

**ICTP**

**NeQuick (ing. hmF2, foF2)**

**NeQuick (ing. hmF2, foF2, Tau)**
Least Square Estimation
Least Square Estimation

Best Linear Unbiased Estimator (BLUE)*

\( x_t \) true model state (dimension \( n \))
\( x_b \) background model state (dimension \( n \))
\( x_a \) analysis model state (dimension \( n \))
\( y \) vector of observations (dimension \( p \))
\( H \) observation operator (dimension \( p \times n \))
\( B \) covariance matrix of background errors \( \varepsilon_b = (x_b - x_t) \) (dimension \( n \times n \))
\( R \) covariance matrix of observation errors \( \varepsilon_o = (y - H[x_t]) \) (dimension \( p \times p \))
\( A \) covariance matrix of analysis errors \( \varepsilon_a = (x_a - x_t) \) (dimension \( n \times n \))

Least Square Estimation

- The following hypotheses are assumed:

  - *Linearized observation operator*: the variations of the observation operator in the vicinity of the background state are linear:
    - \( \forall \, \mathbf{x} \) close enough to \( \mathbf{x}_b \), \( H(\mathbf{x}) - H(\mathbf{x}_b) = H(\mathbf{x} - \mathbf{x}_b) \) where \( H \) is a linear operator

  - *Non-trivial errors*: \( \mathbf{B} \) and \( \mathbf{R} \) are positive definite matrices.

  - *Unbiased errors*: the expectation of the background and observation errors is zero:
    - \( E[\mathbf{x}_b - \mathbf{x}_t] = E[\mathbf{y} - H[\mathbf{x}_t]] = 0 \)
Least Square Estimation

• The following hypotheses are assumed (cont.):

  • **Uncorrelated errors**: observation and background errors are mutually uncorrelated.
    
    • \( \mathbb{E}[(\mathbf{x}_b - \mathbf{x}_t)(\mathbf{y} - H[\mathbf{x}_t])^T] = 0 \)

  • **Linear analysis**: we look for an analysis defined by corrections to the background which depend linearly on background observation departures.

  • **Optimal analysis**: we look for an analysis state which is as close as possible to the true state in an r.m.s. sense (i.e. it is a minimum variance estimate).
Least Square Estimation

The optimal least-square estimator (BLUE analysis) is defined by

\[
x_a = x_b + K (y - H[x_b])
\]

\[
K = BH^T (HBH^T + R)^{-1}
\]

\[
A = (I-KH)B
\]

The BLUE analysis is equivalently obtained as a solution to the variational optimization problem:

\[
x_a = \text{Arg min}_x J
\]

\[
J(x) = (x - x_b)^T B^{-1} (x - x_b) + (y - H[x])^T R^{-1} (y - H[x])
\]

\[
= J_b(x) + J_o(x)
\]

where \( J \) is called the cost function of the analysis

\( J_b \) is the background term; \( J_o \) is the observation term

If the background and observation errors pdfs are Gaussian, then \( x_a \) is also the maximum likelihood estimator of \( x_t \)
Least Square Estimation

In our case:

\( y \sim \text{TEC} \)

\( x_a \sim \text{retrieved electron density} \)

\( x_b \sim \text{background electron density} \)

\( H \sim \text{“crossing lengths” in “voxels”} \)

\[ \text{e.g. } \text{bckg}_i \text{TEC}_i = H_i x_b = \sum_j H_{ij} x_{bj} \]

\( R_{ij} = c_R \delta_{ij} y_i^2 \)

\( B_{ij} = c_B x_{bi} x_{bj} \exp[-(z_{ij}/L_z)^2] \exp[-(\alpha_{ij}/L_\alpha)^2] \) (measurements are independent)

\( z_{ij} \sim \text{height difference between voxels } i \text{ and } j \)

\( \alpha_{ij} \sim \text{angular (great circle) distance between voxels } i \text{ and } j \)

\( L_z \sim \text{correl. distance in vert. direction (may depend on height)} \)

\( L_\alpha \sim \text{correl. (angular) distance in hor. direction (may depend location, ...)} \)
Data used

• For the assimilation
  • Ground-based GNSS-derived slant TEC data
  • Radio-Occultation-derived TEC data (calibrated TEC values along the LEO–to–GPS link below the LEO orbit)

• For the validation
  • Manually scaled ionosonde derived foF2 and hmF2
  • Ground-based GNSS-derived slant TEC data (not used for assimilation)
LISN: 3 days data (2011/03/11-12-13)
Assimilation effect

Background model
(before the assimilation)

Cross section
19:33UT; -64.75°E
from -40°N to 10°N

Analysis
(after the assimilation)
Ground-based & RO TEC DA

NeQuick used as a background with f10.7 input

Ne profiles at Ionosonde location

Ground-based only
TEC is assimilated

Ground-based & RO TEC is assimilated
TEC DA with NeQuick pre-adaptation

Ne(h) at ray perigee

TEC residuals

Background

Analysis

The Abdus Salam International Centre for Theoretical Physics
Thank you for your attention
References


References


References


References

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