Gaia relativistic astrometry models in comparison

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2 Relativistic modeling for Gaia astrometry
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   - Gaia observables
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   - RAMOD
   - GREM

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   - Reconstruction of astrometric parameters on the celestial sphere

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Relativity in modern astronomy

Relativity is already widely used (mostly as corrections) in astronomy

- VLBI, HIPPARCOS
- Planetary radar ranging and Lunar Laser ranging
- Satellite orbit determination

New generation of very accurate missions and observations

- Gaia → from mas to μas
- ACES
- LLR with APOLLO → from cm to mm
- Cassini, BepiColombo → from m to cm

More accurate → more sensitive to Solar System gravitational field

A fully relativistic treatment of these observations is required
Impact on astrometric measurements

- **Astrometry**: angular measurement of light direction
- **Aberration** (see AV): special relativity
- **Light deflection**: relativistic deflection by Solar System gravitational field

<table>
<thead>
<tr>
<th>body</th>
<th>light bending ($\mu$as)</th>
<th>$\psi_{max}(1\ \mu$as) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>$1.75 \times 10^6$</td>
<td>180°</td>
</tr>
<tr>
<td>Mercury</td>
<td>83</td>
<td>9’</td>
</tr>
<tr>
<td>Venus</td>
<td>493</td>
<td>4.5°</td>
</tr>
<tr>
<td>Earth</td>
<td>574</td>
<td>123°</td>
</tr>
<tr>
<td>Mars</td>
<td>116</td>
<td>25’</td>
</tr>
<tr>
<td>Jupiter</td>
<td>16270</td>
<td>90°</td>
</tr>
<tr>
<td>Saturn</td>
<td>5780</td>
<td>17°</td>
</tr>
<tr>
<td>Uranus</td>
<td>2080</td>
<td>71’</td>
</tr>
<tr>
<td>Neptune</td>
<td>2533</td>
<td>51’</td>
</tr>
</tbody>
</table>

[Klioner, 2003]

Relativistic models for space era

- **Gaia**: global map of the Milky way
- **AGP**: tests of General Relativity
- **Theia**: sub-\(\mu\)as astrometry
## Relativistic light propagation

<table>
<thead>
<tr>
<th>Post-Minkowskian</th>
<th>Post-Newtonian</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM</td>
<td>PN</td>
</tr>
<tr>
<td>$h_{\mu \nu}(x) = \sum_n G^n h_{\mu \nu}^{(n)}(x)$</td>
<td>$h_{\mu \nu}(x) = \sum_n \frac{1}{c^n} \tilde{h}_{\mu \nu}^{(n)}(x)$</td>
</tr>
</tbody>
</table>

Light propagation solutions use different approximations:

- **1PN**: Kopeikin and Schafer (1999), Blanchet et al (2001), Klioner (2003), Chauvineau et al (2005), ...

**HUGE dataset** (\( \sim 10^{12} \) observations) = analytical solutions
Relativistic astrometric models for Gaia

GREM
[Klioner]

RAMOD
[de Felice, Crosta, Vecchiato, ...]

TTF
[Teyssandier, Linet, ...]
Gaia’s processing: multiple pipelines and approaches

Why multiple pipelines and models?
- First fully-relativistic astrometric mission;
- No external verification possible at Gaia accuracy;
- Errors in the derivation or in the implementation of the relativistic formulae hard to detect;
- Systematic errors would affect multiple fields.

- Independent pipelines and groups within Gaia DPAC;
- Data analysis by independent models and pipelines and cross-check
Short summary of the problem

- What we solve for:
  - star parameters $(\alpha, \delta, \omega, \mu_{\alpha}, \mu_{\delta})$;
  - additional parameters $p$ - e.g., corrections).
- What we have (see AV talk):
  - observables ($\Phi$, along-scan, and $\zeta$, across-scan);
  - $\cos \phi = \mathcal{F}(x_S, x_A, x_C, x_G)$
- Main ingredients (see AV talk):
  - description of the observer;
  - model for light propagation.

Every model should describe same set of observations as function of solve-for parameters
Modeling of astrometric observations

Light propagation

Null-geodesics equations

\[ \frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0, \quad k^\alpha k_\beta = 0 \]

Observer definition

Gaia position, velocity, attitude at observation

\[ \vec{x}(t), \vec{v}(t), q_\alpha(t) \]

Spacetime metric and frames:

IAU 2000 Resolutions on Relativity
[Soffel et al., 2003] (see BB and CLPL talks)

IAU framework introduced at different levels in the processing.
Time Transfer Functions (TTF)

- Light propagation between 2 points as "simple" integral along straight line
- Avoids explicit computation of null-geodesic path
- Use TTF properties to define light direction at observation

**Light direction triple**

\[
(\hat{k}_i)_{A/B} \equiv \left( \frac{k_i}{k_0} \right)_{A/B} = -c \frac{\partial T_e/r}{\partial x^i_B} \left[ 1 - \frac{\partial T_e/r}{\partial t_B} \right]^{-1}
\]

**Projection on obs. ref. frame:**

\[
n^{(i)} = -\frac{\lambda^{0(i)}_{(i)} + \lambda^{j(i)}_{(i)} \hat{k}_{(i)j}}{\lambda^{0(0)}_{(0)} + \lambda^{j(0)}_{(0)} \hat{k}_{(0)j}}
\]

- General **closed form** equation exact at 2PM and well adapted for
  - numerical resolution for any weak field metric
  - systematic modeling of relativistic observables
Relativistic Astrometric MODel (RAMOD)

- Based on general weak-field assumptions for the metric \( g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \).
- Strictly follows measurement protocol of relativity.
- Uses a projected tangent four-vector \( \bar{\ell}^{\alpha} \) and geodesic equations:

\[
 k^{\alpha} \rightarrow \bar{\ell}^{\alpha}
\]

\[
 \frac{d\bar{\ell}^{\alpha}}{d\sigma} = F^{\alpha}_{\sigma} \partial_{p} h_{\mu\nu}
\]

- Projection of \( \bar{\ell}^{\alpha} \) on observer tetrad \( \lambda_{\beta}^{\alpha} \) at observation.
- Analytical solutions up to the \((v/c)^{3}\) order (currently).
Gaia RElativistic Model (GREM)

- Completely compatible and strictly related to IAU 2000.
- "Relativistic theory of reference systems" to define an algorithm "quite similar to traditional methods" (KK92).
- As a consequence, the algorithm provides the following steps:
  1. take out the aberration ($s \rightarrow n$)
  2. take out the light deflection ($n \rightarrow \sigma \rightarrow k$);
  3. take out the parallax ($k \rightarrow l$);
  4. model the proper motion ($l \rightarrow l(t)$).

GREM

[Klioner 2003]
[Klioner & Kopeikin 1992]
## Models summary

<table>
<thead>
<tr>
<th>Model</th>
<th>Observer</th>
<th>Light deflection</th>
<th>Ref. Frames</th>
<th>Pipeline</th>
</tr>
</thead>
<tbody>
<tr>
<td>GREM</td>
<td>aberration parallax</td>
<td>differential $k^\alpha$</td>
<td>at each step</td>
<td>AGIS</td>
</tr>
<tr>
<td>RAMOD</td>
<td>comoving tetrad</td>
<td>differential $\bar{\ell}^\alpha$</td>
<td>BCRS SRS</td>
<td>GSR</td>
</tr>
<tr>
<td>TTF</td>
<td>comoving tetrad</td>
<td>integral $\hat{k}_i$</td>
<td>BCRS SRS</td>
<td>GSR</td>
</tr>
</tbody>
</table>

Observable $\Phi$ and $\zeta$ are the same for all models. Gaia crosscheck requires to understand relations among models.
Linear System of Equation: \( \mathbf{b} = A\mathbf{x} \), sparse, overdetermined

\[
\mathbf{x} = (A^T A)^{-1} A^T \mathbf{b}
\]

GSR: Iterative

AGIS: Block Iterative

GSR approach: iterative

AGIS approach: block iterative
Comparison of Gaia relativistic models

- **Analytical model**
  - *are equations solving for the same theoretical values?*

- **Implementations**
  - *are the implemented modelings equivalent?*
  - *impact of "technical factors" (numerical noise, implementation errors, ...)?*
    - **Computed observables and residuals**
    - **Determination of star coordinates and sphere reconstruction**

*Allows to correctly interpret discrepancies between results*
Analytical/formal equivalence

**Observer modeling:**
- GREM (IAU-like) description of the observer’s reference system equivalent to the tetrad formulation (RAMOD, TTF) [Klioner, 2004];
- GREM aberration correction equivalent to RAMOD tetrad approach [Crosta and Vecchiato, 2010].

**Light propagation:**
- equivalence of formulations for light deflection (TTF, GREM and RAMOD) at Gaia accuracy \(v/c^2\) [Bertone et al., 2014].

✓ Relations proven for IAU metric chosen for Gaia (\(\mu\)as accuracy)
Comparison of Gaia relativistic models

Compare processing steps and results

- director cosines

\[ n^{(i)} = -\frac{\lambda_{(i)}^0 + \lambda_{(i)}^j \hat{k}_j}{\lambda_{(0)}^0 + \lambda_{(0)}^j \hat{k}_j} \]

- abscissae (GSR/AGIS)

\[ \cos \phi = \frac{n^{(1)}}{\sqrt{1 - n^{(3)}}} \]

\[ \eta = \phi - \Gamma/2 \]

- linearize and solve

\[ \cos \phi = F(x_S, x_A, x_C, x_G) \]

Based on simulated data (CU2), we compare:

- Observable \( \phi \) residuals
- Least square solution: source coordinates \( x_S \) and Gaia’s attitude \( x_A \)
Computed $\Phi$: angle btw observation and Gaia axes

Based on same catalog and synthetic observations
AGIS vs GSR processing
Differences below $\mu$as accuracy
Computed $\Phi$: angle btw observation and Gaia axes

- Based on same catalog and synthetic observations
- AGIS vs GSR processing
- Differences below $\mu\text{as}$ accuracy
Comparison of Gaia relativistic models

Summary for stars of different magnitudes G

- Residuals differences (relative %) of RAMOD and TTF implementations in GSR (simulated $\phi/AL$ and $\zeta/AC$)

Noise budget vs magnitude G
Reconstruction of stellar parameters (S+A+I)

- Relative differences (%) w.r.t. best solution for each case
Reconstruction of Gaia attitude

- AGIS Run: $2.3 \times 10^6$ stars, GSR Run: $0.9 \times 10^6$ stars;
- Combined solution with star parameters;
- Heavily depends on time coverage (rescaling only partially possible).
Conclusions

- GREM: relativistic version of classical astrometric approach (ref. systems)
- RAMOD/TTF: fully relativistic definition of astrometric observable
- **Equivalence check** at analytical level (both observer and light deflection) and analysis of simulated data (input known).
- *(At Gaia level)* no good or bad model/approach: more intuitive for astrometrists ”obliged” to take into account GR (GREM) or for relativists analysing astrometric data (RAMOD/TTF).
- *(Beyond Gaia)* RAMOD/TTF: general approach applicable to any metric; GREM: depends on new definition of reference systems and frames.

Exercise tomorrow (with A. Vecchiato, B. Bucciarelli, and S. Bertone)

- Astrometric reduction in a simplified python3 implementation;
- Calculation of the abscissae $\phi$ based on synthetic catalog and observation epochs, residual analysis, stars-only solution;
- Check-out code and exercise https://github.com/steo85it/pygsr
Selected References

GREM

RAMOD

TTF